Evolution of density perturbations in decaying vacuum cosmology

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We study cosmological perturbations in the context of an interacting dark energy model, in which the cosmological term decays linearly with the Hubble parameter, with concomitant matter production. A previous joint analysis of the redshift-distance relation for type Ia supernovas, barionic acoustic oscillations, and the position of the first peak in the anisotropy spectrum of the cosmic microwave background has led to acceptable values for the cosmological parameters. Here we present our analysis of small perturbations, under the assumption that the cosmological term, and therefore the matter production, are strictly homogeneous. Such a homogeneous production tends to dilute the matter contrast, leading to a late-time suppression in the power spectrum. Nevertheless, an excellent agreement with the observational data can be achieved by using a higher matter density as compared to the concordance value previously obtained. This may indicate that our hypothesis of homogeneous matter production must be relaxed by allowing perturbations in the interacting cosmological term.

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I. INTRODUCTION

The cosmological constant problem has acquired a renewed importance since several independent observations have been pointing to the presence of a negative pressure component in the cosmic fluid [1]. From the point of view of quantum field theories, the natural candidate for such a dark energy is the quantum vacuum. Since, at the macroscopic level, it has the symmetry of the background, its energy-momentum tensor has the form $T_\mu^\nu = \Lambda g_\mu^\nu$, where Λ is a scalar function of coordinates. This leads, in the case of an isotropic and homogeneous space-time and comoving observers, to the equation of state $p_\Lambda = -\rho_\Lambda = -\Lambda$, where Λ may be, in general, a function of time. In the case of a constant Λ, the vacuum contribution plays the role of a cosmological constant in Einstein's equations.

However, the estimation of the vacuum energy density by quantum field theories in the flat space-time leads, after some regularization procedure, to a very huge result when compared to the observed value. A possible way out of this difficult is to argue that such a result is valid only in a flat background, in which the very Einstein equations predict a null total energy-momentum tensor. Therefore, the huge vacuum density should be canceled by a bare cosmological constant, like in a renormalization process. Now, if we could obtain the vacuum density in the Friedmann-Lemaitre-Robertson-Walker (FLRW) space-time, after the subtraction of the Minkowskian result it would remain an effective time-dependent Λ term, which decreases with the expansion.

The idea of a time-dependent cosmological term has found different phenomenological implementations [2], being a subject of renewed interest in recent years [3–5]. A general feature of all those approaches is the production of matter, concomitant with the vacuum decay in order to assure the covariant conservation of the total energy [6]. Indeed, in the FLRW space-time, the Bianchi identities lead to the conservation equation

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \tag{1}$$

where ρ_T and p_T stand for the total energy density and pressure, respectively, and $H = \dot{a}/a$ is the Hubble parameter. By writing $\rho_T = \rho_m + \Lambda$ and $p_T = p_m - \Lambda$ (where ρ_m and p_m are the energy density and pressure of matter), the above equation reduces to

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -\dot{\Lambda}, \tag{2}$$

which shows that, in the case of a varying Λ, matter is not independently conserved.[1]

An important point to be clarified in this kind of model is the homogeneity of matter production. Of course, in a strictly homogeneous space-time the production is homogeneous, since ρ_m and Λ depends only on time. But, in the presence of density perturbations, is the new matter produced homogeneously, or just where matter already exists [8]? In the case of a homogeneous production, the new matter tends to dilute the density perturbations, leading to a

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[1]Properly speaking, we should also consider the pressure and energy associated to the very process of matter production, that is, the energy-momentum tensor of the interaction between matter and vacuum. In this sense, decaying vacuum models do not differ essentially from interacting dark energy models [7], with the scalar function Λ replaced by a scalar field interacting with matter. Nevertheless, if the vacuum decays into nonrelativistic particles, as we will consider here, the interaction term can be neglected, and the above decomposition may be considered a good approximation.

suppression of the density contrast. In some models, this suppression is strong enough to impose very restrictive observational limits to them [9].

In this paper we will analyze the evolution of density perturbations in a particular, spatially flat, cosmological model with vacuum decay [10,11]. It can be based on a phenomenological prescription for the variation of Λ with time [12], given by $\Lambda \approx (H+m)^4 - m^4$, where m is a characteristic energy that can be identified with the scale of the QCD vacuum condensation, the latest cosmological vacuum transition. Although it can be corroborated by holographic arguments [12,13], based on the thermodynamics of de Sitter space-times, here we will take it just as a phenomenological ansatz. In the limit of very early times, we have $\Lambda \approx H^4$, which provides a nonsingular inflationary solution [12].

In the opposite limit of large times we have $\Lambda = \sigma H$, with $\sigma \approx m^3$. This scaling law for the vacuum density was also suggested in [3], on the basis of different arguments. It leads to a cosmological scenario in qualitative agreement with the standard one [10], with an initial radiation era followed by a long phase dominated by dust. This dust phase tends asymptotically to a de Sitter universe, with the deceleration/acceleration transition occurring some time before the present epoch. On the other hand, a quantitative analysis has shown a good accordance with supernova observations, leading to age and matter density parameters inside the limits imposed by other independent observations [11].

Since the radiation phase we obtain is indistinguishable from the standard one, our analysis will be initially focused on the evolution of density perturbations of nonrelativistic matter in the dust-dominated phase, considering wavelengths inside the horizon.[2] In this way, it will be possible to make use of a generalization of the Newtonian linear treatment of the problem, which includes the effects of matter production [14]. We will show that, even in the case of a homogeneous vacuum decay, the contrast suppression is important only for late times, not affecting the process of galaxy formation. On the other hand, it dominates for future times, and we will discuss how this behavior can possibly alleviate another problem related to the cosmological term: the cosmic coincidence problem.

Subsequently, a relativistic analysis will be performed, in order to construct the matter power spectrum. Again, the hypothesis of homogeneous matter production will be used, leading as well to a consequent power suppression.

[2]As already commented, we will assume that the vacuum is decaying into nonrelativistic particles, in order to avoid any conflict with cosmic microwave background (CMB) observations and with the observed coldness of dark matter. We will also suppose that only dark matter is produced, since the baryon content is well constrained by nucleosynthesis. Evidently, these assumptions cannot be verified without a microscopic theory of the vacuum-matter interaction.

A second interesting difference as compared to the ΛCDM model is a shift of the spectrum turnover to the left, that is, to smaller wave numbers. The late-time suppression is not very sensitive to the value used for the matter density, a feature that can already be noted from the Newtonian analysis. On the other hand, the correction of the turnover position, by taking a higher matter density, displaces all the spectrum to the right, compensating the late-time power suppression. In this way we can obtain an excellent fit of data, but with a higher matter density in comparison with the standard case.

The article is organized as follows. In next section we review the main features of our interacting model. In Sec. III we perform the Newtonian analysis of evolution of density perturbations in the matter era. In Sec. IV the matter power spectrum is constructed, on the basis of a simplified relativistic calculation. In Sec. V the reader can find our concluding remarks.

II. THE MODEL

The Friedmann equations in the spatially flat case are given by (1) and $\rho_T = 3H^2$. Let us take $\rho_T = \rho_m + \Lambda$, $p_T = p_m - \Lambda$, and $p_m = (\gamma - 1)\rho_m$, with constant γ. Let us also take the ansatz $\Lambda = \sigma H$, with σ constant and positive. We obtain the evolution equation

$$2\dot{H} + 3\gamma H^2 - \sigma\gamma H = 0. \tag{3}$$

The solution, for $\rho_m, H > 0$, is given by [10]

$$a = C[\exp(\sigma\gamma t/2) - 1]^{2/(3\gamma)}, \tag{4}$$

where a is the scale factor, C is an integration constant, and a second one was taken equal to zero in order to have $a = 0$ for $t = 0$.

In the radiation phase, taking $\gamma = 4/3$ and the limit of early times ($\sigma t \ll 1$), we have

$$a \approx \sqrt{2C^2\sigma t/3}. \tag{5}$$

This is the same scaling law we obtain in the standard case, leading to $H \approx 1/2t$. In the same limit we then have $\rho_m = \rho_T - \Lambda = 3H^2 - \sigma H \approx 3H^2 = \rho_T$. By using (5) we then obtain

$$\rho_T \approx \rho_m \approx \frac{\sigma^2 C^4}{3a^4} \approx \frac{3}{4t^2}, \tag{6}$$

i.e., the same variation law for radiation one obtains in the standard model, which shows that, during the radiation era, both the cosmological term and the matter production can be dismissed.

On the other hand, in the matter era we obtain, by doing $\gamma = 1$,

$$a = C[\exp(\sigma t/2) - 1]^{2/3}. \tag{7}$$

Taking again the limit of early times, we have

$$a \approx C(\sigma t/2)^{2/3}, \tag{8}$$

as in the Einstein-de Sitter solution. It is also easy to see that, in the opposite limit $t \to \infty$, (7) tends to the de Sitter solution.

With the help of (7), and by using $\Lambda = \sigma H$ and $\rho_m = 3H^2 - \sigma H$, it is straightforward to derive the matter and vacuum densities as functions of the scale factor. One has

$$\rho_m = \frac{\sigma^2 C^3}{3a^3} + \frac{\sigma^2 C^{3/2}}{3a^{3/2}}, \tag{9}$$

$$\Lambda = \frac{\sigma^2}{3} + \frac{\sigma^2 C^{3/2}}{3a^{3/2}}. \tag{10}$$

In these expressions, the first terms give the standard scaling of matter (baryons included) and vacuum densities, being dominant in the limits of early and very late times, respectively. The second ones are owing to the process of matter production, being important at an intermediate time scale.

With (9) and (10) we obtain, for the total energy density and pressure,[3]

$$\rho_T = \frac{\sigma^2}{3}\left[\left(\frac{C}{a}\right)^{3/2} + 1\right]^2, \tag{11}$$

$$p_T = -\sqrt{\frac{\sigma^2}{3}}\rho_T^{1/2}. \tag{12}$$

From (7) we can also derive the Hubble parameter as a function of time in the matter era. It is given by

$$H = \frac{\sigma/3}{1 - \exp(-\sigma t/2)}. \tag{13}$$

With this expression, and by using (7) and (9), it is not difficult to obtain the present age of the universe, given, in terms of the age parameter, by

$$H_0 t_0 = \frac{2\ln\Omega_{m0}}{3(\Omega_{m0} - 1)}, \tag{14}$$

where $\Omega_{m0} = \rho_{m0}/3H_0^2$ is the relative matter density at present.

Finally, with the help of (7) and (13), we can express H as a function of the redshift $z = a_0/a - 1$, which leads to

$$H(z) = H_0[1 - \Omega_{m0} + \Omega_{m0}(z + 1)^{3/2}]. \tag{15}$$

[3]These are the same expressions we obtain for a generalized Chaplygin gas (characterized by the equation of state $p_{ch} = -A/\rho_{ch}^{\alpha}$ [15]), if we choose $\alpha = -1/2$ and $A = \sqrt{\sigma^2/3}$ (see [16] for a detailed discussion about this and other curious equivalences between dark energy models). Note, however, that the oscillations in the evolution of density perturbations characteristic of a Chaplygin gas [17] are not present in our case, as we will see below.

With this function we have analyzed the redshift-distance relation for type Ia supernovas [11], obtaining data fits as good as with the flat ΛCDM model. With the Supernova Legacy Survey (SNLS) [18]—the most confident survey we have so far—the best fit is given by $h = 0.70 \pm 0.02$ and $\Omega_{m0} = 0.32 \pm 0.05$ (with 2σ), with a reduced χ-square $\chi_r^2 = 1.01$ [here, $h \equiv H_0/(100\ \mathrm{km/s\,Mpc})$]. On the other hand, a joint analysis of the Legacy Survey, baryonic acoustic oscillations, and the position of the first peak of CMB anisotropies has led to the concordance values $h = 0.69 \pm 0.01$ and $\Omega_{m0} = 0.36 \pm 0.01$ (with 2σ), with $\chi_r^2 = 1.01$ [19]. With these results one can obtain, from (14), a universe age $t_0 \approx 15.0$ Gyr, inside the interval allowed by age estimations of globular clusters [20].

III. NEWTONIAN EVOLUTION OF DENSITY PERTURBATIONS

The Newtonian equation for the evolution of density perturbations in a pressureless fluid can be generalized in order to account for matter production [14]. In this generalized form, it is given by

$$\frac{\partial^2\delta}{\partial t^2} + \left(2H + \frac{\Psi}{\rho_m}\right)\frac{\partial\delta}{\partial t} - \left[\frac{\rho_m}{2} - 2H\frac{\Psi}{\rho_m} - \frac{\partial}{\partial t}\left(\frac{\Psi}{\rho_m}\right)\right]\delta = 0. \tag{16}$$

Here, $\delta = \delta\rho_m/\rho_m$ is the density contrast of the pressureless matter, and Ψ is the source of matter production, defined as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = \Psi. \tag{17}$$

In the case of a constant Λ, $\Psi = 0$, and (16) reduces to the usual nonrelativistic equation for the linear evolution of the contrast. In our case, on the other hand, $\Psi = -\dot{\Lambda} = -\sigma\dot{H}$, as can be seen from (2).

Equation (16) is derived on the basis of two main assumptions [14]. The first one is that the produced particles have negligible velocities as measured by observers comoving with the cosmic fluid. This is a reasonable hypothesis, since we are dealing with a nonrelativistic phase of universe expansion, when H (and so Λ) varies slowly enough. The second assumption is that the vacuum component Λ is strictly homogeneous, which means that matter production is homogeneous as well. This stronger hypothesis is totally *ad hoc* at the present stage of the model development and, as we will see, leads to a suppression of the contrast at large times.

In order to solve (16) for our case, it is convenient to introduce the new variable

$$x = \exp(-\sigma t/2). \tag{18}$$

After calculating ρ_m, H, and Ψ as functions of x with the help of (7), (9), and (13), Eq. (16) takes the form

$$3x^2(x-1)^2\delta'' + 4x(x-1)\delta' - 2(3x-2)\delta = 0, \tag{19}$$

where the prime means derivative with respect to x. It is possible to show that, in the limit of early times, it reduces to the evolution equation for the contrast in the Einstein-de Sitter model, as should be.

The general solution of (19) can be written as

$$\delta = \frac{x}{x-1}\left\{C_1 + C_2\left[\frac{2}{3}\beta(x, 1/3, 2/3) + x^{1/3}(x-1)^{2/3}\right]\right\}, \tag{20}$$

where C_1 and C_2 are integration constants to be determined by initial conditions, and $\beta(x, a, b)$ is the incomplete beta function, defined as

$$\beta(x, a, b) = \int_0^x y^{a-1}(1-y)^{b-1}dy. \tag{21}$$

This β function can be expanded in a Laurent series around $x = 1$, leading to

$$\beta(x, 1/3, 2/3) \approx \beta(1, 1/3, 2/3) - \tfrac{3}{2}(x-1)^{2/3}. \tag{22}$$

In this way, with the help of (18) and (22) we can expand (20) around $t = 0$, obtaining

$$\delta \approx \frac{D_1}{t} + D_2 t^{2/3}, \tag{23}$$

which is precisely the general solution obtained in the Einstein-de Sitter model, as expected. The new arbitrary constants are given by

$$D_1 = -\frac{2}{\sigma}\left[C_1 + \frac{2}{3}\beta(1, 1/3, 2/3)C_2\right], \tag{24}$$

$$D_2 = \frac{(4\sigma)^{2/3}}{15}C_2. \tag{25}$$

If, in the early time approximation (23), we want to retain just the growing mode, proportional to $t^{2/3}$, we must choose $D_1 = 0$. Then, our general solution (20) reduces to

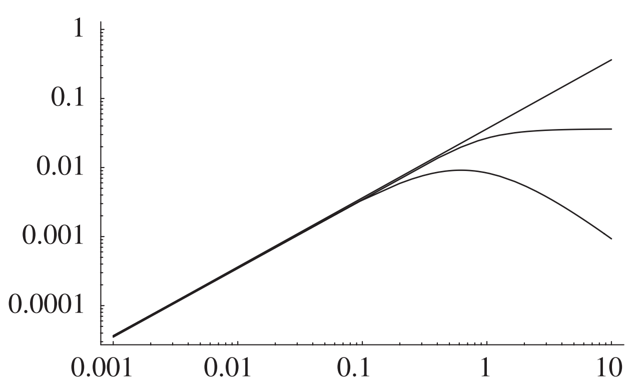

FIG. 1. The density contrast as a function of the scale factor.

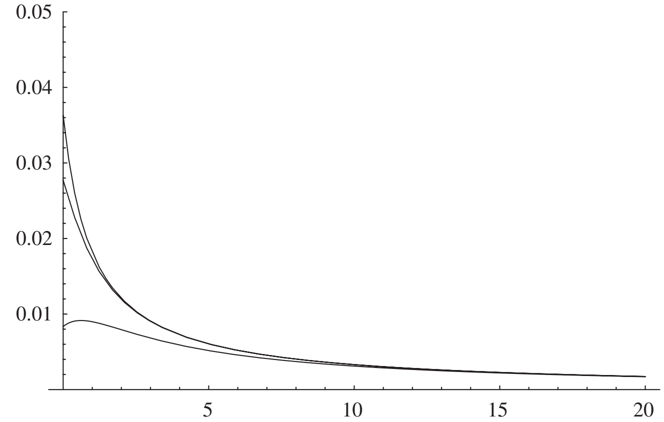

FIG. 2. The density contrast as a function of the redshift.

$$\frac{\delta}{C_2} = \frac{2x[\beta(1, 1/3, 2/3) - \beta(x, 1/3, 2/3)]}{3(1-x)} - \frac{x^{4/3}}{(1-x)^{1/3}}. \tag{26}$$

The above solution can be expressed as functions of t or a, with the help of (7) and (18). It can also be expressed as a function of the redshift, by using the relation

$$x = \frac{\Omega_{m0}(1+z)^{3/2}}{1 - \Omega_{m0} + \Omega_{m0}(1+z)^{3/2}}, \tag{27}$$

which can be derived with the help of (7), (9), and (13).

Figures 1 and 2 show the density contrast (26) as a function of a and z, respectively. We have taken $a_0 = 1$, and used for the matter density parameter the best-fit value we have obtained from the SNLS analysis [11], $\Omega_{m0} = 0.32$. The integration constant C_2 was chosen so that for the time of last scattering ($z \approx 1100$) one has $\delta \approx 10^{-5}$, as imposed by anisotropy observations of the cosmic microwave background [21]. For the sake of comparison, we have also plotted the evolution of the density contrast in the Einstein-de Sitter solution and in the spatially flat ΛCDM model with $\Omega_{m0} = 0.27$.

In our case the density contrast grows monotonically with time until $z \approx 0.6$, after which it decreases monotonically, tending to zero in the limit $t \to \infty$. The consequences of such a suppression at large times will be discussed in our Conclusions, where a possible relation with the cosmic coincidence problem will be outlined. The important point here is that the evolution of δ in our case is indistinguishable from its behavior in the ΛCDM case until $z \approx 5$, that is, along the entire era of galaxy formation. On the other hand, the late-time suppression leads to a present contrast approximately $1/3$ of the standard one, a difference that will be manifest in the power spectrum, as we will see now.

IV. THE POWER SPECTRUM

The shape of the spectrum depends on several parameters. But one of the most important is given by the moment of equilibrium between radiation and matter, $\Omega_R = \Omega_m$, where Ω_R and Ω_m are the respective density parameters (relative to the critical density). In the ΛCDM model, we have

$$\Omega_R = \frac{\Omega_{R0}}{a^4} = \Omega_{R0}(1+z)^4, \tag{28}$$

$$\Omega_m = \frac{\Omega_{m0}}{a^3} = \Omega_{m0}(1+z)^3, \tag{29}$$

where Ω_{R0} and Ω_{m0} are the density parameters for radiation and matter today. The redshift at equilibrium is then given by

$$1 + z_{\rm eq} = \frac{\Omega_{m0}}{\Omega_{R0}}. \tag{30}$$

Following [22], we fix, for the ΛCDM model,

$$\Omega_{m0}h^2 = 0.127, \qquad \Omega_{R0}h^2 = 4.1 \times 10^{-5}. \tag{31}$$

This implies

$$1 + z_{\rm eq} = 3097. \tag{32}$$

Remark that this value, as a matter of fact, is independent of h.

Now, we can analyze the moment the perturbations enter in the horizon. This is obtained by inspecting the perturbed equations. In general, it can be written as

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left\{v_s^2\frac{k^2}{a^2} - \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2\right\}\delta = 0. \tag{33}$$

In this equation, δ is the density contrast, and $v_s^2 = \frac{\partial p}{\partial \rho}$ represents the sound velocity in unities of c (the velocity of light). The presence of a first derivative term is related to the *friction* due to the expansion of the universe, while the two last terms describe the interplay between the pressure, that avoids the collapse, and the gravitational attraction, that drives the collapse. When the first of these terms dominates, the perturbation does not grow; when the second one dominates, the perturbation increases. Ignoring numerical factors of order of unities, related to the sound velocity, equation of state etc., the condition that separates both regimes is

$$k = \frac{a}{d_H}, \qquad d_H = \frac{c}{H} = \frac{ca}{\dot{a}}. \tag{34}$$

In this expression, d_H is the Hubble radius. Of course, this is just an estimation.

For the ΛCDM model, we have

$$d_H = \frac{c}{H_0}\{\Omega_{m0}(1+z)^3 + \Omega_{R0}(1+z)^4 + \Omega_{\Lambda 0}\}^{-1/2}, \tag{35}$$

where $\Omega_{\Lambda 0}$ is the density parameter for the cosmological term today. Hence, we have

$$[(1+z)kl_{H0}]^2 = \Omega_{m0}(1+z)^3 + \Omega_{R0}(1+z)^4 + \Omega_{\Lambda 0}, \tag{36}$$

where l_{H0} is the Hubble's radius today, $l_{H0} = 3000h^{-1}$ Mpc. In general, for large values of z the term $\Omega_{\Lambda 0}$ can be ignored. In doing so, and using the expression above for $z = z_{\rm eq}$, we find the formula (7.39) of Ref. [23],

$$k_{\rm eq} = \sqrt{\frac{2}{\Omega_{R0}}}\frac{\Omega_{m0}}{l_{H0}}. \tag{37}$$

Using, besides the values of Ω_{m0} and Ω_{R0} already quoted, also $h = 0.7$, we obtain

$$k_{\rm eq} = 0.013. \tag{38}$$

We notice that, using the BBKS transfer function for the ΛCDM model [24], the turning point is also located at $k = 0.013$.

Now, the observations cover scales from $k_{\rm min}h^{-1} = 0.010$ until $k_{\rm max}h^{-1} = 0.185$. Using the parameters above, we find that these modes entered in the Hubble horizon at

$$k_{\rm min} \to z_1 = 2077; \qquad k_{\rm max} \to z_2 = 59\,143. \tag{39}$$

That is, essentially, all modes entered in the radiation dominate era.

Turning to the present interacting model, the main modifications are the following:

(1) The expression governing the moment the modes enter in the Hubble horizon is given by

$$[kl_{H0}(1+z)]^2 = \frac{1}{\Omega_{m0}+\Omega_{\Lambda 0}}[\Omega_{\Lambda 0} + \Omega_{m0}(1+z)^{3/2}]^2 + \Omega_{R0}(1+z)^4, \tag{40}$$

with $\Omega_{m0} + \Omega_{\Lambda 0} \approx 1$. This is an approximate expression obtained from (15) by adding a conserved radiation density to the Friedmann equation $3H^2 = \rho_T$.[4]

(2) An inspection of (15) for high z, when Λ and the matter production are dismissable, shows that $\Omega_m(z) = \Omega_{m0}^2(1+z)^3$. In other words, we have the same scaling of conserved matter as in the standard model, but with an extra factor Ω_{m0}. This is owing to the matter production between $t(z)$ and t_0: in order to have the same matter density today, we need a smaller density at high redshifts. As a

[4]Note that the inclusion of conserved radiation changes the dynamics, and, consequently, the production of matter, $\Lambda(z)$ and $\rho_m(z)$ also change. Therefore, the exact generalization of (15) requires a reanalysis of the dynamics. Nevertheless, as $\Omega_{R0} \approx 10^{-4} \ll 1$, when the vacuum and the matter production begin to have importance, the radiation is negligible, and vice versa. In this way, (40) can be considered a very good approximation. Indeed, a numerical analysis in the range $0 < z < 10^4$ has shown that the difference between (40) and the exact $H(z)$ is as small as 0.01%.

consequence, the redshift of equilibrium between matter and radiation is now given by $z_{\rm eq} = \Omega_{m0}^2/\Omega_{R0}$, while for the correspondent wave number we obtain, instead of (37),

$$k_{\rm eq} = \sqrt{\frac{2}{\Omega_{R0}}}\frac{\Omega_{m0}^2}{l_{H0}}. \tag{41}$$

Note the extra factor Ω_{m0} as compared to the corresponding ΛCDM expression. As this factor is smaller than unity, this means that the turnover of the spectrum is moved to the left, that is, to smaller k's as compared to the standard model.

(3) The matter density parameter and the Hubble parameter are not the same as before. In the subsequent analysis we will use $\Omega_{m0} = 0.32$ and $h = 0.7$ (the type Ia supernovas best fitting [11]).

Now, the results are the following:

(1) The equilibrium occurs at $z_{\rm eq} = 2263$, which implies $k_{\rm eq} = 0.007$;

(2) The mode $k_{\rm min}$ enters in the Hubble horizon at $z_1 = 3469$, while the mode $k_{\rm max}$ at $z_2 = 81\,404$.

As already noticed, the results indicate that the spectrum is displaced to the left, implying that there is a power suppression with respect to the ΛCDM model. Moreover, there is, as we have seen in the previous section, an additional power suppression during the matter dominated phase. Hence, essentially, we must expect that the power spectrum displays, in what concerns matter agglomeration, an expressive power suppression in comparison with the ΛCDM model.

However, we can displace the spectrum to the right, instead of displace it to the left, if the values of Ω_{m0} and/or h are increased. For example, for $\Omega_{m0} = 0.48$ and $h = 0.73$, the $k_{\rm eq}$ occurs at 0.016, with $z_{\rm eq} = 5094$. Moreover, $k_{\rm min}$ enters in the Hubble horizon at $z_1 = 2589$ and $k_{\rm max}$ at $z_2 = 80\,020$. The substantial displacement to the right of $k_{\rm eq}$ compensates the smaller growing of perturbations during the matter dominated phase. So, the general features of the power spectrum are reproduced for larger values of Ω_{m0} as compared to the ΛCDM model.

A precise derivation of the spectrum is a very tough calculation, since the Einstein-Boltzmann coupled system must be considered. A complete analysis for the ΛCDM model leads to the so-called BBKS transfer function [24], which gives the spectrum today as a function of a given primordial spectrum. For the scale invariant spectrum, favored by the primordial inflationary scenario, the BBKS transfer function is given by

$$P_m(k) = |\delta_m(k)|^2 = AT(k)\frac{g^2(\Omega_{m0})}{g^2(\Omega_T)}k, \tag{42}$$

where A is a normalization of the spectrum (which can be fixed by the spectrum of anisotropy of the cosmic microwave background radiation), $T(k)$ is given by

$$T(k) = \frac{\ln(1+2.34q)}{2.34q}[1 + 3.89q + (16.1q)^2 + (5.64q)^3 + (6.71q)^4]^{-1/4}, \tag{43}$$

$$q = \frac{k}{h\Gamma}\ \mathrm{Mpc}^{-1}, \qquad \Gamma = \Omega_{dm0}he^{-\Omega_{b0}-(\Omega_{b0}/\Omega_{dm0})}, \tag{44}$$

and where Ω_{m0}, Ω_{dm0}, Ω_{b0}, and Ω_T are, respectively, the present density parameters of pressureless (baryonic + dark) matter, dark matter, baryons, and the total energy. The function $g(\Omega)$ is defined by

$$g(\Omega) = \frac{5}{2}\Omega\left[\Omega^{4/7} - \Omega_{\Lambda 0} + \left(1+\frac{\Omega}{2}\right)\left(1+\frac{\Omega_{\Lambda 0}}{70}\right)\right]^{-1}. \tag{45}$$

The transfer function defined above represents the fitting of the complete numerical evaluation.

A simplified version of the transfer function, which keeps all its essential features, can be obtained by integrating the perturbed equations for the coupled system containing radiation and pressureless matter, from a very high redshift until today [25,26]. The starting point is given by the Einstein equations and the conservation law for the energy-momentum tensor:

$$R_{\mu\nu} = 8\pi G\sum_i\left\{T^i_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^i\right\}, \tag{46}$$

$$T^{\mu\nu}_{i} = 0, \tag{47}$$

where the indice denotes the ith fluid component. One of them will be radiation. The other one will be the pressureless matter in the ΛCDM case, or the vacuum-matter interacting fluids in our case (remember that, in our case, the pressureless matter is not independently conserved, since it interacts with vacuum). Introducing the perturbations, $g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}$, $\rho_i = \rho^0_i + \delta\rho$, $p_i = p^0_i + \delta p_i$, with $(g^0_{\mu\nu}, \rho^0_i, p^0_i)$ being the background solutions, and imposing the synchronous coordinate condition $h_{\mu 0} = 0$, we end up with the following set of coupled equations:

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} = \rho_m\delta_m + 2\rho_R\delta_R, \tag{48}$$

$$\dot{\delta}_m - \frac{\dot{\Lambda}}{\rho_m}\delta_m = \frac{\dot{h}}{2}, \tag{49}$$

$$\dot{\delta}_R + \frac{4}{3}\left\{\frac{v}{a} - \frac{\dot{h}}{2}\right\} = 0, \tag{50}$$

$$\dot{v} = \frac{k^2}{4a}\delta_R, \tag{51}$$

where $h = h_{kk}/a^2$, δ_m and δ_R are the density contrast for matter and radiation, respectively, v is connected with the peculiar velocities of the perturbed radiative fluid, and in the ΛCDM case $\dot{\Lambda}$ is, evidently, zero.

We now eliminate the variable $\dot{h}$ using (49), divide all the expressions by H_0^2, and rewrite the resulting equations in terms of the redshift z, which becomes the new dynamical variable. In the ΛCDM case the system of equations is reduced to

$$\delta_m'' - \frac{g[z]}{f[z]}\frac{\delta_m'}{1+z} = \frac{3}{2f[z]}\{\Omega_{m0}(1+z)\delta_m + 2\Omega_{R0}(1+z)^2\delta_R\}, \tag{52}$$

$$\delta_R' - \frac{4}{3}\left\{\frac{\upsilon}{\sqrt{f[z]}} + \delta_m'\right\} = 0, \tag{53}$$

$$\upsilon' = -\left(\frac{kl_{H0}}{2}\right)^2 \frac{\delta_R}{\sqrt{f[z]}}, \tag{54}$$

where the primes indicate derivative with respect to the redshift z. The background functions $f[z]$ and $g[z]$ are given by

$$f[z] = \frac{\dot{a}^2}{a^2} = \Omega_{m0}(1+z)^3 + \Omega_{R0}(1+z)^4 + \Omega_{\Lambda 0}, \tag{55}$$

$$g[z] = \frac{\ddot{a}}{a} = -\frac{1}{2}\Omega_{m0}(1+z)^3 - \Omega_{R0}(1+z)^4 + \Omega_{\Lambda 0}. \tag{56}$$

Integrating, for example, from $z = 10^8$ (when the initial spectrum is supposed to be scale invariant, i.e., δ_m, $\delta_R \propto \sqrt{k}$) until today, $z = 0$, we can reproduce the BBKS transfer function with about 10% of precision.

We can perform the same calculation for the present model, finding the following set of perturbed equations:

$$\delta_m'' - \left\{\frac{\Omega_\Lambda'}{\Omega_m} + \frac{g_1[z]}{f_1[z]}\frac{1}{(1+z)^2}\right\}\delta_m' + \left\{\frac{g_1[z]}{f_1[z]}\frac{\Omega_\Lambda'}{\Omega_m}\frac{1}{(1+z)^2} - \frac{\Omega_\Lambda''}{\Omega_m} + \frac{\Omega_m'\Omega_\Lambda'}{\Omega_m^2}\right\}\delta_m = \frac{3}{2}\frac{1}{f_1[z](1+z)^4}\{\Omega_m\delta_m + 2\Omega_R\delta_R\}, \tag{57}$$

$$\delta_R' - \frac{4}{3}\left\{\frac{\upsilon}{(1+z)\sqrt{f_1[z]}} + \delta_m' - \frac{\Omega_\Lambda'}{\Omega_m}\delta_m\right\} = 0, \tag{58}$$

$$\upsilon' = -\left(\frac{kl_{H0}}{2}\right)^2 \frac{\delta_R}{(1+z)\sqrt{f_1[z]}}. \tag{59}$$

In these equations, we use the following definitions:

$$f_1[z] = \dot{a}^2 = \frac{1}{(\Omega_{\Lambda 0} + \Omega_{m0})(1+z)^2}\{\Omega_{\Lambda 0} + \Omega_{m0}(1+z)^{3/2}\}^2 + (1+z)^2\Omega_{R0}, \tag{60}$$

$$g_1[z] = \ddot{a} = -\frac{(1+z)^2}{2} f_1'[z], \tag{61}$$

$$\Omega_m(z) = \frac{\Omega_{\Lambda 0}\Omega_{m0}}{\Omega_{\Lambda 0} + \Omega_{m0}}\left\{\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}(1+z)^3 + (1+z)^{3/2}\right\}, \tag{62}$$

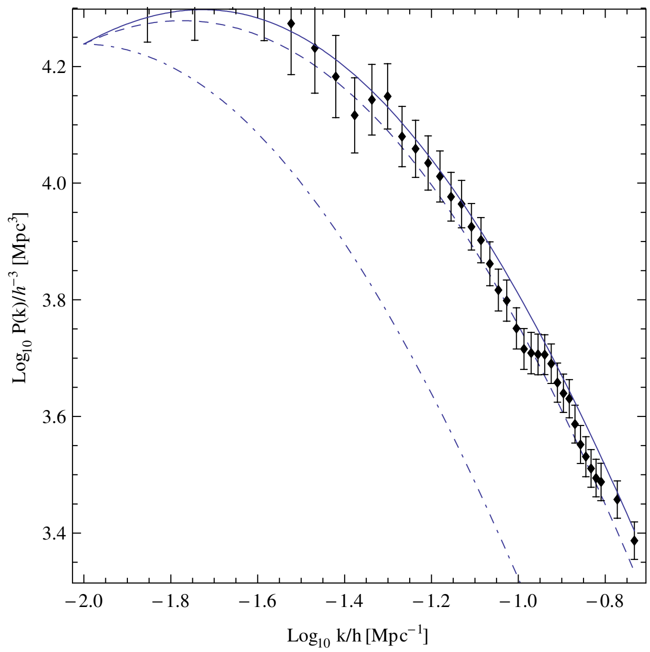

FIG. 3 (color online). The matter power spectra as given by the BBKS transfer function (dashed line), the approximative numerical analysis used here for ΛCDM (continuous line), and for the interacting model (dot-dashed line). The data come from the 2dFGRS galaxy survey program [27]. It has been used $\Omega_{m0} = 0.36$ for the interacting model.

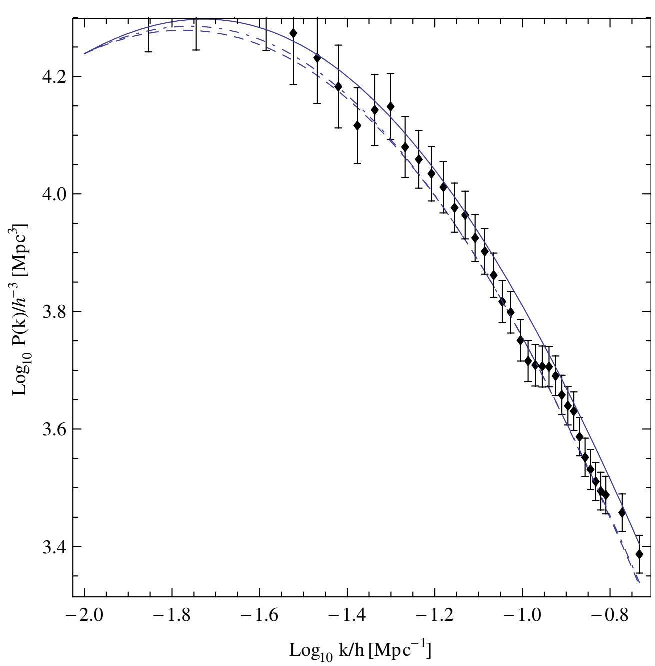

FIG. 4 (color online). The matter power spectra as given by the BBKS transfer function (dashed line), the approximative numerical analysis used here for ΛCDM (continuous line), and for the interacting model (dot-dashed line). The data come from the 2dFGRS galaxy survey program [27]. It has been used $\Omega_{m0} = 0.48$ for the interacting model.

$$\Omega_\Lambda(z) = \frac{\Omega_{\Lambda 0}^2}{\Omega_{\Lambda 0} + \Omega_{m0}} \left\{ 1 + \frac{\Omega_{m0}}{\Omega_{\Lambda 0}} (1+z)^{3/2} \right\}. \quad (63)$$

In Figs. 3 and 4 we display the results for the exact transfer function for the ΛCDM model (dashed line), the corresponding numerical approximation (continuous line), and the approximative transfer function for the present model (dot-dashed line). The observational data come from the 2dFGRS galaxy survey program [27]. In the case of the interaction model we used, in Fig. 3, $\Omega_{m0} = 0.36$, the concordance value obtained from the joint analysis of type Ia supernovas, baryonic acoustic oscillations, and CMB [19]. In Fig. 4, on the other hand, we have used $\Omega_{m0} = 0.48$. We see that in the first case there is a substantial suppression of power, while in the second case, where the dark matter parameter has been increased, the agreement is excellent.

Hence, concerning the matter power spectra, the interacting model with homogeneous matter production requires an almost double quantity of dark matter with respect to the ΛCDM model.

V. CONCLUSIONS

In spite of the physical plausibility of a time-dependent cosmological term, a complete theoretical development of this idea, including the microscopic details of the vacuum-matter interaction, is still lacking. On the other hand, macroscopic approaches depend on some phenomenological hypothesis, leading some times to diverse prescriptions for the vacuum decay.[5] For this reason, a careful comparison with current observations is very important, playing the role of corroborating or ruling out the different models.

We have already analyzed the supernova observations [11], obtaining good fits and cosmological parameters in accordance with other independent tests, as the age of globular clusters and dynamical limits to the matter density [28]. Other precise tests, as the position of the first acoustic peak of the cosmic microwave background and the baryonic acoustic oscillations have also been performed [19], showing a good concordance when jointed to the supernova analysis.

In the present paper we have studied the evolution of matter density perturbations, in particular, the contrast suppression associated to the process of matter production. We have shown that, even in the case of a homogeneous production, the evolution of the contrast is the same as in the standard recipe along the entire era of galaxy formation, diverging from the later only for $z < 5$.

On the other hand, the suppression would be dominant for future times, and this may have an interesting relation with another problem related to the cosmological term, namely, the approximate coincidence between the present densities of matter and dark energy. Indeed, we can see from Figs. 1 and 2 that the matter contrast has its maximum just before today, when matter and vacuum give similar contributions to the total density. The largest structures formed until now tend to disaggregate in the future, and their existence then coincides with the time of approximate equality between the matter and vacuum densities.

This could alleviate the cosmic coincidence problem, if galaxies also follow such a process. However, we should remember that galaxies have left the linear regime of growth a long time ago, and that now their evolution is nonlinear, driven essentially by their self-gravitation. Therefore, an explanation of the cosmic coincidence in the terms above will depend on a nonlinear study of density perturbations in the context of the present model. Only such an investigation would tell us whether the contrast suppression described here can affect smaller structures like galaxies.

Its also important to have in mind that the homogeneity of the matter production, implicit in the derivation of solution (26) and in our simplified relativistic treatment, is just an *ad hoc* hypothesis, to be verified from both the theoretical and observational viewpoints. For a constant, noninteracting vacuum term it is certainly true, but not necessarily in the present case. Any inhomogeneity of the vacuum density around matter distributions may lead to an inhomogeneous production, reducing in this way the contrast suppression. This would allow us to fit the observed power spectrum with a smaller matter density, closer to the concordance value obtained in [19]. Whether the matter contrast will still have a maximum around the present time, with the discussed implications for the coincidence problem, is a matter of investigation. A relativistic study of this case, that is, with $\delta\Lambda \neq 0$, is already in progress.

[5]For example, the linear dependence of Λ on the Hubble parameter we use here contrasts with the quadratic dependence used in Ref. [5]. In that work, the quadratic dependence is due to the computation of quantum effects of matter field in a cosmological background, which leads to a running cosmological term. The authors also used the matter power spectrum data to constrain the fundamental parameters of the quantum model.

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