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Origin of primordial magnetic fields

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Magnetic fields of intensities similar to those in our galaxy are also observed in high redshift galaxies, where a mean field dynamo would not have had time to produce them. Therefore, a primordial origin is indicated. It has been suggested that magnetic fields were created at various primordial eras: during inflation, the electroweak phase transition, the quark-hadron phase transition (QHPT), during the formation of the first objects, and during reionization. We suggest here that the large-scale fields $\sim\mu$G, observed in galaxies at both high and low redshifts by Faraday rotation measurements (FRMs), have their origin in the electromagnetic fluctuations that naturally occurred in the dense hot plasma that existed just after the QHPT. We evolve the predicted fields to the present time. The size of the region containing a coherent magnetic field increased due to the fusion of smaller regions. Magnetic fields (MFs) $\sim 10\ \mu$G over a comoving ~ 1 pc region are predicted at redshift $z \sim 10$. These fields are orders of magnitude greater than those predicted in previous scenarios for creating primordial magnetic fields. Line-of-sight average MFs $\sim 10^{-2}\ \mu$G, valid for FRMs, are obtained over a 1 Mpc comoving region at the redshift $z \sim 10$. In the collapse to a galaxy (comoving size ~ 30 kpc) at $z \sim 10$, the fields are amplified to $\sim 10\ \mu$G. This indicates that the MFs created immediately after the QHPT (10^{-4} s), predicted by the fluctuation-dissipation theorem, could be the origin of the $\sim\mu$G fields observed by FRMs in galaxies at both high and low redshifts. Our predicted MFs are shown to be consistent with present observations. We discuss the possibility that the predicted MFs could cause non-negligible deflections of ultrahigh energy cosmic rays and help create the observed isotropic distribution of their incoming directions. We also discuss the importance of the volume average magnetic field predicted by our model in producing the first stars and in reionizing the Universe.

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I. INTRODUCTION

The origin of galactic and extragalactic magnetic fields is one of the most challenging problems in modern astrophysics [e.g., [1,2]]. Magnetic fields on the order of $\sim\mu$G are detected in galaxies as well as in clusters of galaxies. It is generally assumed that the coherent large-scale $\sim\mu$G magnetic fields observed in disk galaxies are amplified and maintained by an $\alpha - \omega$ dynamo, which continuously generates new fields by the combined action of differential rotation (ω) and helical turbulence (α). However, the dynamo mechanism needs seed magnetic fields and sufficient time in order to amplify them.

There have been many attempts to explain the origin of seed fields. One of the most popular is that they are generated by the Biermann mechanism [3]. It has been suggested that this mechanism acts in diverse astrophysical systems, such as large-scale structure formation [4–6], cosmological ionizing fronts [7], and formation of supernova remnants of the first stars [8]. Outflows is an additional means of filling protogalaxies with magnetic fields. For example, in Sec. III G, we discuss outflows of magnetic fields from extragalactic jets, as suggested in [9].

Another suggestion for the origin of seed fields is that they were created by different mechanisms in the very early Universe, before galaxy formation took place. For example, such fields may have been created during the quark-hadron phase transition (QHPT), when the Universe was at a temperature $T_{\rm QHPT} \cong 10^{12}$ K (Sec. III A), during the electroweak phase transition (Sec. III B), or in the inflation era (Sec. III C).

One major difficulty with most scenarios for the creation of magnetic fields in the very early primordial Universe ($\ll 1$ sec), such as those discussed in Secs. III A, III B, and III C, is the small coherence lengths of the fields at redshifts $z \lesssim 10$. The coherence length is limited by the radius of the horizon at the time of the creation of the magnetic field. When expanded to the present time, the coherence length is too small to explain the existing observed large coherent magnetic fields on the order of the size of galaxies.

In this paper, we suggest that the observed magnetic fields have their origin in the electromagnetic fluctuations in the hot dense plasmas of the very early Universe. This is a natural way to create magnetic fields and circumvents the problem of small coherence lengths. The fluctuation-dissipation theorem predicts very large magnetic fields in the equilibrium plasma immediately after the QHPT. We evolve these fields to a redshift $z \sim 10$, when galaxies were beginning to form and find them to be sufficiently strong to

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explain the magnetic field observations in both high and low redshift galaxies.

We investigate the magnitude of the present magnetic fields in galaxies and the intergalactic medium created by the plasma fluctuations shortly after the QHPT, when the plasma properties are well understood. The magnetic fields, created by the plasma fluctuations before the QHPT, are poorly understood and we leave their evaluation for a future investigation.

Using the fluctuation-dissipation theorem (FDT), Opher and Opher [10–12] studied the magnetic fluctuations as a function of frequency in the primordial nucleosynthesis era and found that they were very large, in particular, at zero frequency. This can be compared with the blackbody prediction which has a zero amplitude magnetic fluctuation at zero frequency.

Tajima *et al.* [13] suggested that the large magnetic fluctuations predicted by the FDT at an early epoch did not dissipate, but continued to exist to the present epoch and now contribute to the dominant magnetic field. This scenario is investigated in detail here. Since the largest magnetic fluctuations in the plasma occurred shortly after the QHPT, we begin our calculations at this epoch.

Primordial magnetic fields can effect the incoming directions of ultrahigh energy cosmic rays (UHECRs) above 3×10^{18} eV. In the last section (Sec. VI) we discuss the possible importance of our predicted primordial magnetic fields on UHECRs.

We review the observations of astrophysical magnetic fields in Sec. II Previous suggestions for creating primordial magnetic fields are given in Sec. III. The creation of magnetic fields in the fluctuations of the hot dense primordial plasma is discussed in Sec. IV. In Sec. V, we discuss our model, based on the analysis in Sec. IV. Our conclusions as well as a discussion of our results are presented in Sec. VI.

II. OBSERVATIONS OF COSMIC MAGNETIC FIELDS

The magnetic fields in our Galaxy have been studied by several methods. Measurements of the Zeeman effect in the 21 cm radio line in galactic HI regions reveal magnetic fields $\simeq 2$–$10\ \mu$G. Similar values for the magnetic fields in other galaxies have been obtained from Faraday rotation surveys.

Observations of a large number of Abell clusters have provided information on magnetic fields in clusters of galaxies [14–16]. The typical magnetic field strength in the cluster is $\sim$1–10 μG, coherent over 10–100 kpc.

High resolution Faraday rotation measurements (FRMs) of high z quasars allow for the probing of magnetic fields in the past. A magnetic field of $\sim\mu$G in a relatively young spiral galaxy at $z = 0.395$ was measured by FRMs from the radio emission of the quasar PKS 229-021, lying behind the galaxy, at $z = 1.038$ [17]. Magnetic fields $\sim\mu$G are also observed in $Ly\alpha$ clouds at redshifts $z \sim 2.5$ [1].

III. PREVIOUS SUGGESTIONS FOR CREATING PRIMORDIAL MAGNETIC FIELDS

There have been various scenarios suggested for the source of primordial magnetic fields. In this section, we review some of the most important ones.

A. Magnetic fields created at the quark-hadron phase transition

In the magnetogenesis scenario at the QHPT, proposed by Quashnock *et al.* [18], an electric field was created behind the shock fronts due to the expanding bubbles of the phase transition. The baryon asymmetry, which was presumed to have already been present, resulted in a positive charge on the baryonic component and a negative charge on the leptonic component of the primordial plasma, so that the charge neutrality of the Universe was preserved. As a consequence of the difference between the equations of state of the baryonic and leptonic fluids, a strong pressure gradient was produced by the passage of the shock wave, giving rise to a radial electric field behind the shock front. Quashnock *et al.* [18] estimated the strength of the electric field to be

$$eE \simeq 15\left(\frac{\epsilon}{0.1}\right)\left(\frac{\delta}{0.1}\right)\left(\frac{kT_{\rm QHPT}}{150\ {\rm MeV}}\right)\left(\frac{100\ {\rm cm}}{l}\right)\frac{\rm keV}{\rm cm}, \quad (1)$$

where ϵ is the ratio of the energy density of the two fluids, $\delta \equiv (l\Delta p/p)$, Δp is the pressure gradient, and l is the average comoving distance between the nucleation sites. They suggested that non-negligible fields were produced when shock fronts collided, giving rise to turbulence and vorticity on scales of order l. It was found that the magnetic field produced on the comoving scale $\sim$1 AU has a present magnitude $\sim 2 \times 10^{-17}$ G.

Cheng and Olinto [19] showed that strong magnetic fields might have been produced during the coexistence phase of the QHPT, during which a baryon excess builds up in front of the bubble wall as a consequence of the difference of the baryon masses in the quark and hadron phases. In this scenario, magnetic fields were generated by the peculiar motion of the dipoles, which arose from the convective transfer of the latent heat released by the expanding walls. The field created at the QHPT was estimated by Cheng and Olinto [19] to be $\simeq 10^{-16}$ G at the present epoch, on a comoving coherence length $\simeq 1$ pc. On a comoving galactic length scale, they estimated the field to be $\simeq 10^{-20}$ G.

Sigl *et al.* [20] predicted a present magnetic field $\simeq 10^{-9}$ G. However, they used very special conditions, such as efficient amplification by hydromagnetic instabilities during the QHPT.

B. Magnetic fields from the electroweak phase transition

There have been some suggestions made for the origin of primordial magnetic fields based on the electroweak phase transition (EWPT). A first order EWPT could possibly have generated magnetic fields [20,21]. During the EWPT, the gauge symmetry broke down from the electroweak group $SU(2)_L \times U(1)_Y$ to the electromagnetic group $U(1)_{\rm EM}$. The transition appears to have been weakly first order, or possibly second order, depending upon various parameters, such as the mass of the Higgs particle [21,22]. If it were first order, the plasma would have supercooled below the electroweak temperature, $\simeq 100$ GeV. Bubbles of broken symmetry would have nucleated and expanded, eventually filling the Universe. At the time of the EWPT, the typical comoving size of the Hubble radius and the temperature were $L_H \approx 10$ cm and $T_H \approx 100$ GeV, respectively. A comoving bubble of size $L_B = f_B L_H$ would have been created with $f_B \simeq 10^{-3}$–10^{-2} [21]. The fluids would have become turbulent when two walls collided. Fully developed MHD turbulence would have led rapidly to equipartition of the field energy up to the scale of the largest eddies in the fluid, assumed to have been comparable to L_B. The magnetic field strength at the EWPT would have been

$$B \simeq (4\pi\epsilon)^{1/2}(T_{\rm EW})T_{\rm EW}^2\left(\frac{v_{\rm wall}}{c}\right)^2 \simeq (7\times 10^{21})\text{–}(2\times 10^{24})\ {\rm G}, \qquad (2)$$

where $\epsilon = g_* a T_{\rm EW}^4/2 \simeq 4\times 10^{11}\ {\rm GeV\,fm^{-3}}$ is the energy density at the time of the EWPT [23].

Magnetic fields could also have arisen in cosmological phase transitions even if they were of second order [24]. In the standard model, the EWPT occurred when the Higgs field ϕ acquired a vacuum expectation value η. To estimate the field strength on larger scales, Vachaspati [24] assumed that ϕ executed a random walk on the vacuum manifold with step size ξ. Over a distance $L = N\xi$, where N is a large number, the field ϕ changes on the average by $N^{1/2}\eta^{-1}$. On a comoving galactic scale, $L = 100$ kpc, at the recombination era ($z \sim 1100$), Vachaspati [24] found a magnetic field $\simeq 10^{-23}$ G.

C. Magnetic fields generated during inflation

Inflation naturally produced effects on large scales, very much larger than the Hubble horizon, due to microphysical processes operating in a causally connected volume before inflation [25]. If electromagnetic quantum fluctuations were amplified during inflation, they could appear today as large-scale coherent magnetic fields. The main obstacle to the inflationary scenario is the fact that in a conformally flat metric, such as the Robertson-Walker, the background gravitational field does not produce relativistic particles if the underlying theory is conformally invariant [26]. This is the case for photons, for example, since classical electrodynamics is conformally invariant in the limit of vanishing fermion masses (i.e., masses much smaller than the inflation energy scale). Several ways of breaking conformal invariance have been proposed. Turner and Widrow [25] considered three possibilities:

(1) introducing a gravitational coupling, such as $RA_\mu A^\mu$ or $R_{\mu\nu}A^\mu A^\mu$, where R is the Ricci scalar, $R_{\mu\nu}$ the Ricci tensor, and A^μ is the electromagnetic field. These terms break gauge invariance and give the photons an effective time-dependent mass. Turner and Widrow [25] showed that for some suitable (though theoretically unmotivated) choice of parameters, such a mechanism could give rise to galactic magnetic fields, even without invoking the galactic dynamo;

(2) introducing terms of the form $R_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa}/m^2$ or $RF^{\mu\nu}F_{\mu\nu}$, where m is some mass scale, required by dimensional considerations. Such terms arise due to one loop vacuum polarization effects in curved space-time. They can account, however, for only a very small primordial magnetic field; and

(3) coupling of the photon to a charged field that is not conformally coupled or anomalous coupling to a pseudoscalar field.

Dolgov and Silk [27] proposed a model invoking a spontaneous breaking of the gauge symmetry of electromagnetism, implying nonconservation of the electric charge in the early evolution of the Universe.

D. Generation of the primordial magnetic fields during the reionization epoch

Gnedin, Ferrara, and Zweibel [7] investigated the generation of magnetic fields by the Biermann battery in cosmological ionization fronts, using simulations of reionization by stars in protogalaxies. They considered two mechanisms: (1) the breakout of ionization fronts from protogalaxies; and (2) the propagation of ionization fronts through high-density neutral filaments. The first mechanism was dominant prior to the overlapping of ionized regions ($z \approx 7$), whereas the second mechanism continued to operate after that epoch as well. After overlap, the magnetic field strength at $z \approx 5$ closely traced the gas density and was highly ordered on comoving megaparsec scales. The present mean field strength was found to be $\approx 10^{-19}$ G in their simulation. Their results corroborate those of Subramanian *et al.* [28].

E. Generation of magnetic fields due to nonminimal gravitational-electromagnetic coupling after recombination

The generation of magnetic fields by nonminimal coupling was investigated by Opher and Wichoski [29]. From general relativity, it can be shown that, if we have a mass spinning at the origin, the space-time metric g_{oi} is equal to the vector product of the angular momentum L and the radial vector r, times $2G/c^3r^3$, where G is the gravitational constant. Opher and Wichoski [29] suggested that the magnetic field created is proportional to the curl of g_{0i}, where the proportionality constant $\sim(G)^{1/2}/2c$ was used, based on the data of the planets in our solar system [30,31].

Angular momentum in galaxies has been previously suggested to have been created by tidal torques between protogalaxies [32–35]. The spin parameter λ is defined as the ratio of the angular velocity of the protogalaxy to the angular velocity required for the protogalaxy to be supported by rotation alone. Numerical simulations find $\lambda \sim 0.05$, while observations of spiral galaxies show $\lambda \sim 0.5$. Since λ is proportional to the square root of the binding energy, it increases by a factor of 10 in the formation of a galaxy due to an increase of the binding energy by a factor of 100 (i.e., the radius of the protogalaxy decreases by a factor of 100).

In their calculations, Opher and Wichoski [29] investigated models in which the angular momentum of a galaxy increased until the decoupling redshift z_d and remained constant thereafter. At the decoupling redshift, the spin parameter was $\lambda \sim 0.05$. They found present galactic magnetic fields $\sim$0.58 μG for a decoupling redshift $z_d = 100$ and noted that galactic magnetic fields $\sim\mu$G could be produced by this mechanism without the need for dynamo amplification.

F. Creation of magnetic fields from primordial supernova explosions

Primordial supernova explosions could also be the origin of magnetic fields in the Universe [8,36,37]. The scenario investigated was a generic multicycle explosive model, in which a Population III object collapsed and then exploded, creating a shock. Matter was swept up by the shock, increasing the density by a factor of 4 (for the case of a strong shock). This matter was heated to a high temperature, which then cooled down. Eventually spheres of radii of approximately half the shell thickness formed and subsequently collapsed into Population III stars. They then exploded, starting a new cycle. The supernova shells produced eventually coalesced. It was assumed that the gradients of temperature and density in the resultant shell were not parallel and that, therefore, a magnetic field was created due to the Biermann mechanism. The rate of change of the magnetic field with time is equal to the vector product of the density gradient and the temperature gradient times $4\pi k_B/\pi e n$, where n is the particle density and k_B is the Boltzmann constant. It was found that this process creates a galactic seed magnetic field $\sim 10^{-16}$ G, which could be later amplified by a dynamo mechanism.

G. The origin of intergalactic magnetic fields due to extragalactic jets

Jafelice and Opher [9] suggested that the large-scale magnetization of the intergalactic medium is due to electric current carrying extragalactic jets, generated by active galactic nuclei at high z. The action of the Lorentz force on the return current expanded it into the intergalactic medium. Magnetic fields created by these currents were identified as the origin of the intergalactic magnetization. They found magnetic fields $\sim 10^{-8}$ G over comoving Mpc regions.

H. Magnetic field generation from cosmological perturbations

Another class of magnetic field generation studies are those based on cosmological perturbations. A recent article on this subject is that of Takahashi *et al.* [38]. They studied the evolution of a three component plasma (electron, proton, and photon), taking into account cosmological perturbations. The collision term between electrons and photons was evaluated up to second order and was shown to induce a magnetic field $\sim 10^{-19}$ G on a 10 Mpc comoving scale at decoupling.

I. Magnetic field generation due to primordial turbulence

Turbulence has been suggested as the primordial source of magnetic fields. Banerjee and Jedamzik [39] has made a detailed study of this scenario. We summarize here their analysis and results and compare them with the analysis and results of the present paper.

It was assumed by Banerjee and Jedamzik [39] that nonstandard out-of-equilibrium stochastic magnetic fields were created at high cosmic temperatures $T \sim$ 100 MeV–100 GeV, corresponding to quark-hadron or electroweak phase transitions. Their numerical simulations were performed using the ZEUS-3D code. Gaussian random fields were used to create the nonstandard initial turbulent fluctuations. A power law with distance l was assumed for the magnetic amplitudes, ($B \propto l^{-n}$, $n = 1$–2). The initial stochastic velocity field was generated in the same way as the initial magnetic field. A correlation length scale L was defined which contains most of the magnetic and fluid kinetic energy. The dissipation of the energy into heat occurs via energy cascading from large eddies ($\sim L$) to small eddies ($\sim l_{\rm diss}$).

Ever since the work of Kolmogorov, it has been known that cascading of energy occurs due to eddies on a scale l breaking up into smaller eddies ($\sim l/2$). Typical energy dissipation times due to the eddy flows from large to small flows are given by the eddy turnover time on the scale L. In the article of Banerjee and Jedamik [39], the turnover time on the scale L is comparable to the Hubble time. Thus, the turbulent energy introduced in the magnetogenesis era is dissipated in one Hubble time. For example, the dissipation time is $\sim 10^{-4}$ s for the quark-hadron transition.

The predicted present magnetic field in this turbulent eddy scenario depends on the turbulent spectrum assumed at the quark-hadron or electroweak phase transitions. Banerjee and Jedamik [39] found the present magnetic field to be correlated with the comoving correlation length L_c: $B \simeq 5 \times 10^{-15} L_c$ G, where L_c is measured in pc. Typically, it was found that $L_c \sim 10^{-2}$ [Eq. (52) in [39]]. Thus, the turbulent eddy scenario, with the large eddy energy transfers to small scales, where energy dissipation rapidly occurs, typically predicts $\sim 10^{-16}$ G on $\sim 10^{-2}$ pc scales. There can be substantial energy transfer to larger scales if the turbulent magnetic field possesses some magnetic helicity [40].

The above can be compared with the magnetogenesis in the present paper, due to the natural fluctuations in thermal equilibrium plasmas. Initially, the magnetic fluctuations had an average size $\bar{\lambda} = 7\pi/3(c/\omega_p)$ [Eq. (15)], where ω_p is the plasma frequency and c is the velocity of light. They have an average intensity $\langle \bar{B}^2 \rangle / 8\pi = (T/2) \times (4\pi/3)/\bar{\lambda}^{-3}$, where T is the temperature. Describing the magnetic fluctuations as dipoles, the magnetic field over a distance l due to the randomly oriented magnetic fields follows a power law: $B = \bar{B}(\bar{\lambda}/l)^{3/2}$. This power-law dependence is similar to the power-law dependence in the turbulent magnetogenesis model, but without a transfer of energy from large to small scales. For a thermal equilibrium plasma, the eddy turnover velocity of size l is the thermal rotation velocity of the mass of plasma with a diameter l. In the power-law spectrum $B = \bar{B}(\bar{\lambda}/l)^{3/2}$ with $l > \bar{\lambda}$, the eddy turnover time is greater than the Hubble time. There is thus negligible energy transfer from the large scale l to the small scale $\bar{\lambda}$. On the small scale $\bar{\lambda}$, dissipation, has already been taken into account by the fluctuation-dissipation theorem.

The present predicted magnetic field in our magnetogenesis model can be compared with the predicted magnetic field of the turbulent eddy model. Whereas, in the turbulent eddy model, a present magnetic field $\sim 10^{-16}$ G over a comoving correlation length $\sim 10^{-2}$ pc is predicted, our model predicts a present magnetic field $\sim 10^{-7}$ G over a comoving length ~ 1 pc. The predicted magnetic field in our model is, thus, 9 orders of magnitude greater (over a comoving length 2 orders of magnitude greater) than that in the turbulent magnetogenesis model. This large difference is due to the fast energy transfer from large to small dissipation scales in a Hubble time in the turbulent magnetogenesis model, which does not occur in our model.

IV. CREATION OF MAGNETIC FIELDS DUE TO THE ELECTROMAGNETIC FLUCTUATIONS IN HOT DENSE EQUILIBRIUM PRIMORDIAL PLASMAS

Thermal electromagnetic fluctuations are present in all plasmas, including those in thermal equilibrium, the level of which is related to the dissipative characteristics of the medium, as described by the FDT [41] [see also Akhiezer *et al.* [42], Dawson [43]], Rostoker *et al.* [44], Sitenko [45]. The spectrum of the fluctuations of the electric field is given by

$$\frac{1}{8}\langle E_i E_j \rangle_{k\omega} = \frac{i}{2} \frac{\hbar}{e^{\hbar\omega/T-1}} (\Lambda_{ij}^{-1} - \Lambda_{ij}^{-1*}), \tag{3}$$

$$\Lambda_{ij}(\omega, \mathbf{k}) = \frac{k^2 c^2}{\omega^2} \left(\frac{k_i k_j}{k^2} - \delta_{ij} \right) + \varepsilon_{ij}(\omega, \mathbf{k}), \tag{4}$$

where $\varepsilon_{ij}(\omega, \mathbf{k})$ is the dielectric tensor of the plasma, ω the frequency, and $\bar{\mathbf{k}}$ is the wave number of the fluctuation. From Faraday's law, $\mathbf{B} = c\mathbf{k}/\omega \times \mathbf{E}$, and setting $\mathbf{k} = k\hat{\mathbf{x}}$, we find for the perpendicular B_2 and B_3 magnetic fluctuations

$$\frac{\langle B_2^2 \rangle_{k\omega}}{8\pi} = \frac{i}{2} \frac{\hbar}{e^{\hbar\omega/T-1}} \frac{c^2 k^2}{\omega^2} (\Lambda_{33}^{-1} - \Lambda_{33}^{-1*}), \tag{5}$$

and

$$\frac{\langle B_3^2 \rangle_{k\omega}}{8\pi} = \frac{i}{2} \frac{\hbar}{e^{\hbar\omega/T-1}} \frac{c^2 k^2}{\omega^2} (\Lambda_{22}^{-1} - \Lambda_{22}^{-1*}), \tag{6}$$

where the subscripts 1, 2, and 3 refer to the x, y, z directions. We then have for the total magnetic fluctuations

$$\frac{\langle B^2 \rangle_{k\omega}}{8\pi} = \frac{i}{2} \frac{\hbar}{e^{\hbar\omega/T-1}} \frac{c^2 k^2}{\omega^2} (\Lambda_{22}^{-1} + \Lambda_{33}^{-1} - \Lambda_{22}^{-1*} - \Lambda_{33}^{-1*}). \tag{7}$$

In order to obtain $\Lambda_{ij}(\omega, \mathbf{k})$ from the equations of motion of the plasma, a multifluid model for the plasma is introduced:

$$m_\alpha \frac{d\mathbf{v}_\alpha}{dt} = e_\alpha \mathbf{E} - \eta_\alpha m_\alpha \mathbf{v}_\alpha, \tag{8}$$

where α is a particle species label and η_α the collision frequency of the species. From a Fourier transformation of the above equation and rearranging terms, the dielectric tensor can be obtained:

$$\epsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij} - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega(\omega + i\eta_\alpha)} \delta_{ij}, \tag{9}$$

where $\omega_{p\alpha}$ is the plasma frequency of the species α. For, an electron-positron plasma, the plasma frequency of the electrons is equal to that of the positrons, $\omega_{pe^+} = \omega_{pe^-}$, and the collision frequencies of the electrons and positrons are equal, $\eta_{e^+} = \eta_{e^-} = \eta$. The dielectric tensor from Eq. (9) then becomes

$$\epsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij} - \frac{\omega_p^2}{\omega(\omega + i\eta)} \delta_{ij}, \tag{10}$$

where $\omega_p^2 = \omega_{pe^+}^2 + \omega_{pe^-}^2$. For electrons, the Coulomb collision frequency is $\eta_e = 2.91 \times 10^{-6} n_e \ln\Lambda T^{-3/2}$ (eV)s^{-1}, where n_e is the electron density.

The collision frequency for the case of an electron-proton plasma, which dominates after the primordial nucleosynthesis era, is $\eta_p = 4.78 \times 10^{-18} n_e \ln\Lambda T^{-3/2}$ (eV)s^{-1}. It describes the binary collisions in a plasma, which we assume to be the dominant contribution to η. We then obtain

$$\Lambda_{ij} = \begin{pmatrix} 1 - \frac{\omega_p^2}{\omega(\omega+i\eta)} & 0 & 0 \\ 0 & 1 - \frac{c^2k^2}{\omega^2} - \frac{\omega_p^2}{\omega(\omega+i\eta)} & 0 \\ 0 & 0 & 1 - \frac{c^2k^2}{\omega^2} - \frac{\omega_p^2}{\omega(\omega+i\eta)} \end{pmatrix}. \tag{11}$$

From Eqs. (7)–(11), the total magnetic field fluctuations as a function of frequency and wave number k were found to be [13]

$$\frac{\langle B^2\rangle_{k,\omega}}{8\pi} = \frac{2\hbar\omega}{e^{\hbar\omega/T} - 1}\eta\omega_p^2 \frac{k^2c^2}{(\omega^2+\eta^2)k^4c^4 + 2\omega^2(\omega_p^2 - \omega^2 - \eta^2)k^2c^2 + [(\omega^2-\omega_p^2)^2 + \eta^2\omega^2]\omega}. \tag{12}$$

V. OUR MODEL

Our model is based on the magnetic fluctuations in the plasma created immediately after the QHPT, which are described by the FDT in the previous section. This plasma was composed primarily of electrons, photons, neutrinos, muons, baryons, and their antiparticles. The baryons were essentially stationary and did not contribute to the fluctuations while the muons also contributed very little and for a very short time. Since neutrinos are essentially massless and act qualitatively like photons, albeit with much smaller cross sections, we assume that they also affect the magnetic fluctuations very little. Therefore we consider only an electron-positron-photon plasma before the electron-positron annihilation era and an electron-proton plasma thereafter.

Most of the electromagnetic fluctuations in the primordial plasma that were created immediately after the QHPT fall into two broad categories: those with large wavelengths ($k \lesssim \omega_{pe}/c$) at near zero frequency ($\omega \ll \omega_{pe}$) and those with very small wavelengths ($k \gg \omega_{pe}/c$) and frequencies greater than ω_{pe}. The modes $k \lesssim \omega_{pe}/c$, denominated "soft" or "plastic" photons by Tajima *et al.* [13], were significantly modified. It is these plastic photons and their magnetic fields in which we are interested.

From Eq. (12), we obtain the strength of the magnetic field whose wavelengths are larger than a size λ,

$$\langle B^2\rangle_\lambda/8\pi = (T/2)(4\pi/3)\lambda^{-3}, \tag{13}$$

which decreases rapidly with wavelength. Thus, the magnetic field in Eq. (13) was concentrated near the wavelength λ. The spatial size λ of the magnetic field fluctuations is related to τ, the lifetime of the fluctuation, by [13]

$$\lambda(\tau) = 2\pi\frac{c}{\omega_p}(\eta_e\tau)^{1/2}. \tag{14}$$

The average size of the magnetic fluctuations was

$$\bar{\lambda} = \frac{\int \lambda[\langle B^2\rangle_\lambda/8\pi]d\lambda}{\int[\langle B^2\rangle_\lambda/8\pi]d\lambda} = \frac{7\pi}{3}(c/\omega_p). \tag{15}$$

Using the model of Tajima *et al.* [13], we assume that a fluctuation predicted by the FDT can be described by a bubble of size $\bar{\lambda}$. It contains a magnetic dipole whose field intensity is given by Eq. (13).

The magnetic bubbles were at the temperature of the plasma. We assume that they touched each other and coalesced in a time $t_{\rm coal} = \bar{\lambda}/v_{\rm bub}$, where $v_{\rm bub}$ was the thermal velocity of the bubble. The coalescence time $t_{\rm coal}$ was found to be much shorter than the lifetime τ of the bubbles in the primordial Universe, for example, $\sim 10^{-5}$ s shortly after $t \sim 10^{-4}$ s. Before the magnetic fields dissipated, the bubbles coalesced with one another. Once a larger bubble was formed, its lifetime, which is proportional to the square of its size, was longer. Larger bubbles lived longer and, thus, had more opportunities to collide with other bubbles. In this way, a preferential formation of larger bubbles occurred.

Magnetic field fluctuations are created immediately after the QHPT as predicted by the FDT, which we evolve to the recombination era and beyond. Magnetic field fluctuations are also predicted to be created by the FDT at the recombination era. Since the created magnetic field fluctuations $\langle B\rangle^2$ are proportional to $Tn^{3/2}$, the evolved magnetic fields from the QHPT at the recombination era are very much greater than the created magnetic fields at the recombination era. The latter source of primordial magnetic fields was thus neglected in our investigation. Tajima *et al.* [13] previously suggested that the evolved primordial fields might continue to exist at the present epoch. No explicit calculation was, however, made. Thus, previously it was not known whether these fields would continue to exist or not to the present era. We show here that these fields do indeed continue to exit and are not destroyed in their evolution by diffusion or reconnection. We also evaluate their structure and intensity as a function of redshift.

We begin our calculations immediately after the QHPT and continue to $z \sim 10$. Magnetic fields were adiabatically amplified at $z \sim 10$ as the baryon matter collapsed to form galaxies.

Although the magnetic energy density of neighboring magnetic dipoles is of the same order as the energy density of the average magnetic field when they are not at the average distance from each other, the magnetic energy density appreciably increases when the neighboring dipoles approach each other. Since the field of a dipole is proportional to r^{-3}, where r is the distance from the dipole, the magnetic energy density of neighboring dipoles is proportional to r^{-6}. Decreasing the separation distance by a factor of 2(4), for example, increases the energy density by a factor 64(4096). Thus, the magnetic energy density of adjacent magnetic bubbles at very short separation distances is very much greater than the average magnetic energy density.

The dipoles tended to align as they interacted due to the intensification of the magnetic interaction energy at shorter interdipole distances. The interaction rate of the dipoles depended on their thermal velocity. We used as the thermal velocity the velocity of the mass of the plasma bubble which is in thermal equilibrium at the temperature of the Universe at a given redshift. When the dipoles were oppositely oriented and interacting, two opposing processes occurred: alignment and reconnection. As the dipoles approached each other, they tended to align in a flip time $\tau_{\rm flip} \sim 10^{-5}$ s shortly after the QHPT at $t \sim 10^{-4}$ s, where $\tau_{\rm flip}$ is the time in which a bubble aligns with a neighboring bubble due to the magnetic torque. We have $\tau_{\rm flip} \propto (I/N_{\rm mag})^{1/2}$, where $N_{\rm mag}$ is the magnetic torque and I is moment of inertia of the bubble. On the other hand, the opposite magnetic fields of the dipoles reconnected in a tearing mode time $\tau_{\rm tear}$. The shortest $\tau_{\rm tear}$ is estimated to be $\bar{\tau}_{\rm tear} \cong 10^{0.20}\tau_A^{1/2}\tau_R^{1/2}$, where $\tau_A = L/v_A$ is the Alfvén time and $\tau_R = 4\pi L^2/c\eta$ is the resistive time [46]. The shortest tearing time shortly after the QHPT was $\sim 10^{15}$ s. Thus, $\tau_{\rm flip} \ll \bar{\tau}_{\rm tear}$ shortly after 10^{-4} s and remains so for all times of interest. Figure 1 plots $\tau_{\rm flip}$, Fig. 2 plots $\tau_{\rm tear}$, and Fig. 3 their ratio, in the time interval $\sim 10^{-4}$–10^2 sec.

The final time plotted in Figs. 1–3, ~ 100 s, is the time in which the magnetic field in a bubble requires the age of the Universe to diffuse away. Magnetic diffusion, inversely proportional to the square of the diameter of the bubble, is only important at early times, when the bubbles were small. An initial magnetic field in a bubble diffused away in a time $\tau_{\rm diff} = 4\pi\sigma L^2$, where L is the diameter of the bubble and σ is the electrical conductivity [47].

In the high temperature regime ($T > 1$ MeV), we followed the treatment of Ahonen and Enqvist [48] who solved numerically the Boltzmann equation in the early Universe. For $T \lesssim 100$ MeV, they found for the conductivity $\sigma \simeq 0.76T$. Since immediately after the QHPT the

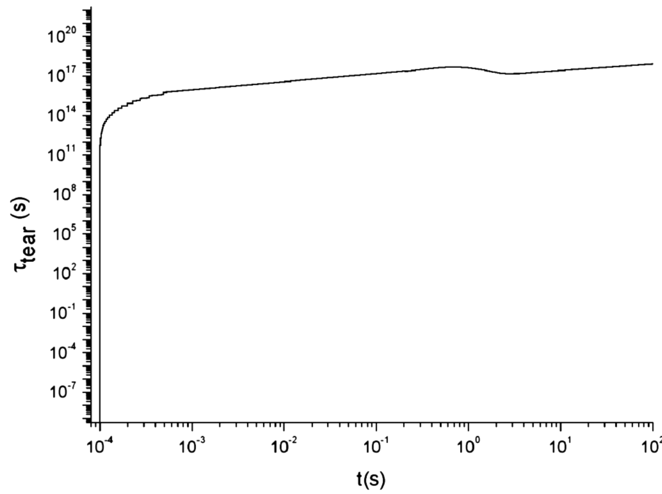

FIG. 2. Evolution of the tearing time $\tau_{\rm tear}$ (s) of the bubbles as a function of time $t(s)$.

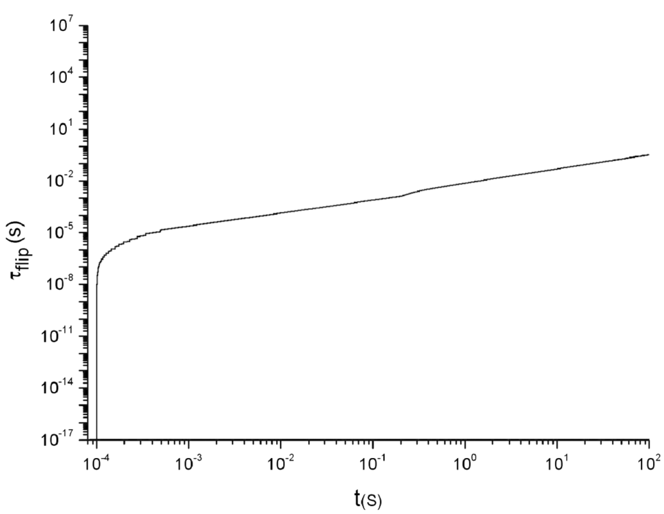

FIG. 1. Evolution of the flip time $\tau_{\rm flip}$ (s) of the bubbles as a function of time $t(s)$.

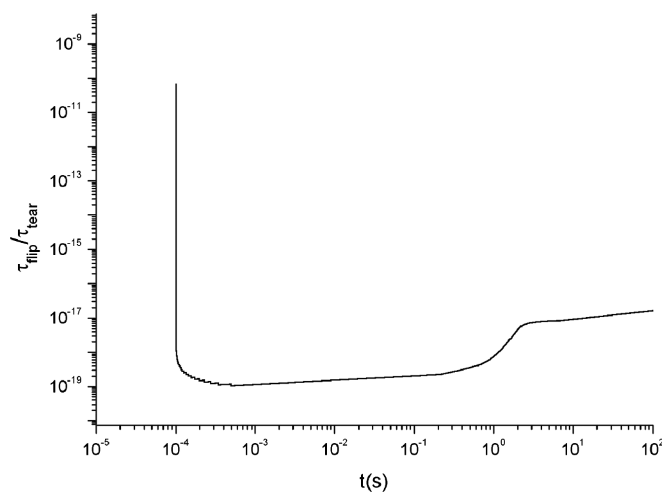

FIG. 3. Ratio of the flip time $\tau_{\rm flip}$ of the bubbles to the tearing time $\tau_{\rm tear}$ as a function of time $t(s)$.

temperature of the Universe was $\sim$100 MeV, we used $\sigma \simeq 0.76T$ for $T > 1$ MeV.

At temperatures $T < 1$ MeV the conductivity can be approximated as

$$\sigma = \frac{m_e}{\alpha \ln\Lambda}\left(\frac{2T}{\pi m_e}\right)^{3/2}, \tag{16}$$

where $\Lambda = (1/6\pi^{1/2})(1/\alpha^{1/2})(m_e^3/n_e)^{1/2}(T/m_e)$, and α, m_e, and n_e are the fine structure constant, the electron mass, and the electron density, respectively [49]. For $L \sim$ 1 AU, $\tau_{\rm diff}$ is equal to the age of the Universe [47]. In our model the bubbles reached a size $\sim$1 AU in a time $\sim$100 s. In Figs. 1–3 $\tau_{\rm flip}$ and $\tau_{\rm tear}$ are thus plotted from the time of the QHPT ($\sim 10^{-4}$ s) to $\sim$100 s.

The magnetic field in a bubble would dissipate before coalescence of the bubble occurred if the magnetic diffusion time was smaller than the coalescence time. In Fig. 4 we plot the ratio of the coalescence time $\tau_{\rm coal}$ to the diffusion time $\tau_{\rm diff}$. It can be seen in Fig. 4 that this ratio remains very much less than unity at early times.

At late times, when the magnetic field flip time (i.e., the time for adjacent dipoles to align) was greater than the Hubble time, the magnetic dipoles remained random. The transition redshift, when random fields began to exist, was $z \sim 10^8$. At this epoch, the comoving size of the bubbles was $\sim$1 pc. In order to explain galactic magnetic fields, we need to evaluate the field over the comoving scale of a protogalaxy, $\sim$1 Mpc, which eventually collapsed to the comoving scale of a galaxy, $\sim$30 kpc.

The magnetic field in a bubble decreased adiabatically as the Universe expanded. Since magnetic flux is conserved, we have

$$B = \frac{B_0}{a^2}, \tag{17}$$

where a is the cosmic scale factor. A ΛCDM model

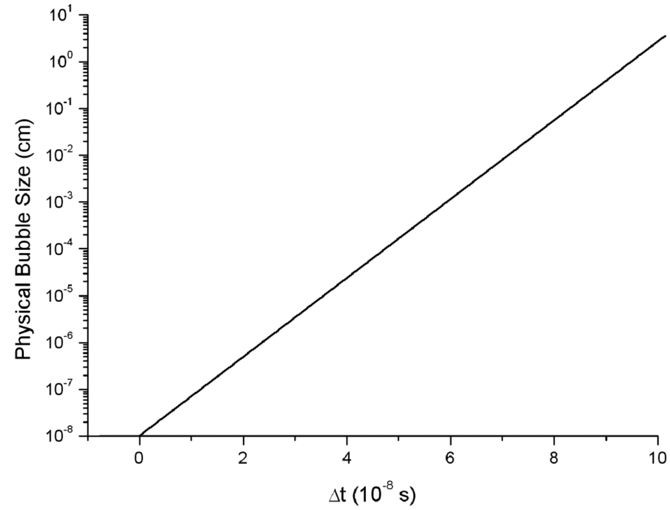

FIG. 5. Initial evolution of the physical size of the magnetic bubbles, created immediately after the QHPT, as a function of time, $t \equiv t_0 + \Delta t$, for $t_0 = 10^{-4}$ s, and $0 < \Delta t(10^{-8}\text{ s}) \leq 10$.

was used to evolve a as a function of time, with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and a Hubble constant $h \equiv H/100\ \mathrm{km\,s^{-1}\,Mpc^{-1}} = 0.72$.

In Figs. 5 and 6, we show the evolution of the size of the bubbles as a function of time, from immediately after the QHPT at 10^{-4} s to a redshift $z \sim 10$ at a time $\sim 10^{16}$ sec. Initially, the size of the bubbles increased rapidly, as shown in Fig. 5. From Fig. 5, we observe that the physical size of a bubble increased from 10^{-8} cm at $t \approx 10^{-4}$ to 1 cm in a time 10^{-7} sec. It continued to increase at this rate until it reached a size $\sim 10^7$ cm. The growth rate then decreased, as shown in Fig. 6. At the redshift $z \sim 10^8$ ($t \sim 3000$ s), the physical size of the bubble was $\sim 10^{10}$ cm (i.e., a comoving size $\sim$1 pc).

The manner in which we extrapolated the field amplitude to cosmological scales followed the phenomenologi-

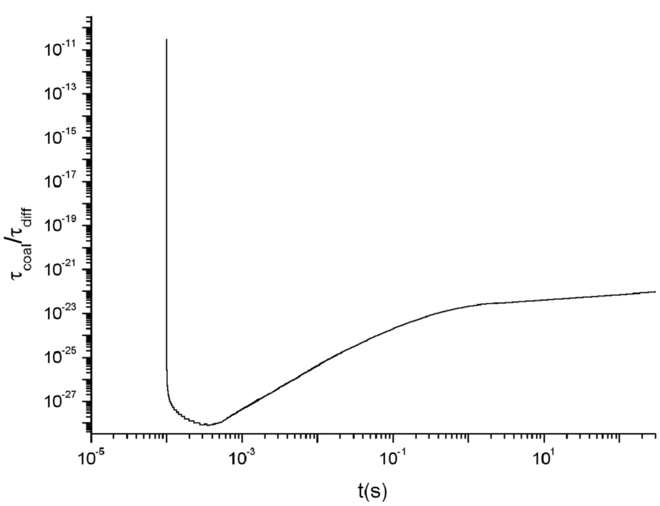

FIG. 4. Ratio of the coalescence time $\tau_{\rm coal}$ of the bubbles to the diffusion time $\tau_{\rm diff}$ as a function of time $t(s)$.

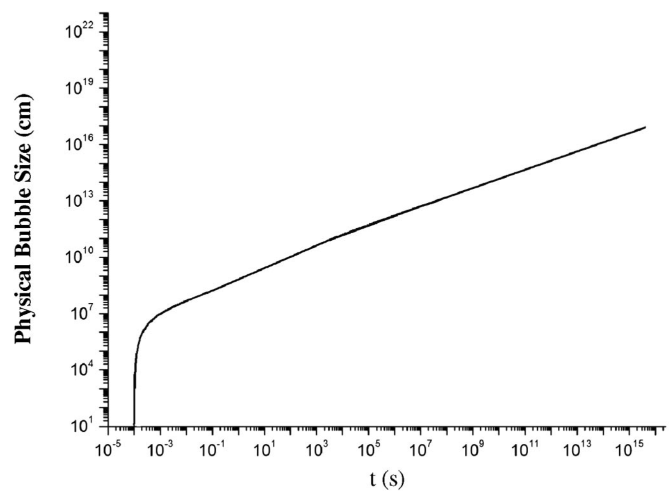

FIG. 6. Evolution of the physical size of the magnetic bubbles as a function of time from $t \sim 0.1$ s.

cal analysis of random distributions of size L, in the review article of Grasso and Rubinstein [47]. Their generic average magnetic field over a distance D at a time t is proportional to $(L/D)^p$, where $p = 1/2$, 1, or 3/2,

$$\langle B(L,t)\rangle_{\rm rms} = B_0\left(\frac{a_0}{a(t)}\right)^2\left(\frac{L}{D}\right)^p. \qquad (18)$$

If we are interested in the volume average magnetic field of a random distribution of dipoles in a sphere of diameter D, and each dipole is in a cell of diameter L, the average magnetic field is proportional to $(L/D)^{3/2}$ and $p = 3/2$ in Eq. (18) [23]. If, however, we are interested in a line-of-sight average magnetic field felt by a cosmic ray particle or a photon (e.g. in Faraday rotation measurements), the average magnetic field is proportional to $(L/D)^{1/2}$, and $p = 1/2$ in Eq. (18).

The non-negligible volume average magnetic field can be important in the formation of the first stars and in reionizing the Universe. The formation of the first objects marks the transformation of the Universe from its smooth initial state to its clumpy current state. In popular cosmological models, the first sources of light began to form at a redshift $z \sim 30$ and reionized most of the hydrogen in the Universe by $z \sim 7$ [50]. In general, it is found difficult to reionize the Universe with a standard Salpeter initial mass function for the first stellar sources formed by a standard fluctuation dark matter spectrum [51–55]. Primordial magnetic fields produce additional fluctuations of baryons by the Lorentz force [56]. The magnetic tension is more effective on small scales where the entanglements of magnetic fields are larger. Tashiro and Sugiyama [56] found that ionizing photons from Population III stars formed in dark halos could easily have reionized the Universe by $z \simeq 10$–20 if the present intensity of the primordial magnetic field is $B_0 \sim$ nG on a comoving scale $\sim$0.1 Mpc. The relevant Lorentz force causing the collapse of baryon matter is proportional to

$$\vec{\nabla}\cdot[(\vec{\nabla}\times\vec{B}_0(\vec{x}))\times\vec{B}_0(\vec{x})] \sim \frac{B_0^2}{D^2} \equiv F. \qquad (19)$$

Thus, Tahiro and Sugiyama [56] found that a value $F \sim 10^{-28}G^2/pc^2$ is important in forming the first objects. In our model a present volume average magnetic field over a comoving scale D is $B_0 \sim 0.1\ \mu$G $(D(pc))^{-3/2}$. We thus have $F = [10^{-14}/D^2]G^2/pc^2$ and obtain a Tashiro and Sugiyama [56] value of F with D $\sim$ kpc. We thus find that a D $\sim$ kpc comoving region in our model produces a Lorentz force which could be important in forming the first stellar sources and in reionizing the Universe. This length is larger than the magnetic Jeans length and the cutoff length due to direct cascade. Their respective wave numbers, given by Tashiro and Sugiyama [56], are $k_{\rm MJ} \sim 32$ Mpc^{-1} B_0^{-1}(nG) and $k_c \sim 102$ Mpc^{-1} B_0^{-1}(nG). Putting our volume average magnetic field B_0 over $D \sim 1$ kpc into these expressions we obtain $k_{\rm MJ} \sim 10$ kpc^{-1} and $k_c \sim 34$ kpc^{-1}. It is to be noted that a sphere of comoving diameter $\sim$1 kpc contains a mass $\sim 10^3 M_\odot$ for a reduced matter density $\Omega_m \sim 0.3$ and Hubble parameter $h \sim 0.72$.

A detailed discussion on average procedures of tangled magnetic fields can be found in Hindmarsh and Everett [57]. Table I shows the growth of the magnetic field in our model and the size of the bubbles down to the redshift $z \sim 10$. The equipartition redshift in Table I was obtained from the relation $(1 + z_{\rm eq}) \approx 2.3 \times 10^4 \Omega_m h^2$ [58]. Table II shows the growth of the line-of-sight average magnetic field over a comoving protogalactic size $L \sim 1$ Mpc.

TABLE I. Size and strength of magnetic fields in bubbles

Epoch	Magnetic field (μG)	Redshift	Time (sec)	Size (cm)
Immediately after the QHPT	10^{22}	6×10^{11}	10^{-4}	10^{-12}
Electron-positron annihilation era	10^{18}	10^{10}	1	10^{8}
Nucleosynthesis era	10^{15}	10^8–10^9	1–500	10^{10}
Equipartition era	2×10^5	3600	10^{12}	3×10^{14}
Recombination era	2×10^2	1100	8×10^{12}	10^{15}
Galaxy formation era	9	$\sim$10	10^{16}	10^{17}

TABLE II. Line-of-sight average magnetic fields in protogalaxies of comoving size $\sim$1 Mpc.

Epoch	Magnetic field (μG)	Redshift	Time (sec)	Size (cm)
Beginning of random fields	9.5×10^{11}	10^8	3×10^3	10^{-12}
Equipartition era	10^4	3600	10^{12}	10^{18}
Recombination era	300	1100	8×10^{12}	4×10^{22}
Galaxy formation era	9×10^{-3}	$\sim$10	10^{16}	10^{23}

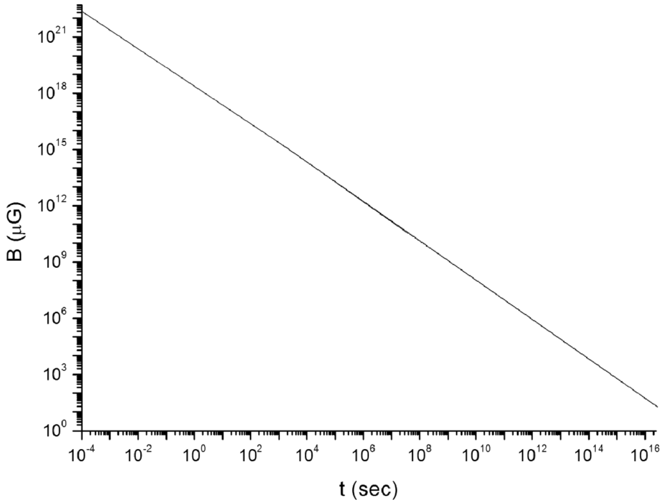

FIG. 7. Evolution of the magnetic field $B(\mu G)$ in the bubbles, created immediately after the QHPT, as a function of time, t (sec).

At $z = 10$, the intensity of the magnetic field in a bubble whose comoving size is ~ 1 pc was ~ 9 μG. Taking the line-of-site average over the comoving scale of 1 Mpc (~ 100 kpc at $z \sim 10$), the rms magnetic field at $z = 10$ was 9×10^{-3} μG. The magnetic field in the bubbles as a function of time is shown in Fig. 7. In Fig. 8, the evolution of the line-of-sight average and volume average magnetic field of comoving size ~ 1 Mpc is shown as a function of time from $t \simeq 3 \times 10^3$ sec, when random fields began to exist, to $z \sim 10$. In the collapse of the comoving 1 Mpc region at $z = 10$ to a galaxy (comoving size ~ 30 kpc), the field is amplified to ~ 10 μG. This indicates that the magnetic fields created immediately after the QHPT could be the origin of the $\sim \mu$G fields observed in galaxies at high and low redshifts.

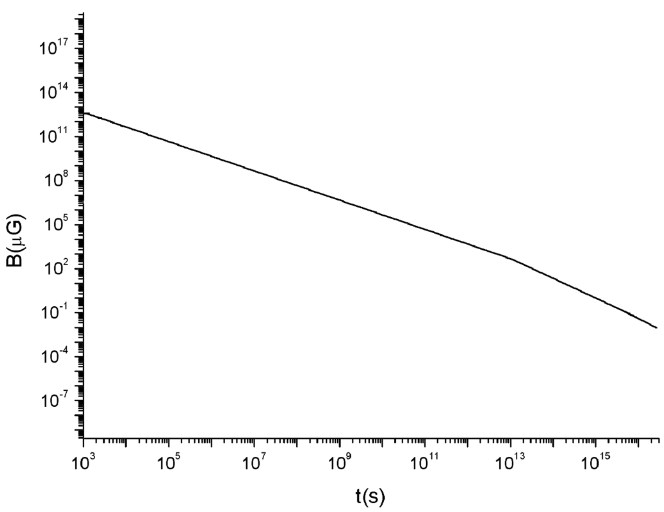

FIG. 8. Evolution of the line-of-sight average magnetic field $B(\mu G)$ of comoving size ~ 1 Mpc as a function of time t (sec) from $t \simeq 3 \times 10^3$ sec, when random fields began to exist, to $t \sim 10^{16}$ sec ($z \sim 10$), when galaxies began to form.

VI. CONCLUSIONS AND DISCUSSION

We showed that the electromagnetic fluctuations in the primordial plasma immediately after QHPT constitute a strong candidate for the origin of primordial magnetic fields in galaxies and clusters of galaxies. We calculated the magnetic field fluctuations in the plasma after this transition and evaluated their evolution with time. Intense magnetic field fluctuations on the order of 10^{16} G existed at $t = 10^{-4}$ sec after the QHPT. These fields formed a spatial linkage due to the process of successive coalescence. We showed that magnetic bubbles created immediately after the transition could survive to $z \sim 10$ and could explain the observed magnetic fields at high and low redshifts determined by FRMs. We found: (1) ~ 10 μG magnetic fields (MFs) over a comoving ~ 1 pc region at a redshift $z \sim 10$; (2) line-of-sight average MFs, important in Faraday rotation measurements, $\simeq 10^{-2}$ μG over a 1 Mpc comoving region at $z \sim 10$ which in the collapse to a galaxy comoving size ~ 30 kpc, are amplified to ~ 10 μG; and (3) volume average MFs over a comoving 1 kpc region that could be important in forming the first stellar sources and in reionizing the Universe [56,59].

We found that the magnetic fields in the bubbles, created originally at the QHPT, had a value ~ 10 μG at the redshift $z \sim 10$ and a size 0.1 pc (Table I). At the present time, these bubbles have a comoving length ~ 1 pc and a field ~ 0.1 μG. We can compare these results with previous calculations of the creation of magnetic fields at the QHPT. Cheng and Olinto [19], for example, found a much smaller magnetic field, $\sim 10^{-10}$ μG, over the same comoving size with their mechanism. Quashnock *et al.* also found a much smaller resultant magnetic field, $\sim 2 \times 10^{-11}$ μG, over a much smaller comoving size, $\sim 10^{-5}$ pc.

It is to be noted that the origin of primordial magnetic fields suggested here is qualitatively different from the other previous suggestions discussed in Sec. III A, III B, III C, III D, III E, III F, and III G. These previous suggestions require special physical initial conditions. Our model, however, does not. The magnetic fields in our model arise from the natural fluctuations in the equilibrium plasma that existed in the primordial Universe, described by the fluctuation-dissipation theorem.

In Sec. III H, we discussed the model of Takahashi *et al.* [38] which, like our model, is based on natural fluctuations that exist in nature. Our model, however, predicts very much larger magnetic amplitudes on a comoving protogalactic scale ~ 1 Mpc. Takahashi *et al.* [38] found a magnetic field $B \sim 10^{-25}$ G on a $\lambda = 10$ Mpc comoving scale. Since their field is $\propto k^3 P(k)$, where k is the wave number ($k = 2\pi/\lambda$), and the fluctuation power spectrum $P(k) \propto k^n$ with $n \sim -2$ for $\lambda < 10$ Mpc, the Takahashi *et al.* [38] prediction is $B \sim 10^{-23}$ G at present for $\lambda \sim 1$ Mpc. This can be compared with our prediction for the same comoving scale, which is many orders of magnitude greater.

Our predicted magnetic fields are consistent with present observations. Extragalactic magnetic fields as strong as $\sim 1\ \mu$G in sheets and filaments in the large-scale galaxy distribution, such as in the local superclusters, are compatible with existing FRMs [1,60–65]. These limits are consistent with our predicted fields. There is mounting evidence from diffuse radiosynchrotron clusters [66] and a few cases of filaments [67,68] that magnetic fields ~ 0.1–$1.0\ \mu$G exist in the low density outskirts of cosmological collapsed objects. These fields may have their origin in the primordial magnetic fields that we predict.

In contrast to the previous models suggested in Sec. III, our model predicts relatively intense magnetic fields over small regions in the intergalactic medium. This prediction may help to solve the long-standing problem of ultrahigh energy cosmic rays (UHECRs) ($> 3 \times 10^{18}$ eV): their extreme isotropy. The UHECRs are extragalactic since their Larmor radii are comparable or greater than the size of the galaxy [69,70]. There are only a few nearby sources that could be the origin of these cosmic rays. However, the observed arrival directions of the UHECRs are highly isotropic [71,72].

The importance of intergalactic magnetic fields in creating an isotropic distribution of UHECRs has been discussed in the literature. However, different articles arrived at opposite conclusions. Whereas Farrar and Piran [73] argue that the magnetic fields created the observed isotropic distribution, Dolag *et al.* [74,75] argue that they are unimportant. Medina-Tanco and Ensslin [76] argue that only weak intergalactic magnetic fields making small angular deflections of the UHECRs may be necessary, since the number of UHECRs sources may be much larger than those that are presently observed and that it is possible that fossil cocoons, so-called radio ghosts, contribute to the isotropization of the UHECR arrival directions [76].

Primordial magnetic fields have been previously assumed to exist, without an explanation for their origin. Dolag *et al.* [74,75], for example, assumed the existence of a homogeneous primordial magnetic field $\sim 10^{-3}\ \mu$G at $z \sim 20$. They made a magnetohydrodynamic simulation of cosmic structure formation that reproduces the positions of known galaxy clusters in the local universe. Protons of energy $\geq 4 \times 10^{19}$ eV were found to have deflections, which do not exceed a few degrees over most of the sky, up to a propagation distance of ~ 500 Mpc. It is difficult to explain, however, an isotropic distribution of UHECRs with their analysis.

Relatively intense magnetic fields have been predicted to exist in filaments in the intergalactic medium. Such a filament might exist between us and the powerful radio galaxy, Cen A. For example, Farrar and Piran [73] suggested that Cen A, at a distance of 3.4 Mpc, could be the source of most UHECRs observed. The extragalactic magnetic field was estimated to be $\sim 0.3\ \mu$G. They argue that this scenario can account for the spectrum of UHECRs down to $\approx 10^{18.7}$ eV, including its isotropy and spectral smoothness.

If our predicted magnetic fields are not spread uniformly over space but, as expected, are concentrated into the web of filaments, predicted by numerical simulations, appreciable deflections of UHECRs propagating along the filaments could occur. The deflection in a distance D of a UHECR with energy $E \equiv E_{20} \times 10^{20}$ eV by magnetic bubbles of size λ and magnetic field B is [73]

$$\delta(\theta) \sim 0.5^0 [D(\mathrm{Mpc})\lambda(\mathrm{Mpc})]^{1/2} B(nG)/E_{20}. \tag{20}$$

We predicted magnetic bubbles with $B \sim 10\ \mu$G and $\lambda \sim 0.1$ pc at $z \sim 10$. Let us assume that λ increased with the cosmic scale factor and that $\lambda \sim 1$ pc at $z \sim 10$, and that the magnetic fields, trapped in the filaments, decreased slightly to $B \sim 1$–$10\ \mu$G. From Eq. (18), with a distance $D \sim 100$ Mpc, we obtain $\delta(\theta) \sim 5^0 - 50^0/E_{20}$. Appreciable deflections could, thus, occur along filaments.

ACKNOWLEDGMENTS

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