

Kinematic constraints on formation of bound states of cosmic strings: Field theoretical approachP. Salmi,^{1,*} A. Achúcarro,^{1,2,†} E. J. Copeland,^{3,‡} T. W. B. Kibble,^{4,§} R. de Putter,^{1,5,||} and D. A. Steer^{6,¶}¹*Lorentz Institute of Theoretical Physics, University of Leiden, The Netherlands*²*Department of Theoretical Physics, University of the Basque Country UPV-EHU, 48080 Bilbao, Spain*³*School of Physics and Astronomy, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom*⁴*Blackett Laboratory, Imperial College, London SW7 2AZ, United Kingdom*⁵*University of California, Berkeley, California 94720, USA*⁶*APC, Université de Paris 7, Batiment Condorcet, 10 Rue Alice Domon et Léonie Duquet, 75205 Paris, France*

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Superstring theory predicts the potential formation of string networks with bound states ending in junctions. Kinematic constraints for junction formation have been derived within the Nambu-Goto thin string approximation. Here we test these constraints numerically in the framework of the Abelian-Higgs model in the Type-I regime and report on good agreement with the analytical predictions. We also demonstrate that strings can effectively pass through each other when they meet at speeds slightly above the critical velocity permitting bound-state formation. This is due to reconnection effects that are beyond the scope of the Nambu-Goto approximation.

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I. INTRODUCTION

There has been a revival of interest in cosmic strings since it was realized that many fundamental string theory models predict so-called *cosmic superstrings*. The leap from string scale to cosmic dimensions is highly nontrivial. At first sight there are many severe problems related to the formation and growth of superstrings to macroscopic scales, such as too high string tension and instability towards breaking up, as pointed out already in [1]. Subsequent advances in fundamental string theory have changed this picture. All these issues can be circumvented and are naturally evaded in many models that possess otherwise desirable aspects from the point of view of providing successful phenomenology. For instance, in the scenarios based on warped compactifications the warp factor can reduce the string tension. Furthermore, in models of brane inflation not only are cosmic strings produced at the end of the inflationary epoch [2,3], but it has been argued in great abundance, too [4].

Considerations inspired by superstring theory suggest a particular kind of cosmic string network that consists of fundamental F-strings, Dirichlet D-strings (or more precisely D1-branes), and *bound states* of these two, known as (p, q) -strings as an abbreviation for p F-strings and q D-strings [5]. The presence of (p, q) -strings brings an additional feature to these networks compared to those made of only one type of solitonic strings. Namely, where two different types of string meet at a point and form a bound state leading away from that point, there is a *junction* in the

network. Another property of cosmic superstrings is that their reconnection probability P can be small, $P \ll 1$ (for a review see [6] and references therein).

A substantial body of work has dealt with modelling of cosmic string networks, which is a challenging task due to the combination of several scales involved and the non-linear nature of the problem. For instance, properties of networks at small scales are still not entirely understood and have been under intense analytic study recently [7–9]. However, it is well established that string networks lose energy efficiently. A scaling string network contributes only a fixed, tiny fraction of the total energy budget of the Universe, a property that has made cosmic strings viable, in contrast to some other defect models, and generated the appeal of cosmic strings from the very start [10].

For the above mentioned reason the effect of junctions and bound states on the evolution of the networks is of profound interest: instead of reaching a scaling regime the network could end in a frozen-out state and start to dominate the energy density of the Universe, which cannot be tolerated in the standard cosmologies. To date, the evolution of networks with junctions has already been studied in several models. Good evidence for scaling was reported in [11] with an $SU(2)/\mathbb{Z}_3$ model of global strings, the junctions being global monopoles. Another field theory study [12] used a model involving two sets of $U(1)$ gauge fields (see also [13]). There bound states are reported to have a significant effect on the network in the absence of long-range interactions, whereas in the case of global strings junctions play a minor role. Other studies include [14–16], which concluded that, even being conservative, the networks can scale also in the presence of junctions.

If the bound states and junctions are of major importance, then a natural question to pose is under which conditions they form. This was examined analytically based on the Nambu-Goto action in [17,18]. It was shown

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that strong kinematic constraints apply to the formation of the bound state. The purpose of this study is to test these constraints in a field theory setup. We work within the Abelian-Higgs model in the type-I regime, investigating when the intersection of strings results in the formation of a bound state (also called a *zipper*). This topic has been addressed already before both analytically [19] and with numerical experiments [20]. Here we revisit it due to the renewed interest in more complicated networks with a spectrum of string tensions and with improved computational resources available. Before presenting the results, we briefly review the outcome from the Nambu-Goto approximation and introduce the model together with the numerical approach.

II. STRING JUNCTIONS

Consider a straight string making an angle α with the positive x -axis and another one with an angle $-\alpha$ (the total angle between the strings being thus 2α). Both strings are on the xy -plane and have a velocity v along the z -axis with opposite directions and have string tensions μ_1 and μ_2 , respectively. Once they intersect, these can potentially form a third string, an “ x -link” which has tension μ_3 (we follow here the notation introduced in [18]; if $\mu_1 = \mu_2$ then the x -link is indeed positioned along the x -axis).

The approach in [17,18] is based on studying the action at a junction where three strings meet. The kinematic constraints follow from the requirement that the total length of the progeny string must increase. The allowed parameter region for the link formation can be determined at least numerically for any combination of string tensions [18]. However, when the colliding strings have the same tension, denoted hereafter by μ_1 , it is possible to express this requirement in a simple closed form

$$\alpha < \arccos\left(\frac{\gamma\mu_3}{2\mu_1}\right), \quad (1)$$

where $\gamma = 1/\sqrt{1-v^2}$ and necessarily $\mu_3 < 2\mu_1$.

Here we want to test this result numerically in a field theory where strings are solitonic objects with a *nonzero width* and see if bound-state formation is the dynamically preferred process.

III. MODEL AND NUMERICAL IMPLEMENTATION

The Abelian-Higgs model is governed by the Lagrangian

$$\mathcal{L} = (\partial_\mu + iqA_\mu)\phi(\partial^\mu - iqA^\mu)\phi^\dagger - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2.$$

The model has vortex solutions [21], in which the scalar field can be expressed, with the help of a function f of the

radial distance r only, as follows:

$$\phi = f(r)e^{in\theta},$$

where n is the winding number. For $n = 1$ the vortex solutions are topologically stable for any value of the scaled coupling, $\beta = \lambda/2q^2 = m_{\text{scalar}}^2/m_{\text{gauge}}^2$, where q is the gauge coupling, $m_{\text{scalar}} = \sqrt{\lambda}\eta$ and $m_{\text{gauge}} = \sqrt{2}g\eta$. The domain $\beta < 1$ is known as the type-I regime. There a string with winding number n has lower energy than n strings with winding number $n = 1$ together. In particular, two $n = 1$ strings can merge to form an $n = 2$ string [22]. In other words, using the previous notation, $\mu_3 < 2\mu_1$, and thus this regime provides a testing ground for the formation of links.

The numerical code used to evolve two boosted strings on a lattice has been reported in [23]. The boundary condition employed here was introduced in [24]: the fields on the boundaries are updated as though the strings moved undisturbed at constant, initial velocities. Of course, this means that the simulations cannot be trusted much beyond the time when the kinks on the strings generated by their interaction reach the boundary. The majority of the simulations were carried out in a computational grid of size 400^3 , setting the lattice spacing dx to be 0.2 (the physical size of the lattice in linear dimension being therefore 80) and the time step $dt = 0.065$ (dx and dt here in units of $(q\eta)^{-1}$). The robustness of the results has been tested by varying the size of the simulation box, lattice spacing, and the initial separation of strings. All the snapshots presented in the following section are from simulations performed on a 600^3 grid, setting $dx = 0.15$.

IV. RESULTS

Local cosmic strings with an equal winding number, here $n = 1$, always *intercommute* when intersecting. However, after intercommutation, strings can come together and merge to form a string of winding number $n = 2$. This occurs when strings meet at relatively small velocity and angle (see [19,20]). Alternatively strings only intercommute and never meet again. In Fig. 1 four snapshots are presented that show the isosurfaces of constant energy density (set to be 0.2 in dimensionless units, roughly 30% of the maximum value in the core for a string with $n = 1$, a value chosen so that strings with winding $n = 1, 2$, and 3 can all be readily identified) when two strings with $n = 1$ collide at velocity $v = 0.1$ and angle $\alpha = 15^\circ$ and the coupling constant is $\beta = 0.125$.

The formation of the bound state in the Abelian-Higgs model is not entirely a straightforward, instantaneous process, as already reported in [20]. The link forms and unforms, the two strings becoming separately visible again. This is not surprising; the process is essentially right-angle scattering which is well established by numerical experiments with vortices [25,26]. Analytically, right-angle scattering after a head-on collision is understood at critical

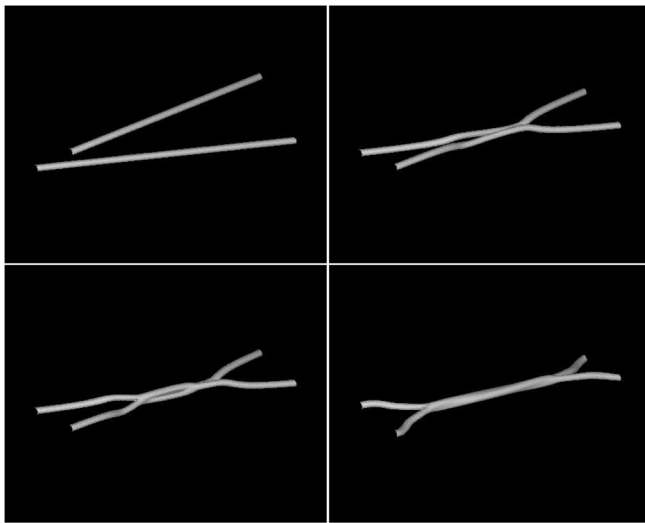


FIG. 1. Snapshots (from left to right and top to bottom) showing constant energy density isosurfaces in a simulation of two strings with $n = 1$ at $\beta = 0.125$ colliding with velocity $v = 0.1$ (top one moving downwards and vice versa), at an angle $\alpha = 15^\circ$, and forming an x -link as indicated by the \circ in Fig. 2.

coupling ($\beta = 1$) [27], in terms of geodesic motion in the moduli space approximation. This approach was introduced for the first time in the context of monopole scattering [28] but has been recently reemployed in studies of strings in more complicated models [29,30]. After intercommutation, here the string segments align almost parallel before coming together. Therefore locally the second collision takes place head-on with effectively zero impact parameter and strings scatter perpendicularly with respect to the incoming direction. Repeated right-angle scatterings are clearly visible in the snapshots; the time scale of attenuation is comparable to the time the simulations can be evolved.

The main result of this study is presented in Fig. 2, where the results from the simulations, standard intercom-

mutation versus link-formation, are shown together with the prediction of (1) for the allowed parameter region in the (α, v) -plane. A decrease in the scalar coupling λ reduces the tension of a string with winding number $n = 2$ relatively more than that of a string with $n = 1$. This allows one to vary the ratio μ_3/μ_1 to a certain extent by reducing the parameter β ; at $\beta = 0.36$ we obtain $\mu_3/\mu_1 \approx 1.88$, whereas $\beta = 0.125$ yields $\mu_3/\mu_1 \approx 1.77$. This already leads to a considerable difference when the limiting curve of the equality in (1) is plotted for $\beta = 0.36$ (left) and $\beta = 0.125$ (right) in Fig. 2. Standard intercommutation events are indicated by crosses and the formation of links by circles. At $\beta = 0.36$ the agreement between the simulations and the analytical prediction is perfect within the precision of the grid used in the (α, v) -parameter space. There is a discrepancy at the largest angles when $\beta = 0.125$; while the formation of an x -link should be allowed, this is not observed in the simulations.

This behavior is to be expected from considerations of energy conservation. In the Nambu-Goto model, all the energy released by shortening of the colliding strings goes into the formation of the linking string. In the field-theory model, by contrast, some energy is radiated away. We therefore expect that it should be slightly harder to form a link, and so there should be a small band where link formation is possible for Nambu-Goto strings, but not for field-theory strings.

In addition, at $\beta = 0.125$ there is a band just above the highest velocity allowing x -links to form suggested by (1) where a link does not form, but strings come together again and intercommute for a second time, events denoted by triangles in Fig. 2. Snapshots of this process are presented in Fig. 3. The end result is thus two strings consisting of the same segments as initially and indistinguishable from the situation where strings had passed through each other apart from some deformation around the interaction point. A similar type of effective nonintercommutation has been

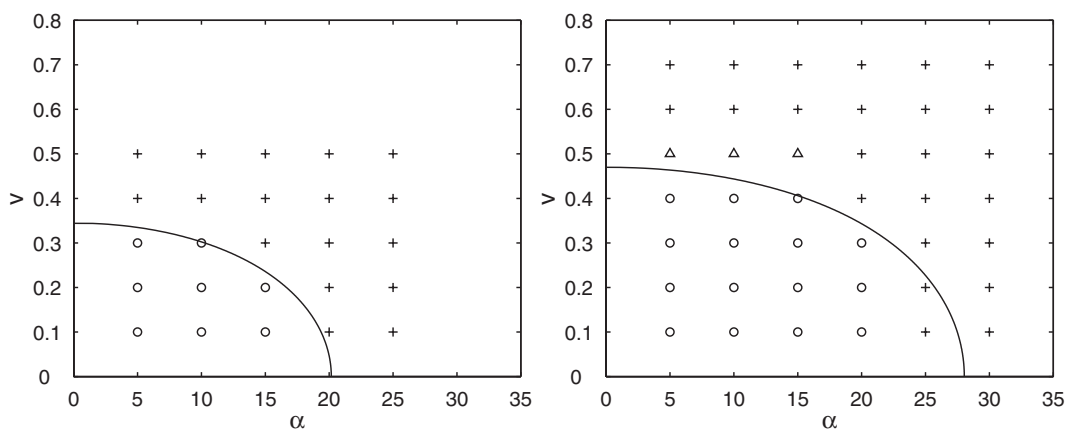


FIG. 2. Kinematic constraints in the (α, v) -plane for $\beta = 0.36$ (left) and $\beta = 0.125$ (right). The solid line shows the curve of equality in (1), + marks the events of single intercommutation, whereas \circ indicates strings merging together to form an x -link, a string with winding number $n = 2$. Events where strings pass through each other by intercommuting twice are shown by Δ .

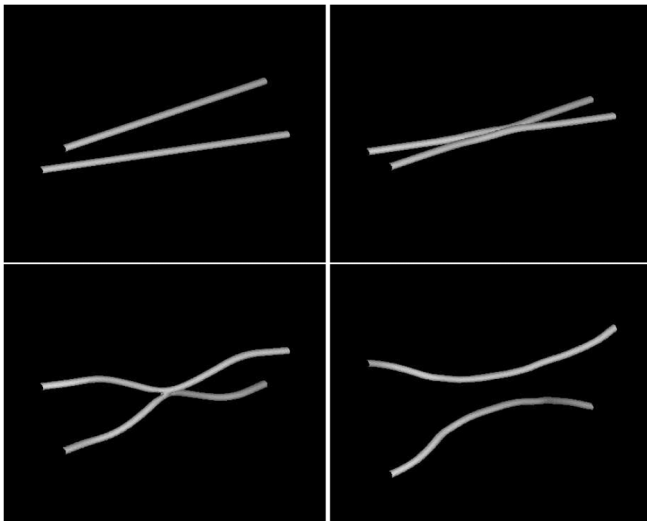


FIG. 3. Snapshots (from left to right and top to bottom) showing constant energy density isosurfaces in a simulation of two strings with winding $n = 1$ at $\beta = 0.125$ colliding with velocity $v = 0.5$, at an angle $\alpha = 10^\circ$, and undergoing double intercommutation, as indicated by the \triangle in right-hand Fig. 2.

reported when strings collide at very high velocities [23,24]. There is no sign of this kind of phenomenon at $\beta = 0.36$. This may be because less energy is radiated away in the field when the scalar mass is larger.

The general case, where all three strings have different tensions can be investigated in the Abelian-Higgs model by colliding strings with different winding numbers. This was done for two strings with windings $n = 2$ and $n = 1$ at $\beta = 0.125$. Though the explicit analytic formula for the asymmetric case [31] is not presented here, the kinematically allowed area is almost degenerate with the one presented in Fig. 2 for the coupling $\beta = 0.125$. A complication in the Abelian-Higgs model is that due to unequal winding numbers, the intercommutation leads to a formation of a bridge between the strings (see also [32]), whose influence on bound-state formation at the very least cannot be entirely neglected. No systematic study in the kinematic parameter space was carried out, but Fig. 4

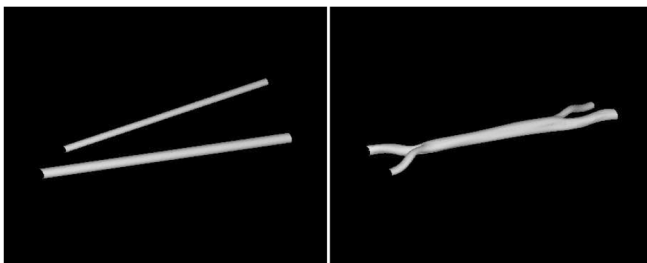


FIG. 4. Snapshots (from left to right) showing constant energy density isosurfaces in a simulation of strings with winding $n = 1$ (initially above) and $n = 2$ at $\beta = 0.125$ colliding with velocity $v = 0.1$ and at an angle $\alpha = 10^\circ$.

shows snapshots of a simulation that provides evidence for a bound-state formation now in the form of a string with winding number $n = 3$.

V. DISCUSSION

We have reported on numerical simulations of string collisions in the Abelian-Higgs model. The objective was to monitor the kinematic parameter space (α, v) and compare the outcome to the analytical prediction. We do not observe x -links outside the area where they are not expected to be kinematically allowed. On the other hand, bound states generically form whenever allowed to appear. This is interesting because such links do not have to form dynamically; there could have been simple intercommutation events instead, but almost the whole region appears to prefer to form x -links.

It is not surprising that the observed discrepancy occurs when strings intersect at large angles—the Nambu-Goto action does not include the effects of intercommutation. As strings in the Abelian-Higgs model always intercommute, effectively the original strings break up, and when the new strings straighten after this reconnection, they do not come together and merge at large angles. This could be different for strings that do not intercommute and exchange partners and it would be interesting to see what a study with strings in two separate gauge fields would yield.

Second, we have demonstrated once more that the “effective” reconnection probability of strings even in the Abelian-Higgs model is not strictly unity. This is well documented by numerical experiments at high collision speeds where a second intercommutation even takes place (see e.g. [23,24]) but as shown here can occur at moderate velocities too (it would be interesting to see if a modification of the moduli space approximation could capture the second intercommutation event). It was argued in [20] that when the bound state with winding $n = 2$ dissolves (which inevitably happens eventually with the boundary conditions introduced in [24]), the original strings re-emerge. This is effectively like no intercommutation taking place. However, the process reported here does not proceed via bound-state formation and dissolution, but rather by repeated intercommutation, and seems therefore to be of a different nature. This is further confirmed by the inequality (1) according to which the bound state is forbidden at velocities where the double intercommutation is observed.

Obviously the use of the Abelian-Higgs model has restricted us in this study to very limited values of the ratio of string tensions μ_3/μ_1 , whereas models inspired by superstring theory would allow a much wider range of values. We have shown that strings in the Abelian-Higgs model can have the same principal features as expected from cosmic superstrings, namely, formation of junctions and effective reconnection probability not strictly unity even at moderately *low velocities*. While network simulations very deep in the type-I regime are hardly feasible to perform,

these results may have implications for theories relying on a very low value of the scalar coupling λ . Such a network can arise e.g. upon breaking a gauge symmetry along a flat direction in supersymmetric theories as has been pointed out recently in [33]. To summarize, as anticipated already in [19], this study demonstrates the richness of strings even in the simple Abelian-Higgs model.

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