

Trapping of strangelets in the geomagnetic field

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Strangelets arriving from the interstellar medium are an interesting target for experiments searching for evidence of this hypothetical state of hadronic matter. We entertain the possibility of a *trapped* strangelet population, quite analogous to ordinary nuclei and electron belts. For a population of strangelets to be trapped by the geomagnetic field, these incoming particles would have to fulfill certain conditions, namely, having magnetic rigidities above the geomagnetic cutoff and below a certain threshold for adiabatic motion to hold. We show in this work that, for fully ionized strangelets, there is a narrow window for stable trapping. An estimate of the stationary population is presented and the dominant loss mechanisms discussed. It is shown that the population would be substantially enhanced with respect to the interstellar medium flux (up to 2 orders of magnitude) due to quasistable trapping.

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I. INTRODUCTION

In a celebrated paper, Witten [1] elaborated on the possibility [2–4] that systems composed of a deconfined Fermi gas of up, down, and strange quarks could have a lower energy per baryon than iron, thus being absolutely stable. This hypothetical state (strange quark matter) could be created by weak interactions introducing the massive s quark, if the energy “cost” of the mass is compensated by the availability of a new Fermi sea associated to this extra flavor, thus lowering the Fermi energy of the u and d quark seas.

Previous works have shown [5] that this stability may be realized for a wide range of parameters of strange quark matter (SQM) in bulk on the basis of the MIT bag model. Calculations also indicate that SQM can be absolutely stable within other models, e.g., shell model [6,7], or not stable at all depending on the adopted model [8]. More recently, studies have indicated that a paired version of SQM, the color-flavor locked (CFL) state, seems to be even more favorable energetically than the unpaired SQM, widening the stability window [9–12].

For the description of *finite* size lumps of strange matter (termed *strangelets*), a few terms have to be added to the bulk one in the free energy. A surface term suffices for $A \gg 10^7$, while other corrections are relevant for the lower masses (see [13] for a recent review). Large lumps will have essentially the same number of quarks of bulk matter, with a small depletion of the massive strange quark result-

ing in a net positive charge. This is a feature also expected for small chunks [6,13,14], which thus resemble heavy nuclei.

Some astrophysical implications of SQM stability would be in the subject of neutron stars’ interior for it could be composed of deconfined quark matter, either being a hybrid star or a strange star (SS) [15–20]; see also [21–23] for recent reviews. The existence of strange stars would imply the presence of strangelets among cosmic ray primaries. The range of energies and masses of strangelets in the interstellar medium (ISM) depends on the injection spectra and acceleration mechanisms, and is not precisely known (see below).

Considering the question of existence of strangelets among cosmic ray primaries, a few injection (production) scenarios have been considered. Witten originally suggested the merging of compact stars as a likely site [1]. In principle, injection spectra and the total mass in the galaxy may be calculated knowing the rate of the events and the total ejected mass in each of them. These estimates are subject to some caveats: for example, while the number of merging systems has been revised upwards [24], numerical work has shown that a substantial ejection of matter is not guaranteed [25], at least in a strange star-black hole system, and the situation is unclear in the case of a fully relativistic SS-SS system, which has only been partially addressed [26] since these calculations had other primary goals. On the other hand, strange matter formation in type II supernova [18] has been preliminarily explored and in these events a small fraction of strange matter may be ejected. A numerical analysis has shown that the pos-

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sible quark matter component of cosmic rays primaries is compatible [27] with models in which strangelets are ejected in either scenario. Another injection scenario, the acceleration from strange pulsars [28], has been recently considered.

In spite of theoretical controversies, it is generally agreed that the ultimate SQM proof must be provided by experiments. The experimental searches of strangelets started some 20 years ago and have been reviewed recently in [29,30]. In addition to direct production of strangelets in heavy ion collisions [31–42], cosmic rays may contain primaries in this state of matter, which could eventually be detected directly or by its interaction with ordinary terrestrial materials [43–46].

Several cosmic ray events have been tentatively identified in the past as primary strangelets (initially the Centauro events [47] and the HECRO-81 experiment [48,49]) because they present abnormal features such as their high penetration in the atmosphere, low charge-to-mass ratio, and exotic secondaries [50]. More recently, at least one event recorded from the AMS-01 experiment [51], a mass spectrometer aboard the shuttle Discovery during a 10-day flight in 1998, has been considered as possible detection of a strangelet. While it is tempting to identify the primary as a strangelet, the inevitable shell effects complicate the analysis and preclude any firm conclusion as yet [52]. It is not clear until today to what extent the anomalous events can be originated by ordinary primaries or if they require a truly exotic origin.

While an uncertain flux from this “contamination” of the ISM is expected [53], we would like to discuss in this paper another likely site to search for strangelets of cosmic origin. Much in the same way heavy nuclei are present in the Earth’s magnetosphere bouncing between magnetic mirror points, strangelets could also become trapped in specific regions of the magnetosphere and their number density increased with respect to the ISM flux, provided some conditions for their capture by planetary magnetic fields are met. This phenomenon is analogous to the Van Allen belts, and was first suggested in a former study [54]. A handful of experiments have probed the magnetosphere by measuring the fluxes of the so-called “anomalous” cosmic ray nuclei, and may already place interesting limits to strangelets as well. Overall the existence and nature of exotic primaries is an important issue. In addition to former and ongoing searches, there will be a mass spectrometer placed at the International Space Station, the AMS-02 experiment [55,56], with one of its goals to help the identification of this exotic component, of crucial importance in testing the validity of the Bodmer-Witten-Terazawa conjecture. We substantiate below the strangelet belt idea, discuss the main features of this population, and advocate for a search of this exotic component at definite sites within existing uncertainties based on these calculations.

II. STATES OF IONIZATION AND ELECTRONIC RECOMBINATION OF STRANGELETS IN THE ISM

As is well known, unpaired (also referred to as “normal” in this work) SQM in bulk contains light u , d and massive s quarks in β equilibrium. Because of the depletion of the more massive s quark, a small fraction of electrons is also present to maintain charge neutrality. On the other hand, SQM in a paired CFL state is automatically neutral, since the equal number of flavors is enforced by symmetry [57]. Actually, a small positive charge is present because of the smaller abundance of s quarks near the surface in CFL strangelets [13]. Therefore it is natural that CFL strangelets will be surrounded by an electronic cloud in order to neutralize its total charge, forming an exotic atom. The same happens for normal strange matter if the strangelet radius is smaller than the electron Compton wavelength, a condition satisfied whenever $A \ll 10^7$.

In the following and throughout the whole analysis presented here, the strangelet rest mass will be assumed to be $\epsilon_0 A \sim (930 \times A)$ MeV, with ϵ_0 the asymptotic value of the energy per baryon of strange quark matter. We will not consider the fact that the energy per baryon number decreases with A in sophisticated model calculations, given that the uncertainties found in other parameter choices are expected to be much larger than the error associated with this approximation. Also the strange quark mass is considered to be $m_s = 150$ MeV and the coupling gap of CFL strange quark matter $\Delta = 100$ MeV in this exploratory study. With these assumptions, the net positive charge of strangelets is given approximately by $Z = 0.125A$ (low baryon number regime) [58] in the MIT bag model approach for normal strange matter and $Z = 0.3A^{2/3}$ for the CFL model [14].

Strangelets from whatever astrophysical injection event would travel through the interstellar medium and become ionized by collisions. A simple analysis to evaluate the degree of ionization of semirelativistic strangelets surrounded by electronic clouds due to these interactions was performed in a Bohr atom approximation. Strangelets are partly neutralized by electrons from the excitation of the vacuum if $Z \gg 100$ [59], but for all cases of interest in this work the baryon number range is such that we do not have to deal with this effect.

We considered a two-body collision (one incident electron and one electron in the strangelet cloud) instead of a multibody problem, which would be much more difficult to handle. The stripping interactions are assumed to be mainly due to electrons with a Maxwellian speed distribution at a temperature of ~ 100 K, an average condition of electrons in the ISM.

The results are shown in Fig. 1 for strangelets with total energy of 1 GeV/ A . Considering the average density in the interstellar medium to be 1 particle/cm³, the mean free

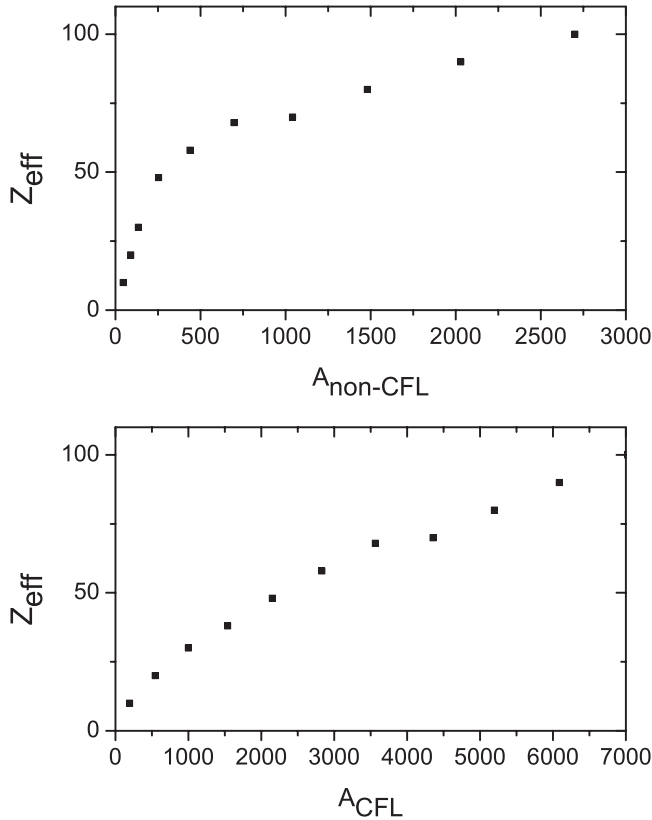


FIG. 1. Strangelet effective charge (1 GeV/A) versus the baryon number A for normal (upper panel) and CFL (lower panel) strange matter after interaction with electrons in the interstellar medium.

path for an electronic collision which may or may not result in ionization is of the order of $10^{15} \text{ cm} \sim 10^{-3} \text{ pc}$, which is very short on astronomical standards.

The ionization degree became stable within a travelled distance of a few pc for 1 GeV/A strangelets. For ultra-relativistic strangelets (i.e., of the type of candidates that would produce a Centauro event [60] $E/A \sim \text{TeV}$) the calculations indicate always full ionization. Furthermore, according to the model proposed by Werner and Salpeter [61] for the radiation flux in the ISM, the influence of the radiation field on ionization of strangelets will be negligible unless the strangelet trajectory crosses a region containing very energetic photons (i.e., the surroundings of a Wolf-Rayet, O and B stars, and/or regions of stellar formation).

We acknowledge that a Bohr atom treatment is a crude approach for the electron distribution around the strangelet, since it does not include quantum corrections as important as the spin-orbit coupling and nonlocal effects, nor relativistic corrections for many electrons bodies ($Z \geq 40$). There is no general expression for these corrections applicable in the case of atoms with many electrons though; the existent models (e.g., Hartree-Fock calculations) are restricted to atoms with few electrons, the same happening

for experimental corrections. In this way, the calculations presented here are rough estimates, showing the general trend of the effects rather than providing precise numerical values.

For low-energy particles the electronic capture can be as important as the ionization process thus far discussed. An approximate cross section for the capture of electrons of velocity v by a charged particle of atomic number Z is given as [62,63]

$$\sigma_c = Z^2 2^{2/3} \alpha^4 \frac{h^2 v^2}{m_e^2 v^2 c^2} \left(\frac{m_e c^2}{h\nu} \right)^{7/2} \times 6.65 \times 10^{-25} \text{ cm}^2, \quad (1)$$

where $h\nu \approx E_e$ for $E_e \gg I$, I and E_e being the electron energies while bound to the nucleus and free in the ISM, respectively, and $m_e c^2$ is the electron rest mass. This form of the cross section for radiative recombination is obtained relating the capture of an electron by a bare nucleus of charge Ze with its capture into the corresponding state of a hydrogen atom. The cross section for the electronic capture by a hydrogen atom is proportional to the energy of the gamma emitted in the process and also to the cross section for the absorption of a quantum of frequency ν by a H^- ion (resulting in emission of an electron of velocity v), and inversely proportional to the momentum of the electron absorbed. In the case of a partially screened nucleus, the cross section is still given approximately by Eq. (1), though a special calculation must be performed to obtain the cross section for capture into an orbital with quantum number n_0 , usually given in tables for ordinary nuclei.

The ‘‘atom’’ or ‘‘ion’’ formed by capturing an electron may also lose this electron in further interactions. For light materials the cross section for electron loss can be approximately expressed for $v > v_0$ [64] as

$$\sigma_l = 8\pi a_0^2 Z^{-2} \left(\frac{v_0}{v} \right)^2, \quad (2)$$

where $a_0 = \hbar^2/m_e e^2 = 0.53 \times 10^{-8} \text{ cm}$ is the Bohr radius and $v_0 = e^2/\hbar$, whereas for intermediate Z materials

$$\sigma_l = \pi a_0^2 Z^{-1} \left(\frac{v_0}{v} \right), \quad (3)$$

because of the screening effect.

A comparison of Eqs. (1)–(3) shows that electronic capture would only be important for high Z strangelets, precisely where this simple picture can no longer be applied due to vacuum excitation effects. That corresponds to a region in baryon number which we believe to be of minimum relevance to the trapped population.

In summary, these results indicate that we can assume total ionization as a good approximation to incoming ISM strangelets that could form an ionization belt in the magnetosphere.

III. CAPTURE OF STRANGELETS IN THE GEOMAGNETIC FIELD

Once in the region dominated by the Earth's magnetic field, and assuming for the moment that they can be captured to perform quasiregular motions in it, we must address the dynamical features of the captured population. The motion of ionized strangelets in the Earth's magnetosphere can be studied by applying the Störmer theory of charges in a dipolar magnetic field. The movement analysis can be described in terms of the geomagnetic latitude and the L parameter, where L is the equatorial distance of a field line to the axis of the dipole measured in units of the Earth's radius.

It is known that the geomagnetic field is not a purely dipolar field. Instead, most realistic magnetic models used for studying it include nearly 50 terms for describing the potential field, from which the magnetic field is obtained in a sum of Legendre functions multiplied by oscillatory coefficients in the azimuthal variable. Since the potential field has a $r^{-(n+1)}$ dependence, the importance of high-order terms decreases rapidly as one moves away from the Earth's surface. In this way, the $n = 1$ term, i.e., the dipole term, is the lowest but dominant one, and most features of the trapped radiation theory are analyzed based on a dipole field.

Charged particles with energy of order of MeV in the inner part of the magnetosphere ($L \ll 10$) rotate with a much higher frequency than that of typical geomagnetic field variation (which varies in time scales of, at most, a few minutes). Under these conditions, the magnetic moment is a conserved quantity (adiabatic invariant). Therefore, particles with high enough magnetic moments become trapped in the dipolar field lines of the geomagnetic field, with mirror points placed near the Earth's poles.

The Swedish physicist, Carl Störmer, motivated by the need to explain the phenomena of the *aurora borealis*, started working with the problem of motion of charged particles in a dipolar field. Poincaré [65] was the first to show that if a charged particle crosses the magnetic field lines at a given angle, it will proceed in a spiral motion around the field lines (in particular, if this angle is a right angle, the motion will be circular). Störmer provided the first detailed mathematical analysis of the trajectories charged particles would make in the geomagnetic field [66]. He showed that for a dipole field the particle following a spiral trajectory would come to a position for which the movement would suffer a reversal in its direction due to the converging magnetic field lines near the poles; i.e., he demonstrated the existence of the so-called mirror point. He also found a solution for the dipole field which describes a special case of an axial symmetric cone that does not allow the access to its interior for particles below a certain rigidity, the so-called "forbidden cone." The geomagnetic cutoff is then a coordinate describing this cone and, consequently, the minimum allowed rigidity, i.e., the

shielding effect of a certain region of the magnetosphere. Almost 50 years passed after the publication of Störmer's first articles on this subject until Fermi [67] derived an approximation for the geomagnetic access of particles at a certain region, given by $R = (A/Q)(E^2 + 1.863E)^{1/2} > 15/L^2$, with R , A , Q , and E being the particle's rigidity (in GV), mass number, effective ionic charge, and kinetic energy (in GeV/nucleon), respectively [68].

In the analysis of charged particles trapped in a magnetic field it is usually considered that the motion of a given particle is a composition of three different motions: the bouncing motion of a guiding center along the magnetic field line, the rotational motion of the particle itself around that guiding center, and the longitudinal drift of the guiding center.

Particles with mirror points that allow penetration in the Earth's atmosphere can be lost via collisions with atoms. The mirror point is usually given in geomagnetic latitude when considering trapping by the Earth's magnetic field. It is located where the projection of the particle's velocity vector along the magnetic field is null, so that the particle reverses its movement in the parallel direction. The mirror point is connected to the pitch angle α , which is defined as the angle the velocity vector of the particle makes with the magnetic field at a given point (Fig. 2). It means that $\alpha = 90^\circ$ at the particle's reflection point. It is usual to study the movement of a trapped particle in the geomagnetosphere by analyzing its equatorial pitch angle, which is the pitch angle at the Earth's magnetic equator. All particles with mirror points placed inside the Earth's radius are obviously lost, meaning that particles with $|\alpha_{\text{eq}}| < \alpha_E$ or $|\pi - \alpha_{\text{eq}}| < \alpha_E$, are inside the Earth's loss cone, where α_{eq} is the equatorial pitch angle and α_E is the equatorial pitch angle a given particle must have in order to have its mirror point located at the Earth's surface.

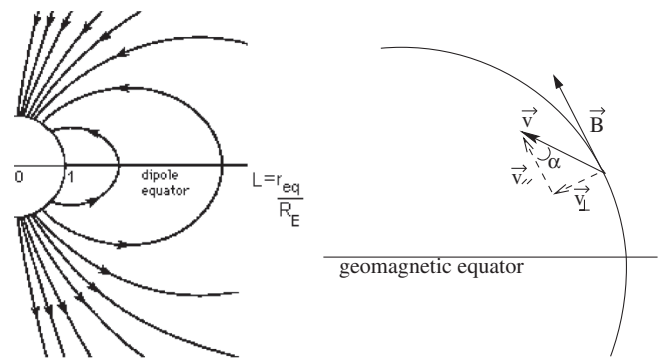


FIG. 2. To the left, schematic view of the dipole lines and the meaning of the parameter L , where r_{eq} is the radial distance from the center of the dipole to a given field line at the geomagnetic equator and R_E is the Earth's radius. To the right, schematic view of the pitch angle, where \vec{v} is the particle velocity with its projections to the parallel and perpendicular direction with respect to the magnetic field \vec{B} .

Taking into account the possibility of interactions between strangelets and particles composing the Earth's atmosphere in order to evaluate the effective position of the strangelet's loss cone, we have considered collisions mainly with the neutral nitrogen molecule (N_2). The probability of interaction of trapped particles which penetrate the atmosphere (suffering collisions losses) can be written as

$$P(s) = 1 - e^{-s/\lambda(s)} \quad (4)$$

at a certain point s , since each process is probabilistically independent, with λ the particle mean free path. Generalizing the previous equation, it is necessary to integrate over the particle path. Assuming that all the strangelets which collide with particles in the atmosphere are eventually removed from the trapped flux, we express the escape probability as

$$P_{\text{esc}} = 1 - e^{-\int_s \sigma[n(s') + s'(dn/ds')]ds'}, \quad (5)$$

where $ds = LR_E \cos\lambda \sqrt{1 + 3\sin^2\lambda} d\lambda$ is the arc along a field line, σ is the particle cross section, and $n(s)$ is the density of particles in the atmosphere at a certain point s of the strangelet's path. Since strangelets are hadrons we may take their relevant interaction cross section to be geometrical ($\sigma \propto A^{2/3}$).

The calculated loss cone for strangelets, assuming an exponential profile of the atmospheric density, is shown in Fig. 3 for different L . It shows a comparison between the values of geomagnetic latitudes obtained through the calculations of the collisional losses due to interaction of strangelets with molecules in the atmosphere and the non-existence of a suitable mirror point due to intersection of

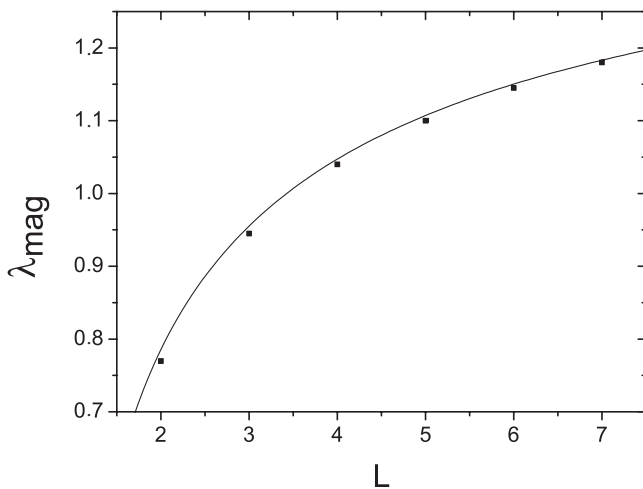


FIG. 3. Loss cone for strangelets in the geomagnetic field. Particles with mirror points placed at geomagnetic latitudes higher than the ones shown with the square dots (coming from the analysis of collisions between strangelets and atmospheric particles) are eventually removed from the trapped population. The full line represents mirror points at the Earth's surface in the dipole field approximation.

the particle's trajectory with the surface of the Earth. As expected, the smaller the equatorial pitch angle, i.e., the farthest the angle the particle makes with the field lines at the geomagnetic equator is from a right angle, the easier it is to remove a trapped particle.

In order for a given particle to penetrate a certain region in the magnetosphere, its energy must be enough to overcome the local geomagnetic cutoff rigidity. Adopting a more detailed expression than the one derived by Fermi, the condition a particle must fulfill to have access to a given region of the magnetosphere can be written as [69]

$$R_{\text{particle}} > \frac{59.6 \cos^4 \lambda}{L^2 [1 + (1 - \cos\gamma \cos^3 \lambda)^{1/2}]^2} \text{ GV}, \quad (6)$$

where λ is the geomagnetic latitude and γ the arrival direction of the particle (east–west).

The condition for a *triplely adiabatic* motion, i.e., the condition for a given ion to perform the motion like described previously, within the picture of the existence of a guiding center, is that the magnetic field intensity must vary very slowly around a cyclotron orbit, imposing a *maximum* energy allowed for stable trapping. The condition that must be imposed for the particle's cyclotron radius at the geomagnetic equator is given by

$$R_C|_{\text{equator}} = \frac{p_{\perp}}{qB} \ll \left. \frac{B}{|\nabla_{\perp} B|} \right|_{\text{equator}}, \quad (7)$$

where q is the effective charge of the particle, B is the magnetic field at a given point, and p_{\perp} and $\nabla_{\perp} B$ are the momentum of the particle and magnetic field variation, respectively, both projected in a direction perpendicular to the field lines at a given point in the magnetosphere.

Figures 4 and 5 show the bounds given by Eqs. (6) and (7) for normal and CFL strangelets, respectively, for $L = 2$ in addition to the minimum baryon number which is required for strangelet stability [7]. The existence of a minimum baryon number is expected in all models of SQM because the energy needed for producing the system increases as the baryon number decreases, until it reaches a value above which the strange matter is unstable. The value adopted has been $A_{\text{min}} = 30$ (shown with a vertical line) and may be trivially altered for any other figure. Strangelets with very high baryon number, though allowed for stable trapping, are not likely to be statistically significant for detection in the magnetosphere due to a substantial decrease of the interstellar flux expected as the baryon number increases.

The upper bound (7) has been enforced in our calculations to a 10% confidence level according to observations of anomalous cosmic rays (ACRs) L -shell distributions [70], and we considered $E_{\perp} \sim E$, which means we are actually *underestimating* the number of particles that could be stably trapped in the geomagnetic field. Obviously, the geomagnetic cutoff curve must be below the adiabaticity criteria for stable trapping to occur. This is not the case for

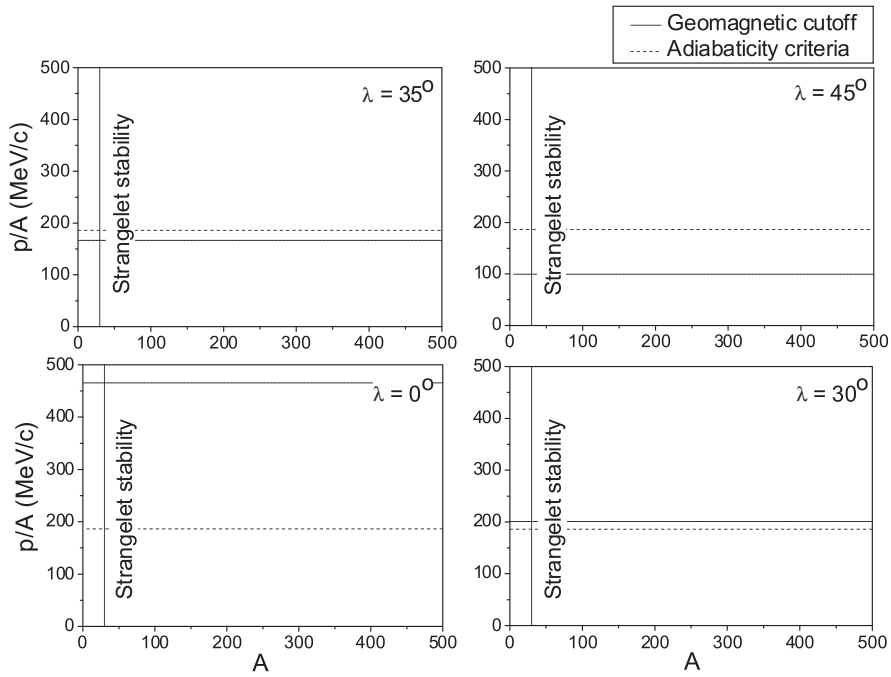


FIG. 4. Restriction curves (6) and (7) for strangelets at $L = 2$ in the baryon number versus momentum plane for normal strangelets coming from the east ($\gamma = \pi$) for different incident geomagnetic latitudes as noted in the upper right side of each plot (reminding the reader that field lines at $L = 2$ penetrate the Earth’s surface at $\lambda = 45^\circ$ in the dipole model).

small geomagnetic latitudes, but there is a narrow “window” in latitude starting slightly above 30° at $L = 2$ for strangelets from the ISM flux to fulfill the conditions of capture and accumulate in regions labeled by the L parameter. However, the number of accumulated particles is still interesting, as shown in the next section.

When this calculation is repeated for the case of CFL strangelets, the region allowed for stable trapping for CFL strangelets has a different shape than that for normal strangelets. This feature is due to the strong dependence of the charge upon A of normal strangelets ($Z \propto A$) resulting in constant values if one considers momentum

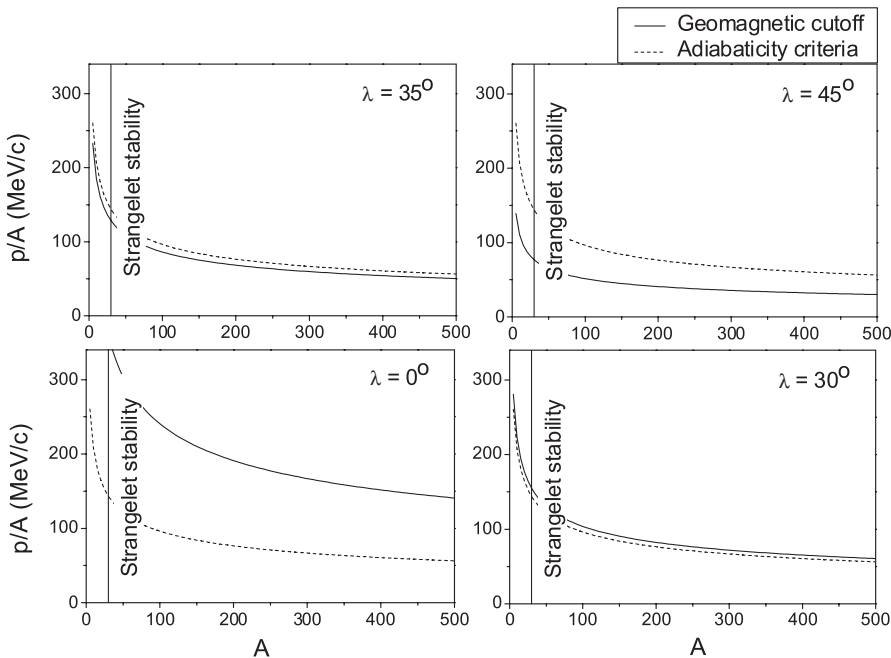


FIG. 5. The same as Fig. 4 for CFL strangelets.

per baryon number, whereas for the CFL strangelets charge is almost independent of A , leading to an $\sim A^{-0.9}$ dependence in the momentum per baryon number variable.

Trapped strangelet population

Even though strangelets can be captured and trapped in the Earth's magnetic field, we must evaluate the possible maintenance of a strangelet population to check whether there is an increase in the flux compared to that expected in the ISM. For this purpose, one must consider forces acting on the trapped particles which can cause a decrease in their population; i.e., loss mechanisms must be taken into account.

In addition to the already analyzed losses by collisions with molecules in the atmosphere, we have considered the *inward drift* driven by asymmetric fluctuations of the geomagnetic field as a dominant mechanism to diminish the trapped strangelet population.

We will not consider in this work direct pitch angle diffusion. Because of their large mass, strangelets are less likely to be scattered appreciably in pitch angle by collisions. The net result of multiple collisions with atmospheric particles would be a reduction in the strangelets' kinetic energy to thermal values and minor changes in their pitch angle. Since we have already considered that particles bouncing at a radial distance from the surface of the Earth below the atmosphere height scale (derived in Sec. II) would be eventually removed, we are in fact replacing a diffusion equation in the $\cos(\alpha_{\text{eq}})$ variable for a constant loss term (a sink function) directly related, though not formally assigned, to pitch angle diffusion.

Radial diffusion must proceed by fluctuations in the third invariant ϕ , which is proportional to L^{-1} , due to changes in the electric or magnetic fields that are more rapid than the particle drift frequency. Because the gyration and bounce periods are much shorter than the drift period, the first and second adiabatic invariants are less likely to be affected by many of these field perturbations.

Guided by the existing calculations and observations for ACR nuclei trapping, we have considered third invariant diffusion due asymmetric fluctuations in the geomagnetic field, which is mainly driven by the solar wind pressure (sudden compression and slow relaxation of the geomagnetic field).

The diffusion coefficient D_{LL} is determined theoretically by taking two consecutive steps [71]. First, one has to evaluate the radial displacement suffered by a particle under the influence of the field disturbance, which is an idealized model of the real disturbances occurring in the geomagnetic field. The following procedure is taken in order to obtain the diffusion coefficient as a function of the statistical features of the disturbances alone. It consists of squaring this displacement and taking the average over several disturbances randomly occurring in time and over all possible particle's initial longitudes.

The diffusion coefficient due to magnetic field fluctuations for equatorially trapped particles, with the assumption of efficient phase mixing [71], can be expressed as

$$D_{LL}^M = \frac{\pi^2}{2} \left(\frac{5}{7}\right)^2 \frac{R_E^2 L^{10}}{B_0^2} \nu_{\text{drift}}^2 P_A(\nu_{\text{drift}}), \quad (8)$$

where $P_A(\nu)$ is the power spectral density of the field variation evaluated at the drift frequency. For off-equatorial particles, the diffusion coefficient presents an exponential decay with latitude.

Already in the case of nuclei, it is known that the complex geometry and inhomogeneities in the geomagnetic field make quantitative calculations ambiguous. The observed values of the diffusion coefficient and their L dependence will change with global magnetic activity, and magnetic disturbances are known to vary appreciably with time. We have assumed a ν^{-2} dependence of the power spectral density for simplicity [71]. The loss of more detailed information associated with this approximation is that the diffusion coefficient becomes independent of the energy of the particle entering the geomagnetic field. In this case, the diffusion coefficients have a strong dependence on the so-called *McIlwain parameter* ($D_{LL} \propto L^{10}$) [72]. This indicates that its influence is most important for particles trapped at higher L shells.

Typical values for changes in the trapped population distribution range from a few hours at $L = 6$ to hundreds of days at $L = 2$. Therefore, if strangelets are captured by the geomagnetic field their density must be higher for lower values of the L parameter, which may result in a substantial increase of this population compared to the ISM flux.

Some other loss mechanisms are of less importance in short time scales, but have influence on long time scales, thus lowering the residence time for trapped particles. This includes electrical drift-resonant interactions between particles and fields, especially in the pulsation frequency or VLF range [73]. Those phenomena are highly affected by the solar wind activity.

The diffusion equation has been employed to study the trapped strangelet flux

$$\frac{\partial f(\mu, J, L)}{\partial t} = \frac{\partial}{\partial L} \left[\frac{D_{LL}}{L^2} \frac{\partial}{\partial L} (L^2 f(\mu, J, L)) \right], \quad (9)$$

where f is the distribution function, D_{LL} is given by Eq. (8), and μ and J are the adiabatic invariant magnetic moment and integral invariant, respectively. The relation between the distribution function and the flux may be given by $j(E, \alpha) = p^2 L^2 f(\mu, J, L)$. A stationary population requires $\partial f / \partial t = 0$; i.e., the assumption that the source and loss terms are instantaneously balanced is valid.

We assume a steady strangelet injection from the interstellar medium at $L = 6$ (the position of the maximum distribution function is very insensitive to the chosen L -shell parameter for this boundary condition) and derive

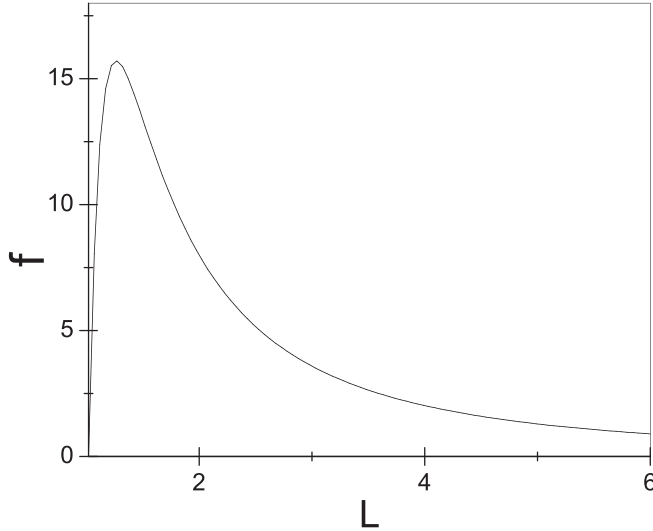


FIG. 6. The distribution function (in arbitrary units) for strangelets trapped in the geomagnetic field as a function of L is obtained as the solution for the differential equation (9) with the boundary conditions $f(L_{\max})$ given by the incoming flux (see text for details) and $f(L_{\min}) = 0$ (corresponding to the height scale of interaction with atmospheric particles). The position of the peak (around $L = 1.3$) does not change with the change in the A/Z relation (CFL and non-CFL strangelets) nor with a change in the energy and A of the strangelets.

the distribution function shape between this maximum and $L \approx 1.05$ where it is null (atmosphere particle interaction height), shown in Fig. 6. We are *not* considering diffusion in pitch angle due to interaction of particles with electromagnetic waves caused by field variations, which alters the first adiabatic invariant.

The calculations were performed adopting two values of the flux from the ISM reaching the outer magnetosphere.

The first one, which will be called “standard,” is the one that assumes the standard cosmic ray dependence on the strangelet flux, $E^{-2.5}$. The total ISM strangelet flux that reaches the Earth as estimated by Madsen [53] for a binary strange star system coalescence scenario is given by

$$F \approx 2 \times 10^5 \text{ m}^{-2} \text{ yr}^{-1} \text{ sr}^{-1} A^{-0.467} Z^{-1.2} \times \max[R_{\text{SM}}, R_{\text{GC}}]^{-1.2} \Lambda, \quad (10)$$

where R_{SM} and R_{GC} are the solar modulation and geomagnetic cutoff rigidities, respectively, and Λ is an uncertain parameter assumed to be of $O(1)$. In this way, the whole flux is fitted with a $E^{-2.5}$ dependence with the constraints of minimum and maximum energy respecting the values $R_{\min} = 5 \text{ MV } A/Z$ and $R_{\max} = 10^6 \text{ GV}$ [53].

The second calculation, which will be called “improved,” considers a more detailed characterization of the differential flux, where for the region of interest in this work (rigidities of few GV), the strangelet flux actually *increases* with a slope of $R^{1.8}$. This flux was obtained from a fit to Ref. [53].

In either way, the flux entering the region of the magnetosphere at L_{\max} has to fulfill the restrictions imposed for stable trapping in the pitch angle and geomagnetic latitude of incidence

$$F_{\text{in}} = \int_{\lambda_{\min}}^{\lambda_{\max}} d\lambda P(\lambda) \int_{\alpha_{\text{loss cone}}}^{\pi/2} d\alpha_{\text{eq}} P(\alpha_{\text{eq}}) \times F. \quad (11)$$

The efficiency factors, $P(\lambda)$ and $P(\alpha_{\text{eq}})$, may be easily identified: $P(\lambda)$ gives the fractional area of the spherical section suitable for trapping discussed previously,

$$P(\lambda) = \frac{2L^2(-\cos\lambda) \int_0^{2\pi} d\phi}{2L^2 \int_0^{\pi/2} \cos\theta \int_0^{2\pi} d\phi},$$

where the factor 2 comes from the symmetry in θ for both hemispheres (north, south). $P(\alpha_{\text{eq}})$ limits the number of particles entering the specific region of the magnetosphere with an appropriate pitch angle to avoid the loss cone as already discussed. We have also assumed an isotropic incoming flux of particles, since there is no compelling reason pointing to any anisotropy in the arrival direction of strangelets, which means that $j_0(\cos\alpha_{\text{eq}}) = \text{const}$ is a reasonable hypothesis:

$$P(\alpha_{\text{eq}}) = 4 \frac{\alpha_{\text{eq}}}{\int_0^{\pi/2} \alpha_{\text{eq}} d\alpha_{\text{eq}}},$$

where the factor 4 stands for the symmetry in the condition for a given particle to belong to the loss cone: $|\alpha_{\text{eq}}| < \alpha_{\text{loss cone}}$ and $|\pi - \alpha_{\text{eq}}| < \alpha_{\text{loss cone}}$.

Solving the differential equation (9) and obtaining the corresponding flux inward (in the $-\hat{z}_r$ direction) for every L , it is possible to determine the mean particle density at a given shell and, in this way, the trapped strangelet flux.

The results are summarized in Tables I and II for $L = 2$ (ACR belt location) and $L = 1.3$ (location of the maximum of the distribution function) for the example of strangelets of $A = 100$ and energy corresponding to $R = 1 \text{ GV}$.

The position of the peak of the distribution function in the geomagnetic field (around $L = 1.3$) is quite robust, it does not appreciably change with the change in the A/Z relation (CFL and normal strangelets), nor with a change in the energy and baryon number of the strangelets. This could be a consequence of the assumption of the power spectral density as being proportional to ν^{-2} , which renders the diffusion coefficient independent of the particle energy. Therefore modifying the energy of the particles does not affect their diffusion properties.

We observe that the trapped population is slightly more favored if strange quark matter is in the CFL state, the

TABLE I. Mean particle flux in units of $\text{part cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} (\text{MeV}/A)^{-1}$ for strangelet stationary population at $L = 1.3$ and $L = 2$ calculated with the standard flux.

	$L = 1.3$	$L = 2$
Normal	1.28×10^{-15}	4.34×10^{-17}
CFL	3.95×10^{-14}	1.34×10^{-15}

TABLE II. Mean particle flux in units of $\text{part cm}^{-2} \text{s}^{-1} \text{sr}^{-1} (\text{MeV}/A)^{-1}$ for strangelet stationary population at $L = 1.3$ and $L = 2$ calculated with the improved flux.

	$L = 1.3$	$L = 2$
Normal	3.83×10^{-14}	1.3×10^{-15}
CFL	1.64×10^{-13}	5.55×10^{-15}

difference between the trapped fluxes for the two species increases with decreasing energy exponent in the incident flux. It happens due to the dependence on the baryonic number of the interstellar flux of strangelets [Eq. (10)]. Since the rigidity interval for stable trapping is the same for both states for it only depends on the geometrical characteristics of the geomagnetic field, the difference on the number of particles trapped strongly depends on the difference in the incoming flux. This dependence of the integrated flux on the number of baryons can be expressed as $F_{\text{ISM}} \propto (0.125)^{-1.2} A^{-1.667}$ and $F_{\text{ISM}} \propto (0.3)^{-1.2} A^{-1.267}$ for normal and CFL strangelets, respectively. In this way the flux of paired CFL strangelets is lower than those without pairing, but only for low baryon number ($A < \sim 13$), that is, in a region where it is believed strangelets are not stable at all. In the stability region the flux for CFL strangelets is always higher than for normal strangelets resulting in a higher trapped density. In this way, the smaller difference seen between strangelets with and without pairing for the improved flux when comparing to that for the standard flux is explained by the smaller difference in the incoming flux due to its softer dependence on the atomic number (the standard flux depends on $E^{-2.5}$, which means that for an analysis made in terms of rigidity, it will depend on the baryonic number, since $R = pc/Ze \propto \sqrt{E}/Z$ and Z is a function of A ; instead, the improved flux was obtained with a direct dependence on the rigidity, $R^{1.8}$).

Additional considerations are relevant for the fate of a trapped population of strangelets. It is well known that the solar wind has a strong influence on the ACR flux upon the Earth. The most abundant ACR heavy ion, oxygen, shows a strong intensity variation with the solar cycle, having its interstellar flux of 8–27 MeV/nucleon lowered up to 2 orders of magnitude during periods of solar maximum activity [74]. During solar minimum, the trapped flux at the Earth’s magnetosphere is of the order of $\sim 5 \times 10^{-4} \text{ particles cm}^{-2} \text{sr}^{-1} \text{s}^{-1} (\text{MeV/nucleon})^{-1}$, corresponding to an enhancement factor of ~ 15 [75], this experimental value being somewhat below the theoretically expected one (higher than 25 [74]). The oxygen component corresponds to about 80% of the trapped ACR, while the C/O , N/O , and Ne/O abundance ratios are < 0.005 , ~ 0.10 – 0.15 , and ~ 0.02 – 0.03 , respectively.

With the results obtained in this study, the trapped flux of strangelets at $L < 2$ would be of order 10^{-14} – $10^{-15} \text{ particles cm}^{-2} \text{sr}^{-1} \text{s}^{-1} (\text{MeV}/A)^{-1}$ at rigidity $R = 1 \text{ GV}$

for strangelets of baryon number $A = 100$. This represents an enhancement factor for trapped flux in the regime of steady-state population compared to the interstellar flux at the same energy and A of order 10 and 10^2 for strangelets trapped at $L = 2$ and $L = 1.3$, respectively, the values for CFL strangelets being about twice the one for normal strangelets ($q \sim 5.5$ and 11 and $q \sim 162$ and 314 for CFL and normal strangelets at $L = 2$ and $L = 1.3$, respectively). These results show that the strangelet flux could be as high as a factor of 10 000 lower than that expected for carbon during periods of maximum solar activity. Although we did not consider the solar modulation in our calculations, it would act significantly over those low-energy strangelets [53], the region of interest in this study. In this manner, it could have an important influence, similar to that detected for oxygen, on the trapped density.

The advantage of a search for trapped strangelets in the geomagnetic field performed during the solar maximum activity whether they are an important component of the radiation belt or are to be measured penetrating the atmosphere towards to surface of the Earth would be the reduced component of ACR, which could reduce dead time losses in the detectors and possibly render a clearer identification of the primaries.

The proposed and widely accepted model for ACR trapping [76] assumes that the high mass-to-charge ratio of singly ionized ACRs enables them to penetrate deeply into the magnetosphere. ACRs with trajectories near a low altitude mirror point interact with particles in the upper atmosphere, losing one or all of their remaining electrons. After stripping, the particle gyroradius is reduced by a factor of $1/Z$, and the ion can become stably trapped. As stated above, the results presented here were obtained assuming fully ionized strangelets, which have just the “right” features to become trapped. However, some fraction of the strangelets should reach the Earth’s atmosphere with an effective charge slightly below their atomic number and suffer a process of interaction similar to ACR’s. Finally, there is also the possibility of quasistable trapping of ions with energies high enough not to obey condition (7), but not too high as to penetrate the magnetosphere without suffering any significant depletion in their incident direction. These two additional mechanisms could result in a further *increase* in the number of trapped strangelets.

When considering $\epsilon_0 = 930 \text{ MeV}$ (see Sec. II), we are overestimating the value of the strangelet’s mass (for the chosen set of parameters) by ignoring the effects of surface and curvature energies and also the shell effects [7,77]. Depending on parameter choice, this asymptotic value can be as low as 850 MeV for normal strangelets and can be even lower for paired SQM [14], depending also on the value adopted for the gap Δ . Since the value usually taken for considering limits of stability for SQM is that the mass should be below 930 MeV, and the interest here is for studying the stable ones (there is no sense in doing this

analysis if SQM is not stable at all), the overall effect of our assumption is the overestimation of the mass of the strangelet. Having a higher mass, the inertia of the calculated strangelets is higher, and the bending the geomagnetic field would cause in the particle's path is smaller, resulting in a bigger gyroradius around the guiding center. It means that the condition of adiabatic motion to hold (that the magnetic field varies vary slowly along a cyclotron orbit) is harder to fulfill in our calculations than if ϵ_0 happens to be smaller. The qualitative effect of this is an *underestimation* of the true number of trapped strangelets, which is on the conservative side.

Also the choices made for the values of the strange quark mass, bag constant, and coupling gap (for CFL matter) would present an influence for the calculations of trapped population. The “windows” for stable trapping shown in Figs. 4 and 5 display the dependence on the atomic number alone. It is so because the other parameters characterizing these particles (B , m_s , Δ) are fixed and the plots are of p/A versus A for specific λ and L . The smaller the Z , the less bending the particle will suffer due to the action of the geomagnetic field. If the atomic number is much lower, the effect is the same as the one explained for the effects of a very high mass: the particle is more likely to be trapped in an unstable orbit with a small time of residence, when compared to that of a particle fulfilling the adiabaticity criteria. In Refs. [14,77], we notice that changing the values of m_s causes the strangelet's charge to change in the region of interest for baryonic number in the following way: when $m_s \rightarrow 0$, the strangelet charge is much lower and our calculations would be overestimated; for $m_s \rightarrow 300$ MeV, the charge reaches a saturation value above the value adopted here (though the work shown in [77] does not take into account the screening effect which could cause the values to change considerably, whereas the value adopted in the calculations for normal strangelets, taken from Ref. [58] considers this effect) and it would result in *more* trapped particles; nevertheless, the value of the strange quark mass is more likely to be in the range $100 \leq m_s \leq 200$ MeV, so the values presented here would be less influenced by this parameter. Also, the values taken of the bag constant and of the gap coupling of CFL SQM appear to be of minor influence in the determination of the strangelet's electric charge. There is room for improving these calculations, i.e., include the modification of the charge produced by screening, improve the calculation of other strangelet features (and within other physical models), etc. The results certainly encourage further exploration of these and other related questions.

IV. CONCLUSIONS

From the analysis presented here we conclude that non-relativistic strangelets with $A < \sim 10^3$ already ionized by collisions with electrons in the ISM could be stably trapped by the geomagnetic field. Assuming the existence of a strangelet contamination in the ISM, its injection in the solar system and given the geomagnetic geometry and the interaction of the magnetic field with the solar wind, it looks quite likely that our planet hosts a strangelet radiation belt. If strangelets are indeed a component of the anomalous cosmic ray belt at $L \sim 2$, we have shown that, even considering the approximations made during the calculations presented here (which mainly affect the averaging of the trapped population behavior), those particles would be present with an enhancement factor comparing with the interstellar flux of the order of 10, and considering the location of the maximum (a *strangelet belt*) at $L \sim 1.3$, the enhancement factor could be as high as $\sim 10^2$ for a stationary population scenario [78]. These exotic baryons could in principle be detectable in the Earth's magnetosphere depending on the chosen parameters for each of the experiments (effective detection area, altitude and type of orbiting, magnetic field for particle depletion, and others). In addition to the already mentioned capture of almost fully ionized strangelets, additional trajectories leading to trapping (but not obeying the adiabatic conditions) may exist, although they must be calculated numerically, and could enhance even further the trapped population, though most probably not affecting substantially the results. Effects that could result in the reduction of the trapped population are the diffusion driven by electric field fluctuations and phenomena directly related to enhanced solar activity, which, though less likely to affect the particles already trapped at low L shells, could have an influence on the particle injection in the outer magnetosphere.

Overall, we believe our estimates to be on the conservative side of the trapped flux, making the search of trapped strangelets a feasible but difficult task. Needless to say, the detection of those trapped particles having low Z/A ratio would be extremely important for determining the properties of cold, dense baryonic matter.

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