Gauge theory duals of cosmological backgrounds and their energy momentum tensors

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We revisit type IIB supergravity backgrounds with null and spacelike singularities with natural gauge theory duals proposed in [S. R. Das, J. Michelson, K. Narayan, and S. P. Trivedi, Phys. Rev. D **74**, 026002 (2006)] and [S. R. Das, J. Michelson, K. Narayan, and S. P. Trivedi, Phys. Rev. D **75**, 026002 (2007)]. We show that for these backgrounds there are always choices of the boundaries of these deformed $\mathrm{AdS}_5 \times S^5$ space-times, such that the dual gauge theories live on *flat* metrics and have space-time dependent couplings. We present a new time dependent solution of this kind where the effective string coupling is always bounded and vanishes at a spacelike singularity in the bulk, and the space-time becomes $\mathrm{AdS}_5 \times S^5$ at early and late times. The holographic energy momentum tensor calculated with a choice of flat boundary is shown to vanish for null backgrounds and to be generically nonzero for time dependent backgrounds.

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I. INTRODUCTION

In two previous papers [1,2] three of us proposed gauge theory duals to a class of time dependent and null backgrounds of IIB supergravity. These solutions are deformations of $\mathrm{AdS}_5 \times S^5$ backgrounds with non-normalizable modes of the metric and the dilaton. The null solutions and their duals were also proposed in [3,4]. It is thus natural to conjecture that the dual gauge theory is deformed by corresponding sources. Generally, the supergravity solutions are singular with spacelike or null singularities where, of course, supergravity breaks down. The idea is to investigate whether the dual gauge theory remains well behaved in this region and possibly provides a way to continue the time evolution beyond the time where the supergravity is singular. Other discussions of similar solutions include [5]. For other approaches to the use of AdS/CFT correspondence to study time dependent backgrounds, see [6].

These solutions have an Einstein frame metric of the form (with the AdS scale $R_{\mathrm{AdS}} = 1$)

$$ds^2 = \frac{1}{z^2}[dz^2 + \tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu] + d\Omega_5^2 \qquad (1)$$

and a dilaton $\Phi(x)$ together with a 5-form field strength

$$F_5 = \omega_5 + \star\omega_5. \qquad (2)$$

This is a solution if

$$\tilde{R}_{\mu\nu} = \tfrac{1}{2}\partial_\mu \Phi \partial_\nu \Phi, \qquad \nabla^2\Phi = 0 \qquad (3)$$

where $\tilde{R}_{\mu\nu}$ is the Ricci tensor for the four dimensional metric $\tilde{g}_{\mu\nu}(x)$.

In this paper, the S^5 part of the metric will remain unaltered and we will not write this out explicitly.

In these coordinates the boundary is at $z = 0$, and as argued in [1,2], it is reasonable to assume that the dual gauge theory lives on a $3 + 1$ dimensional space-time with metric $\tilde{g}_{\mu\nu}(x)$ and has a space-time dependent coupling $g_{\mathrm{YM}}(x) = e^{\Phi(x)/2}$. Of particular interest are solutions where

$$\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu = e^{f(x)}[-2dx^+dx^- + d\vec{x}^2]. \qquad (4)$$

Now, because of the space-time dependence of the coupling constant, the Yang-Mills theory is not conformally invariant in the sense of conformal coordinate transformations. However, the theory is still Weyl invariant under Weyl transformations of the metric and corresponding transformations of the fields. One might therefore hope that the overall factor in (4) can be removed by a Weyl transformation leaving us with a gauge theory on flat space with space-time dependent coupling. Such a step would be, however, subtle in the quantum theory because of a possible Weyl anomaly.

Null solutions with the conformal factor depending on a single null coordinate are of special interest from several points of view. First, for such solutions, with $f = f(x^+)$ and $\Phi = \Phi(x^+)$ the Eq. (3) becomes

$$\tfrac{1}{2}(f')^2 - f'' = \tfrac{1}{2}(\Phi')^2. \qquad (5)$$

This means that we have a one function worth of solutions. We can pick any $f(x^+)$ and solve for $\Phi(x^+)$. In particular, we may look for solutions where the dilaton is bounded everywhere. Indeed an interesting solution is given by

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$$e^{f(x^+)} = (\tanh x^+)^2 \qquad e^{\Phi(x^+)} = g_s \left|\tanh\left(\frac{x^+}{2}\right)\right|^{\sqrt{8}}. \quad (6)$$

This solution asymptotes to $AdS_5 \times S^5$ with string coupling g_s at $x^+ \to \pm\infty$. Both the metric and the effective string coupling e^Φ drops to zero at $x^+ = 0$ which is the location of the null singularity. This is good: things appears to be controlled. What makes this kind of background appealing is the fact that the Weyl anomaly in the gauge theory living in the metric (4) identically vanishes for such null backgrounds. Therefore, we could perform a Weyl transformation without bothering about the anomaly, and obtain a dual theory which is on flat space. In this dual theory, the coupling is always bounded, vanishing at $x^+ = 0$. One would expect that the Yang-Mills theory is well behaved for such profiles. A careful argument along the above lines was given in [2].

This provides a clean formulation of the problem in terms of light front evolution. Consider the strong 't Hooft coupling regime of the gauge theory

$$g_{YM}^2 = g_s \to 0 \qquad N \to \infty \qquad g_{YM}^2 N = \text{finite and large.} \quad (7)$$

Then at $x^+ \to -\infty$ we start with a gauge theory in its ground state. Since the 't Hooft coupling is large, supergravity in $AdS_5 \times S^5$ provides an accurate description in this regime. Now turn on a source leading to x^+ dependence of the effective coupling, $g_{YM}^2 \to g_{YM}^2 e^{\Phi(x^+)}$. In the dual supergravity this means that we have turned on a *non-normalizable* dilaton mode $\Phi(x^+)$. As x^+ increases, the system evolves. In the gauge theory side the effective coupling decreases, but remains large so long as $|x^+|$ is large enough. In this regime, therefore, the supergravity description remains good and that is what is described by the supergravity solution. As we approach $x^+ = 0$, the gauge theory coupling becomes weak, becoming exactly zero at $x^+ = 0$. A gauge theory with weak coupling is not well described by dual supergravity—stringy effects are important. Indeed, if we look at the evolution on the supergravity side, and extrapolate the solution to small x^+—where supergravity should no longer be valid—we encounter a singularity. However, because the gauge coupling is weak, one would imagine that the gauge theory description remains good—maybe even well approximated by perturbation theory. This gauge theory then describes the region which appears to be singular if the supergravity solution is extrapolated to the regime where it should not have been in the first place. Because of null dependence of the coupling, the gauge theory has several properties which indicate that the theory is actually well behaved at $x^+ = 0$. A null isometry ensures absence of particle production, and in a light cone gauge the kinetic terms are standard so that only positive powers of the x^+ dependent coupling appear in the nonlinear terms [2]. Since the coupling in fact vanishes at $x^+ = 0$ one might hope that perturbation theory is reliable in this region and may be even used to extend the time evolution through $x^+ = 0$ to positive x^+.

In contrast to the null solutions, the time-dependent solutions found in [1] do not have such nice features. These solutions have boundary metrics which are Kasner cosmologies, e.g.

$$\begin{aligned} ds^2 &= \frac{1}{z^2}\left[dz^2 - dt^2 + \sum_{i=1}^{3} t^{2p_i} dx^i dx^i\right] \\ \sum_i p_i &= 1 \\ e^{\Phi(t)} &= |t|^{\sqrt{2(1-\sum p_i^2)}}. \end{aligned} \quad (8)$$

The string coupling—and therefore the Yang-Mills coupling—still goes to zero at the spacelike singularity at $t = 0$, but diverges at early or late times. As we shall see below it still turns out that the Weyl anomaly of the boundary theory in these coordinates vanishes. However, because of the divergence of the Yang-Mills coupling, it is unclear whether the gauge theory makes sense. Furthermore, as discussed below, time-dependent backgrounds (unlike null backgrounds) have curvature singularities at $z = \infty$ for any time. For the solution (8), at any fixed time t the Ricci scalar diverges as $\frac{z^2}{t^3}$. At early and late times, however, the divergence goes away.

While the results stated above are suggestive, there are several causes for serious concern. The first issue concerns the choice of boundary described above. In this choice, the null or spacelike singularity extends all the way to the boundary. While a Weyl transformation removes this and brings us to a flat boundary metric, the conformal factor required is clearly singular: this is certainly an uncomfortable situation. Furthermore, both for the null and the spacelike cases, the behavior of the Yang-Mills coupling is *nonanalytic* [7], casting serious doubts about a smooth time evolution through this point. In fact, it turns out that if the conformal factor $e^{f(x^+)}$ is chosen to vanish at $x^+ = 0$ in an analytic fashion, $e^{\Phi(x^+)}$ has this nonanalytic behavior. On the other hand if $e^{\Phi(x^+)}$ is analytic, $e^{f(x^+)}$ becomes nonanalytic. Finally, the Kasner-like solutions with spacelike singularities do not appear to lead to a controlled dual theory.

In this paper we take some steps in solving some of these problems. We show that for the solutions with brane metrics conformal to flat space, one can always choose a foliation such that the boundary is flat. To show this, we use the well-known fact that in asymptotically AdS spacetimes, a Weyl transformation of the boundary theory corresponds to a special class of coordinate transformations in the bulk—the Penrose-Brown-Henneaux transformations. We find these exact transformations for any null solution of this kind and for the Kasner solutions described above. For other time dependent solutions, we find these transformations in a systematic expansion around the (new) boundary. Thus in these new coordinates the boundary theory is explicitly defined on flat space and the nontrivial feature is in the time (or null time) dependence of the coupling. Furthermore, for the null solutions of the type (4), we do not have to worry about the function $f(x^+)$ anymore and choose the function $\Phi(x^+)$ to have a nice analytic behavior at $x^+ = 0$.

We then describe new supergravity solutions with spacelike singularities which asymptote to $AdS_5 \times S^5$ at early and late times. The couplings are bounded everywhere and vanish at the spacelike singularity. This opens up the possibility of posing questions about spacelike singularities in the gauge theory dual in a fashion analogous to our formulation of null singularities. At this time, however, we are still unable to arrive at such a clean formulation for spacelike singularities.

We then compute the energy momentum tensors of these solutions using standard techniques of holographic RG [8–13]. We find that the energy-momentum tensor vanishes for all null backgrounds for both foliations. For time-dependent backgrounds, the trace anomaly vanishes in the coordinates of (1), while the energy momentum tensor in the foliation leading to a flat boundary metric is nonzero. In fact the answer diverges at the time when the coupling constant vanishes. While a nonzero energy momentum may be interpreted as particle production, one cannot attach any significance to the divergence since this happens at the place where the supergravity description is invalid.

While this paper was being written, [14] appeared on the archive, which has some overlap with our Sec. II.

II. NULL COSMOLOGIES IN BETTER COORDINATES

In this section we will rewrite the supergravity solutions of [1] in new coordinates leading to a choice of the boundary with a flat metric.

A. PBH transformations

It is well known that Weyl transformations in the boundary theory correspond to special coordinate transformations in the bulk—the Penrose-Brown-Henneaux (PBH) transformations [15,16]. Any asymptotically AdS spacetime may be written in a standard coordinate system of the Feffermann-Graham form

$$ds^2 = \frac{1}{\bar{\rho}^2} d\bar{\rho}^2 + \frac{1}{\bar{\rho}^2} \tilde{g}_{\mu\nu}(x, \bar{\rho}) dx^\mu dx^\nu. \tag{9}$$

Now consider the coordinate transformations [8,17,18]

$$\bar{\rho} \rightarrow \bar{\rho} e^{-\sigma(x,\bar{\rho})} \qquad x^\mu \rightarrow x^\mu + a^\mu(x, \bar{\rho}) \tag{10}$$

which keep this form of metric invariant. For infinitesimal transformations this is ensured by requiring σ to be a function of x alone, and

$$\frac{1}{\bar{\rho}} \partial_{\bar{\rho}} a^\mu = -\tilde{g}^{\mu\nu} \partial_\nu \sigma. \tag{11}$$

The transformation of the metric $\tilde{g}_{\mu\nu}$ is given by

$$\delta \tilde{g}_{\mu\nu}(x, \bar{\rho}) = 2\sigma(x, \bar{\rho})(1 - \tfrac{1}{2}\bar{\rho}\partial_{\bar{\rho}})\tilde{g}_{\mu\nu}(x, \bar{\rho}) + \nabla_{(\mu} a_{\nu)}(x, \bar{\rho}). \tag{12}$$

The expression (12) explicitly shows that this transformation includes a Weyl transformation of the metric $\tilde{g}_{\mu\nu}$.

Consider now a metric of the form

$$ds^2 = \frac{1}{z^2}[dz^2 + e^{f(x)} \eta_{\mu\nu} dx^\mu dx^\nu]. \tag{13}$$

Our aim is to perform a PBH transformation to remove the conformal factor $e^{f(x)}$ in the boundary metric. However we need to do this for finite PBH transformations.

B. Null cosmologies in new coordinates

When the conformal factor $f(x)$ is a function of a single null coordinate x^+, i.e. when the original metric is of the form

$$ds^2 = \frac{1}{z^2}[dz^2 + e^{f(x^+)}(-2dx^+ dx^- + d\vec{x}^2)] \tag{14}$$

it turns out that it is easy to figure out the correct finite PBH transformations. These are given by the following

$$\begin{aligned} z &= w e^{f(y^+)/2} \\ x^- &= y^- - \tfrac{1}{4} w^2 (\partial_+ f) \\ x^+ &= y^+ \qquad \vec{x} = \vec{y}. \end{aligned} \tag{15}$$

In these coordinates the metric becomes

$$\begin{aligned} ds^2 &= \frac{1}{w^2}\bigg[dw^2 - 2dy^+ dy^- + d\vec{y}^2 + \frac{1}{4} w^2 [(f')^2 - 2f''](dy^+)^2 \bigg] \\ &= \frac{1}{w^2}\bigg[dw^2 - 2dy^+ dy^- + d\vec{y}^2 + \frac{1}{4} w^2 (\Phi')^2 (dy^+)^2 \bigg] \end{aligned} \tag{16}$$

where in the second line we have used (5).

The new coordinates provide a new foliation of the space-time. The boundary $w = 0$ is naively the same as the original boundary $z = 0$. However, it is well known that AdS/CFT requires an infrared cutoff in the bulk which corresponds to a ultraviolet cutoff in the dual gauge theory. For any such finite cutoff ϵ, the boundary $w = \epsilon$ is not the same as $z = \epsilon$, and becomes flat in the $\epsilon \rightarrow 0$ limit. Consequently the dual Yang-Mills theory lives on a flat space with a x^+ dependent coupling.

Notice that in these coordinates, there is only one function $\Phi(x^+)$ which we are free to choose. In particular, e^Φ can be chosen to be bounded and vanishing at $x^+ = 0$ in an *analytic* fashion. The proposed dual will then have a coupling which is bounded and vanishes at $x^+ = 0$ in a smooth fashion.

For such solutions, $\partial_+\Phi$ will, however, diverge at $x^+ = 0$. This means that the bulk space-time will be as usual singular. This may be seen by looking at the behavior of geodesics as in [1,2]. In the new coordinates, these geodesics are

$$w = z_0 F(y^+) \qquad y^- = y_0^- - \frac{1}{4} z_0^2 \frac{d}{dy^+}(F(y^+)^2 \tag{17}$$

where $F(y^+) = e^{-f(y^+)/2}$. The affine parameter along such geodesics is given by

$$\lambda = \int^{y^+} \frac{1}{F(y^+)^2} dy^+. \tag{18}$$

It is easy to find functions $\Phi(x^+)$ so that the singularity at $y^+ = 0$ is reached in finite affine parameter. For such solutions $F(y^+)$ must diverge at $y^+ = 0$ The magnitude of the tidal acceleration between two such geodesics separated along a transverse direction is given by (see equation (2.10) of [2])

$$|a| = (F(y^+))^3 F''(y^+) \tag{19}$$

and would diverge as well. The form of the metric (16), however, shows that this singularity weakens as we approach the boundary $w = 0$ leaving a flat boundary metric.

III. KASNER-TYPE SOLUTIONS

Similar considerations apply to time dependent solutions of the form of (8). We will concentrate on solutions with $p_1 = p_2 = p_3 = \frac{1}{3}$. Redefining the time coordinate, this solution may be written in the form

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \frac{2t}{3}(-dt^2 + (dx^1)^2 + \cdots (dx^3)^2)\right]$$
$$e^{\Phi(t)} = |t|^{\sqrt{3}}. \tag{20}$$

This solution has a spacelike singularity at $t = 0$.

Since the boundary metric on $z = 0$ is conformally flat, there should be a PBH transformation which leads to a foliation with a flat boundary. This is indeed true. The solution for $t > 0$ becomes

$$ds^2 = \frac{1}{\rho^2}\left[d\rho^2 - \frac{(16T^2 - 5\rho^2)^2}{256T^4} dT^2 + \frac{(16T^2 - \rho^2)^{4/3}(16T^2 + 5\rho^2)^{2/3}}{256T^4}((dx^1)^2 + \cdots (dx^3)^2)\right] \tag{21}$$

where the new coordinates (ρ, T) are related to the coordinates (z, t) in the region $\rho < 4T$ by the transformations

$$z = \frac{32\rho T^{5/2}}{\sqrt{6}} \frac{1}{16T^2 - \rho^2} \qquad t = T\left(\frac{16T^2 + 5\rho^2}{16T^2 - \rho^2}\right)^{2/3}. \tag{22}$$

The dilaton may be written down in new coordinates by substituting (22) in (20).

It is clear that in this new foliation defined by slices of constant ρ, the boundary $\rho = 0$ has a flat metric. However this coordinate system has a coordinate singularity at $\rho = 4T$, but may be extended beyond this point.

The arguments in the previous section then indicate that there is a dual field theory which lives in a flat space-time, but with a time-dependent coupling which vanishes at $T = 0$. Unlike the null solutions, the coupling *diverges* at early or late times, and we cannot make any careful argument about the behavior of this dual theory.

As noted in the introduction, these solutions have a curvature singularity at any finite time, though the singularity goes away at early and late times. The bulk Ricci scalar is given by

$$\mathcal{R}_5 = -\left(\frac{9z^2}{4t^3} + 20\right). \tag{23}$$

In the global geometry the Poincare horizon is a product of a null plane times a S^2. This singularity appears at *one* point on this null plane. The rest of the Poincare horizon is nonsingular.

IV. NEW CLASS OF TIME DEPENDENT SOLUTIONS

A necessary condition for a well-defined dual theory is that the coupling should be bounded at all times. This motivates us to search for new solutions which have spacelike singularities of this type. We will present such solutions in this section.

These solutions are special cases of a class of time-dependent solutions whose boundary metrics are FRW universes. The Einstein frame metric[1] is given by

$$ds^2 = \frac{1}{z^2}\left[dz^2 + A(t)\left[-dt^2 + \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin\theta^2 d\phi^2)\right]\right] \tag{24}$$

with $k = 0, \pm 1$, and

$$\Phi(t) = \pm\sqrt{3}\int \frac{dt}{A(t)} \tag{25}$$

where

[1]These solutions can be derived from a generic ansatz with diagonal metric, and imposing that the dilaton is a (spatially homogeneous) function $\Phi(t)$ of time t alone.

$$A(t) = C_1 \sin(2\sqrt{k}t) + C_2 \cos(2\sqrt{k}t). \tag{26}$$

The solutions with $k = -1$ are particularly interesting. If we choose $A(t) = |\sinh(2t)|$, the dilaton becomes

$$e^{\Phi(t)} = g_s |\tanh t|^{\sqrt{3}} \tag{27}$$

so that the coupling is bounded and vanishes at $t = 0$. There is a spacelike singularity at $t = 0$. In the following we will restrict our attention to the "big crunch" part of the space-time, i.e. for $t < 0$. In this case we use $A(t) = |\sinh(2t)|$.

The boundary metric is in fact conformal to parts of Minkowski space. This is seen by defining new coordinates (for $t < 0$)

$$r = \frac{R}{\sqrt{\eta^2 - R^2}} \qquad e^{-t} = \sqrt{\eta^2 - R^2}. \tag{28}$$

The solution now becomes

$$\begin{aligned} ds^2 &= \frac{1}{z^2}\bigg[dz^2 + \left|1 - \frac{1}{(\eta^2 - R^2)^2}\right| \\ &\quad \times [-d\eta^2 + dR^2 + R^2 d\Omega_2^2]\bigg] \\ e^{\Phi} &= \left|\frac{\eta^2 - R^2 - 1}{\eta^2 - R^2 + 1}\right|^{\sqrt{3}}. \end{aligned} \tag{29}$$

The $t > 0$ part of the solution also becomes this metric after a coordinate transformation obtained by reversing the sign of t in (28). In these coordinates it is clear that as $t \to -\infty$, i.e. $\eta^2 - R^2 \to \infty$, the space-time is AdS and e^{Φ} asymptotes to a constant.

The coordinate transformation (28) is valid in the region $\eta^2 - R^2 > 0$, and $\eta^2 - R^2 = 1$ are the two spacelike singularities. Even though we started with the form of the metric (24) we could extend the solution beyond part this part of Minkowski space in the standard manner. In this extended solution, there are timelike singularities at $R^2 - \eta^2 = 1$. As is evident, the dilaton shows a singular behavior at the location of these singularities, even though the value of e^{Φ} goes to zero. In the following we will be interested in the solution in the regions $(\eta^2 - R^2) > 1$, i.e. the space-time described by the metric (24).

Like the Kasner solutions, these solutions generically have curvature singularities at $z = \infty$. In the big crunch region, The bulk Ricci scalar is given by

$$\mathcal{R}_5 = -\left(20 - 3\frac{z^2}{(\sinh(2t))^3}\right), \tag{30}$$

where t is as in (24) with $A(t) = -\sinh(2t)$ in this $t < 0$ region of the space-time. The global nature of this singularity is similar to the Kasner-type solution. In particular, at early times $t \to -\infty$ there is no such singularity.

We can, therefore, view these backgrounds in the same way as the null backgrounds. For $t < 0$ the space-time is pure $AdS_5 \times S^5$ in the infinite past. As time evolves one generates a *spacelike singularity* at $t = 0$ which extends to the boundary defined at $z = 0$. However, since the boundary metric is conformal to flat space, we can choose a different foliation by performing a PBH transformation and choose a boundary which is completely flat. (In this case, we have not been able to find the exact PBH transformations, but—as detailed in the Appendix—the PBH transformation may be found in an expansion around the boundary). The gauge theory defined on this latter boundary is on flat space with a time-dependent coupling constant which vanishes at the location of the bulk singularity. The source in the gauge theory evolves the initial vacuum state. On the supergravity side, a (timelike) singularity develops at $z = \infty$. While we do not have a clear idea of the meaning of this singularity in the gauge theory it is reasonable to presume—in view of the usual AdS/CFT duality—that this should manifest itself in the infrared behavior. Finally, as the time evolves, the gauge coupling goes to zero—this manifests itself as a spacelike singularity in the bulk in a region where supergravity itself breaks down.

The analysis of this dual gauge theory appears to be more complicated than the dual gauge theory for null backgrounds. One issue is related to the fact that the gauge theory Lagrangian has an overall factor of $e^{-\Phi}$. When Φ depends only on a null direction, it was shown in [2] that a choice of light cone gauge, together with a field redefinition, converts the kinetic terms in the action into standard form for constant couplings. All factors of couplings then appear in the nonlinear terms as positive powers of $e^{\Phi(x^+)}$, which vanish at the location of the bulk singularity. This allowed us to arrive at some clean conclusions about the behavior of the gauge theory. In [14] a different gauge choice was used—this again made analysis of the gauge theory easier. For time dependent backgrounds, we have not been able to find a gauge choice and a field redefinition which leads to a similar simplification. Nevertheless we expect that the theory is amenable to perturbative analysis near $t = 0$ where the gauge coupling becomes weak.

V. THE HOLOGRAPHIC STRESS TENSOR

In this section we use the standard techniques of covariant counterterms [8–12,23] to calculate the holographic stress tensor. The gravity-dilaton action in five dimensional space $\mathcal{M}$, with boundary $\partial\mathcal{M}$ is given by,

$$\begin{aligned} I_{\rm bulk} + I_{\rm surf} &= \frac{1}{16\pi G_5}\int_{\mathcal{M}} d^5x\sqrt{-g}\left(R^{(5)} + 12 - \frac{1}{2}(\nabla\Phi)^2\right) \\ &\quad - \frac{1}{8\pi G_5}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}\Theta. \end{aligned} \tag{31}$$

Where the second term is the Gibbons-Hawking boundary term, $h_{\mu\nu}$ is the induced metric on the boundary and Θ is the trace of the extrinsic curvature[2] of the boundary $\partial\mathcal{M}$.

The above action is divergent. Therefore, one might use one of the known techniques to regularize such action. Here we choose to work with the covariant counterterm method since we are interested in calculating the boundary energy momentum and its trace. To have a finite action one can add the following counterterms

$$I_{\rm ct} = -\frac{1}{8\pi G_5}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}\bigg[3 + \frac{\mathcal{R}}{4} - \frac{1}{8}(\nabla\Phi)^2 - \log(\rho_0)a_{(4)}\bigg] \quad (32)$$

where ρ_0 is a cutoff on the radial coordinate ρ which has to be taken to zero at the end of the calculation. $\mathcal{R}$ is the Ricci scalar for h. The term proportional to $\log(\rho_0)$ is required to cancel a logarithmic divergence in the action (31). However this term does not contribute to the renormalized energy momentum tensor.

Now the total action is given by $I = I_{\rm bulk} + I_{\rm surf} + I_{\rm ct}$. Using this action one can construct a divergence free stress energy tensor [9]:

$$\begin{aligned} T^{\mu\nu} &= \frac{2}{\sqrt{-h}}\frac{\delta I}{\delta h_{\mu\nu}} \\ &= \frac{1}{8\pi G_5}\bigg[\Theta^{\mu\nu} - \Theta h^{\mu\nu} - 3h^{\mu\nu} + \frac{1}{2}G^{\mu\nu} \\ &\quad - \frac{1}{4}\nabla^\mu\Phi\nabla^\nu\Phi + \frac{1}{8}h^{\mu\nu}(\nabla\Phi)^2\bigg]. \end{aligned} \quad (33)$$

Here $G_{\mu\nu}$ and ∇ are the Einstein tensor and covariant derivative with respect to h. In the regime where the supergravity approximation is valid, the vev of the CFT's energy momentum tensor $\langle T^{\mu\nu}\rangle$ is related the above stress tensor by the following equation

$$\sqrt{-\tilde{g}}\tilde{g}_{\mu\nu}\langle T^{\nu\sigma}\rangle = \lim_{z\to 0}\sqrt{-h}\,h_{\mu\nu}T^{\nu\sigma} \quad (34)$$

where we have used the notation of Eq. (9).

The energy-momentum tensors calculated in the holographic RG approach correspond to operators in the dual field theory which are regularized using the specific boundary metric used to perform the bulk calculation.

A. Conformally flat boundary

Let us first consider bulk metrics of the form (1). This means we use a cutoff defined in terms of the radial coordinate z. Using the above expression for the stress tensor, one can easily show that for a any solution with conformally flat boundary, i.e. of the form of Eq. (4), the stress tensor vanishes. Let us see how this result is obtained. First, the extrinsic curvature for a solution with a conformally flat boundary is

$$\Theta_{\mu\nu} = -h_{\mu\nu}. \quad (35)$$

The extrinsic curvature terms in the expression then cancel with the term proportional with the induced metric. Using (3) and its contraction, one can see directly that the last three terms exactly cancel leading to the vanishing of the stress tensor. As a result, the trace anomaly vanishes. Comparing this result with the known results in the literature one finds the following. Our result does not match with the field theory calculation of trace anomaly in [19]. The reason is that in this calculation only terms up to quadratic order in the dilaton were included and all higher orders have been ignored. But this result agrees with the holographic anomaly expression calculated in [10] since their expression contains these terms which are crucial to have a vanishing anomaly.

B. Flat boundary

We will now consider the energy momentum tensor which is defined by a choice of foliation which leads to a *flat* boundary metric. This is of course a different regularization and would lead to a different answer which would give us the energy momentum tensor of the gauge theory defined on flat space in an appropriate regime.

1. Null solutions

It is easy to check by a direct calculation that for the solutions with null singularities, the energy momentum tensor continues to vanish.

2. Kasner-type solutions

Now consider the Kasner-type solution in new coordinates (21). Using the above expression for the holographic stress tensor, one gets the following

$$T^\nu_\mu = \frac{\rho^4}{1024\pi G_5 T^4}\,{\rm diag}(9, 13, 13, 13) + O(\rho^6). \quad (36)$$

The energy momentum tensor of the CFT as in (34) is given by

$$\langle T^\nu_\mu\rangle = \frac{N^2}{512\pi^2 T^4}\,{\rm diag}(9, 13, 13, 13), \quad (37)$$

which has the following nonvanishing trace:

$$\langle T^\mu_\mu\rangle = \frac{3N^2}{32\pi^2}\frac{1}{T^4}, \quad (38)$$

here we have used

$$G_5 = \frac{\pi}{2N^2}. \quad (39)$$

The trace computed here agrees with the holographic trace

[2] $\Theta_{ab} = -\frac{1}{2}(\nabla_a n_b + \nabla_b n_a)$, where n^a in the unit normal vector to the surface $z =$ constant and pointing to the boundary $\partial\mathcal{M}$[22]

anomaly found in [10]. The nonzero energy momentum tensor can be possibly interpreted as particle production.

3. FRW solutions

To calculate the energy momentum tensor for the new FRW solution with $k = -1$ it is convenient to work with the following coordinate system. These coordinates allow the conformal factor to depend only on one coordinate, while keeping the boundary metric Minkowski. The coordinate transformations are as follows:

$$\tau^2 = \eta^2 - R^2, \qquad r^2 = \frac{\eta + R}{\eta - R}. \tag{40}$$

This puts the metric in the form

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}\left(1 - \frac{1}{\tau^4}\right) \times \left[-d\tau^2 + \frac{\tau^2}{r^2}dr^2 + \frac{\tau^2}{4}\left(r - \frac{1}{r}\right)^2 d\Omega_2^2\right]. \tag{41}$$

The dilaton in these coordinates is given by

$$\Phi(\tau) = \sqrt{3}\ln\left[\frac{\tau^2 - 1}{\tau^2 + 1}\right]. \tag{42}$$

One can use these coordinates to do a PBH transformations as explained in the appendix and obtain another form of this solution with Minkowski boundary. In this form the stress energy tensor is given by

$$T_\mu^\nu = \frac{\rho^4}{4\pi G_5(\bar{T}^4 - 1)^4} \times \mathrm{diag}(12 - 3\bar{T}^4, 4 + 9\bar{T}^4, 4 + 9\bar{T}^4, 4 + 9\bar{T}^4) + O(\rho^6) \tag{43}$$

where the coordinate $\bar{T}$ is defined in the appendix. Using (34) and (39), the energy momentum tensor of the CFT is given by

$$\langle T_\mu^\nu \rangle = \frac{N^2}{2\pi^2(\bar{T}^4 - 1)^4} \times \mathrm{diag}(12 - 3\bar{T}^4, 4 + 9\bar{T}^4, 4 + 9\bar{T}^4, 4 + 9\bar{T}^4), \tag{44}$$

which has the following nonvanishing trace:

$$\langle T_\mu^\mu \rangle = \frac{12N^2}{\pi^2}\frac{(\bar{T}^4 + 1)}{(\bar{T}^4 - 1)^4}. \tag{45}$$

Again this trace agrees with the calculation in [10]. Note that the energy-momentum tensor vanishes at early times. This reinforces our claim that at early times we have started with the vacuum state of the dual gauge theory, with a source which vanishes at $\bar{T} \to -\infty$. At later times, the source produces a nonzero energy momentum tensor as well as a nonzero expectation value of the operator dual to the dilaton. In other words, the Heisenberg picture state is the vacuum of the CFT. It is tempting to interpret the nonzero stress tensor as an effect of particle production. Once again the stress tensor diverges at the singularity $\bar{T} = 1$. However this is precisely the place where the holographic calculation cannot be trusted.

The real question is whether the gauge theory is well behaved in this region. For null backgrounds, this appears to be so [14,20,21]. For time dependent backgrounds, this is not clear at the moment, particularly because of bulk singularities at $z = \infty$ which signify that there are infrared problems in the gauge theory. These issues are under investigation.

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APPENDIX: PBH TRANSFORMATIONS

In the coordinates displayed in (23), the spacelike singularity extends to the boundary. However in the form (29) the boundary metric is conformal to flat space. This suggests that there should be PBH transformations, which leads to a flat boundary metric. In the case of FRW solutions however, we have not yet been able to find the exact PBH transformations. We will show below how to find these transformations systematically in the neighborhood of the boundary and obtain them to the order which is required for our analysis of the energy-momentum tensor in the next section.

Let us show how can we obtain such coordinate transformation for a solution with a conformally flat boundary on the following form

$$ds^2 = \frac{1}{z^2}[dz^2 + f(t)\eta_{\mu\nu}dx^\mu dx^\nu]. \tag{A1}$$

We chose the conformal factor to depend only on one coordinate since this will be sufficient to deal with the cases under consideration in this work. One can generalize such a procedure to cases with a general conformal factor $f(x^\mu)$ and boundaries other than Minkowski. We define the following coordinate transformations

$$t(\rho, T) = \sum_{n=0,2,..} a_i(T)\rho^i \qquad z(\rho, T) = \rho \sum_{n=0,2,..} s_i(T)\rho^i. \tag{A2}$$

One can choose $a_0(T) = T$, then expanding all metric components in ρ, they have the following form in new coordinates

$$\begin{aligned} g_{\rho\rho} &= \frac{1}{\rho^2} + \frac{4}{s_o^2}[s_0 s_2 - a_2^2 f] + \frac{4}{s_0^3}[2s_4 s_0^2 - a_2^3 \dot{f} s_0 \\ &\quad + 2a_2^2 f s_2 - 4a_2 f a_4 s_0]\rho^2 + O(\rho^4) \\ g_{\rho T} &= \frac{1}{s_0^2}[\dot{s}_0 s_0 - 2a_2 f]\frac{1}{\rho} + \frac{1}{s_0^3}[\dot{s}_2 s_0^2 + s_2\dot{s}_0 s_0 - 2a_2 f \dot{a}_2 s_0 \\ &\quad - 2a_2^2 \dot{f} s_0 + 4f a_2 s_2 - 4f a_4 s_0]\rho + O(\rho^3) \\ g_{TT} &= \frac{-f}{s_0^2}\frac{1}{\rho^2} - \frac{1}{s_0^3}[a_2 s_0 \dot{f} - 2f s_2 + 2\dot{a}_2 f s_0 - \dot{s}_0^2 s_0] \\ &\quad - \frac{1}{2s_0^4}[2s_0^2 \dot{f} a_4 + s_0^2 \ddot{f} a_2^2 - 4\dot{f} a_2 s_0 s_2 + 4\dot{f} a_2 s_0^2 \dot{a}_2 \\ &\quad - 4f s_4 s_0 + 6f s_2^2 - 8f \dot{a}_2 s_2 s_0 + 4f s_0^2 \dot{a}_4 + 2f s_0^2 \dot{a}_2^2 \\ &\quad + 4\dot{s}_0^2 s_2 s_0 - 4\dot{s}_0 \dot{s}_2 s_0^2]\rho^2 + O(\rho^4) \\ g_{ii} &= \frac{f}{s_0^2}\frac{1}{\rho^2} - \frac{1}{s_0^3}[2s_2 f - a_2 s_0 \dot{f}] + \frac{1}{2s_0^4}[2s_0^2 \dot{f} a_4 + s_0^2 \ddot{f} a_2^2 \\ &\quad - 4\dot{f} a_2 s_0 s_2 - 4f s_4 s_0 + 6f s_2^2]\rho^2 + O(\rho^4). \end{aligned} \tag{A3}$$

To keep the PG form of the metric and to get a Minkowski boundary, one imposes the following conditions

$$g_{\rho\rho} = \frac{1}{\rho^2}, \qquad g_{\rho T} = 0, \qquad g_{\mu\nu} = \eta_{\mu\nu}\frac{1}{\rho^2} + O(1), \tag{A4}$$

this guarantees the existence of such a coordinate system, at least, close to the new boundary. These conditions lead to

$$\begin{aligned} s_0(T) &= f(T)^{1/2}, \qquad s_2(T) = \frac{\dot{f}(T)^2}{16 f(T)^{3/2}} \\ s_4(T) &= \frac{\dot{f}(T)^4}{256 f(T)^{7/2}}, .. \qquad a_2(T) = \frac{\dot{f}(T)}{4f(T)}, \\ a_4(T) &= 0. \end{aligned} \tag{A5}$$

Applying the above procedure to Kasner-type solutions in (20) with $f(t) = \frac{2}{3}t$, one can obtain the following coordinate transformations

$$\begin{aligned} z(\rho, T) &= \frac{\sqrt{6T}\rho}{3}\left[1 + \frac{\rho^2}{16T^2} + \frac{\rho^4}{256T^4}\right] + O(\rho^7), \\ t(\rho, T) &= T + \frac{\rho^2}{4T} + O(\rho^6). \end{aligned} \tag{A6}$$

The metric in these coordinates has a Minkowski boundary and has the following form

$$\begin{aligned} ds^2 &= \left[\frac{1}{\rho^2} + O(\rho^4)\right]d\rho^2 \\ &\quad - \left[\frac{1}{\rho^2} - \frac{5}{8T^2} + \frac{25}{256T^4}\rho^2 + O(\rho^4)\right]dT^2 \\ &\quad + \left[\frac{1}{\rho^2} + \frac{1}{8T^2} - \frac{7}{256T^4}\rho^2 + O(\rho^4)\right]d\bar{x}^2 \end{aligned} \tag{A7}$$

which agrees with the exact coordinate transformation in (22) and the metric (21) upon expanding it in powers of ρ.

Before applying this procedure to calculate the FRW solution with $k = -1$ let us use the coordinate system given in (38).

$$\tau^2 = \eta^2 - R^2, \qquad r^2 = \frac{\eta + R}{\eta - R}. \tag{A8}$$

This puts the metric in the form

$$\begin{aligned} ds^2 &= \frac{dz^2}{z^2} + \frac{1}{z^2}\left(1 - \frac{1}{\tau^4}\right) \\ &\quad \times \left[-d\tau^2 + \frac{t^2}{r^2}dr^2 + \frac{\tau^2}{4}\left(r - \frac{1}{r}\right)^2 d\Omega_2^2\right]. \end{aligned} \tag{A9}$$

Following the above procedure one can obtain the PG form of this solution with Minkowski boundary. The coordinate transformations and the metric are

$$\begin{aligned} z(\rho, \bar{T}) &= \frac{\sqrt{\bar{T}^4 - 1}\rho}{\bar{T}^2}\left[1 + \frac{\rho^2}{\bar{T}^2(\bar{T}^4 - 1)^2} + \frac{\rho^4}{\bar{T}^4(\bar{T}^4 - 1)^4}\right] \\ &\quad + O(\rho^7), \\ \tau(\rho, \bar{T}) &= \bar{T} + \frac{\rho^2}{(\bar{T}^4 - 1)\bar{T}} + O(\rho^6) \end{aligned} \tag{A10}$$

$$\begin{aligned} ds^2 &= \left[\frac{1}{\rho^2} + O(\rho^4)\right]d\rho^2 - \left[\frac{1}{\rho^2} - \frac{10\bar{T}^2}{(\bar{T}^4 - 1)^2} \right. \\ &\quad \left. + \frac{25\bar{T}^4}{(\bar{T}^4 - 1)^4}\rho^2 + O(\rho^4)\right]d\bar{T}^2 \\ &\quad + \left[\frac{1}{\rho^2} + \frac{2\bar{T}^2}{(\bar{T}^4 - 1)^2} + \frac{(\bar{T}^4 - 8)}{(\bar{T}^4 - 1)^4}\rho^2 + O(\rho^4)\right] \\ &\quad \times \left[\frac{\bar{T}^2}{r^2}dr^2 + \frac{\bar{T}^2}{4}\left(r - \frac{1}{r}\right)^2 d\Omega_2^2\right]. \end{aligned} \tag{A11}$$

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