Magnetic strings in QCD as non-Abelian vortices

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Lattice studies indicate existence of magnetic strings in QCD vacuum. We argue that recently found non-Abelian strings with rich world-sheet dynamics provide a pattern which fits the strings observed on the lattice. In particular, within this pattern we explain the localization of the monopole-antimonopole pairs on the magnetic string world sheet and the negative contribution of the magnetic strings into the vacuum energy and gluon condensate. We suggest the D2 brane realization of the magnetic string which explains the temperature dependence of its tension.

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I. INTRODUCTION

An explanation of the QCD vacuum structure remains a challenging problem. Recently, progress has been made in the lattice studies and their interpretation [1]. In particular, the essential contribution from the 2D surfaces (strings) and 3D volumes (domain walls) with some unusual properties to the vacuum characteristics has been found. For our purposes, the key properties of the magnetic strings observed on the lattice can be summarized as follows (for references see Sec. IV below and reviews [1]):

(i) The tension of the magnetic strings vanishes below the critical temperature and they percolate through the vacuum, forming a kind of a vacuum condensate.
(ii) The world sheet of magnetic string is populated by monopole-antimonopole pairs.
(iii) Above the temperature of the deconfinement phase transition magnetic string becomes tensionful.

Most recently, it was argued that

(i) The magnetic strings become a component of Yang-Mills plasma [2] and the first measurements indicate surprisingly that
(ii) The contribution of the strings to the gluon condensate and 4D bulk vacuum energy is opposite in sign compared to its total value [3].

A natural question concerns the very existence of strings with such properties in the continuum theory. The goal of this note is to argue that non-Abelian magnetic strings found recently in the SUSY gauge theories naturally provide the desired pattern. We are not aiming to prove rigorously that non-Abelian strings populate the QCD vacuum. However, our considerations clearly indicate that this kind of object fits the lattice data perfectly.

The non-Abelian strings which are essentially twisted Z_N strings with orientational moduli were first found in SUSY context [4,5]. However, later it was recognized that they do exist in non-SUSY theories as well [6] (see [7–9] for reviews). The key property of the non-Abelian strings which distinguishes them from other objects discussed in this context is highly nontrivial world-sheet theory which in the simplest examples can be identified with the $CP(N-1)$ sigma model. Moreover, it was found that kinks on the world sheet are nothing else but the 4D monopoles "trapped" by the string [10,11]. In the non-supersymmetric case $CP(N-1)$ world-sheet theory is in the confinement phase [12] so that only kink-antikink pairs exist which parallels the lattice QCD observations. It was also argued recently that non-Abelian strings could play an essential role in the Seiberg duality [13,14].

As is mentioned above, the very recent lattice data indicates that magnetic strings contribute to the vacuum energy and gluon condensate above the critical temperature with the unexpected sign [3]. On the other hand, it was a found long time ago [15] that vacuum energies in 4D gauge theories and the 2D $CP(N-1)$ sigma model have opposite signs. We argue that this old observation provides a pattern for an interpretation of the recent lattice data [3].

The lattice data suggests that the tension of the magnetic string is zero below the deconfinement temperature T_c and the question is whether the non-Abelian strings share this property. To get insight into the problem, we will use the brane realization of the non-Abelian string as a D2 brane in a particular supergravity background. Within this picture we argue that interpretation of the magnetic string as wrapped D2 brane explains the temperature dependence of the tension. In the simple model, we discuss the background geometry formed by N_c D4 branes wrapped around one compact dimension. Since both D4 and D2 branes share coordinates, one could expect the effect of dissolving of D2 branes inside D4 branes due to the tachyons in the spectrum of D2-D4 strings. Since D4 branes are substituted by the background geometry, one could expect that there exists a counterpart of dissolving phenomena in the gravity background. We will argue that in the dual gravity picture below the deconfinement temperature D2 branes yield tensionless strings which can condense indeed. It is this effect that corresponds to the dissolving phenomena at large N_c. The change of the background above the critical temperature results to the tensionful strings and an interesting phenomenon that the properties of the time- and space-oriented magnetic strings become different.

The paper is organized as follows. In Sec. II we explain the construction of the non-Abelian string solution in a simple model. In Sec. III we argue that the non-Abelian string pattern explains the negative contributions to the vacuum energy and the gluon condensate and temperature dependence of the magnetic-string tension. In Sec. IV we briefly compare our picture with available lattice data on the magnetic strings. Section V involves a brief discussion of the results obtained and some unsolved issues.

II. NON-ABELIAN STRINGS

Here we review the simplest model which can be used to analyze non-Abelian strings. The gauge group of the model is $SU(N) \times U(1)$. Besides $SU(N)$ and $U(1)$ gauge bosons, the model contains N scalar fields charged with respect to $U(1)$ which form N fundamental representations of $SU(N)$. It is convenient to write these fields as $N \times N$ matrix $\Phi = \{\varphi^{kA}\}$ where k is the $SU(N)$ gauge index while A is the flavor index,

$$\Phi = \begin{pmatrix} \varphi^{11} & \varphi^{12} & \cdots & \varphi^{1N} \\ \varphi^{21} & \varphi^{22} & \cdots & \varphi^{2N} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi^{N1} & \varphi^{N2} & \cdots & \varphi^{NN} \end{pmatrix}. \tag{1}$$

The action of the model has the form

$$S = \int d^4x \Big\{ \frac{1}{4g_2^2}(F^a_{\mu\nu})^2 + \frac{1}{4g_1^2}(F_{\mu\nu})^2 + \mathrm{Tr}(\nabla_\mu\Phi)^\dagger(\nabla^\mu\Phi) + \frac{g_2^2}{2}[\mathrm{Tr}(\Phi^\dagger T^a\Phi)]^2 + \frac{g_1^2}{8}[\mathrm{Tr}(\Phi^\dagger\Phi) - N\xi]^2 + \frac{i\theta}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\Big\}, \tag{2}$$

where T^a stands for the generator of the gauge $SU(N)$,

$$\nabla_\mu\Phi \equiv \left(\partial_\mu - \frac{i}{\sqrt{2N}}A_\mu - iA^a_\mu T^a\right)\Phi, \tag{3}$$

and θ is the vacuum angle. The action (2) represents a truncated bosonic sector of the $N = 2$ SUSY model. The last term in the second line forces Φ to develop a vacuum expectation value (VEV) while the last but one term forces the VEV to be diagonal,

$$\Phi_{\mathrm{vac}} = \sqrt{\xi}\,\mathrm{diag}\{1, 1, \ldots, 1\}. \tag{4}$$

We assume that the parameter ξ is large,

$$\sqrt{\xi} \gg \Lambda_4, \tag{5}$$

where Λ_4 is the scale of the four-dimensional theory (2). That is, we are in the weak-coupling regime since both couplings g_1^2 and g_2^2 are frozen at a large scale.

The vacuum field (4) results in spontaneous breaking of both gauge and flavor $SU(N)$'s. A diagonal global $SU(N)$ survives

$$U(N)_{\mathrm{gauge}} \times SU(N)_{\mathrm{flavor}} \to SU(N)_{\mathrm{diag}}, \tag{6}$$

yielding color-flavor locking in the vacuum.

Within this model, there exists a topologically stable string solution. The topological considerations unify the Z_N center of the $SU(N)$ with the elements $\exp(2\pi ik/N) \in U(1)$. In other words,

$$\pi_1(SU(N) \times U(1)/Z_N) \neq 0, \tag{7}$$

and this nontrivial topology amounts to winding of just one element of Φ_{vac}. For instance, asymptotically

$$\Phi_{\mathrm{string}} = \sqrt{\xi}\,\mathrm{diag}(1, 1, \ldots, e^{i\alpha(x)}), \qquad x \to \infty. \tag{8}$$

These strings can be called elementary Z_N strings; their tension is $1/N$th of that of the Abrikosov-Nielsen-Olesen (ANO) string. The ANO string can be viewed as a bound state of N Z_N strings.

The Z_N string solution can be written as follows [5]:

$$\begin{aligned} \Phi &= \begin{pmatrix} \phi(r) & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \phi(r) & 0 \\ 0 & 0 & \cdots & e^{i\alpha}\phi_N(r) \end{pmatrix}, \\ A_i^{SU(N)} &= \frac{1}{N}\begin{pmatrix} 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & -(N-1) \end{pmatrix} \times (\partial_i\alpha)[-1 + f_{NA}(r)], \\ A_i^{U(1)} &= \frac{1}{N}(\partial_i\alpha)[1 - f(r)], \qquad A_0^{U(1)} = A_0^{SU(N)} = 0, \end{aligned} \tag{9}$$

where $i = 1, 2$ labels coordinate in the plane orthogonal to the string axis and r and α are the polar coordinates in this plane. The profile functions $\phi(r)$ and $\phi_N(r)$ determine the profiles of the scalar fields, while $f_{NA}(r)$ and $f(r)$ determine the $SU(N)$ and $U(1)$ fields of the string solutions, respectively. These functions satisfy the following boundary conditions:

$$\phi_N(0) = 0, \qquad f_{NA}(0) = 1, \qquad f(0) = 1, \tag{10}$$

at $r = 0$, and

$$\begin{aligned} \phi_N(\infty) &= \sqrt{\xi}, \qquad \phi(\infty) = \sqrt{\xi}, \\ f_{NA}(\infty) &= 0, \qquad f(\infty) = 0 \end{aligned} \tag{11}$$

at $r = \infty$. These profile functions satisfy the following first-order differential equations:

$$
\begin{aligned}
r\frac{d}{dr}\phi_N(r) - \frac{1}{N}(f(r) + (N-1)f_{NA}(r))\phi_N(r) &= 0,\\
r\frac{d}{dr}\phi(r) - \frac{1}{N}(f(r) - f_{NA}(r))\phi(r) &= 0,\\
-\frac{1}{r}\frac{d}{dr}f(r) + \frac{g_1^2 N}{4}[(N-1)\phi(r)^2 + \phi_N(r)^2 - N\xi] &= 0,\\
-\frac{1}{r}\frac{d}{dr}f_{NA}(r) + \frac{g_2^2}{2}[\phi_N(r)^2 - \phi(r)^2] &= 0.
\end{aligned}
\tag{12}
$$

The tension of this elementary string is

$$T_1 = 2\pi\xi \tag{13}$$

while the tension of the ANO string is

$$T_{\rm ANO} = 2\pi N\xi \tag{14}$$

which confirms its composite nature.

The elementary strings are essentially non-Abelian since, besides trivial translational moduli, they give rise to moduli corresponding to the spontaneous breaking of a non-Abelian symmetry. Indeed, while the "flat" vacuum is $SU(N)_{\rm diag}$ symmetric, the solution (9) breaks this symmetry down. This means that the world-sheet theory of the elementary string moduli is the $CP(N-1)$ sigma model.

To obtain the non-Abelian string solution from the Z_N string (9), we apply the diagonal color-flavor rotation preserving the vacuum (4). Consider the singular gauge where the scalar fields have no winding at infinity, while the string flux comes from the vicinity of the origin. In the singular gauge we have

$$
\begin{aligned}
\Phi &= U\begin{pmatrix} \phi(r) & 0 & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & \cdots & \phi(r) & 0\\ 0 & 0 & \cdots & -\phi_N(r)\end{pmatrix}U^{-1},\\
A_i^{SU(N)} &= \frac{1}{N}U\begin{pmatrix} 1 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & \cdots & 1 & 0\\ 0 & 0 & \cdots & -(N-1)\end{pmatrix}U^{-1}\\
&\quad\times(\partial_i\alpha)f_{NA}(r),\\
A_i^{U(1)} &= -\frac{1}{N}(\partial_i\alpha)f(r), \qquad A_0^{U(1)} = A_0^{SU(N)} = 0,
\end{aligned}
\tag{15}
$$

where U is a matrix $\in SU(N)$. This matrix parametrizes orientational zero modes of the string associated with flux embedding into $SU(N)$. The orientational moduli encoded in the matrix U were first observed in [4,5].

Turn now to the world-sheet description of the non-Abelian string. It is important that there are two independent contributions from "space" and "internal" terms. The space-time action does not reduce entirely to the Nambu-Goto term which is only the first approximation term. The corresponding tension is proportional to ξ. To obtain the kinetic term in the "internal" action we follow the standard logic in the derivation of the low-energy action in the moduli approximation. That is, we substitute our solution, which depends on the moduli n^l, into the action, assuming that the fields acquire dependence on the coordinates x_k via $n^l(x_k)$. Then we arrive at the $CP(N-1)$ sigma model (for details see [8]),

$$S_{CP(N-1)}^{(1+1)} = 2f\int dt\,dz\{(\partial_k n^*\partial_k n) + (n^*\partial_k n)^2\}, \tag{16}$$

where the coupling constant f is determined by the normalization condition defined in terms of the string profile functions:

$$f = \frac{2\pi}{g_2^2}. \tag{17}$$

That is, the two-dimensional coupling constant is determined by the four-dimensional non-Abelian coupling.

The relation between the four-dimensional and two-dimensional coupling constants (17) is obtained on the classical level. In quantum theory both couplings run and hence we have to specify the scale at which the relation (17) holds. The two-dimensional $CP(N-1)$ model is an effective low-energy theory valid for description of internal string dynamics at low energies, much lower than the inverse thickness of the string which, in turn, is given by $g_2\sqrt{\xi}$. Therefore, $g_2\sqrt{\xi}$ plays the role of a physical ultraviolet cutoff in (16). Below this scale, the coupling f runs according to its two-dimensional renormalization-group flow.

The sigma model (16) is asymptotically free; hence, at large distances it gets into the strong coupling regime. The running coupling constant as a function of the energy scale E at one loop is given by

$$4\pi f = N\ln\left(\frac{E}{\Lambda_{CP(N-1)}}\right) + \cdots, \tag{18}$$

where $\Lambda_{CP(N-1)}$ is the dynamical scale of the $CP(N-1)$ model. As was mentioned above, the UV cutoff of the sigma model at hand is determined by $g_2\sqrt{\xi}$. Hence,

$$\Lambda_{CP(N-1)}^N = g_2^N\xi^{N/2}e^{-(8\pi^2/g_2^2)}. \tag{19}$$

In the bulk theory, due to the VEV's of the scalar fields, the coupling constant is frozen at $g_2\sqrt{\xi}$. There are no logs in the bulk theory below this scale and the logs of the world-sheet theory take over.

III. MAGNETIC STRINGS VERSUS NON-ABELIAN STRINGS

A. Monopole pairs on the world sheet

Let us show that the pattern of the non-Abelian strings provides an explanation of the properties of the magnetic strings observed on the lattice. First, we would like to note that from the discussion above it is clear that monopole pairs are present on the non-Abelian string indeed.

The world-sheet theory is the nonsupersymmetric σ-model which has a single vacuum state and the spectrum consists of kink-antikink bound states [12]. These bound states can be identified with monopole-antimonopole bound states from the four-dimensional viewpoint. The IR scale Λ_{CP} is generated in the world-sheet theory and can be related to the scale in the bulk theory. The masses of the bound states in the theory are of order Λ_{CP} and they cannot be found exactly since world-sheet theory is in the strong coupling regime. In the SUSY setup, one can consider massive flavors yielding the quasiclassical picture of the bound states. In the non-SUSY case we have no such simple argumentation. Note however that the monopoles in the Higgs phase on the string world sheet are smoothly related to the t'Hooft-Polyakov monopoles via the continuous deformation in the parameter space (see a recent discussion in [16]).

If one introduces a θ term in the bulk theory then due to the Witten effect monopoles acquire nonzero electric charge and become dyons. A similar phenomenon happens on the world sheet as well. The θ term penetrates into the world-sheet theory and a kink in the world-sheet theory acquires global charge.

B. Vacuum energy and gluon condensate

In view of the recent lattice measurements [3] of contribution of magnetic strings into Yang-Mills plasma energy, we will consider the energy issue in the context of the non-Abelian strings. There are two contributions to the energy associated with dynamics in space-time and internal space, respectively. These contributions above the critical temperature can be treated separately. Our basic observation is that the contribution from the internal, $CP(N-1)$ part is in fact negative and opposite in sign to its total value, in agreement with the lattice data.

First, note that vacuum energy (at vanishing temperature) in the Yang-Mills theory is related to the conformal anomaly,

$$E_{\rm vac}^{\rm YM} = \frac{1}{4}\langle 0|\theta_{\mu\mu}^{\rm YM}|0\rangle = \langle 0| - \frac{b_0\alpha_S}{32\pi}\,{\rm Tr}G^2|0\rangle, \tag{20}$$

where b_0 is the beta function coefficient. A similar relation holds in the $CP(N-1)$ model as well. Namely,

$$E_{\rm vac}^{CP} = \frac{1}{2}\langle 0|\theta_{\mu\mu}^{CP}|0\rangle = \frac{N}{8\pi}\Lambda_{CP}^2. \tag{21}$$

The gluon condensate $\langle {\rm Tr}G^2\rangle$ gets a contribution from the non-Abelian strings since the internal tension is proportional to inverse gauge coupling,

$$\langle {\rm Tr}G^2\rangle_{\rm tot} \propto \frac{d}{d(1/g^2)}\log Z \propto \langle {\rm Tr}G^2\rangle_{\rm YM} + C_{CP}\langle {\rm Tr}G^2\rangle_{CP}. \tag{22}$$

The two-dimensional contribution from the non-Abelian strings comes from the vacuum expectation value of two-dimensional conformal anomaly in the $CP(N-1)$ model which has the opposite sign [15] compared to the total value

$$\langle {\rm Tr}G^2\rangle_{CP} \propto \langle 0|\theta_{\mu\mu}^{CP}|0\rangle = \frac{N}{8\pi}\Lambda_{CP}^2. \tag{23}$$

The value of the dimensionful constant C_{CP} is determined by the density of the strings and we cannot estimate its value at the moment. Let us emphasize that above the critical temperature the world-sheet theory on the magnetic string is supposed to be in the confinement phase from the four-dimensional viewpoint. Let us emphasize that we consider only the nonperturbative contributions to the vacuum energy which is determined by the nonperturbative contribution to the conformal anomaly. Increasing the temperature we could reach the temperature of the phase transition on the magnetic-string world sheet. At this point the magnetic-string contribution into the vacuum energy shall vanishes.

C. Low-energy theorems and dilaton

Condensation of the magnetic string, observed on the lattice, asks for consideration of the backreaction of a single non-Abelian string on the bulk fields. Below we use the low-energy theorems in the $CP(N-1)$ model to argue that the scalar mode on the world sheet contributes negatively to the mass squared of the corresponding mode in four dimensions, contrary to the pseudoscalar case.

In the bulk theory, for any operator A there holds the dilatation Ward identity,

$$i\int d^4x\langle 0|\theta_{\mu\mu}^{\rm YM}(x)A(0)|0\rangle = -d_A\langle 0|A|0\rangle, \tag{24}$$

where d_A is the canonical dimension of the operator A. This equation follows from the very fact that the theory is asymptotically free. Similar arguments apply to the world-sheet theory and the corresponding dilatation Ward identity reads [15]

$$i\int d^2x\langle 0|\theta_{\mu\mu}^{CP}(x), A(0)|0\rangle = -d_A\langle 0|A|0\rangle, \tag{25}$$

where we consider the correlator of the $\theta_{\mu\mu}$ with an arbitrary operator in the $CP(N-1)$ sigma model.

Some of the operators A are of special interest. Consider first the operator $A = {\rm Tr}G^2$ so that the corresponding low-energy theorem reads as

$$i\int d^4x\langle 0|{\rm Tr}G^2(x), {\rm Tr}G^2(0)|0\rangle = S_{\rm YM}(0) \propto \langle 0|{\rm Tr}G^2|0\rangle. \tag{26}$$

In YM theory the right-hand side is positive and if we assume one-particle saturation of $S(0)$ this sign corresponds to a positive mass squared. This particle naturally could be related to the dilaton ϕ because of the standard coupling of the dilaton, $e^{\phi}\,\mathrm{Tr}G^2$. On the other hand, it is clear from the arguments above that this correlator has a contribution from the string of the form

$$i\int d^2x\langle 0|\theta^{CP}_{\mu\mu}(x),\theta^{CP}_{\mu\mu}(0)|0\rangle = S_{CP}(0). \qquad (27)$$

The low-energy theorem yields $S_{CP}<0$ [15] which corresponds to a tachyonic contribution to the mass of the particle in the intermediate state. The total mass squared of the scalar is positive while the stringy contribution is negative.

Let us compare the bulk–world-sheet interplay of the dilaton dynamics with the similar consideration concerning axion [17]. It was shown in [17] that the two-dimensional axion due to the mixing with photon is responsible for deconfinement on the world sheet. The reason is that because of this mixing the world-sheet photon becomes massive and linear confinement disappears. On the other hand, the non-Abelian string does not cause a strong modification of the bulk dynamics and results only on the axion-emission halo around the string.

One can consider the correlator of topological-charge densities:

$$\frac{d^2\log Z}{d^2\theta}=\int d^4x\langle 0|\mathrm{Tr}G\tilde{G}(x),\mathrm{Tr}G\tilde{G}(0)|0\rangle = P_{\mathrm{YM}}(0), \quad (28)$$

which can be saturated by the axion in the intermediate state (we have no quarks). Since the four-dimensional θ term penetrates into the world-sheet theory, the $P(0)$ has the two-dimensional contribution

$$i\int d^2x\langle 0|F(x)F(0)|0\rangle = P_{CP}(0), \qquad (29)$$

where $F=\epsilon_{\mu\nu}\partial_\mu A_\nu$ and A_μ is the auxiliary Abelian gauge field in the $CP(N-1)$ model which acquires the mass at the quantum level. The contribution of the P_{CP} to the total $P(0)$ depends on the density of the non-Abelian strings but the crucial point now is the sign of this contribution. Namely, it is known [15] that the value of $P_{CP}(0)$ is positive and has the same sign as the total correlator. That is, we have opposite influence of the non-Abelian strings on the dynamics of the dilaton and axion. The string tends to decrease the mass of scalar and acts oppositely in the pseudoscalar case.

D. Magnetic-string tension from brane perspective

Let us discuss the brane realization of the magnetic string. To begin with, consider the weak-coupling non-Abelian string. In the $N=2$ SQCD case, the non-Abelian string is perfectly identified as a D2 brane stretched between two NS5 branes placed at large distance ξ in some direction. According to the standard logic, the tension of the non-Abelian string then turns out to be proportional to ξ so that the quasiclassical analysis is reasonable.

The geometry of the non-SUSY QCD is not established well enough. However, the natural starting point is the geometry provided by the set of D4 branes wrapped around one compact dimension [18]. We shall consider the pure gauge sector and do not discuss the chiral matter in this note.

We shall assume the large N_C limit and consider the supergravity approximation. In this approximation the geometry looks as $M_{10}=R_{3,1}\times D\times S^4$ and the corresponding metric reads as

$$ds^2=\left(\frac{u}{R_0}\right)^{3/2}(-dt^2+\delta_{ij}dx^idx^j+f(u)dx_4^2) + \left(\frac{u}{R_0}\right)^{-3/2}\left(\frac{du^2}{f(u)}+u^2d\Omega_4^2\right)e^{\Phi}=\left(\frac{u}{R_0}\right)^4,$$
$$F_4=\frac{3N_c\epsilon_4}{4\pi},\qquad f(u)=1-\left(\frac{u_\Lambda}{u}\right)^3, \qquad (30)$$

where $R_0=(\pi g_sN_c)^{1/3}$ and $R=\frac{4\pi}{3}(\frac{R_0^3}{u_\Lambda})^{1/2}$. The coupling constant of Yang-Mills theory is related to the radius of the compact dimension R as follows:

$$g^2_{\mathrm{YM}}=\frac{8\pi^2g_sl_s}{R}.$$

At zero temperature theory is in the confinement phase and in the (u,x_4) coordinates we have the geometry of a cigar with the tip at $u=u_\Lambda$. The D4 branes are located along our $\mathrm{D}=4$ geometry and are extended along the x_4 coordinate. Let us emphasize that for the magnetic string we discuss the target space looks as $M_{10}\times CP(N-1)$ and involves an "internal" part.

Let us turn to our proposal for the magnetic string within the brane setup. Assuming the non-Abelian string to constitute a correct pattern for the magnetic string implies that the magnetic string at the strong coupling regime is the probe D2 brane wrapped around S_1 parametrized by x_4 and its tension is therefore proportional to the effective radius $R(u)$. Because of the cigar geometry this wrapping is topologically unstable and the D2 brane shrinks to the tip where its tension vanishes. This is the large N_c counterpart of the effect of dissolving of p-brane inside $p+2$-brane [19]. We see that in this way one immediately reproduces the observed property of tensionlessness of the magnetic string at zero temperature.

We can also check that there arises the correct θ-term in the magnetic-string world-sheet Lagrangian. To trace the θ term let us consider the CS term on the D2 world volume,

$$L_{CS}=\int d^3xC_1\wedge F, \qquad (31)$$

where C_1 is the Ramond-Ramond (RR) one-form field. Taking into account that $\theta=\int dx_4C_1$, we reproduce the standard θ-term in the $CP(N-1)$ model $\theta\int d^2xF$.

Consideration of the finite $-N_C$ case is much more subtle. Let us consider the finite N_C D4 brane wrapped around x_4 coordinate and a single D2 brane wrapped around x_4 as well. What could be the mechanism for the magnetic-string condensation for finite N_C? A natural conjecture is that the phenomenon of dissolving of the p-brane inside the $(p+2)$-brane [19] takes place here. Indeed, in our D2-D4 system we have proper brane dimensions and the tachyonic mode of the D2-D4 open string could lead to the D2 brane condensation providing an additional magnetic field in four-dimensional theory via the CS term on the D4 world sheet,

$$L_{CS} = \int d^5 x C_3 \wedge F, \tag{32}$$

induced by the D2 brane RR field. However, it is not clear how this D2-D4 string tachyonic mode could disappear at the critical temperature and this point needs a careful consideration. Possible stabilization mechanisms of the p-brane inside the $(p+2)$-brane which could be relevant also in our context were discussed in [20] and are based on the account of RR fields in the bulk.

E. Temperature dependence of the magnetic string

A crucial test of our proposal concerns the temperature dependence of the magnetic string. We have argued above that at zero and small temperatures the cigar geometry in the (x_4, u) plane amounts to the vanishing tension of the magnetic string since the radius of the circle D2 brane wrapped around shrinks to zero. However, the magnetic string becomes tensionful above the critical temperature T_c of the deconfinement phase transition. How does the change of two regimes happens?

The key point is that in the nonzero temperature case there are two backgrounds with similar asymptotic topology of $R^3 \times S^1_\tau \times S^1 \times S^4$, where τ is the Wick-rotated time coordinate $\tau = it$, $\tau \propto \tau + \beta$. One background corresponds to the analytic continuation of the metric described above while the second background corresponds to interchange of τ and x_4, that is the warped factor is attached to the τ coordinate and the cigar geometry emerges in the (τ, u) plane instead of the (x_4, u) plane which now exhibits the cylinder geometry, see Fig. 1. It was shown in [21] by calculation of the free energies that above T_c the second background dominates.

Thus, above $T = T_c$ one gets the geometry of the cylinder in (x_4, u) and of the cigar in (τ, u), so that the wrapping around x_4 is topologically stable now and the magnetic-string tension is proportional to the cylinder radius. Moreover, by construction the D2 brane is wrapped around the x_4 coordinate but the other two coordinates of the D2 brane can fill different dimensions. If both coordinates of the magnetic string are transverse to the time direction, it does not feel the instability in the (τ, u) cigar geometry and behaves as the S-string. On the other hand, if the magnetic string is wrapped around the τ coordinate, it is unstable in the cigar geometry and shrinks along the τ coordinate to zero. That is, the magnetic string extended in the time direction loses one physical dimension above T_C and, speaking somewhat loosely, looks as a "particle." One could say that the vanishing tension below the critical temperature is "traded for" a lost dimension above the critical temperature.

Let us emphasize that we have discussed above the "space" tension corresponding to the linear density of the energy which jumps at the phase transition point. On the other hand, the "internal" tension defining the $CP(N-1)$ part of the action of the magnetic string $T_{\text{int}} = 1/g^2$ could change smoothly at any temperature in accordance with the asymptotic freedom.

Note that the discussion in this section is somewhat similar to the consideration in [22] of the role of the instantons in the similar geometry which are represented by Euclidean D0 branes wrapped around x_4. In that case it was argued that the single instanton is ill defined below T_c because of the D0-brane instability in the cigar geometry while above T_c it is well defined due to the geometry of the cylinder. The change of the instanton role at the transition point corresponds to the change from the Witten-Veneziano to t'Hooft mechanisms of the solution to the U(1) problem.

IV. LATTICE DATA

A. Lattice strings at zero temperature

In this section we will provide a short guide to the literature on lattice measurements relevant to the theoretical issues discussed in this note.

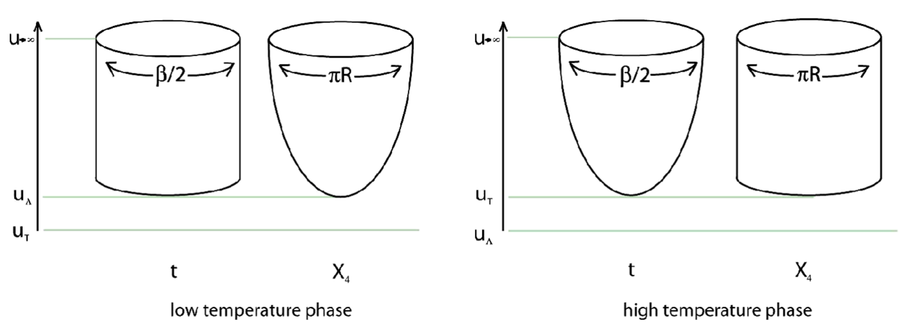

FIG. 1 (color online). Compact coordinates below and above the phase transition temperature.

Magnetic strings were introduced first in the context of the confinement studies, as confining field configurations and are known mostly as "center vortices," for review and references see [23]. In particular, it was found that the vortices percolate in the vacuum, i.e. form an infinite cluster, or a kind of condensate. Also, the total area of the vortices is in physical units,

$$(\mathrm{Area})_{\mathrm{total}} \sim \Lambda^2_{\mathrm{QCD}} V_{\mathrm{total}}, \tag{33}$$

where V_{total} is the total volume of the lattice.

For confinement, the transverse size of the strings is not crucial and the strings were mostly thought of as "thick vortices." The fact that they are actually thin, two-dimensional surfaces was discovered as a result of measurements of the distribution of non-Abelian action associated with the vortices [24]. The action turned to be singular in the continuum limit,

$$(\mathrm{Action})_{\mathrm{lattice}} \sim (\mathrm{Area})_{\mathrm{total}}/a^2, \tag{34}$$

where a is the lattice spacing, $a \to 0$ in the continuum limit. Moreover, the non-Abelian field living on the surface is aligned, or trapped to the surface. It is these thin strings which are relevant to our discussion. Moreover, the strings are closed in the vacuum state but can be open on an external 't Hooft line, for argumentation and references see [1]. Hence, the name of "magnetic strings."

Note the physical string tension is not directly related to the lattice action but is to be rather calculated as a difference between lattice action and entropy factors (see, e.g., [25]). It is difficult to directly check the strength of the cancellation between this two contributions. The fact that the physical tension for the lattice magnetic strings is vanishing in the confining phase follows from the very existence of an infinite, or percolating cluster of surfaces. Indeed, if the tension were not zero only finite clusters could be observed, by virtue of the uncertainty principle.

Lattice monopoles, in turn, are identified as closed trajectories, or particles (for review see [26]). Their lattice algorithmic definition is independent of the definition of the surfaces, or strings. Nevertheless, the lattice simulations reveal that the monopole trajectories lie in fact on the surfaces [24,27]. The non-Abelian fields associated with the monopoles are also singular [28] and are aligned with the surfaces [24].

All the lattice data on the magnetic strings are obtained with the standard Wilson action and in fact refer mostly to pure Yang-Mills cases. There is no direct explanation of the data within the Yang-Mills theory itself. One can check, however, that the singular non-Abelian fields are just of the type which is in no contradiction with the asymptotic freedom [29].

B. Lattice strings in the deconfinement phase

We also considered strings at nonzero temperature and here we will provide references to the lattice measurements at temperatures above the deconfinement phase transition.

The basic observation made on the lattice [23,30] is that at temperatures above the phase transition the strings become time oriented. The four-dimensional infinite percolating cluster is dissolved and does not exist any longer. However, the percolation is not eliminated altogether. Namely, in three-dimensional slices the four-dimensional strings are projected to lines. In the case of the magnetic strings, the properties of these lines depend crucially on whether one considers equal-time or equal-space-coordinate slices. In case of equal-time slices the lines, which are intersections of the strings and of the 3D spaces, continue to percolate. In case of the equal-space-coordinate slices, there is no percolation at all.

Clearly, these lattice data are reproduced by the phenomenon of a "missed dimension" discussed in detail in Sec. III E in the brane language.

V. DISCUSSION

During the past 30 years, the derivation of the Mandelstam's qualitative explanation of the confinement via the dual Meissner effect was the main goal of the nonperturbative QCD studies. The recent lattice data suggests that probably the picture is to be modified and condensation of the tensionless magnetic strings takes place in QCD vacuum, instead of the condensation of the magnetic monopoles. If fact, there is no deep contradiction between two scenarios. Indeed the magnetic strings observed on the lattice support the monopoles at their world sheets. In other words, condensation of the strings implicitly assumes the condensation of the monopoles. The monopoles become, however, particles living on a string, or in 2D instead of ordinary particles living in 4D.

In this paper we conjecture that the strings observed on the lattice follow the pattern of the non-Abelian strings with their rich world-sheet structure supporting monopole-antimonopole pairs. We have argued that this picture explains qualitatively basic facts about the magnetic strings observed on the lattice in the pure Yang-Mills case. Moreover, it turns out that the interpretation of the magnetic strings as wrapped D2 branes fits perfectly with their properties. In particular, within D2 brane interpretation we explain the existence of the magnetic-string condensate below the critical temperature and nonzero tension above the critical temperature.

We have focused on the pure Yang-Mills theory; however, generalization to QCD with fundamental quarks seems possible. In particular, it is interesting to investigate the role of the magnetic strings in the chiral properties of the theory. In the brane setup we can add N_f flavor branes and analyze the dynamics in the Sakai-Sugimoto model [31] (see also [32] for the earlier papers). We have discussed magnetic strings that are wrapped D2 branes only. However, there are other wrapped D4 and D6 branes in this setup which have an interesting interpretation. These issues shall be discussed elsewhere.

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