

Horizons of radiating black holes in Einstein-Gauss-Bonnet gravity

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A Vaidya-based model of a radiating black hole is studied in a 5-dimensional Einstein gravity with Gauss-Bonnet contribution of quadratic curvature terms. The structure and locations of the apparent and event horizons of the radiating black hole are determined.

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I. INTRODUCTION

The Vaidya metric [1], which has the form

$$ds^2 = -\left[1 - \frac{2m(v)}{r}\right]dv^2 + 2\epsilon dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad \epsilon \pm 1 \quad (1)$$

is a solution of Einstein's equations with spherical symmetry for a null fluid (radiation) source described by energy-momentum tensor $T_{ab} = \mu l_a l_b$, l_a being a null vector field. For the case of an ingoing radial flow, $\epsilon = 1$ and $m(v)$ is a monotone increasing mass function in the advanced time v , while $\epsilon = -1$ corresponds to an outgoing radial flow, with $m(v)$ being in this case a monotone decreasing mass function in the retarded time v . Also, several solutions in which the source is a mixture of a perfect fluid and null radiation have been obtained in later years [2–8]. The Vaidya-based metric is today commonly used for the study of Hawking radiation, the process of black hole evaporation [9], and study of the dynamical evolution of the horizon associated with radiating black holes [10–14].

In recent years, motivated by development in the string theory, there has been renewed interest in the theories of gravity in higher dimensions. As a possibility, the Einstein-Gauss-Bonnet gravity, as low energy limit of the string theory, is of particular interest because of its special features. In this paper, we consider the 5D action with the Gauss-Bonnet terms for gravity and give a model of the gravitational collapse of a null fluid including the perturbative effects of quantum gravity. The Gauss-Bonnet terms are the higher curvature corrections to general relativity and naturally arise as the next leading order of the α -expansion of heterotic superstring theory, where α is the inverse string tension [15].

The aim of this brief report is to study how the location, character, and evolution of event horizon (EH) and apparent horizon (AH) get modified in the presence of the Gauss-Bonnet term. The calculations are based on the

recently introduced exact solution for a radiating Vaidya solution in the Einstein-Gauss-Bonnet gravity [16,17].

II. VAIDYA SOLUTION IN EINSTEIN GAUSS-BONNET GRAVITY

We begin with a review of Einstein Gauss-Bonnet gravity and Vaidya radiating black hole solution in it. The gravitational part of the 5-dimensional (5D) action that we consider is

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa_5^2} (R - 2\Lambda + \alpha L_{GB}) \right] + S_{\text{matter}}, \quad (2)$$

where R and Λ are the 5D Ricci scalar and the cosmological constant, respectively. $\kappa_5 \equiv \sqrt{8\pi G_5}$, where G_5 is the 5D gravitational constant. The Gauss-Bonnet Lagrangian is the combination of the Ricci scalar, Ricci tensor R_{ab} , and Riemann tensor $R_{b\rho\sigma}^a$ as

$$L_{GB} = R^2 - 4R_{ab}R^{ab} + R_{ab\rho\sigma}R^{ab\rho\sigma}. \quad (3)$$

In the 4-dimensional space-time, the Gauss-Bonnet terms do not contribute to the field equations. α is the coupling constant of the Gauss-Bonnet terms. This type of action is derived in the low-energy limit of heterotic superstring theory [15]. In that case, α is regarded as the inverse string tension and positive definite and we consider only the case with $\alpha \geq 0$ in this paper. We consider a null fluid as a matter field, whose action is represented by S_{matter} in Eq. (2).

From the action (2) we derive the following field equations:

$$G_{ab} - \alpha H_{ab} = T_{ab}, \quad (4)$$

where

$$G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R, \quad (5)$$

$$H_{ab} = 2[RR_{ab} - 2R_{a\alpha}R_b^\alpha - 2R^{\alpha\beta}R_{\alpha ab\beta} + R_a^{\alpha\beta\gamma}R_{b\alpha\beta\gamma}] - \frac{1}{2}g_{ab}L_{GB}. \quad (6)$$

The energy-momentum tensor of a null fluid is

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$$T_{ab} = \mu l_a l_b, \quad (7)$$

where μ is the nonzero energy density and l_a is a null vector such that

$$l_a = \delta_a^0, \quad l_a l^a = 0. \quad (8)$$

Expressed in terms of Eddington advanced time coordinate (ingoing coordinate) v , the metric of general spherically symmetric space-time

$$ds^2 = -A(v, r)^2 f(v, r) dv^2 + 2A(v, r) dv dr + r^2 d\Omega_3^2, \quad (9)$$

$d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi^2 d\psi^2$. Here A is an arbitrary function. We wish to find the general solution of the Einstein equation for the matter field given by Eq. (7) for the metric (9), which contains two arbitrary functions. It is the field equation $G_1^0 = 0$ that leads to $A(v, r) = g(v)$. This could be absorbed by writing $d\tilde{v} = g(v)dv$. Hence, without loss of generality, the metric (9) takes the form

$$ds^2 = -f(v, r) dv^2 + 2dv dr + r^2 d\Omega_3^2, \quad (10)$$

The Einstein field equations take the form

$$f' - \frac{2}{r}(1-f) + \frac{4\alpha}{r^2}(1-f)f' = 0, \quad (11)$$

$$f'' + \frac{4}{r}f' + \frac{2}{r^2}(1-f) + \frac{4\alpha}{r^2}[f''(1-f) + f'^2] = 0, \quad (12)$$

$$\mu = \frac{3}{2} \frac{\dot{f}}{r} + \frac{6\alpha}{r^3} \dot{f}(1-f). \quad (13)$$

Then, f is obtained by solving only the (11). This equation is integrated to give the general solution as

$$f = 1 + \frac{r^2}{4\alpha} \left[1 \pm \sqrt{1 + \frac{8\alpha m(v)}{r^4}} \right], \quad (14)$$

where $m(v)$ is an arbitrary function of v . The special case in which m is a nonzero constant we call the GB-Schwarzschild solution, of which the global structure is presented in [18].

There are two families of solutions which correspond to the sign in front of the square root in Eq. (14). We call the family which has the minus (plus) sign the minus- (plus +) branch solution. From the (r, v) component of (4), we obtain the energy density of the null fluid as

$$\mu = \frac{3}{2r^3} \dot{m} \quad (15)$$

for both branches, where the dot denotes the derivative with respect to v . In order for the energy density to be non-negative, $\dot{m} \geq 0$ must be satisfied. In the general relativistic limit $\alpha \rightarrow 0$, the minus-branch solution reduces to

$$f = 1 - \frac{m(v)}{r^2}, \quad (16)$$

which is the 5D Vaidya solution in Einstein gravity [19]. It may be noted that, in 5D Einstein gravity, the density is still given by Eq. (15). There is no such limit for the plus-branch solution. In the static case $\dot{m} = 0$, this solution reduces to the solution which was independently discovered by Boulware and Deser [20] and Wheeler [21].

The Kretschmann scalar ($\text{KS} = R_{abcd}R^{abcd}$, R_{abcd} is the 5D Riemann tensor) and Ricci scalar ($\text{R} = R_{ab}R^{ab}$, R_{ab} is the 5D Ricci tensor) for the metric (10) reduces to

$$\text{KS} = f'^2 + \frac{6}{r^4} f'^2 + \frac{12}{r^4} (1-f)^2, \quad (17)$$

and

$$\text{R} = f'' + \frac{6}{r} f' - \frac{6}{r^2} (1-f), \quad (18)$$

which diverges at $r = 0$ and hence the singularity is a scalar polynomial [22]. Radial (θ and $\phi = \text{const.}$) null geodesics of the metric (10) must satisfy the null condition

$$2 \frac{dr}{dv} = 1 + \frac{r^2}{4\alpha} \left[1 \pm \sqrt{1 + \frac{8\alpha m(v)}{r^4}} \right], \quad (19)$$

The nature (a naked singularity or a black hole) of the collapsing solutions can be characterized by the existence of radial null geodesics coming out from the singularity. It has been shown that a timelike naked singularity is formed, which does not appear in the general relativistic case [17].

III. RADIATING BLACK HOLE HORIZONS

In this section, we study the structure and location of the EH's and AH's in the presence in Einstein-Gauss-Bonnet gravity and compare it with that in the general relativity case by use of the solution obtained in the previous section. We consider the minus-branch solution in order to compare with the general relativistic case. The line element of the radiating black hole in Einstein-Gauss-Bonnet gravity has the form (10) with f given by Eq. (14) and the energy-momentum tensor (7). The luminosity due to loss of mass is given by $L_M = -dM/dv$, $L_M < 1$ measured in the region where d/dv is timelike. In order to further discuss the physical nature of our solutions, we introduce their kinematical parameters. Following York [10] a null-vector decomposition of the metric (10) is made of the form

$$g_{ab} = -\beta_a l_b - l_a \beta_b + \gamma_{ab}, \quad (20)$$

where

$$\beta_a = -\delta_a^v, \quad l_a = -\frac{1}{2} f(v, r) \delta_a^v + \delta_a^r, \quad (21)$$

$$\gamma_{ab} = r^2 \delta_a^\theta \delta_b^\theta + r^2 \sin^2(\theta) \delta_a^\phi \delta_b^\phi + r^2 \sin^2(\theta) \sin^2(\phi) \delta_a^\psi \delta_b^\psi, \quad (22)$$

$$\begin{aligned} l_a l^a &= \beta_a \beta^a = 0, & l_a \beta^a &= -1, \\ l^a \gamma_{ab} &= 0, & \gamma_{ab} \beta^b &= 0, \end{aligned} \quad (23)$$

with $f(v, r)$ given by Eq. (14). The optical behavior of null geodesics congruences is governed by the Raychaudhuri equation

$$\frac{d\Theta}{dv} = K\Theta - R_{ab} l^a l^b - \frac{1}{2}\Theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab}, \quad (24)$$

with expansion Θ , twist ω , shear σ , and surface gravity K . The expansion of the null rays parametrized by v is given by

$$\Theta = \nabla_a l^a - K, \quad (25)$$

where the ∇ is the covariant derivative and the surface gravity is

$$K = -\beta^a l^b \nabla_b l_a. \quad (26)$$

The AH is the outermost marginally trapped surface for the outgoing photons. The AH can be either null or spacelike, that is, it can “move” causally or acausally. The apparent horizons are defined as surface such that $\Theta \simeq 0$ which implies that $f = 0$. Using Eqs. (14), (21), and (26)

$$K = \frac{r}{4\alpha} \left[1 - \sqrt{1 + 8\alpha \frac{m(v)}{r^4}} \right] + \frac{\frac{2m(v)}{r^3}}{\sqrt{1 + 8\alpha \frac{m(v)}{r^4}}}. \quad (27)$$

Then Eqs. (14), (21), (25), and (27) yield the expansion parameter

$$\Theta = \frac{3}{2r} \left[1 + \frac{r^2}{4\alpha} \left[1 - \sqrt{1 + 8\alpha \left(\frac{m(v)}{r^4} \right)} \right] \right]. \quad (28)$$

From the Eq. (28) it is clear that AH is the solution of

$$\left[1 + \frac{r^2}{4\alpha} \left[1 - \sqrt{1 + 8\alpha \left(\frac{m(v)}{r^4} \right)} \right] \right] = 0. \quad (29)$$

i.e.,

$$r_{\text{AH}} = \sqrt{m(v) - 2\alpha}. \quad (30)$$

In the relativistic limit $\alpha \rightarrow 0$ then $r_{\text{AH}} \rightarrow \sqrt{m(v)}$. Hence our solution reduces to the solution [14,19] in 5D spacetime. One sees that $g_{vv}(r_{\text{AH}} = \sqrt{m(v) - 2\alpha}) = 0$ implies that $r = \sqrt{m(v) - 2\alpha}$ is timelike surface. For an outgoing null geodesic $r = \sqrt{m(v) - 2\alpha}$, \dot{r} is given by Eq. (19). It is clear that the presence of the coupling constant of the Gauss-Bonnet terms α produces a change in the location of the AH. Such a change could have a significant effect in the dynamical evolution of the black hole horizon.

$$\dot{r} = \frac{r\dot{r}}{4\alpha} \left(1 - \sqrt{1 + 8\alpha \frac{m(v)}{r^4}} \right) + \frac{\frac{L}{2r^2} + \frac{4m(v)\dot{r}}{r^3}}{\sqrt{1 + 8\alpha \frac{m(v)}{r^4}}} \quad (31)$$

At the timelike surface $r = \sqrt{m(v) - 2\alpha}$, $\dot{r} = 0$ and $\dot{r} > 0$ for $L > 0$. Hence the photon will escape from the $r = \sqrt{m(v) - 2\alpha}$ and reach an arbitrary large distance.

On the other hand, the EH is a null three-surface which is the locus of outgoing future-directed null geodesic rays that never manage to reach arbitrarily large distances from the black hole and are determined via Raychaudhuri equation. It can be seen to be equivalent to the requirement that

$$\left[\frac{d^2 r}{dv^2} \right]_{\text{EH}} \simeq 0. \quad (32)$$

An outgoing radial null geodesic satisfies

$$\frac{dr}{dv} = \frac{1}{2} \left[1 + \frac{r^2}{4\alpha} \left[1 - \sqrt{1 + 8\alpha \left(\frac{m(v)}{r^4} \right)} \right] \right]. \quad (33)$$

Then Eqs. (27) and (28) can be used to put Eq. (32) in the form

$$K\Theta_{\text{EH}} \simeq \left[\frac{3}{2r} \frac{\partial f}{\partial v} \right]_{\text{EH}} \simeq -\frac{3}{2r_{\text{EH}}^3} \frac{L_M}{\sqrt{1 + 8\alpha \left(\frac{m(v)}{r_{\text{EH}}^4} \right)}}, \quad (34)$$

where the expansion is

$$\Theta_{\text{EH}} \simeq \frac{3}{2r_{\text{EH}}} \left[1 + \frac{r_{\text{EH}}^2}{4\alpha} \left[1 - \sqrt{1 + 8\alpha \left(\frac{m(v)}{r_{\text{EH}}^4} \right)} \right] \right]. \quad (35)$$

For the null vectors l_a in Eq. (21) and the component of the energy-momentum tensor yields

$$R_{ab} l^a l^b = \frac{3}{2r} \frac{\partial f}{\partial v}. \quad (36)$$

The Raychaudhuri equation, with $\sigma = \omega = 0$ [10]:

$$\frac{d\Theta}{dv} = K\Theta - R_{ab} l^a l^b - \frac{1}{2}\Theta^2. \quad (37)$$

Thus neglecting Θ^2 , Eqs. (34), (36), and (37), imply that

$$\left[\frac{d\Theta}{dv} \right]_{\text{EH}} \simeq 0. \quad (38)$$

The event horizon in our case are therefore placed by Eq. (38). Following [11,14], the solution can be immediately written

$$r_{\text{EH}} = \sqrt{m^*(v) - 2\alpha}, \quad (39)$$

where

$$m^*(v) = m(v) - \frac{L}{K}. \quad (40)$$

Thus the expression of the apparent horizon is exactly same as its counterpart Eq. (30) with the mass replaced

by the effective mass m^* . The region between the AH and the EH is defined as a *quantum ergosphere* [10]. All these results are consistent with known results in 5D space-time for the $\alpha \rightarrow 0$ case.

IV. CONCLUSION

We have examined Vaidya radiating black holes in Einstein-Gauss-Bonnet gravity. The structure and location of the AH and EH are determined. We have pointed out exact location of these horizons. It is clear that presence of

the coupling constant of the Gauss-Bonnet terms α produces a change in the location of these horizons. Such a change could have a significant effect in the dynamical evolution of the black hole horizon. In particular, our results in the limit $\alpha \rightarrow 0$ reduced exactly to vis-à-vis 5D relativistic case.

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