Detecting neutrino magnetic moments with conducting loops

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It is well established that neutrinos have mass, yet it is very difficult to measure those masses directly. Within the standard model of particle physics, neutrinos will have an intrinsic magnetic moment proportional to their mass. We examine the possibility of detecting the magnetic moment using a conducting loop. According to Faraday's law of induction, a magnetic dipole passing through a conducting loop induces an electromotive force in the loop. We compute this electromotive force for neutrinos in several cases, based on a fully covariant formulation of the problem. We discuss prospects for a real experiment, as well as the possibility to test the relativistic formulation of intrinsic magnetic moments.

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I. INTRODUCTION

Neutrinos are known to have very small masses, though it is not known yet what those masses are [1]. It appears to be difficult to probe masses much below the eV-level with current techniques, so new techniques are desirable. We would like to take advantage of the *linear* relation between the mass of an elementary particle and its magnetic dipole moment. For example, the standard model prediction is $\mu_\nu = \frac{3eG_F m_\nu}{8\pi^2\sqrt{s}}$ [2], and if new physics enters at scale Λ, then $\mu_\nu \sim \frac{16\pi}{\alpha}\frac{m_e m_\nu}{\Lambda^2}$ on general grounds [3]. Our suggestion is to exploit Faraday's law of induction which states that a change in magnetic flux through a conducting loop induces a current in that loop. Neutrinos in the lab are entirely left-handed, so the passage of a beam of neutrinos through a conducting loop will result in pulses in the loop. The purpose of this paper is to estimate the basic size and shape of such pulses.

We consider a conceptual experiment consisting of a small conducting ring through which a neutral elementary particle passes with a known trajectory and momentum. If that particle possesses an intrinsic magnetic dipole moment, then the passage of the particle through the loop induces an electromotive force (EMF) due to the change of magnetic flux through the surface bounded by the ring. The EMF is proportional to the magnetic dipole moment. Experimental methods based on induced currents have been successfully applied for measuring the magnetic moments of neutrons and various nuclei [4,5] and in searches for magnetic monopoles [6–8].

II. LORENTZ TRANSFORMATIONS OF THE MAGNETIC DIPOLE MOMENT

If a particle at rest has an intrinsic electric dipole moment $\boldsymbol{d}$, and it is placed in an external electric field $\boldsymbol{E}$, its potential energy is $-\boldsymbol{d}\cdot\boldsymbol{E}$. Similarly, a particle with a magnetic dipole moment $\boldsymbol{\mu}$ has a potential energy $-\boldsymbol{\mu}\cdot\boldsymbol{B}$ when it is placed in an external magnetic field $\boldsymbol{B}$. We need to express these terms in covariant form, using the antisymmetric 4-tensor for the electric and magnetic fields, $F_{\mu\nu}$. The covariant generalization of the electric and magnetic three-vectors $(\boldsymbol{d}, \boldsymbol{\mu})$ is thought to be another antisymmetric tensor [9,10]:

$$S^{\mu\nu} = \begin{pmatrix} 0 & cd_x & cd_y & cd_z \\ -cd_x & 0 & -\mu_z & \mu_y \\ -cd_y & \mu_z & 0 & -\mu_x \\ -cd_z & -\mu_y & \mu_x & 0 \end{pmatrix}, \tag{1}$$

where c is the speed of light. The quantity

$$U = -\boldsymbol{d}\cdot\boldsymbol{E} - \boldsymbol{\mu}\cdot\boldsymbol{B} = \tfrac{1}{2}S^{\mu\nu}F_{\mu\nu} \tag{2}$$

is a Lorentz scalar, which coincides with the sum of dipole interaction energies.

The transformation equations for the electric and magnetic dipole moments under a Lorentz boost are now easily obtained on the basis of Eqs. (1) and (2):

$$\begin{aligned} \boldsymbol{\mu} &= \gamma\boldsymbol{\mu}' + (1-\gamma)\frac{\boldsymbol{v}\cdot\boldsymbol{\mu}'}{v^2}\boldsymbol{v} + \gamma\boldsymbol{v}\times\boldsymbol{d}', \\ \boldsymbol{d} &= \gamma\boldsymbol{d}' + (1-\gamma)\frac{\boldsymbol{v}\cdot\boldsymbol{d}'}{v^2}\boldsymbol{v} - \frac{\gamma}{c^2}\boldsymbol{v}\times\boldsymbol{\mu}', \end{aligned} \tag{3}$$

where γ is the Lorentz factor and $\boldsymbol{v}$ is the relative velocity of the coordinate frames. If we assume that the electric dipole moment is zero, and the velocity $\boldsymbol{v}$ has an x-component only, then from Eq. (3) we have $\mu_x = \mu'_x$, $\mu_y = \gamma\mu'_y$, and $\mu_z = \gamma\mu'_z$, where the primed quantities refer to the rest frame of the particle. This is the starting point for our calculation.

III. THE INDUCED EMF IN THE RING

We introduce two coordinate systems (see Fig. 1) to calculate the induced EMF when the particle passes

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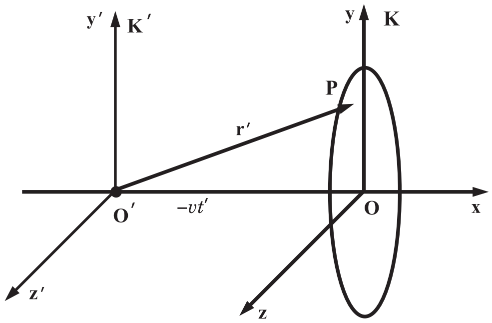

FIG. 1. Coordinate frames K and K', associated with the ring and particle, respectively.

through the center of a ring of radius b, and its motion is perpendicular to the plane of the ring. At $t = t' = 0$ the origin of the laboratory coordinate system (K), attached to the ring, coincides with the origin of the coordinate system (K') attached to the particle. From the Lorentz transformations we have $x' = \gamma(x - v t)$ and $r' = \sqrt{\gamma^2(x - v t)^2 + y^2 + z^2}$.

The magnetic field of the dipole in K' is

$$\mathbf{B}'(\mathbf{r}') = \frac{\mu_0}{4\pi}\frac{1}{r'^3}[3(\boldsymbol{\mu}' \cdot \hat{\mathbf{r}}')\hat{\mathbf{r}}' - \boldsymbol{\mu}'], \tag{4}$$

where μ_0 is the permeability of the vacuum, and $\mathbf{r}'$ ($\hat{\mathbf{r}}'$) is a (unit) radius vector in K'. The electromagnetic field-tensor components in the laboratory frame are obtained by performing the Lorentz field transformations with $\boldsymbol{E}' = 0$.

We assume that the particle moves uniformly through the ring and its energy loss is negligible. When the particle passes through the ring there is an induced current in the ring. Thus, there will be a force [see Eq. (2)] on the particle as it approaches the ring

$$\boldsymbol{F} = \boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \boldsymbol{B}) = \mu_x \frac{\partial B_x}{dx}\hat{\mathbf{x}}. \tag{5}$$

The work done on the particle by this force is negligible compared to the energy of the particle even for a strong constant current in the ring ($W = -\mu_0 \mu_x I/b$). When the particle passes through the center, the induced current reverses its direction, and therefore the particle continues to dissipate energy, consistent with the conservation of energy.

To calculate the induced EMF, we parameterize the path of the integration in counterclockwise direction from positive x [11]

$$\varepsilon = \oint_c \mathbf{E} \cdot d\ell = \int_0^{2\pi} \mathbf{E}(\boldsymbol{\rho}(\theta)) \cdot \frac{\mathbf{d}\boldsymbol{\rho}}{d\theta} d\theta, \tag{6}$$

where $\boldsymbol{\rho}(\theta) = b\cos\theta\hat{\mathbf{y}} + b\sin\theta\hat{\mathbf{z}}$ is the parameterization of the curve for parameter $\theta \in [0, 2\pi]$. We have the following geometric relations

$$\frac{d\boldsymbol{\rho}(\boldsymbol{\theta})}{d\theta} = -b\sin\theta\hat{\mathbf{y}} + b\cos\theta\hat{\mathbf{z}}, \qquad y^2 + z^2 = b^2. \tag{7}$$

Integration yields

$$\varepsilon = \frac{3}{2}\mu_0 \mu_x b^2 \frac{\gamma^2 v^2 t}{(\gamma^2 v^2 t^2 + b^2)^{5/2}}. \tag{8}$$

We can calculate the induced EMF by a second method. Define the flux $\varphi = \int \mathbf{B} \cdot \mathbf{d}\sigma$ and calculate the time derivative $d\varphi/dt$,

$$\frac{d\varphi}{dt} = \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\boldsymbol{\sigma} = \int_0^{2\pi}\int_0^b \frac{\partial \mathbf{B}}{\partial t} \cdot \left(\frac{\partial \mathbf{r}}{\partial \rho} \times \frac{\partial \mathbf{r}}{\partial \theta}\right) d\rho d\theta, \tag{9}$$

where $\mathbf{r}(\rho, \theta) = \rho\cos\theta\hat{\mathbf{y}} + \rho\sin\theta\hat{\mathbf{z}}$ is the parameterization of the surface; $\theta \in [0, 2\pi]$ and $\rho \in [0, b]$ are the angular and radial parameters. From the right-hand rule and the direction of $d\ell$, it follows that $d\boldsymbol{\sigma}$ (the element of surface area of the ring) should point in $\hat{\boldsymbol{x}}$ direction. Indeed, $(\frac{\partial \mathbf{r}}{\partial \rho} \times \frac{\partial \mathbf{r}}{\partial \theta}) = \rho\hat{\mathbf{x}}$. Therefore,

$$\frac{d\varphi}{dt} = \int_0^{2\pi}\int_0^b \frac{\partial B_x}{\partial t}\rho d\rho d\theta, \tag{10}$$

and integration yields

$$\frac{d\varphi}{dt} = -\frac{3}{2}\mu_0 \mu_x b^2 \frac{\gamma^2 v^2 t}{(\gamma^2 v^2 t^2 + b^2)^{5/2}}, \tag{11}$$

which is consistent with Eq. (8).

To calculate the induced EMF when the particle does not pass through the center of the ring, we need to modify the parameterization of the curve along which the line integral [Eq. (6)] is calculated. We will continue to assume that the particle trajectory is perpendicular to the plane of the ring. The parametrization of the curve describing the conducting loop is now (Fig. 2)

$$\boldsymbol{\rho}(\theta) = (b\cos\theta - h\cos\alpha)\hat{\mathbf{y}} + (b\sin\theta - h\sin\alpha)\hat{\mathbf{z}}. \tag{12}$$

From symmetry it follows that the induced EMF is independent of α, so we set $\alpha = 0$. We have the following geometric relations:

$$\frac{d\boldsymbol{\rho}(\boldsymbol{\theta})}{d\theta} = -b\sin\theta\hat{\mathbf{y}} + b\cos\theta\hat{\mathbf{z}}, \quad y^2 + z^2 = b^2 + h^2 - 2bh\cos\theta. \tag{13}$$

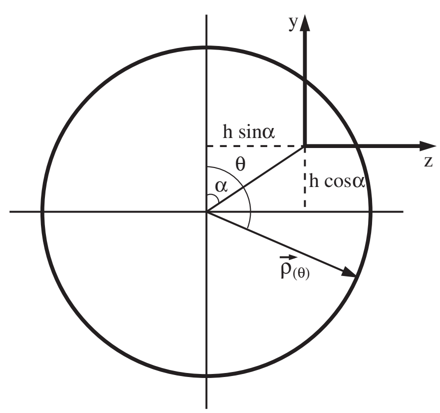

FIG. 2. Coordinates for off-axis passage, (h, α).

The induced EMF can be computed by inserting Eqs. (12) and (13) into Eq. (6). There is no analytical solution, however, so the integral has been evaluated numerically.

In the general case, the particle moves in arbitrary direction toward the ring. The velocity $\boldsymbol{v}$ defines the x-axis. We define the orientation of the ring by two angles ϕ and β. Let us define the coordinate system (ℓ, m, n), where m and n axes define the plane of the ring and ℓ is directed along the normal to the plane. The origins of the (ℓ, m, n) and (x, y, z) coordinate systems coincide. Performing two rotations in three dimensions (by angle ϕ relative to ℓ and by angle β relative to x), we can relate the coordinates of a point in (ℓ, m, n) to the (x, y, z) coordinate system. We can define the m-axis to be along the line of the nodes (the intersection line of the yz plane and the plane of the ring) [12], so that one of the Eulerian angles is zero. The other two Eulerian angles have simple geometric meanings, namely, ϕ is the angle between the x-axis and the normal of the ring (ℓ-axis), and β is the angle between the y-axis and the line of nodes (m-axis). We can parameterize ℓ, m, n similarly as in Sec. III:

$$\begin{aligned}\boldsymbol{\rho}(\theta) = {} & -\sin\phi(b\sin\theta - h\sin\alpha)\hat{\mathbf{x}} + [\cos\beta(\cos\theta - h\cos\alpha) \\ & + \cos\phi\sin\beta(b\sin\theta - h\sin\alpha)]\hat{\mathbf{y}} \\ & + [\cos\beta\cos\phi(b\sin\theta - h\sin\alpha) \\ & - \sin\beta(b\cos\theta - h\cos\beta)]\hat{\mathbf{z}}. \end{aligned} \tag{14}$$

IV. THE INDUCED CURRENT IN THE RING

To find the current in the ring, one must solve the following differential equation

$$\frac{d\phi}{dt} + L\frac{dI}{dt} + IR = 0, \tag{15}$$

where R is the resistance of the ring, L is its inductance, and

$$\varepsilon = -\left[\frac{d\phi}{dt} + L\frac{dI}{dt}\right]. \tag{16}$$

For $R \neq 0$, the second term in Eq. (15) is negligible compared to the third, and one finds trivially that the current is simply the EMF divided by the resistance.

For superconductors, $R = 0$, and it follows from Eq. (16) that the EMF is zero. From Eq. (15) we obtain $d(\phi + LI)/dt = 0$, which states that the magnetic flux through a superconducting loop is constant in time. One says that the magnetic field lines are "frozen" in the ring, a result which can also be obtained directly from the field equations. We have already calculated $d\phi/dt$—see Eq. (11). Solving this differential equation we have

$$I(t) = -\frac{\mu_0\mu_x b^2}{2L}\frac{1}{(c^2\gamma^2t^2 + b^2)^{3/2}}, \tag{17}$$

where the initial current is taken to be zero. The negative sign follows from the orientation of the ring that we chose in calculating Eq. (11).

V. RESULTS AND DISCUSSION

In the simple case in which the particle passes normally through the center of the ring, the EMF does not depend on the transverse components μ_y and μ_z of the magnetic dipole moment. The only contribution to the EMF comes from μ_x, the component of magnetic moment in the direction of motion.

One should investigate Eq. (8) for the maximum and minimum values of the EMF. By taking the derivative of the equation over time we have

$$\varepsilon_{\max} = \pm\frac{3}{4}\left(\frac{4}{5}\right)^{5/2}\mu_0\mu_x c\frac{\gamma}{b^2}, \tag{18}$$

corresponding to the times

$$t_{\max} = \pm\frac{b}{2c\gamma}, \tag{19}$$

which depend on the geometry of the configuration and the energy of the particle.

We present the induced EMF in the ring as a function of time in Fig. 3. For this example, we took an electron neutrino (ν_e) with energy 10 GeV and a ring with diameter of 1 cm. We assume that neutrino has a mass of $M_\nu = 1$ eV and the magnetic moment is $\mu_x = 10^{-10}\mu_B$, where μ_B is the Bohr magneton. This mass and magnetic moment are close to the current upper limits [13]. The solid curve represents the induced EMF when the particle passes through the center and is perpendicular to the surface of the ring. The dashed and dotted curves represent the induced EMF when the particle passes through the ring with distances from its center of 0.4b and 0.6b, respectively.

As seen in Fig. 3, the induced EMF from a single relativistic neutrino is extremely small, on the order of 10^{-17} V, and the pulse is extremely brief, on the order of 10^{-21} s. For off-axis trajectories, the effective radius of the

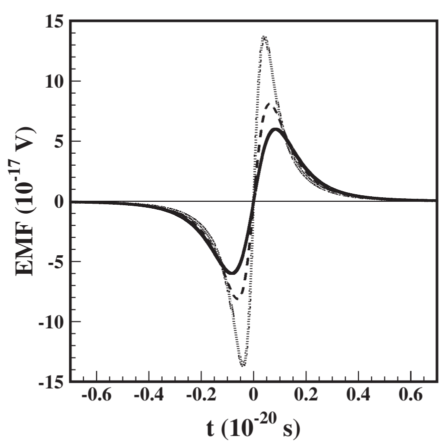

FIG. 3. Induced EMF in the ring when particle passes through the ring on-axis (solid line) and off-axis (dashed line for $h = 0.4b$ and dotted line for $h = 0.6b$).

ring appears smaller and as a result the induced EMF is larger (dashed and dotted curves in Fig. 3) and occurs at shorter times [8].

An example for the EMF in the general case is presented in Fig. 4, for which $h = 0.6b$, $\phi = 0.1$ nrad, $\beta = 0$, and $\alpha = \pi/2$. Because of the tilt, there is an asymmetry between the maximum and minimum EMF which might be useful in an experiment. However, in this case the EMF decreases very rapidly with ϕ.

Could a practical detector be built to observe this signal? Certainly special techniques would be required to detect such a small and brief pulse. It is not easy to increase the signal, since increasing γ or decreasing b will increase $\varepsilon_{\max}$ but decrease $t_{\max}$; and in fact, $\int_0^\infty \varepsilon(t)dt = 1.5\mu_0\mu_x/b$, independent of γ. If the loop has resistance R, then the power dissipated in the loop for all time is

$$W_{\text{tot}} = \frac{5\pi}{128}\left(\frac{3}{2}\mu_0\right)^2 \frac{\gamma v \mu_x^2}{b^3 R}. \quad (20)$$

If the magnetic moment is proportional to the mass, $\mu_x = AM_\nu$, as in the standard model [2], then $W_{\text{tot}} \approx A^2 E_\nu M_\nu / b^3 R$, which suggests that M_ν might be measured from this technique. However, for our on-axis example depicted in Fig. 3, and for $R = 1\Omega$, $W_{\text{tot}} \approx 10^{-53}$ W, which is exceedingly small.

The case of a superconducting ring is also interesting. The current $I(t)$ was presented in Eq. (17). It is very interesting that the current does not change direction as a function of time. Its maximum magnitude, however, is extremely small, on the order of 10^{-45} A.

Neutrinos produced at an accelerator are bunched in time, so one might try to increase the signal from the coherent contributions of a large number of neutrinos [$N_\nu = \mathcal{O}(10^6)$], effectively boosting μ_x by that number [cf. Eqs. (18) and (20)]. However, the neutrinos would have to arrive at the loop with time differences small compared to $t_{\max}$, and since the bunching of neutrinos occurs on the scale of 10^{-12} s, which is very long compared to $t_{\max} \approx 250$ fm$/c$, this seems impractical.

Apart from any possibility of measuring the magnetic moment of the neutrino, we would like to mention the opportunity of investigating experimentally the transformation of the magnetic moment of a particle under Lorentz boosts. Equation (8) predicts a dramatic variation of the EMF as a function of the particle's velocity, and Eq. (20) indicates that $W_{\text{tot}} \propto \beta\gamma$, which rises linearly with $v \ll c$ and then increases much more rapidly for $v > 0.6c$. The neutron has a magnetic moment of $-1.91\mu_N$, where μ_N is the nuclear magneton, which is roughly 2000 times smaller than μ_B. The neutron's magnetic moment is therefore a factor 10^7 times larger than the moments we have considered for the neutrino. However, for neutron energies $E_n = 10$ GeV, $\gamma \approx 10$, which is much less than the factor 10^{10} assumed for the neutrino, so the signal is again quite small: $\varepsilon_{\max} = 7 \times 10^{-19}$ V, $t_{\max} = 0.8$ ps, and $W_{\text{tot}} = 10^{-48}$ W. For nonrelativistic neutrons, the signal would be even smaller. Nonetheless, a confirmation of Eq. (8) or Eq. (18) would imply that the Ansatz for the relativistic formulation of magnetic and electric dipole moments, as given in Eqs. (1) and (2), is correct.

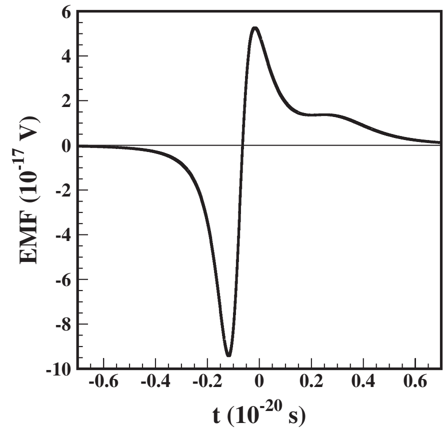

FIG. 4. Induced EMF when the particle trajectory makes an angle $\phi = 0.1$ nrad to the normal of the ring.

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