Inflation-produced magnetic fields in nonlinear electrodynamics

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We study the generation of primeval magnetic fields during inflation era in nonlinear theories of electrodynamics. Although the intensity of the produced fields strongly depends on characteristics of inflation and on the form of electromagnetic Lagrangian, our results do not exclude the possibility that these fields could be astrophysically interesting.

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I. INTRODUCTION

All galaxies seem to be permeated by magnetic fields with intensities of order $B_{galactic} \simeq 10^{-6}$ G [1].

To explain the galactic magnetism, generally one needs the presence of seed magnetic fields prior to protogalaxy collapse. When a protogalaxy collapses to form a galactic disk, magnetic fields suffer an amplification (mainly due to magnetic flux conservation) of order $\mathcal{A}_{pg} \simeq 10^4$ [2]. Moreover, due to magnetohydrodynamic turbulence effects and differential rotation of galaxy, seed fields can be further amplified. This last mechanism, know as "galactic dynamo" [3], can be very efficient and, in principle, allow extremely weak seeds to reproduce the properties of presently-observed galactic fields. Dynamo action produces an exponential amplification $\mathcal{A}_{dyn} \simeq e^{\Gamma \Delta t}$, where the growth rate Γ is a model-dependent quantity, and $\Delta t = t_f - t_i$ is the time during which the dynamo operates. The minimum and maximum values of Γ that can be found in the literature are $\Gamma \simeq 0.45$ Gyr^{-1} [4] and $\Gamma \simeq 5$ Gyr^{-1} [5]. In a spatially-flat Friedman universe, the time interval is [6]
$$\Delta t = (2/3H_0 \Omega_\Lambda^{1/2}) \ln[(\omega_f + \sqrt{1 + \omega_f^2})/(\omega_i + \sqrt{1 + \omega_i^2})].$$
Here, $H_0 = 100h$ km sec^{-1} Mpc^{-1} is the Hubble constant ($h \simeq 0.73$ [7]), $\omega_{i,f} = (1 + z_{i,f})^{-3/2}(\Omega_\Lambda / \Omega_m)^{1/2}$, z is the redshift, and $\Omega_m \simeq 0.28$ and $\Omega_\Lambda \simeq 0.72$ [7] are the actual energy densities, in units of the critical density $\rho_{cr} = 3H_0^2/8\pi G$, associated to matter and cosmological constant. Un upper bound on the value of redshift at which the dynamo begins to operate, z_i, is given by the redshift $z_{pg} \simeq 50$ at which a protogalaxy separates from the Hubble flow to then collapse [6]. If dynamo is efficient during galactic disk formation or if its amplification becomes effective only after that, is still not clear. In the latter case, since astronomical observations indicate that disk

galaxies at $z \simeq 3$ are still in progress of being formed [8], one should take this value of redshift as a conservative bound on z_i. A lower bound on z_f is $z_f \simeq 0.4$, since microgauss magnetic fields have been detected in galaxies at that redshift [9].

The galactic magnetism can be then explained as the result of the amplification of comoving seed fields as strong as $B_{seed} \gtrsim 10^{-6}(1 + z_{pg})^{-2} \mathcal{A}_{pg}^{-1} \mathcal{A}_{dyn}^{-1}$G, where the factor $(1 + z_{pg})^{-2}$ takes into account the adiabatic scaling of the magnetic field from the protogalaxy collapse until today. Taking $(\Gamma, z_i, z_f) = (5, 50, 0.4)$ we have $B_{seed} \gtrsim 10^{-33}$ G, while for $(\Gamma, z_i, z_f) = (0.45, 3, 0.4)$ we get $B_{seed} \gtrsim 10^{-15}$ G. In order to have an efficient galactic dynamo, however, the seed magnetic field must be correlated on comoving scales of order 10 kpc. We observe, also, that without dynamo amplification a comoving seed field as strong as $B_{seed} \gtrsim 10^{-14}$ G is needed to explain galactic magnetism. In this case, however, the field must be correlated on comoving scales of order of linear dimensions of a protogalaxy, that is 1 Mpc.

Surprisingly, there are compelling indications of existence of large-scale, microgauss magnetic fields in galaxy clusters [6,10,11]. If confirmed, this would indicate that the entire universe is magnetized.

Essentially, there are two possible classes of mechanisms to produce cosmic fields depending on when they are generated [12]: Astrophysical mechanisms acting during large-scale structure formation [13], and mechanisms acting in the primordial universe, during [14–21] or before [22] inflation. However, we can admit the existence of strong fields in the primordial universe provided that their presence does not spoil predictions of the standard cosmological model, such as that of big bang nucleosynthesis (BBN) [23], large-scale-structure formation (LSS) [24], and cosmic microwave background (CMB) [25]. It turns out that limits coming from LSS and CMB are more stringent than those from BBN. Putting together the limits found in Refs. [24,25] it results that, for comoving scales in the range 400 pc $\lesssim \lambda \lesssim d_H(t_0)$, where $d_H(t_0) \sim H_0^{-1} \sim$ 4000 Mpc is the present size of the universe, the maximum

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strength allowed to comoving primordial fields is $B \sim 10^{-9}$ G.

In the ambit of generation of cosmological fields in the early universe, the mechanisms operating during inflation are particularly attractive since they produce large-scale correlated fields. Magnetic fields created after inflation, instead, suffer from a "small-scale problem," that is their comoving correlation length is much smaller than the characteristic scale of the observed cosmic fields [however, if magnetohydrodynamic turbulence operates during their evolution, an enhancement of correlation length can occur (see, e.g., Ref. [26])].

It is worth noting that, due to conformal invariance of standard (Maxwell) electrodynamics and to the fact the spacetime described by the Robertson-Walker metric is conformally flat, magnetic fields generated at inflation are vanishingly small. For this reason, all generating models proposed in the literature repose on the breaking of conformal invariance of Maxwell theory. This has been attained, for instance, by nonminimally coupling the photon with gravity [14], introducing interactions of photons with scalar, pseudoscalar, or vector fields (such as inflaton [15], dilaton [16], pseudo-Goldstone bosons [17], axion [18], or "graviphoton" [19]), taking into account the so-called quantum conformal anomaly [20], and so on [21].

In this paper, we study the possibility to generate seed magnetic fields during inflation in nonlinear theories of electrodynamics (NLE) described by the general action

$$S = \frac{1}{4\pi} \int d^4x \sqrt{-g} \mathcal{L}(F), \qquad (1)$$

where $F = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the electromagnetic field strength tensor, and $g = \det\|g_{\mu\nu}\|$ is the determinant of the metric tensor.

Since the standard Maxwell theory is a good theory for low energies, we shall assume that nonlinear Lagrangians reduce to the Maxwell one, $\mathcal{L}(F) \simeq -F$, in the limit of small fields F.

A considerable amount of interest has emerged in the last few years in cosmological effects of nonlinear electrodynamics [27,28]. This is due principally to the fact that some theories of NLE are able to produce inflation [28,29], a period of cosmic acceleration [28,30], and can also avoid the problem of initial singularity [28,31,32].

A natural candidate for NLE theories could be the Euler-Heisenberg nonlinear effective Lagrangian induced by charged scalar fields minimally coupled to the electromagnetic field.

In general, nonlinear electrodynamic theories are non-conformally invariant. As we shall see in the next section, depending on the actual form of the Lagrangian, astrophysically interesting magnetic fields can be generated during inflation.

II. GENERATION OF SEED FIELDS IN NLE

A. Equations of motion

We will work in a flat universe described by a Robertson-Walker metric, $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$, where $a(\eta)$ is the expansion parameter and η is the conformal time related to the cosmic time t through $d\eta = dt/a$. Introducing the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ in the usual way as $F_{0i} = -a^2 E_i$, $F_{ij} = \epsilon_{ijk} a^2 B_k$ (Latin indices range from 1 to 3), and varying the action (1) with respect to A_μ, we get the equations of motion:

$$\frac{\partial(a^2\mathbf{E})}{\partial\eta} - \nabla \times (a^2\mathbf{B}) = -\frac{\partial \ln \mathcal{L}_F}{\partial\eta} a^2\mathbf{E} + \nabla \ln \mathcal{L}_F$$
$$\times a^2\mathbf{B}, \qquad (2)$$

and $\nabla \cdot (\mathcal{L}_F \mathbf{E}) = 0$, together with the Bianchi identities: $\partial_\eta(a^2\mathbf{B}) + \nabla \times (a^2\mathbf{E}) = 0$ and $\nabla \cdot \mathbf{B} = 0$. Here, subscript on $\mathcal{L}$ denotes differentiation, and spatial derivatives are taken with respect to comoving coordinates.

We are interested to the evolution of electromagnetic fields outside the horizon, that is to modes whose physical wavelength is much greater then the Hubble radius d_H, $\lambda_{\text{phys}} \gg d_H$, where $\lambda_{\text{phys}} = a\lambda$, $d_H \sim H^{-1}$, and λ is the comoving wavelength. Since $\eta a \sim H^{-1}$, introducing the comoving wave number $k = 2\pi/\lambda$, the above condition reads $|k\eta| \ll 1$. Observing that the first Bianchi identity gives $\mathbf{B}(k\eta) \sim k\eta\mathbf{E}(k\eta)$, we have that $\mathbf{B}^2$ is negligible with respect to $\mathbf{E}^2$, and we can write $\partial_\eta \ln \mathcal{L}_F \simeq -(\partial_\eta \mathbf{E}^2/2)(d\ln \mathcal{L}_F/dF)$. Moreover, we can neglect the second term with respect to the first one both in the left- and right-hand side of Eq. (2). In fact, it results $|\nabla \times a^2\mathbf{B}|/|\partial_\eta(a^2\mathbf{E})| \sim |k\eta|^2 \ll 1$, and $|\nabla \ln \mathcal{L}_F \times a^2\mathbf{B}|/|(\partial_\eta \ln \mathcal{L}_F)a^2\mathbf{E}| \sim |k\eta|^2 \ll 1$. Finally, multiplying Eq. (2) by $\mathbf{E}$, and solving with respect to $F \simeq -\frac{1}{2}\mathbf{E}^2$, we get

$$(\mathcal{L}_F)^2 F \propto a^{-4}. \qquad (3)$$

Knowing the form of nonlinear Lagrangian (see below), from the above equation one gets the evolution law for the electric field outside the horizon. Consequently, using the first Bianchi identity, one finds how superhorizon magnetic fields scale in time during inflation (see subsection II D).

B. Initial electromagnetic spectrum

During inflation, all fields are quantum mechanically exited. Because the wavelength λ associated to a given fluctuation grows faster then the horizon, there will be a time, say t_1, when this mode crosses outside the horizon itself. After that, this fluctuation cannot collapse back into the vacuum being not causally self-correlated, and then "survives" as a classical real object [33]. The energy associated to a given fluctuation is subjected to the uncertainty relation, $\Delta E \Delta t \gtrsim 1$. Therefore, the energy density in the volume ΔV, $\mathcal{E} = \Delta E/\Delta V$, is approximately given by

$\mathcal{E} \sim H^4$, where H is the Hubble parameter. Here, we used the fact that at the horizon crossing $\Delta t \sim H^{-1}$ and $\Delta V \sim H^{-3}$ [33]. When a comoving length λ crosses the horizon it results $|k\eta| \simeq 1$, and then from the first Bianchi identity we get $\mathbf{B}^2(\lambda) \simeq \mathbf{E}^2(\lambda)$. Therefore, since $F = -\frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)$, at the crossing the nonlinear terms in the electromagnetic Lagrangian are negligible, and the energy density is simply given by $\mathcal{E} \simeq (1/8\pi)(\mathbf{E}^2 + \mathbf{B}^2)$. Consequently, the spectra of the quantum mechanically generated electric and magnetic fluctuations are given by

$$\frac{\mathbf{B}^2(\lambda)}{4\pi}\bigg|_{t_1} \sim \frac{\mathbf{E}^2(\lambda)}{4\pi}\bigg|_{t_1} \sim H^4\big|_{t_1} = \left(\frac{8\pi}{3}\right)^2 \frac{\rho_{\rm tot}(\lambda)}{m_{\rm Pl}^4}\bigg|_{t_1}, \quad (4)$$

where in the last equality we used the Friedman equation $H^2 = (8\pi/3)\rho_{\rm tot}/m_{\rm Pl}^2$. Here, $\rho_{\rm tot}$ is the total energy density during inflation and $m_{\rm Pl} \simeq 10^{19}$ GeV the Planck mass.

In the following we shall consider both the case of de Sitter inflation and the case of "power-law inflation". It is useful to write the expansion parameter as $a(\eta) \propto \eta^s$. During de Sitter inflation $s = -1$ and the total energy is a constant,[1] $\rho_{\rm tot} \equiv M^4$. For power-law inflation described by the equation of state $p_{\rm tot} = \gamma\rho_{\rm tot}$ with $-1 < \gamma < -1/3$, we have $s = 2/(1 + 3\gamma)$ and the total energy decreases as a power of the expansion parameter, $\rho_{\rm tot} \propto a^{-3(1+\gamma)}$. Therefore, the total energy, when a comoving length λ crosses outside the horizon, depends on λ [see Eq. (13)].

C. Plasma effects

The effects of a conducting plasma in the early universe on the evolution of magnetic fields are taken into account by adding to the electromagnetic Lagrangian the source term $J^\mu A_\mu$ [14]. Here, the external current J^μ, expressed in terms of the electric field, has the form $J^\mu = (0, \sigma_c\mathbf{E})$, where σ_c is the conductivity. Plasma effects introduce, in the left-hand side of Eq. (2), the extra term $-a\sigma_c(a^2\mathbf{E})$. In the limit of high conductivity, $\sigma_c \to \infty$, one finds $\mathbf{E} \to 0$ and, consequently, from the first Bianchi identity, it follows that the magnetic field is frozen into the plasma and evolves adiabatically, $a^2\mathbf{B} \sim$ const. More precisely, the solution of Eq. (2), for modes outside the horizon and when plasma effects are taken into account, is $(\mathcal{L}_F)^2F \propto a^{-4}\exp(-2\int d\eta a\sigma_c)$. Observing that $\int d\eta a\sigma_c \sim \sigma_c/H$, we get that the magnetic field can be considered as frozen into the plasma when $\sigma_c \gg H$. After inflation, the universe enters in the so-called reheating phase, during which the energy of the inflaton is converted into ordinary matter. In this paper, we shall restrict our analysis to the case of instantaneous reheating, that is after inflation the universe enters the radiation dominated era. In this era, the con-

ductivity is approximatively equal to $\sigma_c \sim T$ [14], while $H \sim T^2/m_{\rm Pl}$ [35], where T is the temperature. Hence, the condition of freezing of the magnetic field becomes $T \ll m_{\rm Pl}$.

The spectrum of gravitational waves generated at inflation is submitted to constraints coming from CMB analysis which requires $\rho_{\rm tot}(\lambda)$ to be less than about $10^{-8}m_{\rm Pl}^4$ on the scale of the present Hubble radius [14]. This, in turns, converts in a upper limit on the value of M, $M \lesssim 10^{-2}m_{\rm Pl}$. (One must impose also that $M \gtrsim 1$ GeV, so that the predictions of BBN are not spoiled [14].) Since the temperature at the end of inflation is $T_{\rm end} = M$, we conclude that after inflation the universe is a good conductor, $\sigma_c \gg H$, and the magnetic field evolves adiabatically, irrespective of when it (eventually) reenters the horizon.

D. Form of NLE Lagrangian

In this paragraph, we consider three models of a nonlinear electromagnetic Lagrangian. In all cases, the Lagrangian depends on a free mass parameter, m, such that in the formal limit $m \to \infty$ we recover the standard Maxwell theory. More precisely, it results $\mathcal{L}(F) \simeq -F$ for $|F| \ll m^4$. In the case of small fields, $|F| \lesssim m^4$, inflation-produced fields are vanishingly small, and then cannot explain the presently-observed fields. For this reason, we shall restrict our analysis to the case of strong fields, $|F| \gtrsim m$.

As a first model, we consider the family of Lagrangians

$$\mathcal{L}(F) = -F + \sum_{i=2}^{n} c_i F^i, \quad (5)$$

where i takes values on the integers, and the coefficients c_j have dimension $[{\rm Mass}]^{4(1-j)}$. We assume that $c_j = m^{4(1-j)}d_j$, where d_j are dimensionless constants of order unity. In a cosmological context, this type of Lagrangian for $n = 2$ has been widely studied in the literature (see, e.g., Refs. [27,28]). In Ref. [36], it has been shown that the Lagrangian[2]

$$\mathcal{L}^{\rm KK}(F) = -F + \Upsilon(b-1)F^2, \quad (6)$$

Υ being a parameter with dimension $[{\rm Mass}]^{-4}$ and b a dimensionless parameter, derives from higher-curvature gravity in Kaluza-Klein theory. In the limit of strong electromagnetic fields, we have $\mathcal{L}(F) \simeq c_n F^n$. In this case, Eq. (3) gives

$$\mathbf{E}^2 \simeq \mathbf{E}_1^2\left(\frac{a}{a_1}\right)^{-4/(2n-1)}, \quad (7)$$

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[1]In de Sitter inflation, the spectrum of electromagnetic fluctuations when crossing the horizon is $|A_\mu| \sim |F_{\mu\nu}|/H \sim H$, that is a scale-invariant spectrum corresponding to the Gibbons-Hawking temperature $T_{\rm GH} = H/(2\pi)$ [34].

[2]The full Lagrangian is $\mathcal{L}^{\rm KK}(F, G) = -F + \Upsilon[(b-1)F^2 - 3G^2/2]$, where $G = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \mathbf{E} \cdot \mathbf{B}$, and $\tilde{F}^{\mu\nu} = (1/2\sqrt{-g})\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual electromagnetic field strength tensor. However, in this paper, we are concerned only with nonlinear theories depending on the invariant F.

where $a_1 = a(t_1)$, and as initial value for the electric field we have taken that at the horizon crossing, $\mathbf{E}_1^2 = \mathbf{E}^2|_{t_1}$. From the first Bianchi identity we get $a^2\mathbf{B} \sim (k\eta)a^2\mathbf{E} +$ const, while from Eq. (7) we have $a^2\mathbf{E} \sim (k\eta)^\xi$ with $\xi \equiv 4s(n-1)/(2n-1)$. Since we are assuming $|k\eta| \ll 1$, if $\xi < -1$ the magnetic field evolves as

$$\mathbf{B}^2 \simeq \mathbf{B}_1^2\left(\frac{a}{a_1}\right)^{2(2n-1-2s)/s(2n-1)} \qquad (8)$$

while, if $\xi \geq -1$, it scales adiabatically. For $n = 1$ we have $\xi = 0$ while for $n \geq 2$ it results $\xi < -1$. Moreover, since $2(2n - 1 - 2s)/s(2n - 1) > -4$ for $n \geq 2$, a "superadiabatic amplification" (i.e. $\mathbf{B}^2$ evolves less slowly than the usual a^{-4}) occurs during inflation.

We now consider a "toy model" described by the Lagrangian

$$\mathcal{L}(F) = -Fe^{-cF}, \qquad (9)$$

where $c = m^{-4}d$, with $d > 0$ a dimensionless constant of order unity. The exponential self-coupling in Lagrangian (9) resembles to the exponential coupling $\propto F_{\mu\nu}F^{\mu\nu}e^{\alpha\phi}$, α being a dimensional constant, between the inflaton (dilaton) ϕ and the electromagnetic field, introduced in Refs. [15,16]. In our case, the scalar field is replaced by the scalar F. In the limit of strong fields, the solution of Eq. (3) is approximately given by $\mathbf{E}^2 \simeq \mathbf{E}_1^2 + m^4 \ln(a_1/a)^{4/d}$. Neglecting the logarithmic term, we have that the second model is equivalent to the first one with $n \to \infty$.

As a third model, we consider the Born-Infeld (BI) Lagrangian[3]

$$\mathcal{L}^{\text{BI}}(F) = m^4\left(1 - \sqrt{1 + \frac{2F}{m^4}}\right), \qquad (10)$$

where, for all field configurations, the condition $2F/m^4 \geq -1$ (which corresponds to $\mathbf{E}^2 \leq m^4$) has to be satisfied. Born and Infeld proposed their theory [37] in order to eliminate the divergence in the energy of a point-charge particle. Indeed, in this theory the self-energy of a pointlike charge is always finite and proportional to the parameter m. The Born-Infeld model also appears in quantized string theory [38] (in this case, $m^2 = 2\pi\alpha'$, where α' is the string tension parameter). Cosmological effects of BI electrodynamics, such as generation of an inflationary phase [29], have been deeply studied in recent years.

Equation (3) gives $\mathbf{E}^2 = m^4[(a/a_1)^4(m^4/\mathbf{E}_1^2 - 1) + 1]^{-1}$. If $\mathbf{E}_1^2 < m^4$, then for $a \gg a_1$ we get the usual behavior $\mathbf{E}^2 \propto a^{-4}$. If $\mathbf{E}_1^2 = m^4$, we have $\mathbf{E}^2 = m^4$ for all times. Therefore, this model is equivalent to the first one with $n \to \infty$, and with the condition $|F| \gtrsim m^4$, that is $\mathbf{E}_1^2 \gtrsim m^4$, replaced by $\mathbf{E}_1^2 = m^4$.

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[3] The full Lagrangian is $\mathcal{L}^{\text{BI}}(F, G) = m^4[1 - \sqrt{1 + 2F/m^4 - G^2/m^8}]$ (see also footnote [2]).

E. Present magnetic fields

We now derive the actual strength of magnetic fields generated during inflation in the nonlinear theories described by the above three Lagrangians.

During de Sitter inflation any weak field, $|F| \lesssim m^4$, is exponentially washed out. We then analyze the case in which electric fields remain strong from the first horizon crossing to the end of inflation. Observing that the electric field is a nonincreasing function of time, we then assume that the quantity $\mathbf{E}^2(\lambda)/m^4|_{t_{\text{end}}}$ is greater then 1, where t_{end} is the time corresponding to the end of inflation. Taking into account Eqs. (4) and (7), we can write the above condition as

$$m \lesssim 10^{20}(10^{24}\lambda_{10\text{ kpc}})^{-\mu}\left(\frac{M}{m_{\text{Pl}}}\right)^{2-\mu} \text{GeV}, \qquad (11)$$

where $\mu = 1/(2n - 1)$, and we used $a_{\text{end}}/a_1 = e^{N(\lambda)} \simeq 10^{24}\lambda_{10\text{ kpc}}M/m_{\text{Pl}}$, $N(\lambda)$ being the number of e-folds elapsing from the crossing of a comoving length λ outside the horizon to the end of inflation [14].

The actual value of the magnetic field follows from Eq. (8) and is $B_{\text{NLE}} \simeq B_1 e^{-\beta N(\lambda)}(a_{\text{end}}/a_0)^2$ where B_1, the magnetic field strength at the horizon crossing, is given by Eq. (4), and $\beta = 1 + 2\mu$. The last term in the above equation takes into account the adiabatic dilution of the magnetic field from the end of inflation until today, $a = a_0$. Using the relation (valid during radiation and matter dominated eras) $a \propto g_{*S}^{-1/3}T^{-1}$, $g_{*S}(T)$ being the number of effectively massless degrees of freedom referring to the entropy density of the universe [35], we arrive to

$$B_{\text{NLE}} \simeq 10^{-4}(10^{24}\lambda_{10\text{ kpc}})^{-\beta}\left(\frac{M}{m_{\text{Pl}}}\right)^{2-\beta} \text{G}, \qquad (12)$$

where we used the values $T_0 \simeq 2.35 \times 10^{-13}$ GeV, $g_{*S}(T_0) \simeq 3.91$, and we assumed $g_{*S}(T_{\text{end}}) \sim 10^2$ [35]. (It is useful to know that $1 \text{ G} \simeq 6.9 \times 10^{-20} \text{ GeV}^2$). Observe that the magnetic field during de Sitter inflation evolves as $B \propto a^{-\beta}$. The case of standard electromagnetic Lagrangian corresponds to $\beta = 2$. Therefore, one finds for $\lambda = 10$ kpc the vanishingly small value $B \simeq 10^{-52}$ G. In nonlinear electrodynamics, taking $M \simeq 10^{-2}m_{\text{Pl}}$ and $\lambda = 10$ kpc, we find that B_{NLE} is an increasing function of n. In particular, we get $B_{\text{NLE}} \simeq 10^{-45}$ G for $n = 2$, $B_{\text{NLE}} \simeq 10^{-33}$ G for $n = 8$, and $B_{\text{NLE}} \simeq 10^{-30}$ G for $n \to \infty$, together with the conditions $m \lesssim 10^8$ GeV, $m \lesssim 10^{14}$ GeV, and $m \lesssim 10^{16}$ GeV, respectively. From the above results, we see that NLE effects are able, in principle, to produce magnetic fields that can seed galactic dynamo.

In the case of power-law inflation, a comoving length λ crosses outside the horizon when [14]

$$\rho_{\text{tot}}(\lambda)|_{t_1} \simeq \left(10^{24}\frac{M}{m_{\text{Pl}}}\lambda_{10\text{ kpc}}\right)^{-2x}M^4, \qquad (13)$$

TABLE I. Actual strength of the magnetic field, B_{NLE}, produced during power-law inflation *via* nonlinear electrodynamic effects at the comoving scale $\lambda = 10$ kpc. The last two rows refer to $\lambda = 1$ Mpc. The index n and the mass m define a particular choice of the nonlinear Lagrangian (see text for details), while M^4 is the total energy density at the end of inflation. The magnetic field depends on the comoving scale λ as $B_{\mathrm{NLE}}(\lambda) \propto \lambda^{-\beta'}$.

n	$M(\mathrm{GeV})$	$B_{\mathrm{NLE}}(\mathrm{G})$	$m(\mathrm{GeV}) \lesssim$	β'
2	10^{16}	10^{-43}	10^{8}	1.6
2	10^{12}	10^{-37}	10^{5}	1.1
2	10^{9}	10^{-32}	10^{3}	0.6
3	10^{16}	10^{-37}	10^{11}	1.3
3	10^{12}	10^{-30}	10^{8}	0.7
3	10^{9}	10^{-26}	10^{6}	0.1
3	10^{6}	10^{-23}	10^{3}	-0.7
∞	10^{16}	10^{-28}	10^{15}	0.9
∞	10^{12}	10^{-20}	10^{13}	0.1
∞	10^{9}	10^{-14}	10^{13}	-0.6
∞	10^{8}	10^{-12}	10^{12}	-0.9

where $M^4 \equiv \rho_{\mathrm{tot}}(\lambda)|_{\mathrm{end}}$ is the total energy density at the end of inflation, and $x \equiv 3(1 + \gamma)/(1 + 3\gamma)$. The bound on graviton production previously discussed translates to [14] $x \geq x_{\mathrm{min}}$, where $x_{\mathrm{min}} \simeq [4 + 2\log_{10}(M/m_{\mathrm{Pl}})]/[29.8 + \log_{10}(M/m_{\mathrm{Pl}})]$. In the following, we assume for simplicity that $x = x_{\mathrm{min}}$, and that the electric field remains strong during inflation. This corresponds to the "best case scenario," or to the minimum dilution of the magnetic field during inflation. Taking into account Eqs. (4) and (7), and using $a_{\mathrm{end}}/a_1 = (M^4/\rho_{\mathrm{tot}}(\lambda)|_{t_1})^{-3(1+\gamma)}$, the condition $\mathbf{E}^2(\lambda)/m^4|_{t_{\mathrm{end}}} \gtrsim 1$ translates into Eq. (11) with the replacement $\mu \rightarrow \mu' = \mu + (1 - \mu)x_{\mathrm{min}}$. The actual strength of the inflation-produced magnetic field follows from Eqs. (4), (8), and (13), and is given by Eq. (12) with β replaced by $\beta' = 1 + 2\mu'$. The standard case of linear electrodynamics corresponds to $\beta' = 2$, and then reduces to that studied for de Sitter inflation.

In Table I, we show the values of B_{NLE} (for the case of power-law inflation) for different values of n and M, together with the condition on m in the order that electromagnetic fields be strong. Looking at the Table, we see that magnetic fields able to seed galactic dynamo or directly explain galactic magnetism can be produced.

III. CONCLUSIONS

Large-scale magnetic fields are ubiquitous in the present universe. Astrophysical observations have proved the existence of microgauss magnetic fields in all types of galaxies (spiral, elliptical, barred and irregular). Remarkably, there are hints of the existence of large-scale, microgauss magnetic fields in galaxy clusters. This sort of "cosmic magnetism" could have been aroused out of quantum electromagnetic fluctuations excited during an inflationary epoch of the universe. However, in the standard, conformally invariant theory of electrodynamics, inflation-produced fields are vanishingly small, and then cannot explain the presently observed fields. Nevertheless, a lot of (sometimes exotic) mechanisms able to break conformal invariance of Maxwell's electrodynamics, and then to produce astrophysically interesting fields, have been proposed in the literature.

In this paper, we have investigated the possibility to generate magnetic fields during the inflation era in nonlinear theories of electrodynamics, in which conformal invariance is naturally broken. We have found that, for a wide range of parameter space of inflationary models, magnetic fields of cosmological interest can be created. In particular, we have shown that magnetic fields able to seed galactic dynamo or to explain directly the galactic magnetism could be a natural consequence of such theories. However, since our results strongly depend on the actual form of the (unknown) nonlinear electromagnetic Lagrangian, they cannot give a definitive answer to the question "Why is our universe magnetized?"

Note added.—After completion of this paper, we noted a recent preprint by Kunze [39], who exploits a very similar idea, but using a different (power-law) parametrization for the nonlinear Lagrangian. We agree with Kunze's results in the common subcase of integer-exponent monomial. We note, however, that we can obtain cosmic magnetic fields of the observed size via power-law inflation (see Table I and related comments), without necessarily invoking a galactic dynamo as in Kunze's paper.

[1] For a review on galactic magnetism see: R. Beck *et al.*, Annu. Rev. Astron. Astrophys. **34**, 155 (1996).

[2] H. Lesch and M. Chiba, Astron. Astrophys. **297**, 305L (1995).

[3] For recent reviews on dynamo mechanisms see, e.g.: A. Brandenburg and K. Subramanian, Phys. Rep. **417**, 1 (2005); A. Shukurov, arXiv:astro-ph/0411739.

[4] K. Ferrière and D. Schmitt, Astron. Astrophys. **358**, 125 (2000).

[5] A. C. Davis, M. Lilley, and O. Tornkvist, Phys. Rev. D **60**, 021301 (1999).

[6] For reviews on cosmic magnetic fields see: L. M. Widrow,

Rev. Mod. Phys. **74**, 775 (2002); , Ref. [10]

[7] G. Hinshaw *et al.*, Astrophys. J. Suppl. Ser. **170**, 288 (2007).

[8] J. D. Lowenthal *et al.*, Astrophys. J. **481**, 673 (1997).

[9] P. P. Kronberg, J. J. Perry, and E. L. H. Zukowski, Astrophys. J. **387**, 528 (1992).

[10] P. P. Kronberg, Rep. Prog. Phys. **57**, 325 (1994); D. Grasso and H. R. Rubinstein, Phys. Rep. **348**, 163 (2001); M. Giovannini, Int. J. Mod. Phys. D **13**, 391 (2004); V. B. Semikoz and D. D. Sokoloff, *ibid.* **14**, 1839 (2005).

[11] For reviews on cosmic magnetism from an observational viewpoint see: J. P. Vallée, New Astron. Rev. **48**, 763 (2004); J. L. Han and R. Wielebinski, Chin. J. Astron. Astrophys. **2**, 293 (2002); arXiv:astro-ph/0209090.

[12] For a review on generation of cosmic magnetic fields in the early universe see, e.g.: A. D. Dolgov, in *From Integrable Models to Gauge Theories*, edited by V. G. Gurzadyan *et al.* (World Scientific, Singapore, 2001), p. 143; arXiv:astro-ph/0306443, Talk given at 17th Les Rencontres de Physique de la Vallee d'Aoste: Results and Perspectives in Particle Physics, La Thuile, Aosta Valley, Italy, 2003.

[13] For generation of magnetic fields through astrophysical mechanisms see, e.g.: R. E. Pudritz and J. Silk, Astrophys. J. **342**, 650 (1989); A. Lazarian, Astron. Astrophys. **264**, 326 (1992); K. Subramanian, D. Narasimha, and S. M. Chitre, Mon. Not. R. Astron. Soc. **271**, L15 (1994); R. M. Kulsrud *et al.*, Astrophys. J. **480**, 481 (1997); G. Davies and L. M. Widrow, *ibid.* **540**, 755 (2000); N. Y. Gnedin, A. Ferrara, and E. G. Zweibel, *ibid.* **539**, 505 (2000); E. R. Siegel and J. N. Fry, *ibid.* **651**, 627 (2006).

[14] M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 2743 (1988).

[15] B. Ratra, Astrophys. J. **391**, L1 (1992).

[16] D. Lemoine and M. Lemoine, Phys. Rev. D **52**, 1955 (1995); M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Rev. Lett. **75**, 3796 (1995).

[17] W. D. Garretson, G. B. Field, and S. M. Carroll, Phys. Rev. D **46**, 5346 (1992).

[18] M. M. Anber and L. Sorbo, J. Cosmol. Astropart. Phys. 10 (2006) 018.

[19] M. Gasperini, Phys. Rev. D **63**, 047301 (2001).

[20] A. Dolgov, Phys. Rev. D **48**, 2499 (1993).

[21] K. Dimopoulos *et al.*, Phys. Rev. D **65**, 063505 (2002); T. Prokopec and E. Puchwein, *ibid.* **70**, 043004 (2004); K. Bamba and J. Yokoyama, *ibid.* **69**, 043507 (2004); **70**, 083508 (2004); A. Ashoorioon and R. B. Mann, *ibid.* **71**, 103509 (2005); O. Bertolami and D. F. Mota, Phys. Lett. B **455**, 96 (1999).For general properties of inflation-produced magnetic fields see: C. G. Tsagas, Phys. Rev. D **72**, 123509 (2005); K. Bamba and M. Sasaki, J. Cosmol. Astropart. Phys. 02 (2007) 030; K. Bamba, Phys. Rev. D **75**, 083516 (2007); J. Cosmol. Astropart. Phys. 10 (2007) 015.

[22] For generation mechanisms of primordial magnetic fields after inflation see, e.g.: J. M. Quashnock, A. Loeb, and D. N. Spergel, Astrophys. J. **344**, L49 (1989); T. Vachaspati, Phys. Lett. B **265**, 258 (1991); B. l. Cheng and A. V. Olinto, Phys. Rev. D **50**, 2421 (1994); G. Baym, D. Bodeker, and L. D. McLerran, *ibid.* **53**, 662 (1996); G. Sigl, A. V. Olinto, and K. Jedamzik, *ibid.* **55**, 4582 (1997);

M. Joyce and M. E. Shaposhnikov, Phys. Rev. Lett. **79**, 1193 (1997); P. Cea and L. Tedesco, Phys. Lett. B **450**, 61 (1999); T. Vachaspati, Phys. Rev. Lett. **87**, 251302 (2001); A. D. Dolgov and D. Grasso, *ibid.* **88**, 011301 (2001); V. I. Demchik and V. V. Skalozub, Eur. Phys. J. C **25**, 291 (2002); Z. Berezhiani and A. D. Dolgov, Astropart. Phys. **21**, 59 (2004); V. B. Semikoz and D. D. Sokoloff, Phys. Rev. Lett. **92**, 131301 (2004); M. Laine, J. High Energy Phys. 10 (2005) 056; A. Diaz-Gil *et al.*, Proc. Sci., LATTICE 2007 (2007) 052 [arXiv:0710.0580].

[23] G. Greenstein, Nature (London) **223**, 938 (1969); J. J. Matese and R. F. O'Connell, Astrophys. J. **160**, 451 (1970); D. Grasso and H. R. Rubinstein, Phys. Lett. B **379**, 73 (1996).

[24] I. Wasserman, Astrophys. J. **224**, 337 (1978); E. j. Kim, A. Olinto, and R. Rosner, *ibid.* **468**, 28 (1996); E. Battaner, E. Florido, and J. Jimenez-Vicente, Astron. Astrophys. **326**, 13 (1997); E. Florido and E. Battaner, *ibid.* **327**, 1 (1997); E. Battaner, E. Florido, and J. M. Garcia-Ruiz, *ibid.* **327**, 8 (1997).

[25] A. Kosowsky and A. Loeb, Astrophys. J. **469**, 1 (1996); J. D. Barrow, P. G. Ferreira, and J. Silk, Phys. Rev. Lett. **78**, 3610 (1997); K. Subramanian and J. D. Barrow, *ibid.* **81**, 3575 (1998); R. Durrer, T. Kahniashvili, and A. Yates, Phys. Rev. D **58**, 123004 (1998); R. Durrer, P. G. Ferreira, and T. Kahniashvili, *ibid.* **61**, 043001 (2000); K. Jedamzik, V. Katalinic, and A. V. Olinto, Phys. Rev. Lett. **85**, 700 (2000); T. R. Seshadri and K. Subramanian, *ibid.* **87**, 101301 (2001); A. Mack, T. Kahniashvili, and A. Kosowsky, Phys. Rev. D **65**, 123004 (2002); L. Campanelli *et al.*, Astrophys. J. **616**, 1 (2004); A. Lewis, Phys. Rev. D **70**, 043011 (2004); A. Kosowsky *et al.*, *ibid.* **71**, 043006 (2005).For effects of magnetic fields on density perturbations in the universe see: J. D. Barrow, R. Maartens, and C. G. Tsagas, Phys. Rep. **449**, 131 (2007), and references therein.

[26] K. Dimopoulos and A. C. Davis, Phys. Lett. B **390**, 87 (1997); D. T. Son, Phys. Rev. D **59**, 063008 (1999); A. Brandenburg, Science **292**, 2440 (2001); G. Sigl, Phys. Rev. D **66**, 123002 (2002); M. Christensson, M. Hindmarsh, and A. Brandenburg, Astron. Nachr. **326**, 393 (2005); L. Campanelli, Phys. Rev. D **70**, 083009 (2004); R. Banerjee and K. Jedamzik, Phys. Rev. Lett. **91**, 251301 (2003); **93**, 179901(E) (2004); Phys. Rev. D **70**, 123003 (2004); L. Campanelli, Phys. Rev. Lett. **98**, 251302 (2007).

[27] G. W. Gibbons and C. A. R. Herdeiro, Classical Quantum Gravity **18**, 1677 (2001).

[28] M. Novello *et al.*, Classical Quantum Gravity **24**, 3021 (2007).

[29] V. V. Dyadichev *et al.*, Phys. Rev. D **65**, 084007 (2002).

[30] P. V. Moniz, Phys. Rev. D **66**, 103501 (2002); E. Elizalde *et al.*, Phys. Lett. B **574**, 1 (2003); M. Novello, S. E. Perez Bergliaffa, and J. Salim, Phys. Rev. D **69**, 127301 (2004).

[31] R. Garcia-Salcedo and N. Breton, Int. J. Mod. Phys. A **15**, 4341 (2000).

[32] V. A. De Lorenci *et al.*, Phys. Rev. D **65**, 063501 (2002).

[33] K. Dimopoulos, arXiv:astro-ph/0105488.

[34] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).

[35] E. W. Kolb and M. S. Turner, *The Early Universe*

(Addison-Wesley, Redwood City, California, 1990).

[36] G. W. Gibbons and C. A. R. Herdeiro, Phys. Rev. D **63**, 064006 (2001).

[37] M. Born and L. Infeld, Proc. R. Soc. A **144**, 425 (1934).

[38] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **163B**, 123 (1985).

[39] K. E. Kunze, arXiv:0710.2435.