Unparticles at heavy flavor scales: *CP* violating phenomena

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Coupling the scale invariant unparticle sector to flavor physics and assuming that it remains scale invariant I investigate its consequences in heavy flavor physics. A characteristic feature of unparticle physics is a *CP*-even phase leading to novel *CP* violating phenomena. The phase is large, based on the assumption that the unparticle sector is strongly self-coupled. I consider the *CP* asymmetry in the leptonic decay $B^+ \to \tau^+ \nu$ and the hadronic decay $B_d \to D^+ D^-$, taking into account constraints of branching ratios and time dependent *CP* asymmetries. It turns out that the *CP* asymmetry can be very large even for small couplings because the unparticle interaction term has a lower scaling dimension than the four-Fermi weak interaction term. *CP* asymmetries in leptonic decays such as $B^+ \to \tau^+ \nu$ are neither experimentally searched for nor predicted by any other model. I show that the novel *CP* violation is consistent with the *CPT* theorem. I identify the *CP* compensating mode in the unparticle sector and explicitly demonstrate the exact cancellation as demanded by the *CPT* theorem.

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I. INTRODUCTION

The possibility of a strongly coupled scale invariant sector, weakly coupled to the standard model (SM), was advocated by Georgi in [1,2]. The operators of the scale invariant theory do not describe single particle excitations but entail a continuous spectrum, hence the name "unparticle." An interesting deconstruction of this spectrum in terms of a particle tower was given in Ref. [3]. By parametrizing a variety of interactions, unparticle phenomena were investigated at various energy scales and domains of particle physics such as electroweak physics [4–6], the breaking of scale invariance due to the coupling of the unparticle to the Higgs vacuum expectation value (VEV) [7,8], collider physics [9–11] (the latter investigates the (un)resonance in the Drell-Yan process due to the breaking of scale invariance by assuming a model proposed in [7]), deep inelastic scattering [12,13], B, D-physics [14–18], light flavor physics [19,20], $g_\mu - 2$ [14,21] lepton flavor violation [22,23], invisible decays [24], cosmology [25] long-range interaction [26] in conjunction with a 5th force [27,28], interaction with the Higgs and mixing with the Higgs VEV [29] and gravity [30].

According to [1] at a very high energy scale $M_{\mathcal{U}} \gg 1$ TeV the particle world could be described by the SM and a strongly self-coupled ultraviolet (UV) sector, interacting with each other via a heavy particle of mass $M_{\mathcal{U}}$ and is described by the effective nonrenormalizable Lagrangian

$$\mathcal{L}^{\mathrm{eff}} \sim \frac{1}{M_{\mathcal{U}}^{d_{\mathrm{UV}}+(d_{\mathrm{SM}}-4)}} O_{\mathrm{SM}} O_{\mathrm{UV}} \to \frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}+(d_{\mathrm{SM}}-4)}} O_{\mathrm{SM}} O_{\mathcal{U}}, \tag{1}$$

and at some energy $\Lambda_{\mathcal{U}}$ the UV sector flows into a *strongly* coupled infrared (IR) fixed point where the UV operator undergoes dimensional transmutation $O_{\mathrm{UV}} \to (\Lambda_{\mathcal{U}})^{d_{\mathrm{UV}}-d_{\mathcal{U}}} O_{\mathcal{U}}$ and the coupling indicated above is $\lambda = c_{\mathcal{U}} (\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^{d_{\mathrm{UV}}+(d_{\mathrm{SM}}-4)}$, with $c_{\mathcal{U}}$ being a matching coefficient expected to be of order one.

From a Lagrangian of the type (1) either real [1] or virtual effects [2] can be investigated from symmetry properties and the scaling dimension $d_{\mathcal{U}}$ alone. The meaning of the real emission of an unparticle is at present unclear or at least model dependent. Virtual effects are described in a transparent way within the formalism of perturbative field theory by the propagator, which can be constructed from the dispersion relation

$$\begin{aligned}\Delta_{\mathcal{U}}(P^2) &\equiv i \int_0^\infty d^4x e^{ip\cdot x} \langle 0|T O_{\mathcal{U}}(x) O_{\mathcal{U}}^\dagger(0)|0\rangle \\ &= \int_0^\infty \frac{ds}{2\pi} \frac{2\,\mathrm{Im}[\Delta_{\mathcal{U}}(s)]}{s - P^2 - i0} + \text{s.t.}\end{aligned} \tag{2}$$

It is assumed that $P^2 \geq 0$ and $P_0 > 0$ and s.t. stands for possible subtraction terms due to nonconvergence in the UV. The imaginary part is related to the local matrix element by the optical theorem

$$2\,\mathrm{Im}[\Delta_{\mathcal{U}}(P^2)] = |\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2 P^{-2} = A_{d_{\mathcal{U}}} (P^2)^{d_{\mathcal{U}}-2}, \tag{3}$$

whose form is dictated by the scaling dimension of $O_{\mathcal{U}}$. The dispersion integral is then elementary [2,4],

$$\begin{aligned}\Delta_{\mathcal{U}}(P^2) &= \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \frac{1}{(-P^2 - i0)^{2-d_{\mathcal{U}}}} \\ &\overset{P^2>0}{\to} \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \frac{e^{-id_{\mathcal{U}}\pi}}{(P^2)^{2-d_{\mathcal{U}}}},\end{aligned} \tag{4}$$

for appropriate $d_{\mathcal{U}}$ to be discussed below. The normalization factor $A_{d_{\mathcal{U}}}$, which is arbitrary up to the requirement $\Delta_{\mathcal{U}}(P^2) \to^{d_{\mathcal{U}}\to 1} 1/P^2$, has been chosen to be $A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{2\pi^{d_{\mathcal{U}}}} \Gamma(d_{\mathcal{U}} + 1/2)(\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}}))^{-1}$ [1]. It is the ana-

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lytic continuation of the phase space volume of $d_{\mathcal{U}}$ massless particles based on the observation that the matrix element $\langle 0|O_{\mathcal{U}}(0)|P\rangle$ behaves as such. This led to the statement that an *unparticle looks like a nonintegral number $d_{\mathcal{U}}$ of massless particles* [1]. The propagator (4) exposes power like scaling, unlike the logarithmic scaling of the trivial UV fixed point of QCD, and a CP-even phase factor $e^{-id_{\mathcal{U}}\pi}$, whose consequences have been investigated in many papers and constitutes the central ingredient to the analysis presented here. The lower bound of values for the scaling dimension of a generic operator is $d_{\mathcal{U}} \geq 1 + j_L + j_R$, where $j_{L(R)}$ is the Lorentz spin, for which the four dimensional conformal group admits unitary representations [31]. This bound assures the IR convergence of the dispersion integral (2). The integral diverges in the UV for $d_{\mathcal{U}} \geq 2$, but on the other hand the theory is described in the UV by the non−scale invariant theory of operators $O_{\rm UV}$, which alters the dispersion integral in the UV. In principle there is no upper boundary but nevertheless in the literature most often the values $1 < d_{\mathcal{U}} < 2$ are assumed without much loss for the phenomenological analyses.

Scale invariance is expected to be broken at lower energies, first by the emergence of the weak scale, by coupling the unparticle to the Higgs VEV for instance [7], and second in concrete realizations discrete parameters, such as the number of colors, might only allow for a near critical behavior. The breaking of scale invariance in the IR will change the nature of the unparticle as a final state in case it does not decay beforehand.

The discussion up to now has been mostly formal based on symmetries. This raises the question of whether there are indeed such theories in four dimension that flow into a nontrivial IR fixed point. In Ref. [1] the (perturbative) Banks-Zaks [32] fixed-point was given as an illustrative example. Walking technicolor constitutes another example, c.f. [33] and references therein, where a scale invariant window is needed in order to suppress flavor-changing neutral currents and contributions to the S-parameter. Very recently it was shown that half of the supersymmetric gauge theories and around a quarter of the nonsupersymmetric gauge theories do indeed flow into a scale invariant phase [34]. Furthermore, it was pointed out that in an appropriate limit the so-called higher dimensional (HEIDI) models, c.f. [35] and references therein, assume the unparticle spectral relation (3) and therefore reproduce the unparticle behavior. The role of the nontrivial anomalous dimension is mimicked through, possibly fractional, flat extra dimensions accessible to SM singlet fields. It is worth pointing out that these models are renormalizable for appropriate ranges of the anomalous dimension.

Unparticle like behavior as in the propagator (4) can be observed in well-known theories as well. For instance the resummation of logarithms due to the emission and absorption of the massless photon in QED leads to an electron propagator $S(p) \sim (0 = \not{p} + m)/(p^2 - m^2 + i0)^{1-\gamma}$ alike (4) [36]. The analogous case of jets in QCD was considered in Ref. [37] which can give rise to large anomalous dimensions in the case where the lower and higher jet scale are widely separated. Another example is the scale invariant and solvable two dimensional Thirring model, where the exact propagator $S(x) \sim \not{x}/(-x^2 + i0)^{1+\gamma}$, which is the two dimensional coordinate space version of the unparticle propagator (4), c.f. Sec. B of the appendix.

I will investigate the effects of the CP-even phase in the propagator through CP violation in B physics. I analyze leptonic decays of the type $B \to \tau\nu$, where the SM and beyond the standard models do *not* predict a CP asymmetry and $B_d \to D^+D^-$, which is further motivated by the unexpectedly large CP asymmetry measured by the Belle collaboration [38].

The effective Lagrangian is parametrized as follows[1]:

$$\mathcal{L}^{\rm eff} = \frac{\lambda^{UD}_{S(P)}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}(\bar{U}(\gamma_5)D)O_{\mathcal{U}} + \frac{\lambda^{\nu l}_{S(P)}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}(\bar{\nu}(\gamma_5)l)O_{\mathcal{U}}$$

where $U = (u, c, t)$, $D = (d, s, b)$, $\nu = (\nu_e, \nu_\mu, \nu_\tau)$, and $l = (e, \mu, \tau)$ are summations over the families. The weak (CP-odd) phases are parametrized as deviation from the phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{\rm ud}$ and analogously the leptons as deviations from the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U_{\nu l}$.

$$\lambda^{UD}_S = e^{i\phi^S_{\rm ud}}|\lambda^{UD}_S|, \qquad \phi^S_{\rm ud} = \arg[V_{\rm ud}] + \delta\phi^S_{\rm ud}. \quad (5)$$

The Lagrangian is a copy of the charged current sector of the SM with scalar instead of vectorial interactions. The unparticle therefore plays the role of the charged Higgs rather than the W boson. This allows me to apply, up to some level, the same tools in the unparticle sector as in the SM.

I shall also investigate how large the impact of unparticles can be without conflicting with branching ratio and indirect CP asymmetry predictions.

The paper is organized as follows. In Sec. II the leptonic decay $B \to \tau\nu$ and $B_d \to D^+D^-$ are investigated followed by a discussion of similar channels. In Sec. III I verify a constraint on CP violation from CPT-invariance; namely, that the partial sum of particle and antiparticle rates, with final states rescattering into each other, are equal. In Sec. IV I present the dimensional analysis of [7] adapted to a weak process. The paper ends with a summary and conclusions in Sec. V.

In this paper I shall adopt $\Lambda_{\mathcal{U}} = 1$ TeV as the scale of the IR fixed point. It is not difficult to rescale the results to

[1]The channel $B^+ \to \tau^+\nu$ is mediated by a $(P \times S) + (P \times P)$ structure whereas $B_d \to D^+D^-$ decays via a $(S \times P)$ interaction. The vector and axial couplings are discussed in the text above and I do not consider tensor couplings since they do not couple to single scalar particles.

a different scale; in the relevant places ($\Lambda_{\mathcal{U}}/1$ TeV) will be shown explicitly in the formulas.

II. A LEPTONIC AND A HADRONIC DECAY

A. Formulas for *CP* violation

In this subsection I am giving the formulas for *CP* violation used in the remaining part. I parametrize a decay amplitude with strong *CP*-even (strong) phases δ_i and *CP*-odd (weak) phases ϕ_i as

$$\bar{\mathcal{A}}(\bar{B} \to X) = A_1 e^{i\delta_1} e^{i\phi_1} + A_2 e^{i\delta_2} e^{i\phi_2}. \tag{6}$$

For a two body decay, as used in this paper, the branching ratio $\mathcal{B}$ and the *CP* averaged branching ratios are given by

$$\begin{aligned} \mathcal{B} &= \mathcal{B}^0 f_\Delta, \qquad f_\Delta = (1 + 2\Delta\cos(\phi_{12} + \delta_{12}) + \Delta^2), \\ \bar{\mathcal{B}} &= \mathcal{B}^0 \bar{f}_\Delta, \qquad \bar{f}_\Delta = (1 + 2\Delta\cos(\phi_{12})\cos(\delta_{12}) + \Delta^2), \\ \Delta &= \frac{|A_2|^2}{|A_1|^2}, \qquad \mathcal{B}^0 = \tau(\bar{B})\frac{1}{16\pi m_B^3}\lambda^{1/2}(m_B^2, m_C^2, m_D^2)|A_1|^2, \end{aligned} \tag{7}$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$, $\phi_{12} = \phi_1 - \phi_2$, and $\delta_{12} = \delta_1 - \delta_2$. The (time dependent) *CP* asymmetry for $B - d$ is defined as

$$\begin{aligned} \mathcal{A}_{CP}(B_d \to f) &\equiv \frac{\Gamma[\bar{B} \to f] - \Gamma[B \to \bar{f}]}{\Gamma[\bar{B} \to f] + \Gamma[B \to \bar{f}]} \\ &= S_f \sin(\Delta M t) - C_f \cos(\Delta M t), \end{aligned} \tag{8}$$

and the direct and the time dependent *CP* asymmetries C and S are then given by

$$\begin{aligned} C &= \frac{2\Delta}{\bar{f}_\Delta}\sin(\delta_{12})\sin(\phi_{12}), \\ S &= \xi_f \frac{-1}{\bar{f}_\Delta}(\sin(\phi_d + 2\phi_1) + 2\Delta\cos(\delta_{12})\sin(\phi_d + \phi_{12}) \\ &\quad + \Delta^2 \sin(\phi_d + 2\phi_2)), \end{aligned} \tag{9}$$

where ξ_f is the *CP* eigenvalue of the final state f.

B. $B^+ \to \tau^+ \nu$

In the standard model charged pseudoscalars decaying to a lepton and a neutrino are of particular interest because of their simple dependence on the pseudoscalar decay constant and the CKM matrix element.

The novelty with an additional unparticle amplitude is a *CP* asymmetry. I will investigate how large this asymmetry can be, remaining consistent with the branching ratio measurement. The unparticle amplitude is the same tree level process as in the SM where the unparticle simply replaces the W boson c.f. Fig. 1. The unparticle is propagating at the scale m_B and I therefore assume that the scale invariant sector extends down to the m_B scale.

The additional unparticle amplitude leads to a slight complication. As a matter of fact in experiment one does

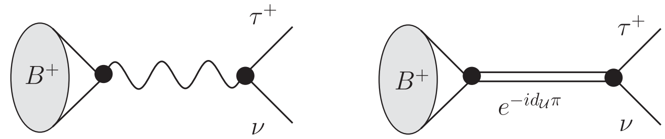

FIG. 1. (left) SM diagram for $B \to \tau\nu$. (right) Unparticle diagram with *CP*-odd phase $e^{id_{\mathcal{U}}\pi}$. The unparticle is denoted by a double line.

not observe the neutrino flavor but an inclusive measurement on the neutrino flavor is performed since the neutrinos are not detectable. In the case where there is only one amplitude, as in the SM, unitarity of the PMNS matrix hides this fact from the final formula. This is not the case for the unparticle amplitude and I shall therefore derive formulas for $B^+ \to \tau^+\nu \equiv \sum_l B^+ \to \tau^+\nu_l$ via $B^+ \to \tau^+\nu_l$. The amplitude is the sum of two incoherent terms of opposite parity in the final state[2]

$$\begin{aligned} \mathcal{A}(B^+ \to \tau^+\nu_l) &= \langle \tau\nu_l | \mathcal{L}_{\rm eff} | \bar{B} \rangle \\ &= i\frac{G_F}{\sqrt{2}} V_{\rm ub}^* U_{\tau\nu_l}^* f_B m_\tau \\ &\quad \times ([\bar{\nu}\tau](1 + \Delta^S_{\tau\nu_l} e^{-id_{\mathcal{U}}\pi} e^{-i\phi_l^S}) \\ &\quad + [\bar{\nu}\gamma_5\tau](1 + \Delta^P_{\tau\nu_l} e^{-id_{\mathcal{U}}\pi} e^{-i\phi_l^P})), \end{aligned} \tag{10}$$

where $\phi_l^D = \delta\phi^P_{\rm ub} - \delta\phi^D_{\tau\nu_l}$ for $D = (S, P)$, $l = (e, \mu, \tau)$. The B-meson decay constant is defined as $m_b\langle 0|\bar{b}i\gamma_5 u|B^+\rangle = f_B m_B^2$, where I neglect isospin breaking effects. The ratio of unparticle to SM amplitude is

$$\begin{aligned} \Delta^D_{\tau\nu_l} &\equiv \frac{|\lambda_D^{\tau\nu_l}|}{|U_{\tau\nu_l}|}\tilde{\Delta}_{\tau\nu} \equiv r_l^D \tilde{\Delta}_{\tau\nu}, \\ \tilde{\Delta}_{\tau\nu} &= \frac{|\lambda_P^{\rm ub}|}{|V_{\rm ub}|}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{m_B^2}{m_b m_\tau}\frac{(G_F/\sqrt{2})^{-1}}{m_B^2}\left(\frac{m_B^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}. \end{aligned} \tag{11}$$

The enhancement factor $\sqrt{2}(G_F m_B^2) \sim 5 \cdot 10^3$ is peculiar to tree level weak processes. I will now make a simplifying assumption in order to simplify the analysis. I impose the left-handed chirality on the unparticle sector, i.e., $\lambda_S^{\tau\nu_l} = \lambda_P^{\tau\nu_l}$ and ($\Delta_{\tau\nu} \equiv \Delta_{\tau\nu}^{(S,P)}$, $\delta\phi_{\tau\nu} \equiv \delta\phi^S_{\tau\nu}$, $r_l \equiv r_l^{(S,P)}$). This means that the amplitudes for opposite parity give the same result and this allows me to combine the two amplitudes into one. The branching fractions to a specific neutrino flavor final state are

[2]The amplitude (10) displays the famous helicity suppression in the SM due to its chiral structure which manifests itself in the fact that the amplitude is proportional to the lepton mass. For a pseudoscalar coupling, as in the charged Higgs model, or the one used here, the helicity suppression is relieved as can be inferred from Eq. (11).

$$\begin{aligned}\mathcal{B}(B^+ \to \tau^+ \nu_l) &= \mathcal{B}^{\rm SM}_{\tau\nu}|U_{\tau\nu_l}|^2 f_{\Delta_{\tau\nu_l}}\\ \bar{\mathcal{B}}(B^+ \to \tau^+ \nu_l) &= \mathcal{B}^{\rm SM}_{\tau\nu_l}|U_{\tau\nu_l}|^2 \bar{f}_{\Delta_{\tau\nu_l}},\end{aligned} \tag{12}$$

with f and $\bar{f}$ as in (7), $\phi_{12} = -\phi_l$, $\delta_{12} = -d_{\mathcal{U}}\pi$. The familiar SM branching fraction reads

$$\mathcal{B}^{\rm SM}_{\tau\nu} = \tau(B^+)\frac{G_F^2}{8\pi}|V_{\rm ub}|^2 f_B^2 m_B m_\tau^2\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tag{13}$$

and does not depend on the neutrino flavor. Please note that in the SM $\mathcal{B}^{\rm SM}_{\tau\nu} = \bar{\mathcal{B}}^{\rm SM}_{\tau\nu}$. The experimentally tractable or neutrino inclusive branching fraction is

$$\begin{aligned}\mathcal{B}(B^+ \to \tau^+ \nu) &= \sum_l \mathcal{B}(B^+ \to \tau^+ \nu_l)\\ &= \mathcal{B}^{\rm SM}_{\tau\nu}\sum_l |U_{\tau\nu_l}|^2(1 + 2r_l\tilde{\Delta}_{\tau\nu}\\ &\quad \times \cos(\phi_l + d_{\mathcal{U}}\pi) + (r_l\tilde{\Delta}_{\tau\nu})^2)\\ &= \mathcal{B}^{\rm SM}_{\tau\nu}(1 + \sum_l |U_{\tau\nu_l}|^2(2r_l\tilde{\Delta}_{\tau\nu}\\ &\quad \times \cos(\phi_l + d_{\mathcal{U}}\pi) + (r_l\tilde{\Delta}_{\tau\nu})^2).\end{aligned} \tag{14}$$

The formula could be further simplified if the r_l were independent of l, which I shall assume shortly below. The CP averaged branching fraction is

$$\begin{aligned}\bar{\mathcal{B}}(B^+ \to \tau^+ \nu) &\equiv \mathcal{B}^{\rm SM}_{\tau\nu}\mathcal{F}\\ &= \mathcal{B}^{\rm SM}_{\tau\nu}(1 + \sum_l |U_{\tau\nu_l}|^2(2r_l\tilde{\Delta}_{\tau\nu}\\ &\quad \times \cos(\phi_l)\cos(d_{\mathcal{U}}\pi) + (r_l\tilde{\Delta}_{\tau\nu})^2)).\end{aligned} \tag{15}$$

The CP asymmetry assumes the following form

$$\begin{aligned}\mathcal{A}_{CP}(\tau\nu) &\equiv \frac{\Gamma(B^- \to \tau^- \bar{\nu}) - \Gamma(B^+ \to \tau^+ \nu)}{\Gamma(B^- \to \tau^- \bar{\nu}) + \Gamma(B^+ \to \tau^+ \nu)}\\ &= \frac{2\tilde{\Delta}_{\tau\nu}}{\mathcal{F}}\sin(d_{\mathcal{U}}\pi)\sum_l \sin(\phi_l) r_l |U_{\tau\nu_l}|^2,\end{aligned} \tag{16}$$

where $\mathcal{F}$ is implicitly defined in Eq. (15). Let me note that the CP violation encountered here is proportional to $\sim{\rm Im}[V^*_{\rm ub}\lambda_P^{\rm ub}U^*_{\tau\nu}\lambda_S^{\tau\nu*}]$, which is hidden in the formula above, and is the product of two quadratic reparametrization invariants. The effect is entirely proportional to the sine of the phase difference between the CKM (PMNS) and the unparticle flavor sector and can therefore not occur in the SM. In order to do a qualitative assessment I shall study the case where there is no flavor dependent perturbation in the neutrino sector and therefore drop the label l. The formulas for the CP averaged branching ratio and the CP asymmetry then simplify to

$$\begin{aligned}\bar{\mathcal{B}}(B^+ \to \tau^+ \nu) &\to \mathcal{B}^{\rm SM}_{\tau\nu}(1 + 2\Delta_{\tau\nu}\cos(\phi)\\ &\qquad \times \cos(d_{\mathcal{U}}\pi) + \Delta^2_{\tau\nu})\\ &\overset{\phi=\pm\pi/2}{\to} \mathcal{B}^{\rm SM}_{\tau\nu}(1 + \Delta^2_{\tau\nu}),\\ \mathcal{A}_{CP}(\tau\nu) &\to \frac{2\Delta_{\tau\nu}\sin(\phi)\sin(d_{\mathcal{U}}\pi)}{1 + 2\Delta_{\tau\nu}\cos(\phi)\cos(d_{\mathcal{U}}\pi) + \Delta^2_{\tau\nu}}\\ &\overset{\phi=\pm\pi/2}{\to} \frac{\pm 2|\Delta_{\tau\nu}||\sin(d_{\mathcal{U}}\pi)|}{1 + \Delta^2_{\tau\nu}},\end{aligned} \tag{17}$$

where in the last step I have simplified the formulas further by setting the weak phase difference to 90(270°).[3] N.B. in the notation used in Eq. (16) $\mathcal{A}_{CP}(\tau\nu) = -C_{\tau\nu}$. This choice maximizes the CP violation for appropriate values for $\Delta_{\nu l}$. Before I am able to constrain the CP violation with the rate I have to give the theoretical and experimental results of the latter.

The following hadronic parameters, $\tau^{B^+} = 1.643$ ps, $f_B = (189 \pm 27)$ MeV a lattice average from [39], and $|V_{\rm ub}| = 3.64(24)\cdot 10^{-3}$ from the fit to the angles of the CKM triangle [39], are used to estimate the SM branching fraction

$$\mathcal{B}(B^+ \to \tau^+ \nu)^{\rm SM}_{\rm theory} = 83(40)\cdot 10^{-6}. \tag{18}$$

I have doubled the uncertainty due to $|V_{\rm ub}|$. This estimate has to be compared with the measurements at the B-factories

units 10^{-6}	$\bar{\mathcal{B}}(B^+ \to \tau^+\nu)$
BaBar[43](223M BB)	90(60)(10)
Belle[44](449M BB)	179(53)(48)
HFAG[40]	132(49) .

(19)

1. Weak phase $\phi = 90(270)°$, flavor independent perturbation neutrino sector

In Fig. 2 (left) the branching fraction (17) is plotted as a function of $\Delta_{\tau\nu}$ with uncertainty taken from the SM estimate (18) at $\Delta_{\tau\nu} = 0$. The shaded band corresponds to the HFAG bounds in Eq. (19). The CP asymmetry is plotted to the right of that figure. The branching ratio does not set limits on the amount of CP violation, demanding the uncertainty bands to be tangent at worst $|\Delta_{\tau\nu}| < 1.8$. Even in the case where the HFAG and theory uncertainty are halved, the value $|\Delta_{\tau\nu}| = 1$, at which the CP asymmetry is maximal, is still consistent.

[3] N.B. $\sin(d_{\mathcal{U}}\pi) < 0$ for $1 < d_{\mathcal{U}} < 2$ as assumed throughout this paper. This is the reason for the absolute values in the equation above.

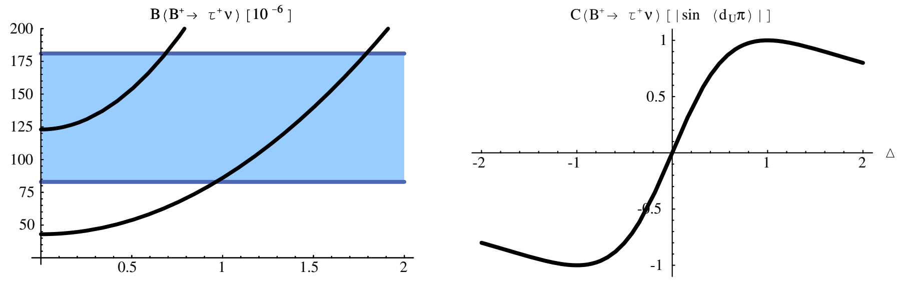

FIG. 2 (color online). A weak phase difference $\phi = 90(270)^\circ$ is assumed here for $\Delta_{\nu\mu}$ positive(negative). (left) Branching fraction (17) as a function of $\Delta_{\tau\nu}$. The black bands correspond to the SM estimate (18) at $\Delta_{\tau\nu} = 0$. The shaded band corresponds to the HFAG bounds in Eq. (19). (right) The *CP* asymmetry as a function of $\Delta_{\tau\nu}$ in units of $|\sin(d_{\mathcal{U}}\pi)|$. The scale $\Lambda_{\mathcal{U}} = 1$ TeV is chosen here. N.B. in the notation used in Eq. (16) $\mathcal{A}_{CP}(\tau\nu) = -C_{\tau\nu}$.

2. Weak phase $\phi \neq 90(270)^\circ$, flavor independent perturbation neutrino sector

In this subsection I shall repeat the analysis for a general weak phase difference and show two dimensional plots in the variables $(\phi, d_{\mathcal{U}})$ for different ratios of effective couplings. The quantity $\Delta_{\tau\nu}$ (11), used in the previous paragraph, depends on the ratio of effective coupling and scaling dimension as follows:

$$\Delta_{\tau\nu} = \rho_{\tau\nu}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{m_B^2}{m_b m_\tau}\frac{(G_F/\sqrt{2})^{-1}}{m_B^2}\left(\frac{m_B^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}$$
$$\simeq 2300\left(2.8\cdot 10^{-5}\frac{\Lambda_{\mathcal{U}}}{1\ \text{TeV}}\right)^{d_{\mathcal{U}}-1}\frac{A_{d_{\mathcal{U}}}}{\sin(d_{\mathcal{U}}\pi)}\rho_{\tau\nu},$$

where

$$\rho_{\tau\nu} \equiv \frac{|\lambda_P^{\text{ub}}\lambda_{(S,P)}^{\tau\nu}|}{|V_{\text{ub}}U_{\tau\nu}|}. \qquad (20)$$

In Fig. 3 (right) the *CP* asymmetry $C_{\tau\nu}$ is plotted as a function of $(\phi, d_{\mathcal{U}})$ for $\rho_{\tau\nu} = (10^0, 10^{-2}, 10^{-4})$. The pattern is clearly regular and the condition for a large asymmetry is $|\Delta_{\tau\nu}| \sim 1$. For smaller values of $\rho_{\tau\nu}$ the amount of possible *CP* violation is decreasing because the condition mentioned above cannot be satisfied. The constraint on the branching fraction, Fig. 3 (left), is defined by the following acceptance function:

$$A(d_{\mathcal{U}}, \phi, \rho) = (1 - r(d_{\mathcal{U}}, \phi, \rho))\Theta(1 - r(d_{\mathcal{U}}, \phi, \rho)),$$
$$r(d_{\mathcal{U}}, \phi, \rho) = \frac{1}{\Delta B}|\mathcal{B}_{\tau\nu}^{\text{SM}}(1 + 2\Delta_{\tau\nu}\cos(\phi)\cos(d_{\mathcal{U}}\pi) + \Delta_{\tau\nu}^2) - \mathcal{B}^{\text{HFAG}}|, \qquad (21)$$

for $\mathcal{B}_{\tau\nu}^{\text{SM}} = 83\cdot 10^{-6}$, $\mathcal{B}^{\text{HFAG}} = 132\cdot 10^{-6}$, and for the quantity ΔB I add the uncertainty of the SM prediction and the HFAG value linearly to $\Delta B \simeq 80\cdot 10^{-6}$. This function assumes values between 0 and 1, where 1 signifies maximal agreement and 0 means that the point is excluded; or in other words I consider predictions with a deviation larger than ΔB as excluded.

For smaller values of $\rho_{\tau\nu}$ the linear term for the branching ratio in Eq. (17) becomes dominant and a regular pattern in $\cos(\phi)$ emerges. Note that since the predicted branching fraction is lower than the central value from experiment, the weak angle $\phi = 180^\circ$ is currently disfavored since it would lower the theory prediction even more.

C. Discussion and remarks on $B \to \mu\nu$, $D \to \mu\nu$, $B_s \to \mu^+\mu^-$, etc.

It has been seen that applying the unparticle scenario to the leptonic decay $B \to \tau\nu$ leads to *CP* violation. There is no experimental data available that gives both the negative and positive charged semileptonic decay rates, i.e., quotes (bounds) on *CP* asymmetry in a semileptonic decay.

The current data on $B \to \tau\nu$ do not allow one to set bounds on the amount of possible *CP* violation. The amount of events at *BABAR* and Belle are of the order $\sim$20. An improvement in theory, in particular, on the *B*-meson decay constant, and the large statistics of a Super *B*-factory would of course improve the situation. Unfortunately the decay $B^+ \to \tau^+\nu$ will not be possible or competitive at CERN LHCb because of the neutrino final state and the intricacies in the τ detection, whether $D(D_s) \to (\tau, \mu)\nu$ decays are possible at LHCb is currently under investigation.

I shall comment on other leptonic modes. They are all described by the same formula (13) for $B \to \tau\nu$ with obvious substitutions for V_{ub}, f_B, m_B, and m_τ. I may also consider the *D*-decays assuming that the scale invariant sector extends to $\sim$2 GeV. The decay $D^+ \to \mu^+\nu$ is measured by CLEO [40]; the $\sim$50 events lead to a 13% accuracy. The decay constant $f_{D^+} = 220(20)$ MeV is taken as an average value of theory determinations from the table in [40] and $|V_{\text{cd}}| = 0.227$ [41]. The Cabibbo allowed decays $D_s^+ \to \mu^+\nu$ are measured as well [41], although with less precision. The decay constant $f_{D_s^+} = 264(36)$ is obtained from an average of $f_{D_s^+}/f_{D^+} = 1.20(5)$ of the table in [40] and $|V_{\text{cs}}| = 0.957(17)(93)$ [41]. A summary of the experimental [41] and theory predictions is

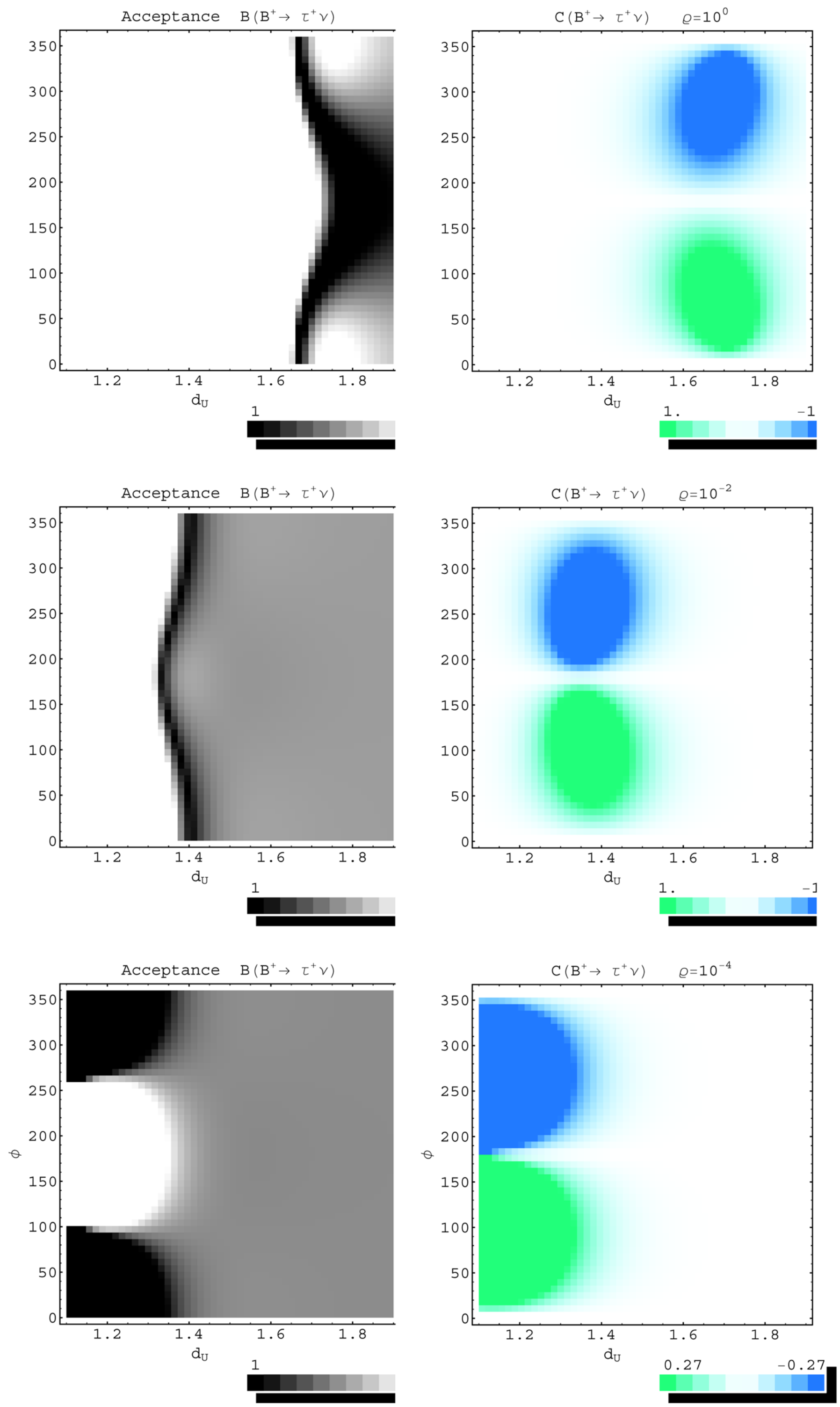

FIG. 3 (color online). A horizontal line of figures corresponds to different fractions of effective couplings $\rho = 10^{0,-2,-4}$ as defined in (21). (left) Constraints on the $(\phi, d_{\mathcal{U}})$ parameter space from the branching fraction. The values in the dark regions are allowed whereas white ones are excluded, c.f. Eq. (21) for a more details. (right) The CP asymmetry as a function of $(\phi, d_{\mathcal{U}})$. The scale $\Lambda_{\mathcal{U}} = 1$ TeV is chosen here.

	$B \to \tau\nu$	$B \to \mu\nu$	$B \to e\nu$
Experiment	$132(49) \cdot 10^{-6}$	$<17 \cdot 10^{-7}$	$<9.8 \cdot 10^{-6}$
Theory	$83(50\%) \cdot 10^{-6}$	$3.7(50\%) \cdot 10^{-7}$	$8.4(50\%) \cdot 10^{-12}$
	$D \to \tau\nu$	$D \to \mu\nu$	$D \to e\nu$
Experiment	$<2.1 \cdot 10^{-3}$	$4.4(7) \cdot 10^{-4}$	$<2.4 \cdot 10^{-5}$
Theory	$1.1(20\%) \cdot 10^{-3}$	$4.3(20\%) \cdot 10^{-4}$	$1.0(20\%) \cdot 10^{-10}$
	$D_s \to \tau\nu$	$D_s \to \mu\nu$	$D_s \to e\nu$
Experiment	$6.4(15) \cdot 10^{-2}$	$6.3(18) \cdot 10^{-3}$	$\cdots$
Theory	$5.5(30\%) \cdot 10^{-2}$	$5.7(30\%) \cdot 10^{-3}$	$1.3(30\%) \cdot 10^{-7}$

The B decays are predicted to 50% due to uncertainties in f_B and $|V_{\rm ub}|$, whereas the $D(D_s)$ decays have a lower uncertainty 20(30)% due to $f_D(f_{D_s})$. The helicity suppression in the SM is apparent from the table.

Repeating the analysis for $D^+ \to \mu^+\nu$, as shown in Fig. 2, I obtain that $|\Delta_{D\to\mu\nu}| < 0.65$ which still allows for a rather large *CP* asymmetry, $|C_{D\to\mu\nu}| < 0.9$.

The prediction of these modes in the SM is solid and a significant deviation would be a clear hint for new physics. In particular one expects larger rates in models where the helicity suppression is relieved. An example is the charged Higgs or the effective Lagrangian used in this paper. The charged Higgs does not predict a significant *CP* asymmetry whereas in unparticle models it is possible and therefore a *CP* asymmetry could be used to discriminate between the models.

I would also like to mention the decay $K^+ \to \mu^+\nu$; the KLOE collaboration reports ~860 000 events and a branching ratio $\mathcal{B}(K^+ \to \mu^+\nu(\gamma)) = 0.6366(9)(15)$ [42]. On the one hand it seems unreasonable that the scale invariant sector could extend to ~500 MeV but on the other this channel has the largest statistics. If one assumes that theory predicts the rate to 5% (10%) this would roughly bound $|\Delta_{K\to\mu\nu}| < 20(30)$ and the *CP* asymmetry to $|C_{K\to\mu\nu}| < 0.4(0.55)$.

Finally a comment about $B_{(d,s)} \to \mu^+\mu^-$. This channel is rare since it is a flavor-changing neutral decay further suppressed by the coupling of the Z and the helicity of final states, $\mathcal{B}(B_{(d,s)} \to \mu^+\mu^-)^{\rm SM} \sim 10^{-10}(10^{-8})$. The branching ratio is not yet measured; the bounds are about one and a half orders of magnitude away from the SM prediction. An analysis along the lines of $B \to \tau\nu$ does not make sense since there are no direct constraints in that channel. A possibility would be to combine it with constraints from $\Delta M_{(d,s)}$, which are measured, as advocated in Ref. [15].

D. $B_d \to D^+D^-$; scale invariant sector at 2 GeV

The decay $B_d \to D^+D^-$ corresponds to a $b \to \bar{c}cd$ transition at the quark level and is color allowed. It has the same quark level transition as $B_d \to J/\Psi\pi_0$ but two complications arise as compared to the latter. First, since it is color allowed it receives sizable contributions from a gluonic penguin [43] and second the final states combine into a sum of isospin $I = 0$ and $I = 1$ waves which have in general different final state interaction phases. Ultimately I will neglect the penguins in my analysis, to be discussed below. My motivation to investigate the $B_d \to D^+D^-$ is driven by the measurement of a large *CP* asymmetry by the Belle collaboration [38]. The SM expectation is $C^{\rm SM}_{D^+D^-} \simeq -0.05$.

	$C_{D^+D^-}$	$S_{D^+D^-}$
BaBar[48](364M BB)	0.11(22)(07)	−0.54(34)(06)
Belle[49](535M BB)	−0.91(23)(06)	−1.13(37)(09)
HFAG	−0.37(17)	−0.75(26)

(22)

It has to be said that the Belle result is somewhat moderated by a significantly lower value from *BABAR* [44] with opposite sign. Note that the central values from Belle also violate the general bound $C^2 + S^2 \leq 1$.

It shall be my goal to see how large a *CP* asymmetry $C_{D^+D^-}$ the unparticles scenario can generate and still be consistent with the branching fraction and the time dependent *CP* asymmetry.

In my analysis the unparticle will replace the W in the tree level amplitude in, c.f. Fig. 4 (left). I therefore assume that the scale invariant sector extends to the D-meson scale ~2 GeV.

I shall first reconsider the situation in the SM before I move on to the unparticles. Writing the amplitude as the sum of the tree and penguin topology

$$\mathcal{A}(B_d \to D^+D^-) = \mathcal{A}_T + \mathcal{A}_P = \mathcal{A}_T(1 - e^{i\delta_{\rm PT}}e^{i\gamma}r_{\rm PT}), \qquad (23)$$

the ratio of penguin to tree amplitude $r_{\rm PT}$ can then be estimated by the Bander-Silverman-Soni mechanism

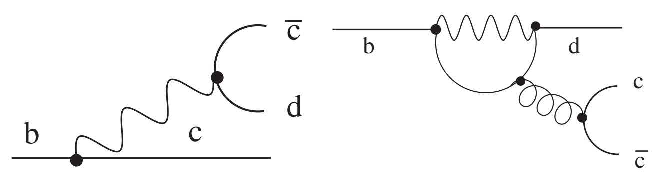

FIG. 4. $b \to d\bar{c}c$ (left) tree diagram, (right) penguin diagram

[43], c.f. [45] or [46] for an updated analysis,

$$\Delta_{\rm PT} \simeq 0.08, \qquad \delta_{\rm PT} \simeq 205°. \tag{24}$$

This allows one to obtain the asymmetries from (9),

$$C^{\rm SM}_{D^+D^-} \simeq -0.05, \qquad S^{\rm SM}_{D^+D^-} \simeq -0.78. \tag{25}$$

Comparing with the experimental results (22) I infer that the SM is in good agreement with the time dependent CP asymmetry $S_{D^+D^-}$. The direct CP asymmetry $C_{D^+D^-} \simeq 0.05$ is about 2 standard deviations lower than the HFAG value 0.37(17). In view of the nonconsistency of the two measurements it is certainly wise to wait for updates from the B-factories. I will in the following neglect the penguin contribution in regard to its moderate size (24) in the SM. I will also neglect the "unparticle penguin." The ratio of the unparticle penguin amplitude to the unparticle amplitude is expected to be of the same size as in the SM, unless the up-type transition is enhanced by the effective couplings. I am therefore implicitly assuming that $|\lambda^{\rm ub}_{(S,P)}\lambda^{\rm ud}_{(S,P)}| \lesssim |\lambda^{\rm cb}_{(S,P)}\lambda^{\rm cd}_{(S,P)}|$.

I will describe the amplitude $B_d \to D^+D^-$ within the naive factorization approximation. Naive factorization describes color allowed modes (topology as in Fig. 4 to the left) like $B \to \pi^+\pi^+$ and $B_d \to D^+\pi^-$ with at least one fast or light meson with an accuracy of around 10–20% level. For $B_d \to D^+D^-$, factorization in general and naive factorization are not expected to hold. The overlap of the emitted D^+-meson with the $B_d \to D^-$ transition is expected to be relatively large. However it is empirically observed that naive factorization still works reasonably well. I shall account for final state interactions, not included in naive factorization, by an isospin analysis of $\bar{B}_d \to D^+D^-$ [45] adapted from $(B, K) \to \pi\pi$. Two out of the three rates from the isospin triangle have been measured,

$$\begin{aligned} \bar{\mathcal{B}}(B_d \to D^+D^-) &= 1.9(6)\cdot 10^{-4}, \\ \bar{\mathcal{B}}(B^+ \to \bar{D}^0D^+) &= 4.8(1)\cdot 10^{-4}, \end{aligned} \tag{26}$$

which allow one to extract

$$\cos((\delta_1 - \delta_0)/2) \simeq \pm 0.63(15), \tag{27}$$

the cosine of the strong phase difference of the isospin 0 and 1 up to a sign. The amplitude for $B_d \to D^+D^-$ is proportional to the latter,

$$\begin{aligned} \mathcal{A}(B_d \to D^+D^-) = {}& \frac{G_F}{\sqrt{2}} V^*_{\rm cb}V_{\rm cd} a_1 f_D((m_B^2 - m_D^2) f_+^{BD}(m_D^2) \\ & + m_D^2 f_-^{BD}(m_D^2)) \cos((\delta_1 - \delta_0)/2) \\ & \times e^{i(\delta_1-\delta_0)/2} \\ \equiv {}& \mathcal{A}^{\rm SM}_{\rm DD}, \end{aligned} \tag{28}$$

where $a_1 = C_2 + C_1/3 \simeq 1$ is the color allowed combination of tree level Wilson coefficients and the D-meson decay constant is defined as $m_c\langle 0|\bar{c}i\gamma_5 d|D^-\rangle = f_D m_D^2$, where I neglect effects due to isospin breaking. The $B \to D$ form factor can be parametrized by use of Lorentz covariance as

$$\langle D|\bar{b}\gamma_\mu c|B\rangle = f_+^{BD}(q^2)(p_B + p_D)_\mu + f_-^{BD}(q^2) q_\mu, \tag{29}$$

with momentum transfer $q = p_B - p_D$. The form factors are related to the famous Isgur-Wise function $f_+^{BD}(q^2) = \sqrt{\frac{m_B+m_D}{4m_Bm_D}}\xi(w)$, $f_-^{BD}(q^2) = -\sqrt{\frac{m_B-m_D}{4m_Bm_D}}\xi(w)$ in the heavy quark limit. Here $w = v\cdot v' = (m_B^2 + m_D^2 - q^2)/(2m_Bm_D)$. Whereas the normalization of the Isgur-Wise function $\xi(1) = 1$ follows from charge normalization in the heavy quark limit the values around maximum recoil are much less known. I shall take the value $f_+^{BD}(0) = 0.54$ from [47] and scale it up to $q^2 = m_D^2$ by use of a single pole model [48], $\xi(w) \sim \sqrt{2/(w+1)}(w_{\max} - w(m^2_{B_c^*}))/(w - w(m^2_{B_c^*}))$. The B_c^*-meson has the correct quantum numbers $J^P = 1^+$ and its mass is the same in the heavy quark limit as $m_{B_c} = 6.29$ GeV [41]. I obtain $f_+(m_D^2) \simeq 0.7$. With $f_D = 220$ MeV I get

$$\bar{\mathcal{B}}(B_d \to D^+D^-)^{\rm SM}_{\rm theory} = 1.7(10)\cdot 10^{-4} \tag{30}$$

as a theory estimate, where the bulk of the uncertainty quoted is due to the isospin final state interaction phases. This estimate has to be compared to the experimental value [41]

$$\bar{\mathcal{B}}(B_d \to D^+D^-)_{\rm PDG} = 1.9(6)\cdot 10^{-4}. \tag{31}$$

The agreement seems accidentally good in regard to the approximations made.

As in the previous section I parametrize the amplitude

$$\mathcal{A}\,(B_d \to D^+D^-) \equiv \mathcal{A}^{\rm SM}_{\rm DD}(1 + \Delta_{\rm DD} e^{-i\phi_{\mathcal{U}}} e^{-i\phi}) \tag{32}$$

with $\mathcal{A}^{\rm SM}_{\rm DD}$ as given in (28) and relative weak phase $\phi \equiv \delta\phi_{\rm cb} - \delta\phi_{\rm cd}$. The ratio of SM to unparticle amplitude is

$$\begin{aligned} \Delta_{\rm DD} = {}& \frac{|\lambda_S^{\rm cb}\lambda_P^{\rm cd}|}{|V_{\rm cb}V_{\rm cd}|}\frac{1}{a_1}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{m_D^2}{m_c(m_b - m_c)} \\ & \times \frac{(G_F/\sqrt{2})^{-1}}{m_D^2}\left(\frac{m_D^2}{\Lambda^2_{\mathcal{U}}}\right)^{d_{\mathcal{U}}-1}. \end{aligned} \tag{33}$$

As in $B \to \tau nu$ there is an enhancement factor $\sqrt{2}(G_F m_D^2)^{-1} \sim 3\cdot 10^4$ which is peculiar to weak tree level processes. Note that, unlike for $B \to \tau\nu$, the negative parity of the D-meson selects only the λ_P^{cd} coupling in the final vertex. The observables are obtained from Eq. (9) with $\xi_{D^+D^-} = 1$, $\phi_d = 2\beta$, $\phi_1 = 0$, $\phi_2 = -\phi$, and $\delta_{12} = d_U\pi$:

$$
\begin{aligned}
\mathcal{B}_{\rm DD} &= \mathcal{B}^{\rm SM}_{\rm DD} f_{\Delta_{\rm DD}}, \qquad \bar{\mathcal{B}}_{\rm DD} = \mathcal{B}^{\rm SM}_{\rm DD} \bar{f}_{\Delta_{\rm DD}}, \\
C_{\rm DD} &= \frac{2\Delta_{\rm DD}}{\bar{f}_{\Delta_{\rm DD}}} \sin[\phi]\sin[d_{\mathcal{U}}\pi], \\
S_{\rm DD} &= \frac{-1}{\bar{f}_{\Delta_{\rm DD}}}(\sin[2\beta] + 2\Delta_{\rm DD}\cos[d_{\mathcal{U}}\pi]\sin[2\beta-\phi] \\
&\quad + \Delta^2_{\rm DD}\sin[2\beta - 2\phi]),
\end{aligned} \tag{34}
$$

and

$$
\mathcal{B}^{\rm SM}_{\rm DD} = \tau(B_d)\frac{G_F^2}{32\pi m_B} a_1^2 f_D^2((m_B^2 - m_D^2) f_+^{BD}(m_D^2) + m_D^2 f_-^{BD}(m_D^2))^2 |V^*_{\rm cb}V_{\rm cd}|^2. \tag{35}
$$

1. Weak phase $\phi = 90(270)°$

In order to look for maximal *CP* violation one may again set the weak phase difference to 90(270°) in the formulas in Eq. (34). In Fig. 5 (left) the branching fraction is plotted as a function of $\Delta_{\rm DD}$ with uncertainty taken from the SM estimate (30) at $\Delta_{\rm DD} = 0$. The shaded band corresponds to the HFAG bounds in Eq. (31). The new feature as compared to the $B \to \tau\nu$ analysis is the constraint from $S_{D^+D^-}$ which corresponds to the figure in the middle. The *CP* asymmetry is plotted to the right of that figure. Once more the branching ratio does not set limits on the amounts of *CP* violation, in fact the uncertainties are very similar as in $B \to \tau\nu$. Demanding the uncertainty bands to be tangential at worst results in $|\Delta_{\rm DD}| < 1.5$. The constraints from $S_{D^+D^-}$ do depend on the scaling dimension. The parameter $d_{\mathcal{U}} = 1.1$, for example, seems slightly disfavored as compared to the value $d_{\mathcal{U}} = 1.9$.

2. Weak phase $\phi \neq 90(270)°$

I investigate the two dimensional parameter space $(\phi, d_{\mathcal{U}})$ for different ratios of effective couplings. These quantities relate to $\Delta_{\rm DD}$ (33) as follows:

$$
\begin{aligned}
\Delta_{\rm DD} &= \rho_{\rm DD}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{m_D^2}{m_c(m_b - m_c)}\frac{(G_F/\sqrt{2})^{-1}}{m_D^2} \\
&\quad \times \left(\frac{m_D^2}{\Lambda^2_{\mathcal{U}}}\right)^{d_{\mathcal{U}}-1} \\
&\simeq 17\cdot 10^3\left(3.5\cdot 10^{-6}\frac{\Lambda_{\mathcal{U}}}{1\ \mathrm{TeV}}\right)^{d_{\mathcal{U}}-1}\frac{A_{d_{\mathcal{U}}}}{\sin(d_{\mathcal{U}}\pi)}\rho_{\rm DD},
\end{aligned}
$$

where

$$
\rho_{\rm DD} \equiv \frac{|\lambda_S^{\rm cb}\lambda_P^{\rm cd}|}{|V_{\rm cb}V_{\rm cd}|}. \tag{36}
$$

In Fig. 6 (right) *CP* asymmetry $C_{D^+D^-}$ is plotted as a function $(\phi, d_{\mathcal{U}})$ for $\rho_{\rm DD} = (10^{0,-2,-4})$. The pattern is very similar in its form to $B \to \tau\nu$. A large asymmetry is obtained for $|\Delta_{\tau\nu}| \sim 1$, which cannot be attained for smaller values $\rho_{\rm DD}$. The constraint on the branching fraction, Fig. 3 (left), and the *CP* asymmetry $S_{D^+D^-}$ are evaluated with the same kind of acceptance function as for $B \to \tau\nu$ (21). The corresponding values for the *CP* asymmetry are $S^{\rm SM}_{D^+D^-} = -\sin(2\beta) = 0.69$, $S^{\rm HFAG} = -0.75$ and $\Delta S = 0.52$ corresponds to 2 standard deviations. The values for the branching fraction are $\mathcal{B}^{\rm SM}_{D^+D^-} = 1.7\cdot 10^{-4}$, $\mathcal{B}^{\rm HFAG}_{D^+D^-} = 1.9\cdot 10^{-4}$ and $\Delta\mathcal{B} = 1.6$ corresponds to linear addition of the theoretical and experimental uncertainty.

A qualitative result that can be inferred from Fig. 3 is that the parameter space of a large positive *CP* asymmetry $C_{D^+D^-}$ is disfavored by the bounds from the $S_{D^+D^-}$. A negative $C_{D^+D^-}$ demands a weak phase $\phi < 180°$ and then the linear and quadratic terms in $S_{D^+D^-}$ add constructively and are in conflict with the consistent result between the SM and experiment in this observable. As for $B \to \tau\nu$ for small $\rho_{\rm DD}$ the linear terms dominate the quadratic ones and a regular pattern in $\cos(\phi)$ and $\sin(2\beta - \phi)$ emerges.

E. Discussion of $B_d \to D^+D^-$ and remarks on U-spin and color related channels

A large *CP* asymmetry $C_{D^+D^-}$ would be a rather puzzling fact, as for instance discussed in Ref. [49]. One is led to suspect that the gluonic penguin $B_d \to D\bar{q}q$ with $q = c$

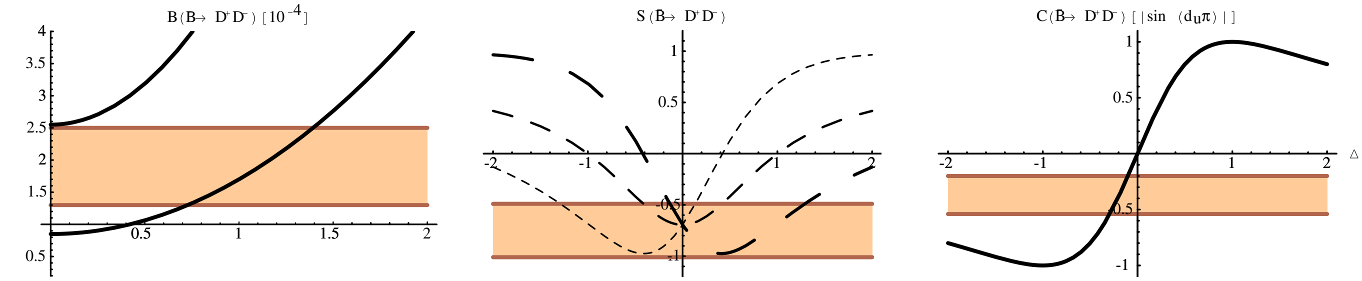

FIG. 5 (color online). A weak phase difference $\phi = 90(270)°$ is assumed here for $\Delta_{\rm DD}$ positive (negative). (left) Branching fraction (34) as a function of $\Delta_{\rm DD}$. The black bands correspond to the SM estimate (30) at $\Delta_{\rm DD} = 0$. The shaded band corresponds to the HFAG bounds in Eq. (31). (middle) Time dependent *CP* asymmetry $S_{D^+D^-}$ as a function of $\Delta_{\rm DD}$ for $d_{\mathcal{U}} = 1.1, 1.5, 1.9$ where the dashes get shorter for larger values of $d_{\mathcal{U}}$. The interpolation between those values is fairly smooth. (right) The *CP* asymmetry as a function of $\Delta_{\rm DD}$ in units of $|\sin(d_{\mathcal{U}}\pi)|$.

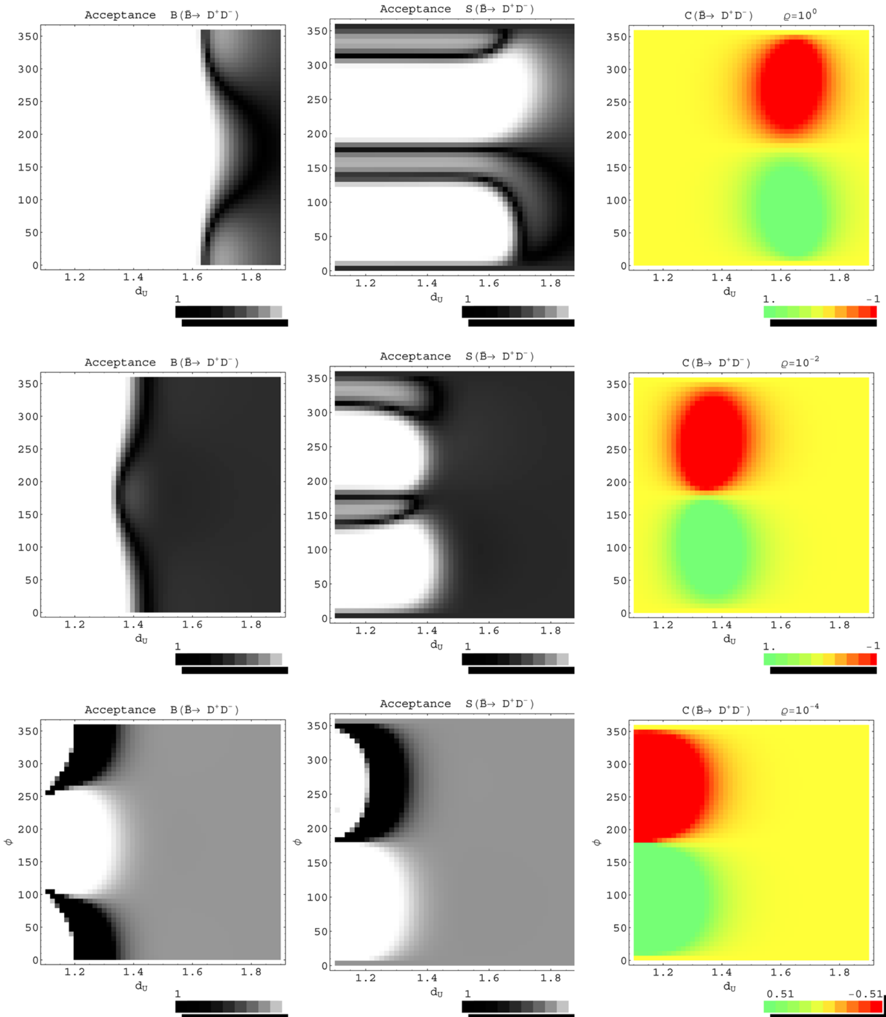

FIG. 6 (color online). The observables with fractions of effective couplings $\rho = 10^{0,-2,-4}$, as defined in (36), are plotted from the top of the figure to the bottom. Constraints on the $(\phi, d_{\mathcal{U}})$ parameter space from (left) the branching fraction and (middle) the CP asymmetry $S_{D^+D^-}$ (middle). The values in the dark regions are allowed whereas white ones are disfavored, c.f. text for more details. (right) The CP asymmetry $C_{D^+D^-}$ as a function of $(\phi, d_{\mathcal{U}})$. The scale $\Lambda_{\mathcal{U}} = 1$ TeV is chosen here.

might be enhanced by new physics. This scenario would or should lead to enhanced penguin amplitudes for $q = (u, d, s)$ as well and enter $B_d \to (\pi\pi, KK)$ in disagreement with the B-factory data.

It has been seen that an unparticle scenario can lead, for appropriate parameters, to enhanced CP violation. One might wonder whether similar results should not also show up in U-spin ($s \leftrightarrow d$) and color related channels. The plots in Fig. 6 indicate that the CP asymmetry S in general does not necessarily receive large contributions. This can be inferred from Eq. (34) or by noting that the unparticles just contribute to a large SM background from $\sin(2\beta)$. I shall therefore focus on the CP asymmetry C. Let me note however that the situation for B_s decays is different since the mixing phase $\phi_s \simeq 0$ ($\phi_d \simeq 2\beta$) in the SM and the contributions of unparticles would be not be shielded by a large SM value.

The color related or color suppressed channel of $B_d \to D^+D^-$ is $B \to J/\Psi\pi_0$. The CP asymmetry has been measured $C_{J/\Psi\pi_0} = -0.11(20)$ [41], which is not conclusive in regard to its size. In the color suppressed modes the nonfactorizable contributions are enhanced due to different combinations of Wilson coefficients (typically ∼2–3 larger than the factorizable amplitude) and have large strong phases. On the practical side it is harder to estimate them reliably in the SM and even more in the unparticle scenario, where the unparticle is dynamical as compared to the contracted W-boson propagator in the SM. The strong phases and the different hierarchy between factorizable and nonfactorizable contributions in the SM and the unparticle scenario[4] make it impossible to draw conclusions without explicit calculations.

[4]A parametric estimate gives that the nonfactorizable contributions in the unparticle scenario are suppressed by a factor $2m_D^2/(m_{J/\Psi}^2 + m_B^2) \sim 0.2$ as compared to the SM.

The U-spin related transitions $b \to \bar{c}cs$ are CKM enhanced and therefore statistics should make them more attractive. In principle there is no reason that generic new physics respects the CKM hierarchy and U-spin. In the unparticle scenario there is no principle that dictates a CKM-like hierarchy in the coefficients $\lambda_{\rm ud}$ in the effective Lagrangian (5). Therefore they are not necessarily of major concern. Let me nevertheless discuss them. The gold plated decay $B \to J/\Psi K_s$ is also color-suppressed. The measurement of the *CP* asymmetry $S_{J/\Psi K_s} = \sin(2\beta)$ has allowed determination of the angle β in the SM, whereas the *CP* asymmetry $C_{J/\Psi K_s} = 0$ is consistent with experiment. This mode is highly consistent with the SM or more precisely with one dominant amplitude. The branching fraction of the color allowed decay $B_d \to D^+ D_s^-$ has been measured but no *CP* asymmetry has been reported, presumably because it does not exhibit *CP* violation in mixing. If the Belle *CP* asymmetry in $C_{D^+D^-}$ gets confirmed a look at the *CP* asymmetry appears mandatory.

In summary the most interesting parallel channel is probably $B_d \to J/\Psi\pi 0$, and the improvement of the measurement in $C_{J/\Psi\pi_0}$ should be watched along with $C_{D^+D^-}$. In the scenario I described I would generically expect a large *CP* asymmetry $C_{D^+D^-}$ to be accompanied by a large asymmetry in $C_{J/\Psi\pi_0}$. It is a serious point of criticism, but on the other hand the experimental result is not conclusive and in theory there might be cancellations between the strong phase $e^{id_{\mathcal{U}}\pi}$ and the phase from the nonfactorizable interactions. The time dependent *CP* asymmetries *S* are shielded by large SM backgrounds for the B_d-meson, whereas in the B_s system the SM expectation is $S \sim 0$ in many cases (e.g., $B_s \to J/\Psi\phi$) and the unparticle scenario might reveal itself.

It has been seen that *CP* violation in $B_d \to D^+D^-$ and $B \to \tau\nu$ can be maximal in the unparticle scenario. After this phenomenological section I shall elaborate on whether a *CP* asymmetry in leptonic decays is possible. Thereafter I shall turn to the question of whether the scale invariance at the TeV-scale or near scale invariance could still be effective at heavy flavor scales ~5 GeV.

III. CONSTRAINTS FROM *CPT* ON (NEW) *CP* VIOLATION

The invariance under *CPT* symmetry imposes constraints on the amount of *CP* violation; it enforces the equality of the partial sum of rates of particles and antiparticles, to be made more precise below. Neither the SM nor any well-known new physics model predict *CP* violation in leptonic decays such as $B \to \tau\nu$ studied in this paper. The aim of this section is to verify explicitly whether the *CP* violation is consistent with the constraints from *CPT*.

Let me note that I expect that *CPT* invariance holds for a theory with a local Hermitian Lagrangian such as in Eq. (5). The explicit verification of *CPT* invariance demands that $\Theta\mathcal{L}(x)\Theta^{-1} = \mathcal{L}(-x)^\dagger = \mathcal{L}(-x)$, where $\Theta = CPT$ denotes the combined *CPT* transformation. The Lagrangian (5) fulfills this requirement provided that $\Theta O_{\mathcal{U}}(x)\Theta^{-1} = O_{\mathcal{U}}^\dagger(-x)$, which I cannot verify explicitly since I do not have equations of motion or a Lagrangian for the unparticle field at hand from where I would infer the transformation under *C*, *P*, and *T*. There also exists a general proof of the *CPT* theorem in the framework of axiomatic field theory [50] based on general principles and axioms such as Lorentz invariance, uniqueness of the vacuum, and causality of field commutators. The unparticle field does obey causality; the commutator for a scale invariant field can be calculated as a function of the its scaling dimension c.f. the appendix.

Summarizing, although I am not able to verify *CPT* invariance I at the same time do not find any indications why it should be violated.

It is well-known that *CPT* symmetry implies equality of the decay rates of particles and antiparticles. In practice there is even a stronger consequence, e.g., [51–53] or [54] where it was applied to charmless *B*-decays. The final state particles can be divided into subclasses of particles which rescatter into each other. It is a fact that the sum of the partial rates of these subclasses for a particle and its antiparticle must be the same. This can be inferred from the following relationship [52] between the weak decay amplitudes of a *B*-meson and its antiparticle $\bar{B}$ to a final state f_x:

$$\langle \bar{f}_x|H_{\rm decay}|\bar{B}\rangle^* = \sum_i \langle f_x|S^\dagger|f_i\rangle\langle f_i|H_{\rm decay}|B\rangle, \qquad (37)$$

where $H_{\rm decay}$ corresponds to the weak transition operator and *S* is the scattering matrix. This relation is derived from the completeness relation $\mathbf{1} = \sum_i |f_i\rangle\langle f_i|$ and the fact that the *CPT*-operator is antiunitary. An equivalent but alternative relation on the level of decay rates can be found in Ref. [51]. From Eq. (37) it is then inferred that all states f_j which rescatter into f_x form a subclass whose partial rates of particles and antiparticles sum to zero,

$$\sum_{i\in I} \Delta\Gamma(B \to f_i) = 0, \qquad \langle f_i|S^\dagger|f_j\rangle \neq 0, \qquad i, j \in I, \qquad (38)$$

where

$$\Delta\Gamma(B \to f) \equiv \Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f}). \qquad (39)$$

The exact relation between the *CP* asymmetry and the difference of decay rates can be inferred from Eq. (8). Whereas the new *CP* asymmetry generated by $\mathcal{A}_{CP}(D^+D^-) \sim \Delta\Gamma(B_d \to D^+D^-)$ may be compensated by $\Delta\Gamma(B_d \to \bar{D}_0 D_0)$ for instance, it is at first sight not clear which mode would compensate for the new *CP* asymmetry in $\mathcal{A}_{CP}(\tau\nu) \sim \Delta\Gamma(B^+ \to \tau^+\nu)$. Among the SM final states there does not seem to be an appropriate candidate. One is led to look in the unparticle sector for a suitable candidate. A firm hint can be gained by counting the coupling constants. Denoting the weak coupling by v and the unparticle coupling by λ (5), the CP asymmetry, which arises due to an interference of the two amplitudes depicted in Fig. 1, is of the order $O(\lambda^2 v^2)$. The processes $B^+ \to \mathcal{U}^+$ with an interference of the two amplitudes depicted in Fig. 7 have the same counting in the coupling constants. One amplitude corresponds to a tree decay and the other one incorporates a virtual correction due to a fermion loop of the τ and the ν. The process $B^+ \to \mathcal{U}^+$ is kinematically allowed since the unparticle has a continuous mass spectrum. It does not proceed at resonance, but rather behaves like a multiparticle final state and is a realization of Georgi's observation that the unparticle field in a final state behaves like a nonintegral number $d_{\mathcal{U}}$ of massless particles.

I shall now explicitly verify the CPT constraint

$$\Delta\Gamma(B^+ \to \tau^+\nu) + \Delta\Gamma(B^+ \to \mathcal{U}^+)_{\tau\nu-\mathrm{loop}} = 0. \quad (40)$$

For the sake of simplicity I shall assume as previously that there is no flavor dependent perturbation in the neutrino sector and that $\lambda_P^{\tau\nu} = -\lambda_S^{\tau\nu}$ in (5). The formula for the first difference can be read off from Eq. (17),

$$\begin{aligned}\Delta\Gamma(B^+ \to \tau^+\nu) &= -4\mathcal{B}_{\tau\nu}^{\mathrm{SM}} \sin(\phi)\sin(d_{\mathcal{U}}\pi)\Delta_{\tau\nu} \\ &= -\sin(\phi)\frac{G_F}{2\sqrt{2}\pi}\frac{m_B}{m_b} m_\tau f_B^2\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \\ &\quad \times |\lambda_{\tau\nu}^S \lambda_{\mathrm{ub}}^P V_{\mathrm{ub}} U_{\tau\nu}| A_{d_{\mathcal{U}}}\left(\frac{m_B^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}. \quad (41)\end{aligned}$$

Note that the cancellation of the phase factor $\sin(d_{\mathcal{U}}\pi)$ by the same factor in the denominator, as previously mentioned, is crucial for the cancellation here since the graphs in Fig. 7 do not involve this factor. The amplitude of the graph in Fig. 7 to the left is

$$\begin{aligned}\mathcal{A}(B^+ \to \mathcal{U}^+)_{\mathrm{Fig.7(left)}} &= \lambda_P^{\mathrm{ub}*}\mathcal{A}_1 \\ &= \frac{\lambda_P^{\mathrm{ub}*}}{\Lambda^{d_{\mathcal{U}}-1}}\frac{m_B^2}{m_b} f_B \langle P|O_{\mathcal{U}}^\dagger|0\rangle \quad (42)\end{aligned}$$

and the amplitude of the graph to the right of Fig. 7 is

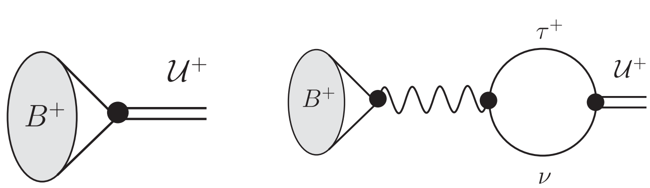

FIG. 7. Decay $B^+ \to \mathcal{U}^+$, the double lines denote an unparticle (left) leading order (right) with virtual $\tau\nu$-loop correction.

$$\begin{aligned}\mathcal{A}(B^+ \to \mathcal{U}^+)_{\mathrm{Fig.7(right)}} &= \lambda_S^{\tau\nu*} V_{\mathrm{ub}}^* U_{\tau\nu}^* \mathcal{A}_2 \\ &= \frac{\lambda_S^{\tau\nu*}}{\Lambda^{d_{\mathcal{U}}-1}} i\frac{G_F}{\sqrt{2}} V_{\mathrm{ub}}^* U_{\tau\nu}^* m_\tau f_B \\ &\quad \times \Pi_{S-P}(m_B^2)\langle P|O_{\mathcal{U}}^\dagger|0\rangle, \quad (43)\end{aligned}$$

where I have factored out the weak parameters and the rest of the amplitude is parametrized in terms of the variables $\mathcal{A}_{(1,2)}$. The fermion loop Π_{S-P} is given by the correlation function

$$\begin{aligned}\Pi_{S-P}(p_B^2 = m_B^2) &= i\int d^4x e^{-ip_B\cdot x}\langle 0|T[\bar{\nu}(1-\gamma_5)\tau](x) \\ &\quad \times [\bar{\tau}(1-\gamma_5)\nu](0)|0\rangle. \quad (44)\end{aligned}$$

The decay rate is calculated from

$$\Gamma = \frac{|\mathcal{A}|^2}{2m_B}\int d\Phi \quad \text{with} \quad \int d\Phi = A_{d_{\mathcal{U}}}(m_B^2)^{d_{\mathcal{U}}-2} \quad (45)$$

being the phase space volume. The difference of decay rates is given by

$$\begin{aligned}\Delta\Gamma(B^+ \to \mathcal{U}^+)_{\tau\nu-\mathrm{loop}} &= 4\sin(\phi)\mathrm{Im}[\mathcal{A}_1^*\mathcal{A}_2] \\ &\quad \times A_{d_{\mathcal{U}}}(m_B^2)^{d_{\mathcal{U}}-2}. \quad (46)\end{aligned}$$

Since $\mathcal{A}_1$ is real only the imaginary part of $\mathcal{A}_2$ will enter. The only strong phase is due to the τ and the ν going on shell in the loop in Fig. 7 (right). Therefore one only needs to know the imaginary part of the fermion loop which is given by

$$\mathrm{Im}[\Pi_{S-P}(m_B^2 + i0)] = \frac{1}{4\pi} m_B^2\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2. \quad (47)$$

Assembling the formulas I get

$$\begin{aligned}\Delta\Gamma(B^+ \to \mathcal{U}^+)_{\tau\nu} &= \sin(\phi)\frac{G_F}{2\sqrt{2}\pi}\frac{m_B}{m_b} m_\tau f_B^2\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \\ &\quad \times |\lambda_S^{\tau\nu}\lambda_P^{ub} V_{\mathrm{ub}} U_{\tau\nu}| A_{d_{\mathcal{U}}}\left(\frac{m_B^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1},\end{aligned} \quad (48)$$

which fulfills the CPT constraint Eq. (40) together with (41).

I have explicitly verified the CPT constraint (38) for the decay $B \to \tau\nu$ with unparticle-SM interactions given by the Lagrangian (5). I do not dare to speculate in any detail on how a decay $B^+ \to \mathcal{U}^+$ might be observed in a laboratory experiment. It can be said though that the unparticle has directed momentum, mass, and charge which it directly inherits from the B-meson. Moreover in the case where there is a CP asymmetry in $B \to \tau\nu$ due to unparticles, it is precisely the CPT constraint (40) which tells one that there is an excess of charged unparticle degrees of freedom produced. Whether a part of this charge could annihilate into neutral particles or decay into charged particles remains unclear since the nature of this degree of freedom remains unknown at this stage. These questions could be addressed once a concrete model realizing the unparticle scenario is known.

IV. BREAKING OF SCALE INVARIANCE—DIMENSIONAL ANALYSIS

I expect scale invariance to be broken at lower energies for two different reasons.

First, in concrete realizations discrete parameters, such as the number of colors, might only allow for a near critical behavior of the coupling constant, as sketched in Fig. 8.

Second, the SM scales will be mediated to the unparticle sectors by the interactions of the type (1). The SM is not scale invariant at the electroweak scale. The logarithmic running and, in particular, the vacuum expectation value of the Higgs, which give masses to the fundamental particles, are responsible for the breaking of scale invariance. It is therefore a legitimate question at what scale the symmetry breaking will be transmitted to the unparticle sector by the effective Lagrangian (1). This will depend on the strength of the coupling and the relevance of the operators in the effective Lagrangian.

The authors of Ref. [7] have addressed this question, which I shall adapt accordingly for the weak sector. Following Ref. [7] I assume that the unparticle field is coupled to the Higgs sector like

$$\mathcal{L}^{\rm eff} = \frac{\lambda_H}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}_0}-2}} |H|^2 O_{\mathcal{U}_0}, \qquad (49)$$

with $\lambda_H = c_{\mathcal{U}}^H (\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^{d_{\rm UV_0}-2}$ in my notation. I have used a new symbol $O_{\mathcal{U}_0}$ for the unparticle operator. This operator is not the same as the one used in Eq. (5) since it has to be electrically neutral. The important quantities for a quantitative analysis are $d_{\mathcal{U}_0}$ and λ_H of which I have limited knowledge and therefore the statements will not be conclusive. In the case where one thinks of the unparticle as being charged under $SU(2)_L$, $O_{\mathcal{U}_0}$ would appear as $\delta\mathcal{L}^{\rm eff} \sim \bar{q}(\gamma_5) q O_{\mathcal{U}_0}$ in addition to the effective Lagrangian (5) and $d_{\mathcal{U}_0} = d_{\mathcal{U}}$ seems unavoidable. In the case where $O_{\mathcal{U}}$ is the only unparticle field then the composite operator $O_{\mathcal{U}_0} = O_{\mathcal{U}} O_{\mathcal{U}}^\dagger$ seems the canonical quantity and can have an anomalous dimension in the range $0 \le d_{\mathcal{U}_0} \le 2 d_{\mathcal{U}}$. The Thirring model at coupling $\lambda = 2\pi$ [55] would be an example saturating the lower bound, whereas supersymmetric QCD at the conformal IR fixed-point [56] constitutes an example saturating the upper bound. In the following I shall quote values for the bounds and the mean value explicitly. The Higgs VEV $\langle |H|^2 \rangle = v^2/2$ is expected to break scale invariance at a scale $\tilde{\Lambda}$

$$\frac{\lambda_H}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}_0}-2}} \frac{v^2}{2} \tilde{\Lambda}^{d_{\mathcal{U}_0}} = \tilde{\Lambda}^4 \Rightarrow \tilde{\Lambda} = \Lambda_{\mathcal{U}} \left(\lambda_H \frac{v^2}{2\Lambda_{\mathcal{U}}^2} \right)^{1/4 - d_{\mathcal{U}_0}}. \qquad (50)$$

For illustrative purposes a possible unparticle scenario is sketched in Fig. 8 where the coupling of the unparticle sector as a function of the energy is sketched. What would this scale be in the cases I have investigated? Besides $\Lambda_{\mathcal{U}}$ there are two unknowns in the equation above, first $d_{\mathcal{U}_0}$ which appears explicitly in my results and $\lambda_H = c_{\mathcal{U}}^H (\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^{d_{\rm UV_0}-2}$. In the latter the matching coefficient will remain unknown but the ratio $(\Lambda_{\mathcal{U}}/M_{\mathcal{U}})$ is encoded in the size of the matching coefficients $\rho_{\rm DD}(\rho_{\tau\nu})$ (11) and (33) and the UV dimensions of the flavor sector. Taking $B \to DD$ as an example the breaking scale is

$$\tilde{\Lambda} = \Lambda_{\mathcal{U}} \left(c_{\mathcal{U}}^H \frac{v^2}{2\Lambda_{\mathcal{U}}^2} \left(\frac{\rho_{\rm DD}}{R_{\rm DD}} \right)^{d_{\rm UV_0} - 2/2(d_{\rm UV}-1)} \right)^{1/4 - d_{\mathcal{U}_0}}, \qquad (51)$$

where $R_{\rm DD} = |c_S^{\rm cb} c_S^{\rm cd}|/|V_{\rm cb} V_{\rm cd}|$. The value $R_{\rm DD} \sim 1$ is natural in case the unparticle sector has the same flavor hierarchy as the SM. For illustrative purposes let me assume for example $\Lambda_U = 1$ TeV, $d_{\mathcal{U}} = 1.2$, $d_{\mathcal{U}_0} = (0, 1.2, 2.4)$, $\rho_{\rm DD} = 10^{-3.5}$, the ratio of amplitudes and the breaking scale for fixed values of UV dimensions become

$$\begin{aligned} &\Delta_{\rm DD} \simeq -0.40 R_{\rm DD}, \\ (d_{\rm UV}, d_{\rm UV_0}) &= (3, 6), \\ &\tilde{\Lambda} \simeq (56, 16, 0.7)\ {\rm GeV} (R_{\rm DD} c_{\mathcal{U}}^H)^{1/(4.0, 2.8, 1.6)}, \\ (d_{\rm UV}, d_{\rm UV_0}) &= (3, 3), \\ &\tilde{\Lambda} \simeq (250, 140, 30)\ {\rm GeV} (R_{\rm DD}^{-1/4} c_{\mathcal{U}}^H)^{1/(4.0, 2.8, 1.6)}. \end{aligned} \qquad (52)$$

The situation is not conclusive, which is not surprising bearing in mind that in the absence of a model there are simply too many unknowns. In the case where both UV dimensions are the same, which should be the case when $O_{\mathcal{U}}$ and $O_{\mathcal{U}_0}$ result from the same structure, a small matching coefficient $c_{\mathcal{U}}^H$ is needed for a sizable effect at the heavy flavor scales. If the UV dimensions differ by a

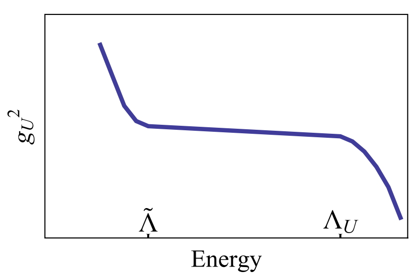

FIG. 8 (color online). A possible scenario, inspired from (walking) technicolor-like theories, of unparticle physics with a nearly scale invariant window between the lower scale $\tilde{\Lambda}$ Eq. (50) and the IR fixed point scale $\Lambda_{\mathcal{U}}$ Eq. (1). The coupling to the Higgs VEV could cause the theory to exit the IR fixed point at the scale $\tilde{\Lambda}$. The ordinate denotes the coupling $g_{\mathcal{U}}^2$ of the unparticle sector.

factor of 2, which can be the case when $O_{\mathcal{U}_0} = O_{\mathcal{U}} O^\dagger_{\mathcal{U}}$, effects are possible for moderate matching coefficient $c^H_{\mathcal{U}}$.

The effect $\Delta_{\rm DD} = -0.40$ appears larger than the analysis or conclusions in Ref. [7] suggest. There are two reasons. First and simply, the CP violating phenomenon investigated in this paper is linear in the ratio of amplitudes, whereas [7] describes a case where the effect is proportional to the square of the amplitude. Second, it was assumed that the SM Lagrangian has dimension four. The crucial point is that the weak Lagrangian has dimension six, $d_{\mathcal{L}_{\rm weak}} = 6$ being suppressed by two powers of the weak scale, whereas the unparticle Lagrangian has dimension $d_{\mathcal{L}_{\rm unp}} = d_{\mathcal{U}} + d_{\rm SM}$. In terms of the effective Lagrangian (5) and $1 < d_{\mathcal{U}} < 2$, $4 < d_{\mathcal{L}_{\rm unp}} < 5$ the unparticle operator is more relevant than the weak operator. This gives rise to an enhancement factor in the amplitudes

$$\sqrt{2}(G_F \mu_{\rm HF}^2)^{-1} = \frac{8m_W^2}{g^2\mu_{\rm HF}^2} \sim 10^3\text{–}10^4, \tag{53}$$

which is explicit in the results of Eqs. (11) and (33). In more physical terms one could state that the weak boson propagates at the high weak scale whereas the unparticle propagates at the low heavy flavor scale.

Adapting the analysis of Ref. [7] I imagine an experiment at a scale $\mu_{\rm HF}$, the unparticle Lagrangian (1) scales as $\mathcal{L}_{\rm eff} \simeq \lambda_S/\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}+(d_{\rm SM}-4)} \mu_{\rm HF}^{d_{\rm SM}+d_{\mathcal{U}}}$, the weak Lagrangian as $\mathcal{L}_{\rm weak} \simeq G_F \mu^6_{\rm HF}$, and the ratio is

$$\Delta \simeq c^S_{\mathcal{U}} \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{d_{\rm UV}-d_{\mathcal{U}}} \left(\frac{\mu_{\rm HF}}{M_{\mathcal{U}}}\right)^{d_{\mathcal{U}}+d_{\rm SM}-4} \left(\frac{G_F^{-1}}{\mu^2_{\rm HF}}\right). \tag{54}$$

Imposing that the energy scale of the experiment is higher than the breaking scale, i.e., $\mu_{\rm HF} \gtrsim \tilde{\Lambda}$, the following bound is obtained[5]:

$$\begin{aligned}\Delta &\lesssim \frac{c^S_{\mathcal{U}}}{c^H_{\mathcal{U}}} \left(\frac{\mu_{\rm HF}}{M_{\mathcal{U}}}\right)^{d_{\rm SM}-2} \left(\frac{2\mu^2_{\rm HF}}{v^2}\right) \left(\frac{G_F^{-1}}{\mu^2_{\rm HF}}\right) \left(\frac{\Lambda_{\mathcal{U}}}{\mu_{\rm HF}}\right)^{d_{\mathcal{U}_0}-d_{\mathcal{U}}} \\ &\simeq \frac{4c^S_{\mathcal{U}}}{c^H_{\mathcal{U}}} \left(\frac{\mu_{\rm HF}}{M_{\mathcal{U}}}\right)^{d_{\rm SM}-2} \left(\frac{\Lambda_{\mathcal{U}}}{\mu_{\rm HF}}\right)^{d_{\mathcal{U}_0}-d_{\mathcal{U}}}.\end{aligned}$$

The term in the middle is easily interpreted. The first factor measures the ratio of the two couplings. The second is a measure between the relevance or dimension of the SM operator that is coupled to the unparticle and the dimension of the Higgs operator. In the third term the scale of the experiment has to compete with the Higgs VEV. The fourth term is peculiar to the weak interactions, as described above, and is due to the fact that the weak process takes place at the weak scale G_F^{-1} and the unparticle propagates at the low scale μ_F. The fifth term is due to the difference of anomalous dimensions of the charged unparticle operator in the effective Lagrangian (5) and the neutral unparticle operator coupling to the Higgs VEV (49), whether it acts as an enhancing or decreasing factor depends on the anomalous dimensions.

Dimensional analysis is not very reliable. A quantitative analysis could be done once a explicit model is proposed of which I have mentioned a few candidates in the introduction.

V. CRITICAL DISCUSSION AND CONCLUSIONS

In this paper I have investigated the consequences of the unparticle scenario in heavy flavor physics. The new feature is a CP-even or strong phase that arises in the propagator as a consequence of the nonintegral scaling dimension. A crucial point is that the phase is sizable, based on the assumption that the unparticle sector is strongly self-coupled. The strong phase together with non-CKM(PMNS) phases in the weak unparticle sector gives rise to novel CP violating phenomena.

CP violation in leptonic decays, such as $B \to \tau\nu$, is unprecedented so far in other models and not searched for in experiment.[6] I have verified in Sec. III that the novel CP violation is consistent with the CPT theorem; namely, the equality of the sum of partial rates, of the subclasses of final states rescattering into each other, of particle and antiparticle. Since $(\tau^+\nu)$ is essentially a class on its own I have inferred that the compensating mode must be due to unparticles. As I have quantitatively verified, the compensating mode is $B^+ \to \mathcal{U}^+$. This might appear surprising at first sight but is possible since the unparticle does not have a definite mass but a continuous spectrum like a multiparticle state which was one of the basic observations in Georgi's first paper [1].

I have investigated the extension of the scale invariant sector to lower energies resorting to dimensional analysis, building up on the work [7]. I have found that effects at the heavy flavor scales are possible provided the coupling of the unparticle field to the Higgs VEV is moderate at the scale $\Lambda_{\mathcal{U}}$. The effects are sizable for two reasons. First the effect of CP violation is linear and not shielded by a large SM background, and second the scaling dimension of the unparticle Lagrangian is more relevant than the one of the weak Lagrangian. Because of the enhancement factor $\sqrt{2}(G_F\mu_H^2) \sim 10^3$–$10^4$ the CP asymmetries in the two channels can be as large as 80% for ratios of unparticle to SM couplings $\rho_{\tau\nu(DD)} \sim 10^{-3}$ (21) and (36).

Bearing in mind the breaking of scale invariance I have chosen decays where the unparticle propagates at a relatively large scale. The two examples I have investigated are the decays $B^+ \to \tau^+\nu$ and $B_d \to D^+D^-$. In doing so I have assumed the scale invariant sector extends to the scale $\sim$5 GeV for the former and to $\sim$2 GeV for the latter.

[5]Setting $c_{\mathcal{U}} \to 1$, the fourth and the fifth term to one and taking the square root of the equation, the bound in Ref. [7] is recovered with $\Delta^2 = \epsilon$.

[6]To the knowledge of the author there is no experimental data available with bounds on CP asymmetries in leptonic decays. Charge symmetry is usually implied in the analysis.

The investigation of $B \to \tau\nu$ was motivated by the fact that *CP* violation in leptonic decays are unprecedented and that it is a channel that has already been measured to some degree. Generic plots for the parameter space of the anomalous dimension and the weak phase difference are shown in Fig. 3. The current experimental data is not yet strong enough to set absolute bounds on the amount of *CP* violation. Comments on flavor related decays are given in Sec. II C. In particular the channel $D \to \mu\nu$ might be of interest since more events have been collected [40] than in $B \to \tau\nu$ [57,58].

The investigation of the nonleptonic decay $B_d \to D^+D^-$ was motivated by the large asymmetry $C_{D^+D^-}$ reported by Belle [38]. I have neglected the penguin contribution and treated the decay in naive factorization. As compared to $B \to \tau\nu$ there is a third observable, the time dependent *CP* asymmetry $S_{D^+D^-}$. The latter agrees rather well with the SM predictions and sets constraints on $C_{D^+D^-}$. It is possible though to find values where the *CP* violation is maximal and satisfies the constraints of the branching ratio and the time dependent *CP* asymmetry. As for $B \to \tau\nu$, plots for generic parameters are shown in Fig. 6. It is encouraging that for small ratios of effective couplings the constraints from $S_{D^+D^-}$ allow for a large negative asymmetry $C_{D^+D^-}$ as reported by Belle whereas the opposite sign seems to be disfavored. This fact is general to any analysis with two amplitudes; the unparticles just provide a scenario with two amplitudes and possible large weak and strong phase differences. The true meaning is that in the case where the decay is described by two amplitudes, the sign of the Belle measurement is more consistent than the opposite sign. Discussions on U-spin and color related decays are given in Sec. II E. Let me emphasize two points from this section once more. Generically one would expect a large asymmetry in $C_{D^+D^-}$ to be accompanied by a large asymmetry in the color related $C_{J/\Psi\pi_0}$. Currently the experimental value $C^{\rm PDG}_{J/\Psi\pi_0} = -0.11(20)$ [41] is not conclusive. It also has to be said that on the theoretical side the channel $B_d \to J/\Psi\pi_0$ is more challenging because the nonfactorizable contributions are enhanced. For B_d decays the time dependent asymmetries are typically proportional to $\sin(2\beta)$ or $\sin(2\alpha)$, the large angles of the B_d triangle, and new physics contributions are therefore hard to see. For B_s decays, the mixing phase is $\phi_s \simeq 0$ and therefore the unparticle scenario could give rise to sizable corrections. This would be particularly interesting for $B_s \to J/\Psi\phi$, which is the main channel at the LHCb to extract the B_s mixing phase ϕ_s.

The drawbacks of the scenario so far is that there is yet no concrete model and that it is not clear to what energies the scale invariant sector extends or what the meaning of the emission of a real unparticle is.[7] The construction of an explicit model of the unparticle scenario would permit one to study these questions in a concrete and quantitative way. Two possible candidates are walking technicolor theories [33] and the higher dimensional models (HEIDI) [35] mentioned in the introduction.

[7]As opposed to a virtual particle, on which I focused throughout this paper.

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APPENDIX: EXPLICIT RESULTS IN COORDINATE SPACE

1. The commutator

The unparticle field is a scale invariant field. The commutator of a scale invariant field is easily obtained, in terms of the scaling dimension $d_{\mathcal{U}}$, from the two point function

$$\begin{aligned} C(x) &= \langle [O_{\mathcal{U}}(x), O_{\mathcal{U}}(0)]\rangle_0 \\ &= \frac{2}{i}\epsilon(x_0)\mathrm{Im}[\langle TO_{\mathcal{U}}(x)O_{\mathcal{U}}(0)\rangle_0] \\ &= \frac{2}{i}\epsilon(x_0)\mathrm{Im}\Big[\int \frac{d^4P}{(2\pi)^4} e^{iPx} i\Delta_{\mathcal{U}}(P^2)\Big] \\ &= \epsilon(x_0)\frac{2^{d_{\mathcal{U}}-3}}{i\pi^2}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \\ &\quad\times \frac{\Gamma(d_{\mathcal{U}})\mathrm{Im}[(-x^2+i0)^{-d_{\mathcal{U}}}]}{\Gamma(2-d_{\mathcal{U}})} \\ &= i\epsilon(x_0)\frac{\Theta(x^2)}{(x^2)^{d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)\pi^{1/2-2d_{\mathcal{U}}}}{\Gamma(2d_{\mathcal{U}})\Gamma(1-d_{\mathcal{U}})}. \end{aligned}$$

The free field case $d_{\mathcal{U}} \to 1$

$$\lim_{d_{\mathcal{U}}\to 1+} C(x) = \frac{-i}{2\pi}\mathrm{sgn}(x_0)\delta(x^2)$$

may be recovered by use of the formula $\lim_{\epsilon\to 0+}\epsilon|z|^{1-\epsilon} = \delta(z)$. Or for any integer n

$$\lim_{d_{\mathcal{U}}\to n+} C(x) \sim -i\mathrm{sgn}(x_0)\delta^{(n-1)}(x^2),$$

it is seen that the commutator has support on the light cone only.

2. The Thirring model—an example with phase factor

The Thirring model belongs to the class of exactly solvable two dimensional models. It is a fermionic model with a vector current-current interaction. The exact solution of the two point function was obtained by Johnson [59] as a function of free fields

$$\langle 0|T\Psi(x)\bar{\Psi}(0)|0\rangle = -ie^{-i4\pi\gamma D_0(x)}G_0(x), \tag{A1}$$

where

$$D_0(x) = \frac{-i}{4\pi}\log(-x^2 + i0), \qquad G_0(x) = \frac{1}{2\pi}\frac{\not{x}}{x^2 - i0} \tag{A2}$$

are the free bosonic and fermionic Greens functions. The anomalous coupling constant is identified in terms of a function of the Thirring model coupling constant λ as $\gamma = (\lambda^2/4\pi^2)(1 - \lambda^2/4\pi^2)^{-1}$ [55]. N.B. $\gamma > 0$ in accordance with $1 < d_{\mathcal{U}} < 2$ and $d_{\mathcal{U}} = 1/2 + \gamma$.

$$\langle 0|T\Psi(x)\bar{\Psi}(0)|0\rangle = \frac{i}{2\pi}\frac{\not{x}}{(-x^2 + i0)^{1+\gamma}} \tag{A3}$$

This corresponds to the form of a fermionic unparticle propagator (4) in coordinate space up to an overall normalization constant. The phase factor arises due to resummation of thresholds at $x^2 > 0$. Note that the overall normalization in a scale invariant theory is a matter of convention and is hidden in the arbitrary scale factor in the logarithm of the free bosonic function $\log[(-x^2 + i0)\mu^2]$. In the notation of Eq. (1) the scale μ is proportional to the fixed point scale $\Lambda_{\mathcal{U}}$. This scale exhibits the phenomenon of dimensional transmutation of the operators $O_{\rm UV}$ in Eq. (1) to the operators $O_{\mathcal{U}}$ in Eq. (1).

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