

# Pseudoscalar and scalar operators of Higgs penguin diagrams in the MSSM and $B \rightarrow \phi K^*, K\eta^{(\prime)}$ decays

Hisaki Hatanaka and Kwei-Chou Yang

*Department of Physics, Chung-Yuan Christian University, Chungli, Taiwan 320, Republic of China*

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We study the effect of  $b \rightarrow s\bar{s}s$  scalar/pseudoscalar operators in  $B \rightarrow K\eta^{(\prime)}, \phi K^*$  decays. In the minimal supersymmetric standard model (MSSM), such scalar/pseudoscalar operators can be induced by the penguin diagrams of neutral-Higgs bosons. These operators can be Fierz-transformed into tensor operators, and the resultant tensor operators could affect the transverse polarization amplitudes in  $B \rightarrow \phi K^*$  decays. A combined analysis of the decays  $B \rightarrow \phi K^*$  and  $B \rightarrow K\eta^{(\prime)}$ , including  $b \rightarrow s\bar{s}s$  scalar/pseudoscalar operators and their Fierz-transformed tensor operators originated from the MSSM, is performed. Our study is based on the following: (1) Assuming that weak annihilations in  $B \rightarrow \phi K^*$  are negligible and the polarization puzzle is resolved by Fierz-transformed tensor operators, it results in too large coefficients of scalar/pseudoscalar operators, such that the resulting  $B \rightarrow K\eta^{(\prime)}$  branching fractions are much larger than observations. (2) When we take the weak annihilations in  $B \rightarrow \phi K^*$  into account, the polarization puzzle can be resolved. In this case, new physics effects are strongly suppressed and no more relevant to the enhancement of the transverse modes in  $B \rightarrow \phi K^*$  decays.

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## I. INTRODUCTION

Recent experimental results for polarization fractions in  $\bar{B}^{0,-} \rightarrow \phi(1020)\bar{K}^{*0,-}$  are

$$f_L(\bar{B}^0 \rightarrow \phi\bar{K}^{*0}) = \begin{cases} 0.506 \pm 0.040 \pm 0.015 & \text{BABAR [1]} \\ 0.45 \pm 0.05 \pm 0.02 & \text{Belle [2]} \\ 0.57 \pm 0.10 \pm 0.05 & \text{CDF [3]} \end{cases},$$

$$f_\perp(\bar{B}^0 \rightarrow \phi\bar{K}^{*0}) = \begin{cases} 0.227 \pm 0.038 \pm 0.013 & \text{BABAR [1]} \\ 0.31_{-0.05}^{+0.06} \pm 0.02 & \text{Belle [2]} \\ 0.20 \pm 0.10 \pm 0.05 & \text{CDF [3]} \end{cases},$$

$$f_L(B^- \rightarrow \phi K^{*-}) = \begin{cases} 0.49 \pm 0.05 \pm 0.03 & \text{BABAR [4]} \\ 0.52 \pm 0.08 \pm 0.03 & \text{Belle [2]} \end{cases},$$

$$f_\perp(B^- \rightarrow \phi) = \begin{cases} 0.21 \pm 0.05 \pm 0.02 & \text{BABAR [4]} \\ 0.19 \pm 0.08 \pm 0.02 & \text{Belle [2]} \end{cases} \quad (1)$$

Here, the polarization fractions  $f_\lambda$  ( $\lambda = L, \parallel, \perp$ ) are given by  $f_\lambda = |A_\lambda^2|/\sum_{\sigma=L,\parallel,\perp}|A_\sigma|^2$ , with polarization amplitudes  $A_L \equiv A_0, A_\parallel$ , and  $A_\perp$  being longitudinal, parallel, and perpendicular modes in the transversity basis, respectively. Experimental results show that  $f_L \sim 0.5$  and  $f_\perp \sim f_\parallel$ . On the other hand, the power-counting estimate in the standard model (SM) tells that the longitudinal mode is dominant [5]. In the SM, the QCD factorization (QCDF) calculation yields [5]  $f_L:f_\parallel:f_\perp = 1 - \mathcal{O}(1/m_b^2):\mathcal{O}(1/m_b^2):\mathcal{O}(1/m_b^2)$ . The experimental results largely deviate from the intuition in the SM. Similar discrepancies have been observed in penguin-dominated  $B^{\pm,0} \rightarrow \rho^{\pm,0}K^{*0}$  decays [6,7]. These discrepancies are referred to as the polarization puzzle/anomaly in  $B \rightarrow VV$  (where  $V$  denotes a vector meson) decays.

Solutions to the puzzle have been discussed within or beyond the standard model [5,8,9]. The recipe of fine-

tuning form factors is proposed in [10]. Effects of final-state interactions are discussed in [11,12]. Sizable annihilation effects are considered in [13–15]. As discussed in [14], the magnitude of annihilation correction is of  $\mathcal{O}[1/m_b^2 \log^2 m_b/\Lambda_h]$ . Furthermore, the effect is destructive to longitudinal, and constructive to transverse modes. Thus we may resolve the polarization puzzles by introducing annihilation effects. We note that, however, the perturbative QCD (pQCD) yields  $f_L \gtrsim 0.75$  even with annihilation effects [8].

The  $b \rightarrow sg$  (where  $g$  denotes a gluon) operator, which enhances the transverse components, was discussed in [16]. However, it was found [13,17] that the contribution due to the operator mainly affects the longitudinal mode.

As for the solutions of the puzzle, the effects of NP-induced tensor operators are discussed in [17] and right-handed currents  $\bar{s}\gamma^\mu(1 + \gamma_5)b\bar{q}\gamma_\mu(1 \pm \gamma_5)q$  are in [18–20]. Because the right-handed currents may decrease the magnitude of  $|A_0|$  and increase  $|A_\perp|$ , it can explain the ratio  $|A_\perp/A_0|$ . However, the resulting  $|A_\parallel| \ll |A_\perp|$  [18] is in contrast with the data  $|A_\parallel| \sim |A_\perp|$ .

New physics (NP) contributions to  $B \rightarrow \phi K^*$  decays due to  $b \rightarrow s\bar{s}s$  tensor operators, first mentioned in [13], are systematically discussed in [17], and later the idea is applied to  $B \rightarrow \rho K^*$  by considering the 4-quark tensor operators related to the processes  $b \rightarrow s\bar{d}d$  and  $s\bar{u}u$  [21]. In the helicity basis,<sup>1</sup> four-quark tensor operators have leading effects to  $\bar{H}_{--}$  (or  $\bar{H}_{++}$ ), but subleading to  $\bar{H}_{00}$ . The possibility of solving the  $B \rightarrow \phi K^*$  polarization puzzle by using four-quark tensor operators is extensively studied in [14,17] and further investigated in [22–26].

<sup>1</sup>Amplitudes in the helicity basis and the transversity basis are related by  $\bar{A}_0 = \bar{H}_{00}, \bar{A}_\parallel = (\bar{H}_{++} + \bar{H}_{--})/\sqrt{2}, \bar{A}_\perp = -(\bar{H}_{++} - \bar{H}_{--})/\sqrt{2}$ .

In [17] the general approach of resolving the polarization anomaly of  $B \rightarrow \phi K^*$  by using four-quark NP operators is studied. There are two types of NP operators which are relevant to solve the polarization anomaly. They are tensor operators with  $\sigma_{\mu\nu}(1 \pm \gamma_5) \otimes \sigma^{\mu\nu}(1 \pm \gamma_5)$  structure. The tensor operator  $\sigma_{\mu\nu}(1 + \gamma_5) \otimes \sigma^{\mu\nu}(1 + \gamma_5)$  results in  $\bar{H}_{00}:\bar{H}_{--}:\bar{H}_{++} = \mathcal{O}(1/m_b):\mathcal{O}(1):\mathcal{O}(1/m_b^2)$ , while  $\sigma_{\mu\nu}(1 - \gamma_5) \otimes \sigma^{\mu\nu}(1 - \gamma_5)$  leads to  $\bar{H}_{00}:\bar{H}_{--}:\bar{H}_{++} = \mathcal{O}(1/m_b):\mathcal{O}(1/m_b^2):\mathcal{O}(1)$ .

The decays  $B \rightarrow PP$  (where  $P$  denotes a pseudoscalar meson) are sensitive to scalar/pseudoscalar four-quark operators whereas  $B \rightarrow VV$  are sensitive to tensor operators. Furthermore, it is known that scalar/pseudoscalar and tensor operators are not independent; scalar/pseudoscalar operators can be Fierz-transformed into tensor operators and vice versa. Therefore, the combined analysis of scalar/pseudoscalar and tensor operators for  $B \rightarrow PP$  and  $B \rightarrow VV$  modes will give more severe constraints about NP scalar/pseudoscalar and tensor operators.

In this paper we focus on the  $b \rightarrow s\bar{s}$  decay processes. We consider the scalar/pseudoscalar operators induced by Higgs penguin diagrams of the MSSM neutral-Higgs bosons (NHB) [26–30], while the tensor operators which are obtained from scalar/pseudoscalar operators by the Fierz transformation. In this NP scenario, such tensor operators contribute to the transverse polarization in  $B \rightarrow \phi K^*$  decays, whereas original scalar/pseudoscalar operators can affect  $B \rightarrow K\eta^{(\prime)}$  decays. We study the consistency of both modes to see the validity of the scenario. One should note that this NP effect is further suppressed by  $m_q/m_s$  in the  $b \rightarrow s\bar{q}q$  channel (with  $q \equiv u$  or  $d$ ) as compared with  $b \rightarrow s\bar{s}$ . Although the recent observations of the sizable transverse fraction in  $B^{\pm,0} \rightarrow \rho^{\pm,0} K^{*0}$  decays may hint at large annihilation effects, the present study for decays  $B \rightarrow \phi K^*$  and  $B \rightarrow K\eta^{(\prime)}$  can offer more severe constraints on the NP.

The organization of the present article is as follows: In Sec. II, we summarize the formulation of MSSM-NHB scalar/pseudoscalar operators and its contributions to  $B \rightarrow K\eta^{(\prime)}$  and  $B \rightarrow \phi K^*$  decays. In Sec. III, we numerically analyze the decays for  $B \rightarrow K\eta^{(\prime)}$  and  $B \rightarrow \phi K^*$ . Section IV is devoted to summary and discussions.

## II. FORMULATION

### A. SM and NP operators

In the SM the effective Hamiltonian relevant to  $B \rightarrow K\eta^{(\prime)}$  and  $B \rightarrow \phi K^*$  decays is given by

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{\text{SM}} = & \frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c} V_{qb} V_{qs}^* (c_1(\mu) O_1^q(\mu) + c_2(\mu) O_2^q(\mu)) \right. \\ & - V_{tb} V_{ts}^* \sum_{i=3}^{10} c_i(\mu) O_i(\mu) + c_{7\gamma}(\mu) O_{7\gamma}(\mu) \\ & \left. + c_{8g}(\mu) O_{8g}(\mu) \right] + \text{H.c.}, \end{aligned} \quad (2)$$

where the operators  $O_{i=1,\dots,10}$  are four-quark operators.  $O_{7\gamma}$  and  $O_{8g}$  are electromagnetic and chromomagnetic dipole operators, respectively.  $\mu$  is the renormalization scale.  $V_{qb}$  and  $V_{qs}$  ( $q = u, c, t$ ) are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The  $b \rightarrow s\bar{s}$  four-quark NP effective Hamiltonian is given by

$$\mathcal{H}_{\text{eff}}^{\text{NP}} = -\frac{G_F}{\sqrt{2}} (V_{tb} V_{ts}^*) \sum_{i=11}^{26} c_i(\mu) O_i(\mu) + \text{H.c.}, \quad (3)$$

where  $O_i$  and  $c_i$  ( $i = 11, \dots, 26$ ) are four-quark NP operators introduced in [17], and corresponding Wilson coefficients,<sup>2</sup> respectively. Explicit forms of  $O_i$  ( $i = 11, \dots, 26$ ) are shown in the following:

(i) right-handed current operators

$$\begin{aligned} O_{11} &= \bar{s}_\alpha \gamma_\mu (1 + \gamma_5) b_\alpha \bar{s}_\beta \gamma^\mu (1 + \gamma_5) s_\beta, \\ O_{12} &= \bar{s}_\alpha \gamma_\mu (1 + \gamma_5) b_\beta \bar{s}_\beta \gamma^\mu (1 + \gamma_5) s_\alpha, \\ O_{13} &= \bar{s}_\alpha \gamma_\mu (1 + \gamma_5) b_\alpha \bar{s}_\beta \gamma^\mu (1 - \gamma_5) s_\beta, \\ O_{14} &= \bar{s}_\alpha \gamma_\mu (1 + \gamma_5) b_\beta \bar{s}_\beta \gamma^\mu (1 - \gamma_5) s_\alpha, \end{aligned} \quad (4)$$

(ii) scalar/pseudoscalar operators

$$\begin{aligned} O_{15} &= \bar{s}_\alpha (1 + \gamma_5) b_\alpha \bar{s}_\beta (1 + \gamma_5) s_\beta, \\ O_{16} &= \bar{s}_\alpha (1 + \gamma_5) b_\beta \bar{s}_\beta (1 + \gamma_5) s_\alpha, \\ O_{17} &= \bar{s}_\alpha (1 - \gamma_5) b_\alpha \bar{s}_\beta (1 - \gamma_5) s_\beta, \\ O_{18} &= \bar{s}_\alpha (1 - \gamma_5) b_\beta \bar{s}_\beta (1 - \gamma_5) s_\alpha, \\ O_{19} &= \bar{s}_\alpha (1 + \gamma_5) b_\alpha \bar{s}_\beta (1 - \gamma_5) s_\beta, \\ O_{20} &= \bar{s}_\alpha (1 + \gamma_5) b_\beta \bar{s}_\beta (1 - \gamma_5) s_\alpha, \\ O_{21} &= \bar{s}_\alpha (1 - \gamma_5) b_\alpha \bar{s}_\beta (1 + \gamma_5) s_\beta, \\ O_{22} &= \bar{s}_\alpha (1 - \gamma_5) b_\beta \bar{s}_\beta (1 + \gamma_5) s_\alpha, \end{aligned} \quad (5)$$

(iii) tensor/axial-tensor operators<sup>3</sup>

$$\begin{aligned} O_{23} &= \bar{s}_\alpha \sigma_{\mu\nu} (1 + \gamma_5) b_\alpha \bar{s}_\beta \sigma^{\mu\nu} (1 + \gamma_5) s_\beta, \\ O_{24} &= \bar{s}_\alpha \sigma_{\mu\nu} (1 + \gamma_5) b_\beta \bar{s}_\beta \sigma^{\mu\nu} (1 + \gamma_5) s_\alpha, \\ O_{25} &= \bar{s}_\alpha \sigma_{\mu\nu} (1 - \gamma_5) b_\alpha \bar{s}_\beta \sigma^{\mu\nu} (1 - \gamma_5) s_\beta, \\ O_{26} &= \bar{s}_\alpha \sigma_{\mu\nu} (1 - \gamma_5) b_\beta \bar{s}_\beta \sigma^{\mu\nu} (1 - \gamma_5) s_\alpha. \end{aligned} \quad (6)$$

Since  $B \rightarrow PP$  ( $B \rightarrow VV$ ) decays are not sensitive to the factorized tensor (scalar/pseudoscalar)  $b \rightarrow s\bar{s}s$  operators, it is a good approximation to use  $O_i$  with  $i = 1, \dots, 22$  ( $i = 1, \dots, 14, 23, \dots, 26$ ) for  $B \rightarrow PP$  ( $B \rightarrow VV$ ). Some of these NP operators are not independent, and can be related with each other by the Fierz transformation:

<sup>2</sup>In [17], CKM factors and Wilson coefficients are not separated.

<sup>3</sup> $\sigma_{\mu\nu}(1 \pm \gamma_5)\sigma^{\mu\nu}(1 \mp \gamma_5)$  type operators  $O_i$  ( $i = 27, \dots, 30$ ) in [17] are found to vanish.

$$\begin{aligned}
O_{19} &= -\frac{1}{2}O_{14}, & O_{20} &= -\frac{1}{2}O_{13}, & O_{21} &= -\frac{1}{2}O_6, \\
O_{22} &= -\frac{1}{2}O_5, & O_{23} &= -4O_{15} - 8O_{16}, \\
O_{24} &= -8O_{15} - 4O_{16}, & O_{25} &= -4O_{17} - 8O_{18}, \\
O_{26} &= -8O_{17} - 4O_{18}.
\end{aligned} \tag{7}$$

Because of the Fierz transformation, we can introduce modified Wilson coefficients  $\bar{c}_i$ , which are defined by

$$\begin{aligned}
\bar{c}_i &= c_i - \frac{1}{2}c_j \\
\text{with } (i, j) &= (5, 22), (6, 21), (13, 20), (14, 19),
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
\begin{pmatrix} \bar{c}_{15} \\ \bar{c}_{16} \\ \bar{c}_{23} \\ \bar{c}_{24} \end{pmatrix} &= M \begin{pmatrix} c_{15} \\ c_{16} \\ c_{23} \\ c_{24} \end{pmatrix}, & \begin{pmatrix} \bar{c}_{17} \\ \bar{c}_{18} \\ \bar{c}_{25} \\ \bar{c}_{26} \end{pmatrix} &= M \begin{pmatrix} c_{17} \\ c_{18} \\ c_{25} \\ c_{26} \end{pmatrix} \\
\text{with } M &= \begin{pmatrix} 1 & 0 & -4 & -8 \\ 0 & 1 & -8 & -4 \\ \frac{1}{12} & -\frac{1}{6} & 1 & 0 \\ -\frac{1}{6} & \frac{1}{12} & 0 & 1 \end{pmatrix}.
\end{aligned} \tag{9}$$

Thus we can replace the Wilson coefficients by the effective ones:

$$\begin{aligned}
\{c_i, \bar{c}_j\} & \quad i = 1, \dots, 4, 7, \dots, 12, 15, \dots, 18, \\
& \quad j = 5, 6, 13, 14,
\end{aligned} \tag{10}$$

for  $B \rightarrow PP$  decays, and

$$\begin{aligned}
\{c_i, \bar{c}_j\} & \quad i = 1, \dots, 4, 7, \dots, 12, \\
& \quad j = 5, 6, 13, 14, 23, \dots, 26,
\end{aligned} \tag{11}$$

for  $B \rightarrow VV$  decays, so that the decay amplitudes can be simplified.

### B. $B \rightarrow K\eta^{(\prime)}$ decay amplitudes

In the SM,  $B \rightarrow K\eta^{(\prime)}$  decay amplitudes are given by [31]

$$A(\bar{B} \rightarrow \bar{K}\eta^{(\prime)}) = \sum_{p=u,c} V_{pb}V_{ps}^* \mathcal{T}_{\bar{K}\eta^{(\prime)}}^p, \tag{12}$$

where

$$\begin{aligned}
\sqrt{2}\mathcal{T}_{B \rightarrow K^-\eta^{(\prime)}}^p &= A_{\bar{K}\eta_q^{(\prime)}}[\delta_{pu}\alpha_2 + 2\alpha_3^p + \frac{1}{2}\alpha_{3,EW}^p + 2\beta_{S3}^p] \\
&+ \sqrt{2}A_{\bar{K}\eta_s^{(\prime)}}[\delta_{pu}\beta_2 + \alpha_3^p + \alpha_4^p - \frac{1}{2}\alpha_{3,EW}^p \\
&- \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p + \beta_{3,EW}^p + \beta_{S3}^p] \\
&+ \sqrt{2}A_{\bar{K}\eta_c^{(\prime)}}[\delta_{pc}\alpha_2 + \alpha_3^p] \\
&+ A_{\eta_q^{(\prime)}\bar{K}}[\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p \\
&+ \alpha_{4,EW}^p + \beta_3^p + \beta_{3,EW}^p],
\end{aligned} \tag{13}$$

$$\begin{aligned}
\sqrt{2}\mathcal{T}_{\bar{B}^0 \rightarrow \bar{K}^0\eta^{(\prime)}} &= A_{\bar{K}\eta_q^{(\prime)}}[\delta_{pu}\alpha_2 + 2\alpha_3^p + \frac{1}{2}\alpha_{3,EW}^p + 2\beta_{S3}^p] \\
&+ \sqrt{2}A_{\bar{K}\eta_s^{(\prime)}}[\alpha_3^p + \alpha_4^p - \frac{1}{2}\alpha_{3,EW}^p \\
&- \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p + \beta_{S3}^p] \\
&+ \sqrt{2}A_{\bar{K}\eta_c^{(\prime)}}[\delta_{pc}\alpha_2 + \alpha_3^p] + A_{\eta_q^{(\prime)}\bar{K}}[\alpha_4^p \\
&- \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p].
\end{aligned} \tag{14}$$

For the  $B \rightarrow PP$  decays  $\alpha_{1,2,3,4,3EW,4EW}$  are defined as

$$\begin{aligned}
\alpha_{1,2} &= a_{1,2}, & \alpha_3^p &= a_3^p - a_5^p, & \alpha_4^p &= a_4^p + r_\chi^{M_2} a_6^p, \\
\alpha_{3,EW}^p &= a_9^p - a_7^p, & \alpha_{4,EW}^p &= a_{10}^p + r_\chi^{M_2} a_8^p,
\end{aligned} \tag{15}$$

with  $r_\chi^K = 2m_K^2/m_b(m_q + m_s)$  and  $r_\chi^{q,s} = h_{\eta^{(q)}}^s/(f_{\eta^{(q)}}^s m_b m_s)$ .  $A_{M_1 M_2}$  are given by

$$A_{M_1 M_2} = i \frac{G_F}{\sqrt{2}} \cdot m_B^2 F_0^{B \rightarrow M_1}(0) f_{M_2}. \tag{16}$$

Contributions from the annihilation diagram are represented by  $\beta_Q$  ( $Q = 3, 4, 3EW, 4EW, S3$ ), where  $\beta_Q = b_Q \cdot B_{M_1 M_2}/A_{M_1 M_2}$  with

$$B_{K\eta^{(q)}} = i \frac{G_F}{\sqrt{2}} f_B f_K f_{\eta^{(q)}}^r, \quad B_{\eta_q^{(q)}K} = i \frac{G_F}{\sqrt{2}} f_B f_K f_{\eta^{(q)}}^q, \tag{17}$$

(where  $r = q$  or  $s$ ), respectively.  $b_{3,4,3EW,4EW}$  are the coefficients due to weak annihilation of penguin operators.  $b_{S3}$  is originated from the singlet penguin contribution which is introduced in [31,32]. Note that following the approximation adopted in [31], we have neglected single weak annihilations  $\beta_{S1}, \beta_{S2}, \beta_{S3,EW}$ , and only keep  $\beta_{S3}$ .

In the above results, we adopt  $a_{i=1,\dots,10}^p$  and  $b_Q^p$ , given by the QCD factorization (QCDF) calculation in [31]. The NP effects due to scalar/pseudoscalar operators can be included by replacing  $c_5$  and  $c_6$  with the effective ones  $c_{5(6)}^{\text{eff}} = c_{5(6)} + \Delta c_{5(6)}$  in the  $\bar{B} \rightarrow \bar{K}\eta_s$  decay amplitudes in the following way:

$$\begin{aligned}
\Delta c_5 &= \frac{1}{2}(-\bar{c}_{16} + \bar{c}_{18} + c_{20} - c_{22}), \\
\Delta c_6 &= \frac{1}{2}(-\bar{c}_{15} + \bar{c}_{17} + c_{19} - c_{21}) \text{ for } \alpha_4^p, \beta_3^p,
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
\Delta c_5 &= \frac{1}{2}(c_{20} - c_{22}), \\
\Delta c_6 &= \frac{1}{2}(c_{19} - c_{21}) \text{ for } \alpha_3^p, \beta_2^p, \beta_{S3}.
\end{aligned} \tag{19}$$

Here  $\bar{c}_{15}, \bar{c}_{16}, \bar{c}_{17}$ , and  $\bar{c}_{18}$  are defined in (9).  $\eta$  and  $\eta'$  mesons states can be regarded as mixed states of  $|\eta_q\rangle \equiv \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle)$  and  $|\eta_s\rangle \equiv |\bar{s}s\rangle$  with a mixing angle  $\phi_\eta$  [32]:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi_\eta & -\sin\phi_\eta \\ \sin\phi_\eta & \cos\phi_\eta \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}. \tag{20}$$

Decay constants  $f_{\eta^{(q)}}^{q,s}$ , pseudoscalar densities  $h_{\eta^{(q)}}^{q,s}$  are defined by

$$\begin{aligned}
\langle \eta^{(l)}(p) | \bar{q} \gamma^\mu \gamma_5 q | 0 \rangle &= -\frac{i}{\sqrt{2}} f_{\eta^{(l)}}^q p^\mu, \\
\langle \eta^{(l)}(p) | \bar{s} \gamma^\mu \gamma_5 s | 0 \rangle &= -i f_{\eta^{(l)}}^s p^\mu, \\
2m_q \langle \eta^{(l)}(p) | \bar{q} \gamma_5 q | 0 \rangle &= -\frac{i}{\sqrt{2}} h_{\eta^{(l)}}^q, \\
2m_s \langle \eta^{(l)}(p) | \bar{q} \gamma_5 q | 0 \rangle &= -i h_{\eta^{(l)}}^s,
\end{aligned} \tag{21}$$

with  $m_q = (m_u + m_d)/2$ . As for explicit forms of  $f_{\eta^{(l)}}^{q,s}$ ,  $h_{\eta^{(l)}}^{q,s}$  and form factors  $F_0^{B \rightarrow \eta^{(l)}}(q^2)$ , we summarize in Appendix A.

### C. $B \rightarrow \phi K^*$ decay amplitudes

Decay amplitudes of  $\bar{B} \rightarrow \phi \bar{K}^*$  can be decomposed as

$$A(\bar{B} \rightarrow \phi \bar{K}^*) = \sum_{h=0,\pm} \bar{H}_{hh}, \tag{22}$$

where

$$\bar{H}_{hh} = \sum_{p=u,c} V_{pb} V_{ps}^* (\mathcal{T}_{\phi K^*,A}^{p,h} + \mathcal{T}_{\phi K^*,B}^{p,h}), \quad (h = 0, \pm), \tag{23}$$

are the amplitudes in the helicity basis. Amplitudes for the emission topology (the  $\mathcal{T}_A$  part) are given by<sup>4</sup>

$$\begin{aligned}
\sum_{p=u,c} V_{pb} V_{ps}^* \mathcal{T}_{\phi K^*,A}^{p,h} &= (-V_{tb} V_{ts}^*) \left\{ A_{(\bar{B}\bar{K}^*,\phi)-}^h \left[ a_3^h + a_4^h + a_5^h - r_\chi^\phi a_6^h - \frac{1}{2} (a_7^h - r_\chi^\phi a_8^h + a_9^h + a_{10}^h) \right] \right. \\
&\quad \left. + A_{(\bar{B}\bar{K}^*,\phi)+}^h [a_{11}^h + a_{12}^h + a_{13}^h - r_\chi^\phi a_{14}^h] + A_{(\bar{B}\bar{K}^*,\phi)T+}^h \left[ a_{23}^h + \frac{1}{2} a_{24}^h \right] + A_{(\bar{B}\bar{K}^*,\phi)T-}^h \left[ a_{25}^h + \frac{1}{2} a_{26}^h \right] \right\},
\end{aligned} \tag{27}$$

with  $r_\chi^\phi$  given by

$$r_\chi^\phi = \frac{2m_\phi}{m_b(\mu)} \frac{f_\phi^T(\mu)}{f_\phi}. \tag{28}$$

Coefficients  $a_i^h$  ( $i = 3, \dots, 10$ ) have been calculated in QCDF [15,31]. However instead of  $c_5$  and  $c_6$ ,  $\bar{c}_5$  and  $\bar{c}_6$  should be used in the calculation of  $a_5$  and  $a_6$  (see (11));

$$a_i^h = \left( \bar{c}_i + \frac{\bar{c}_{i\pm 1}}{N_c} \right) + \mathcal{O}(\alpha_s), \quad \text{with } i = 23, 24, 25, 26, \tag{29}$$

where the radiative corrections are negligible. We summarized the explicit form of  $a_i^h$  for  $i = 11, \dots, 14$  due to the right-handed four-quark operators in Appendix C.

In (27), coefficients  $A_{(BV_1, V_2)\pm}^h$  and  $A_{(BV_1, V_2), T\pm}^h$  are given by

$$\begin{aligned}
A_{(\bar{B}\bar{K}^*,\phi)\mp}^h &\equiv \frac{G_F}{\sqrt{2}} \langle \phi(q, \varepsilon_1(h)) | \bar{s} \gamma^\mu (1 - \gamma_5) s | 0 \rangle \langle \bar{K}^*(p', \varepsilon_2(h)) | \bar{s} \gamma_\mu (1 \mp \gamma_5) b | \bar{B}(p) \rangle \\
&= \frac{G_F}{\sqrt{2}} \left\{ i f_\phi m_\phi \left[ \frac{-2i}{m_B + m_{K^*}} \epsilon_{\mu\nu\alpha\beta} \varepsilon_1^{\mu*} \varepsilon_2^{\nu*} p^\alpha p'^\beta V(q^2) \right] \right. \\
&\quad \left. \mp i f_\phi m_\phi \left[ (m_B + m_{K^*}) (\varepsilon_1^* \cdot \varepsilon_2^*) A_1(q^2) - (\varepsilon_1^* \cdot p) (\varepsilon_2^* \cdot p) \frac{2A_2(q^2)}{m_B + m_{K^*}} \right] \right\},
\end{aligned} \tag{30}$$

<sup>4</sup>The coefficient “1/2” in front of  $a_{24,26}^h$  can be realized as follows. Take  $O_{23}$  as an example. Because, under the Fierz transform,  $O_{23}$  can be written by

$$O_{23} = (1/2) \bar{s}_\alpha \sigma_{\mu\nu} (1 + \gamma_5) b_\beta \bar{s}_\beta \sigma^{\mu\nu} (1 + \gamma_5) s_\alpha - 6 \bar{s}_\alpha (1 + \gamma_5) b_\beta \bar{s}_\beta (1 + \gamma_5) s_\alpha, \tag{24}$$

therefore, in the factorization limit, we obtain

$$\begin{aligned}
\langle \phi \bar{K}^* | O_{23} | \bar{B} \rangle &= \langle \phi | \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) s | 0 \rangle \langle \bar{K}^* | \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b | \bar{B} \rangle + \frac{1}{2} \langle \phi | \bar{s}_\beta \sigma^{\mu\nu} (1 + \gamma_5) s_\alpha | 0 \rangle \langle \bar{K}^* | \bar{s}_\alpha \sigma_{\mu\nu} (1 + \gamma_5) b_\beta | \bar{B} \rangle \\
&= \left( 1 + \frac{1}{2N_c} \right) \langle \phi | \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) s | 0 \rangle \langle \bar{K}^* | \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b | \bar{B} \rangle.
\end{aligned} \tag{25}$$

Note that the second term in the right-hand side of (24) gives no contribution since the local scalar current cannot couple to  $\phi$ . Similarly, in the factorization limit we have

$$\langle \phi \bar{K}^* | O_{24} | \bar{B} \rangle = \left( \frac{1}{N_c} + \frac{1}{2} \right) \langle \phi | \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) s | 0 \rangle \langle \bar{K}^* | \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b | \bar{B} \rangle. \tag{26}$$

The same procedure can be applied to the matrix elements containing  $O_{25}$  and  $O_{26}$ .

$$\begin{aligned}
A_{(\bar{B}\bar{K}^*,\phi),T\pm}^h &\equiv \frac{G_F}{\sqrt{2}} \langle \phi(q, \varepsilon_1(h)) | \bar{s} \sigma_{\mu\nu} s (1 \pm \gamma_5) | 0 \rangle \langle \bar{K}^*(p', \varepsilon_2(h)) | \bar{s} \sigma^{\mu\nu} (1 \pm \gamma_5) b | \bar{B}(p) \rangle \\
&= \frac{G_F}{\sqrt{2}} f_\phi^T \left\{ 8 \varepsilon_{\mu\nu\rho\sigma} \varepsilon_1^{\mu*} \varepsilon_2^{\nu*} p^\rho p'^\sigma T_1(q^2) \mp 4i T_2(q^2) [(\varepsilon_1^* \cdot \varepsilon_2^*)(m_B^2 - m_{K^*}^2) - 2(\varepsilon_1^* \cdot p)(\varepsilon_2^* \cdot p)] \right. \\
&\quad \left. \pm 8i T_3(q^2) (\varepsilon_1^* \cdot p)(\varepsilon_2^* \cdot p) \frac{m_\phi^2}{m_B^2 - m_{K^*}^2} \right\}, \tag{31}
\end{aligned}$$

or in the explicit forms

$$\begin{aligned}
A_{(\bar{B}\bar{K}^*,\phi)\mp}^0 &= \mp \frac{G_F}{\sqrt{2}} (if_\phi m_\phi) (m_B + m_{K^*}) [a A_1(m_\phi^2) - b A_2(m_\phi^2)], \\
A_{(\bar{B}\bar{K}^*,\phi)-}^\pm &= \frac{G_F}{\sqrt{2}} (if_\phi m_\phi) \left[ (m_B + m_{K^*}) A_1(m_\phi^2) \mp \frac{2m_B p_c}{m_B + m_{K^*}} V(m_\phi^2) \right], \\
A_{(\bar{B}\bar{K}^*,\phi)+}^\pm &= \frac{G_F}{\sqrt{2}} (if_\phi m_\phi) \left[ -(m_B + m_{K^*}) A_1(m_\phi^2) \mp \frac{2m_B p_c}{m_B + m_{K^*}} V(m_\phi^2) \right], \\
A_{(\bar{B}\bar{K}^*,\phi)T\pm}^0 &= \mp \frac{G_F}{\sqrt{2}} 4 (if_\phi^T) m_B^2 [h_2 T_2(m_\phi^2) - h_3 T_3(m_\phi^2)], \\
A_{(\bar{B}\bar{K}^*,\phi)T+}^\pm &= -\frac{G_F}{\sqrt{2}} 4 (if_\phi^T) m_B^2 [\pm f_1 T_1(m_\phi^2) - f_2 T_2(m_\phi^2)], \\
A_{(\bar{B}\bar{K}^*,\phi)T-}^\pm &= -\frac{G_F}{\sqrt{2}} 4 (if_\phi^T) m_B^2 [\pm f_1 T_1(m_\phi^2) + f_2 T_2(m_\phi^2)], \tag{32}
\end{aligned}$$

with  $a = (m_B^2 - m_\phi^2 - m_{K^*}^2)/(2m_\phi m_{K^*})$ ,  $b = (2m_B^2 p_c^2)/(m_\phi m_{K^*} (m_B + m_{K^*}))$  and

$$\begin{aligned}
f_1 &= 2p_c/m_B, \\
f_2 &= (m_B^2 - m_{K^*}^2)/m_B^2, \\
h_2 &= \frac{1}{2m_{K^*} m_\phi} \left[ \frac{(m_B^2 - m_\phi^2 - m_{K^*}^2)(m_B^2 - m_{K^*}^2)}{m_B^2} - 4p_c^2 \right], \\
h_3 &= \frac{1}{2m_{K^*} m_\phi} \left( \frac{4p_c^2 m_\phi^2}{m_B^2 - m_{K^*}^2} \right). \tag{33}
\end{aligned}$$

Here we have used decay constants and form factors defined in Appendix B.

Weak annihilation contributions (the  $\mathcal{T}_B$ -part) to  $\bar{B}^0 \rightarrow \phi \bar{K}^{*0}$  and  $B^- \rightarrow \phi K^{*-}$  decay amplitudes in the helicity basis can be given by

$$\begin{aligned}
\sum_{p=u,c} V_{pb} V_{ps}^* \mathcal{T}_{\phi K^{*0},B}^{p,h} &= B_{\phi K^*} (-V_{tb} V_{ts}^*) \left[ b_3^h - \frac{1}{2} b_{3EW}^h + b_5^h \right], \\
\sum_{p=u,c} V_{pb} V_{ps}^* \mathcal{T}_{\phi K^{*-},B}^{p,h} &= B_{\phi K^*} \{ (-V_{tb} V_{ts}^*) [b_3^h + b_{3EW}^h + b_5^h] \\
&\quad + V_{ub} V_{us}^* \cdot b_2^h \}, \tag{34}
\end{aligned}$$

where

$$B_{\phi K^*} \equiv i \frac{G_F}{\sqrt{2}} f_B f_{K^*} f_\phi \tag{35}$$

and

$$\begin{aligned}
b_3^h &= \frac{C_F}{N_c^2} [c_3 A_1^{i,h} + \bar{c}_5 (A_3^{i,h} + A_3^{f,h}) + N_c \bar{c}_6 A_3^{f,h}], \\
b_{3EW}^h &= \frac{C_F}{N_c^2} [c_9 A_1^{i,h} + c_7 (A_3^{i,h} + A_3^{f,h}) + N_c c_8 A_3^{f,h}], \\
b_2^h &= \frac{C_F}{N_c^2} c_2 A_1^{i,h}, \\
b_5^h &= -\frac{C_F}{N_c^2} [c_{11} A_2^{i,h} + \bar{c}_{13} (A_3^{i,h} + A_3^{f,h}) + N_c \bar{c}_{14} A_3^{f,h}], \tag{36}
\end{aligned}$$

with  $h = 0, -, +$ . Building blocks  $A_{1,3}^{i(f),h}$  can be found in the appendix of [15].

#### D. Scalar and pseudoscalar operators in the MSSM

In the MSSM, scalar and pseudoscalar operators can be induced by NHB penguin diagrams. We refer to such an operator as the MSSM-NHB scalar/pseudoscalar operators. In [28],  $b \rightarrow s \bar{\ell} \ell$  (where  $\ell$  denotes a charged lepton) scalar/pseudoscalar operators  $Q_{1,2}^{(f)}$  induced by the MSSM-NHB penguin diagrams are considered.  $b \rightarrow s \bar{q} q$  scalar/pseudoscalar operators can be obtained by replacing Higgs- $\bar{\ell} \ell$  vertex with Higgs- $\bar{q} q$  vertex [29]. They are

$$\begin{aligned}
\mathcal{O}_{15} &= \bar{s}(1 + \gamma_5)b \sum_q \frac{m_q}{m_b} \bar{q}(1 + \gamma_5)q, \\
\mathcal{O}_{16} &= \bar{s}_i(1 + \gamma_5)b_j \sum_q \frac{m_q}{m_b} \bar{q}_j(1 + \gamma_5)q_i, \\
\mathcal{O}_{17} &= \bar{s}(1 - \gamma_5)b \sum_q \frac{m_q}{m_b} \bar{q}(1 - \gamma_5)q, \\
\mathcal{O}_{18} &= \bar{s}_i(1 - \gamma_5)b_j \sum_q \frac{m_q}{m_b} \bar{q}_j(1 - \gamma_5)q_i, \\
\mathcal{O}_{19} &= \bar{s}(1 + \gamma_5)b \sum_q \frac{m_q}{m_b} \bar{q}(1 - \gamma_5)q, \\
\mathcal{O}_{20} &= \bar{s}_i(1 + \gamma_5)b_j \sum_q \frac{m_q}{m_b} \bar{q}_j(1 - \gamma_5)q_i, \\
\mathcal{O}_{21} &= \bar{s}(1 - \gamma_5)b \sum_q \frac{m_q}{m_b} \bar{q}(1 + \gamma_5)q, \\
\mathcal{O}_{22} &= \bar{s}_i(1 - \gamma_5)b_j \sum_q \frac{m_q}{m_b} \bar{q}_j(1 + \gamma_5)q_i,
\end{aligned} \tag{37}$$

where  $q = u, d, s, c$ .<sup>5</sup> The Wilson coefficients  $C_i(\mu)$  of  $\mathcal{O}_i$  with  $i = 15, \dots, 22$  at  $\mu = m_W$  are given by [28,29]:

$$\begin{aligned}
C_{15}(m_W) &= \frac{e^2}{16\pi^2} (C_S + C_P), \\
C_{17}(m_W) &= \frac{e^2}{16\pi^2} (C'_S - C'_P), \\
C_{19}(m_W) &= \frac{e^2}{16\pi^2} (C_S - C_P), \\
C_{21}(m_W) &= \frac{e^2}{16\pi^2} (C'_S + C'_P), \\
C_i &= 0 \quad (i = 16, 18, 20, 22),
\end{aligned} \tag{38}$$

and four-quark tensor operators are not directly induced. Here

$$\begin{aligned}
C_S^{(i)} &= \frac{4}{3\lambda_t} \frac{g_s^2}{g_2 \sin^2 \theta_W} \frac{m_b^2}{m_{H^0}^2} \frac{\cos \alpha^2 + r_s \sin^2 \alpha}{\cos^2 \beta} \\
&\quad \times \frac{m_{\bar{g}}}{m_b} f'_b(x) \delta_{23}^{dLL(RR)} \delta_{33}^{dLR(LR^*)}, \\
C_P^{(i)} &= \mp \frac{4}{3\lambda_t} \frac{g_s^2}{g_2 \sin^2 \theta_W} \frac{m_b^2}{m_{A^0}^2} (r_p + \tan^2 \beta) \\
&\quad \times \frac{m_{\bar{g}}}{m_b} f'_b(x) \delta_{23}^{dLL(RR)} \delta_{33}^{dLR(LR^*)},
\end{aligned} \tag{39}$$

with  $r_s = m_{H^0}^2/m_{h^0}^2$ ,  $r_p = m_{A^0}^2/m_{Z^0}^2$ ,  $x = m_{\bar{q}}^2/m_{\bar{g}}^2$  and  $\lambda_t \equiv V_{tb}V_{ts}^*$ .  $g_2$  and  $g_s$  are gauge couplings for the weak and strong interactions, respectively.  $m_{h^0}$ ,  $m_{H^0}$ , and  $m_{A^0}$  are

masses of neutral-Higgs bosons  $h^0$ ,  $H^0$ , and  $A^0$ , respectively.  $\alpha$  is the neutral-Higgs mixing angle and  $\tan \beta$  is the ratio of the two Higgs vacuum expectation values,  $m_{\bar{g}}$  and  $m_{\bar{q}}$  are the gluino mass and common squark mass, respectively. Factors  $\delta_{23}^{dLL}$ ,  $\delta_{23}^{dRR}$ , and  $\delta_{33}^{dLR}$  are down-type left-light second-third, right-right second-third, and left-right third generation squark mixing parameters, respectively. The loop function  $f'_b(x)$  is defined as  $f'_b(x) \equiv (x^2/2)\partial^2 f_{b0}(x)/\partial x^2$  and  $f_{b0}(x)$  is defined in [29].  $f'_b(x)$  is given by

$$f'_b(x) = -\frac{x(-1+x^2-2x \log x)}{2(x-1)^3}. \tag{40}$$

Because  $O_i$  ( $i = 15, \dots, 22$ ) and  $\mathcal{O}_i$  are related by

$$\frac{m_s}{m_b} O_i \subseteq \mathcal{O}_i, \tag{41}$$

the Wilson coefficients  $c_i(\mu)$  for  $O_i(\mu)$  with  $i = 15, \dots, 26$  at  $\mu = m_W$  are given by

$$\begin{aligned}
c_{15}(m_W) &= \frac{m_s}{m_b} C_{15} = D(A - B)\xi, \\
c_{17}(m_W) &= \frac{m_s}{m_b} C_{17} = D(A - B)\xi', \\
c_{19}(m_W) &= \frac{m_s}{m_b} C_{19} = D(A + B)\xi, \\
c_{21}(m_W) &= \frac{m_s}{m_b} C_{21} = D(A + B)\xi', \\
c_i &= 0 \quad \text{for } i = 16, 18, 20, 22, 23, 24, 25, 26,
\end{aligned} \tag{42}$$

where

$$\begin{aligned}
D &\equiv \frac{1}{12\pi^2} \frac{1}{\lambda_t} \frac{e^2 g_s^2}{g^2 \sin^2 \theta_W} f'_b(m_{\bar{q}}^2/m_{\bar{g}}^2) m_s m_{\bar{g}}, \\
A &\equiv \frac{1}{m_{H^0}^2} \left( \frac{\cos^2 \alpha + (m_{H^0}^2/m_{h^0}^2) \sin^2 \alpha}{\cos^2 \beta} \right), \\
B &\equiv \frac{1}{m_{A^0}^2} \left( \frac{m_{A^0}^2}{m_{Z^0}^2} + \tan^2 \beta \right), \\
\xi &\equiv \delta_{23}^{dLL} \delta_{33}^{dLR}, \quad \xi' \equiv \delta_{23}^{dRR} \delta_{33}^{dLR*}.
\end{aligned} \tag{43}$$

Since the Wilson coefficients for  $(\bar{s}b)(\bar{d}d)$  scalar and pseudoscalar operators are suppressed by  $m_d/m_s$ , we neglect contributions for operators with  $q = d$  and we consider only  $b \rightarrow s\bar{s}s$  type operators. In (43), we note that  $D$  is almost positive and real because  $f'_b(x) < 0$ ,  $\text{Re}\lambda_t < 0$  and the imaginary part of  $\lambda_t$  is negligibly small.

There are three enhancement factors:  $\tan \beta$ ,  $1/m_{A^0}$ , and  $1/m_{H^0}$  in Eq. (43). Therefore, if  $\delta_{33}^{dLR}$  is sizable and neutral Higgses,  $A^0$  and  $H^0$ , are sufficiently light and  $\tan \beta$  is large, then the Wilson coefficients for scalar/pseudoscalar operators can be large enough. In the present paper, for simplicity, we neglect the mixing between scalar/pseudoscalar and tensor operators through renormalization-group equations (RGEs) since such a mixing effect is small. Thus we

<sup>5</sup>Precisely speaking, in the two doublet Higgs model, couplings of the light neutral Higgs  $h^0$  to up-type quarks are suppressed by  $\tan \beta$  compared with the down-type quarks. Therefore we can neglect contributions of up-type quarks.

assume  $c_i(m_b) \propto c_i(m_W)$  and we use the same symbols  $A, B, D, \xi$ , and  $\xi'$  at  $\mu \sim m_b$ .

Under the existence of the MSSM-NHB scalar/pseudoscalar operators (43),  $\Delta c_{5,6}$ , given in (18) and (19), in  $B \rightarrow K \eta_s$  decays are rewritten by

$$\Delta c_6 = \begin{cases} DB(|\xi|e^{\pm i\phi} - |\xi'|e^{\pm i\phi'}), & \text{for } \alpha_4, \beta_3, \\ \frac{1}{2}DB\left(2 - \frac{B-A}{B}\right)(|\xi|e^{\pm i\phi} - |\xi'|e^{\pm i\phi'}), & \text{for } \alpha_3, \beta_2, \beta_{S3}, \end{cases} \quad \Delta c_5 = 0, \quad (44)$$

where  $\xi^{(\prime)} = |\xi^{(\prime)}| \exp(i\phi^{(\prime)})$ . As for the Wilson coefficients in  $B \rightarrow \phi K^*$  decays, we use the replacements (11). They are given by

$$\begin{aligned} \bar{c}_6 - c_6 &= -\frac{1}{2}D(A+B)\xi' = \frac{1}{2}\left(\frac{B-A}{B} - 2\right)DB|\xi'|e^{i\phi'}, & \bar{c}_{14} &= -\frac{1}{2}D(A+B)\xi = \frac{1}{2}\left(\frac{B-A}{B} - 2\right)DB|\xi|e^{i\phi}, & \bar{c}_5 &= c_5, \\ c_{11} = c_{12} = \bar{c}_{13} &= 0, & \bar{c}_{23} &= \frac{1}{12}D(A-B)\xi, & \bar{c}_{24} &= -\frac{1}{6}D(A-B)\xi, & \bar{c}_{25} &= \frac{1}{12}D(A-B)\xi', & \bar{c}_{26} &= -\frac{1}{6}D(A-B)\xi'. \end{aligned} \quad (45)$$

As for  $a_{23-26}$ , we parametrized the following coefficients that appear in (27):

$$\begin{aligned} a_{23} + \frac{1}{2}a_{24} &\approx \frac{1}{8N_c} \frac{B-A}{B} DB|\xi|e^{i(\delta \pm \phi)}, \\ a_{25} + \frac{1}{2}a_{26} &\approx \frac{1}{8N_c} \frac{B-A}{B} DB|\xi'|e^{i(\delta' \pm \phi')}, \end{aligned} \quad (46)$$

where the  $\alpha_s$  corrections are negligible. However, we still parametrize the strong phases  $\delta$  and  $\delta'$  here. Actually the strong phases are consistent with zero in the fit. Here and below the helicity labels are omitted for  $a_{23-26}$  since these coefficients very weakly depend on their helicities.

The factor  $(B-A)/B$  depends on the details of the neutral-Higgs sector. In Fig. 1, we have plotted the value of  $(B-A)/B$  for various  $m_A$  and  $\tan\beta$  in the MSSM. In the MSSM,  $(B-A)/B$  is always smaller than 1 and  $-0.1 \leq (B-A)/B \leq 0.3$  for  $\tan\beta \geq 1$ . When  $(B-A)/B \sim 0.2$ , the ratio  $|(a_{23(25)} + \frac{1}{2}a_{24(26)})/\Delta c_6| \approx 0.02(B-A)/B \approx \mathcal{O}(10^{-2})$ .

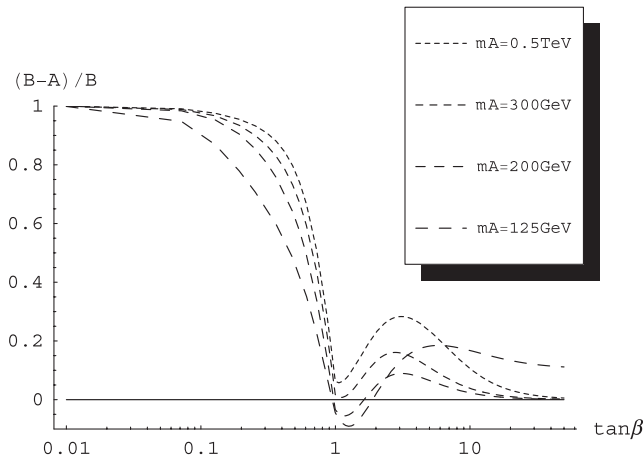


FIG. 1.  $(B-A)/B$  for various  $\tan\beta$  and  $m_A$ .

### III. NUMERICAL ANALYSIS

#### A. Numerical inputs

We summarize input parameters in Table I. As for  $B \rightarrow K^*$  vector and tensor form factors, we follow the light-cone sum-rule (LCSR) results [35], defined as

$$F(q^2) = F(0) \exp(c_1 q^2/m_B^2 + c_2 q^2/m_B^2), \quad (47)$$

for  $F \equiv A_0, A_1, A_2, V, T_1, T_2, T_3$ , where  $c_1$  and  $c_2$  are listed in Table III of [17]. We use  $m_{B^*} \approx m_{B^0} = 5.279$  GeV. We parametrize  $\lambda_B$  to be  $m_B/\lambda_B \equiv \int_0^1 dy \Phi_{B1}(y)/y$ , where  $\Phi_{B1}(y)$  is one of the two  $B$  meson light-cone distribution amplitudes with  $y$  being the momentum fraction carried by the light spectator quark in the  $B$  meson. As for the renormalization scale we use  $\mu = m_b/2$ . In the calculation of the hard spectator and the weak annihilation, we adopt  $\mu_h = \sqrt{\Lambda_h \cdot \mu} \sim 1$  GeV (with  $\Lambda_h = 0.5$  GeV) corresponding to the hadronic scale.

We also include hadronic uncertainty parameters defined in [15,36]:  $X_H^M, X_A^M$  ( $M = K, \eta^{(\prime)}, K^*$ ), and  $X_L^{K^*}$ . For simplicity, in the fit, we assume that  $X_{A,H}^K = X_{A,H}^{\eta^{(\prime)}} \equiv X_{A,H}^{(K\eta^{(\prime)})}$  in  $B \rightarrow K \eta^{(\prime)}$  decays, and parameterize them as [36]

$$X_{A,H}^{(K\eta^{(\prime)})} = [1 + \rho_{A,H} \exp(i\phi_{A,H})] \log\left(\frac{m_B}{\Lambda_h}\right), \quad (48)$$

$$0 \leq \rho_{A,H} \leq 1.$$

In  $B \rightarrow \phi K^*$  decays we have fixed  $X_L = m_B/\Lambda_h$  and  $X_H = \log(m_B/\Lambda_h)$  (i.e.,  $\rho_H = \rho_L = 0$ ), because in these decays the branching ratios are very insensitive to them.

As for NP effects, we use  $DB|\xi|, DB|\xi'|, \phi, \phi', \delta, \delta'$ , and  $(B-A)/B$  as the independent parameters. We have constrained weak and strong phases to be  $|\phi^{(\prime)}| \leq \pi$  and  $|\delta^{(\prime)}| \leq \pi/2$ , respectively. In the fit, for simplicity we consider the two scenarios: (i) NP-(A) for which  $\xi' = 0$  and (ii) NP-(B) for which  $\xi = 0$ .

TABLE I. Input parameters.

Decay constants [5,31]	$f_\pi = 131$ MeV, $f_K = 160$ MeV, $f_B = 210$ MeV, $f_q = (1.07 \pm 0.02)f_\pi$ , $f_s = (1.34 \pm 0.06)f_\pi$ , $f_{K^*} = 218$ MeV, $f_{K^*}^T = 175$ MeV, $f_\phi = 221$ MeV, $f_\phi^T = 175$ MeV
$B$ meson parameter [31]	$\lambda_B = 200_{-0}^{+250}$ MeV
$\eta - \eta'$ mixing angle [31]	$\phi_\eta = 39.3^\circ \pm 1.0^\circ$
CKM parameters [33]	$A = 0.807 \pm 0.018$ , $\lambda = 0.2265 \pm 0.0008$ , $ V_{ub}/V_{cb}  = 0.881_{-0.010}^{+0.011}$ , $\phi_3(\gamma) = (67.4_{-4.5}^{+2.8})^\circ$ , $\sin 2\phi_1(\sin 2\beta) = 0.688_{-0.024}^{+0.025}$
$B$ meson lifetimes [34]	$\tau_{B^0} = 1.530$ ps, $\tau_{B^-} = 1.638$ ps
$B \rightarrow P$ form factors [31]	$F_0^{B \rightarrow \pi}(0) = 0.28$ , $F_0^{B \rightarrow K}(0) = 0.34$

TABLE II. World averages of observables for  $B \rightarrow K\eta^{(\prime)}$  are shown in the second column [34,37–46]. Upper limits are at 90% CL. In the third and fourth columns we have shown best-fit values for combined fit with corrections received from  $B \rightarrow \phi K^*$  annihilation. Corresponding best-fit parameters are shown in Table IV and in Fig. 2. Best-fit values of  $B \rightarrow \phi K^*$  are shown in Table III.

Observable	Experiment	Combined Fit with $\phi K^*$ ann.	
		NP-(A)	NP-(B)
$\mathcal{B}(B^+ \rightarrow \eta' K^+) \times 10^6$	$69.7_{-2.7}^{+2.8}$	$68.9 \pm 2.2$	$68.9 \pm 2.2$
$\mathcal{B}(B^0 \rightarrow \eta' K^0) \times 10^6$	$64.9 \pm 3.5$	$66.3 \pm 2.1$	$66.3 \pm 2.1$
$\mathcal{B}(B^+ \rightarrow \eta K^+) \times 10^6$	$2.2 \pm 0.3$	$2.2 \pm 0.3$	$2.2 \pm 0.3$
$\mathcal{B}(B^0 \rightarrow \eta K^0) \times 10^6$	$(< 1.9)$	$1.5 \pm 0.3$	$1.5 \pm 0.3$
$A_{CP}(B^+ \rightarrow \eta' K^+)$	$0.031 \pm 0.021$	$0.031 \pm 0.021$	$0.030 \pm 0.018$
$A_{CP}(B^0 \rightarrow \eta' K^0)$	$0.09 \pm 0.06$	$0.02 \pm 0.01$	$0.02 \pm 0.01$
$A_{CP}(B^+ \rightarrow \eta K^+)$	$-0.29 \pm 0.11$	$-0.30 \pm 0.11$	$-0.31 \pm 0.10$
$A_{CP}(B^0 \rightarrow \eta K^0)$	(N.A.)	$-0.20 \pm 0.11$	$-0.14 \pm 0.07$
$-\eta_{CP} S_{K\eta'}(B^0 \rightarrow \eta' K^0)$	$0.61 \pm 0.07$	$0.70 \pm 0.03$	$0.70 \pm 0.01$
$-\eta_{CP} S_{K\eta}(B^0 \rightarrow \eta K^0)$	(N.A.)	$0.89 \pm 0.17$	$0.73 \pm 0.03$

## B. Experimental data

In the fit for  $\bar{B} \rightarrow \bar{K}\eta^{(\prime)}$  decays, we use 7 observables including 3 averaged branching fractions, 3 direct  $CP$  violations, and the  $\bar{B}^0 \rightarrow \bar{K}^0\eta'$  indirect  $CP$  violation  $-\eta_{CP} S_{K\eta'}$ . Here  $S_{K\eta^{(\prime)}}$  is defined by

$$S_{K\eta^{(\prime)}} = \frac{2 \text{Im}(\lambda_f)}{1 + |\lambda_f|^2}, \quad \lambda_f = \frac{q}{p} \frac{A(\bar{B}^0 \rightarrow K_{S,L}^0 \eta^{(\prime)})}{A(B^0 \rightarrow K_{S,L}^0 \eta^{(\prime)})}, \quad (49)$$

with  $q/p \simeq e^{-2i\phi_1}$  for  $B_q^0$ , and  $\eta_{CP}$  is the  $CP$  eigenvalue of  $|K_{S,L}^0 \eta^{(\prime)}\rangle$ . The value of  $-\eta_{CP} S_{K\eta^{(\prime)}}$  should be close to  $\sin 2\phi_1$  in the SM. The experimental data for  $B \rightarrow K\eta^{(\prime)}$  are listed in Table II.

For the  $B \rightarrow \phi K^*$  decays we have 20 observables, which include  $\bar{B}^{0,-} \rightarrow \phi \bar{K}^{*0,-}$  branching fractions ( $\mathcal{B}$ ), polarization fractions ( $f_\parallel, f_\perp$ ), and  $CP$  asymmetries ( $A_{CP}^{\text{tot}}, A_{CP}^0, A_{CP}^\perp$ ), phases of polarized modes ( $\phi_\parallel, \phi_\perp$ ), and phase differences ( $\Delta\phi_\parallel, \Delta\phi_\perp$ ), where  $A_{CP}^{\text{tot}} = (\sum_\lambda |\bar{A}_\lambda|^2 - \sum_\lambda |A_\lambda|^2) / (\sum_\lambda |\bar{A}_\lambda|^2 + \sum_\lambda |A_\lambda|^2)$ ,  $A_{CP}^\perp = (|\bar{A}_\perp|^2 - |A_\perp|^2) / (|\bar{A}_\perp|^2 + |A_\perp|^2)$ ,<sup>6</sup>  $\phi_\lambda = \arg(A_\lambda/A_0)$ ,  $\Delta\phi_\lambda = \frac{1}{2} \times$

<sup>6</sup>In *BABAR* measurements [47], instead of  $A_{CP}^\perp$ , the asymmetries of  $f_\lambda^{+1}$  and  $f_\lambda^{-1}$  are defined.  $f_\lambda^{+1}$  and  $f_\lambda^{-1}$  are the polarization fractions measured in  $\bar{B}$  and  $B$  decays, respectively.

$\arg(\bar{A}_\lambda/\bar{A}_0 \cdot A_0/A_\lambda)$  with  $\lambda = 0, \parallel, \perp$ . The experimental data for the  $B \rightarrow \phi K^*$  decays are shown in Table III.

## C. Combined fits

If we ignore the annihilation effects in  $B \rightarrow \phi K^*$  decays, the resulting  $\chi_{\text{min}}^2 \simeq 170$  is too huge; i.e. we cannot have a reliable fitting result. This is the fact that if the  $B \rightarrow \phi K^*$  polarization anomaly was mainly due to the tensor operators induced by the Fierz transformation, then the NP effects would lead to too large  $B \rightarrow K\eta^{(\prime)}$  branching ratios as compared with the data.

Once the  $B \rightarrow \phi K^*$  annihilation effects are included, we can see that the  $\chi_{\text{min}}^2$  is drastically small. In Table IV, we have summarized the best-fit values of the  $\chi_{\text{min}}^2$  and parameters.<sup>7</sup> The resulting  $\delta$  and  $\delta'$  are consistent with zero, which are also consistent with the fact that the  $\alpha_s$ -corrections to the tensor operators are negligible. The

<sup>7</sup>The errors of parameters in Table IV are obtained from the error matrix (covariance matrix) at the global minimum of  $\chi^2$ . The error matrix is the inverse matrix of the curvature matrix of chi-square function with respect to its free parameters. The errors of best-fit values in Tables II and III are estimated from the same error matrices for each NP scenario.



TABLE III. World averages and best-fit values of observables for  $\bar{B}^0 \rightarrow \phi \bar{K}^{*0}$  (upper) and  $B^- \rightarrow \phi K^{*-}$  (lower) [1–4,34,37]. In the third and fourth column, best-fit values for combined fit of  $B \rightarrow K\eta^{(\prime)}$  and  $B \rightarrow \phi K^*$  for NP-(A) and NP-(B) scenarios, including the contributions from  $B \rightarrow \phi K^*$  annihilations, are shown. Corresponding best-fit parameters are shown in Fig. 2 and Table IV. Best-fit values for  $B \rightarrow K\eta^{(\prime)}$  are shown in Table II.

Observable	Experiment	Combined Fit with $\phi K^*$ ann.	
		NP-(A)	NP-(B)
$\mathcal{B}_{\text{tot}} \times 10^6$	$9.5 \pm 0.8$	$9.4 \pm 0.6$	$9.3 \pm 0.6$
	$10.0 \pm 1.1$	$10.1 \pm 0.8$	$9.4 \pm 0.6$
$f_L$	$0.491 \pm 0.032$	$0.491 \pm 0.025$	$0.493 \pm 0.024$
	$0.50 \pm 0.05$	$0.499 \pm 0.028$	$0.486 \pm 0.025$
$f_{\perp}$	$0.252 \pm 0.031$	$0.247 \pm 0.013$	$0.239 \pm 0.021$
	$0.20 \pm 0.05$	$0.243 \pm 0.010$	$0.241 \pm 0.022$
$A_{CP}^{\text{tot}}$	$-0.01 \pm 0.06$	$-0.03 \pm 0.05$	$0.00 \pm 0.01$
	$-0.01 \pm 0.08$	$0.02 \pm 0.03$	$0.00 \pm 0.00$
$A_{CP}^0$	$0.02 \pm 0.07$	$-0.03 \pm 0.03$	$0.01 \pm 0.01$
	$0.17 \pm 0.11$	$0.05 \pm 0.07$	$0.00 \pm 0.00$
$A_{CP}^{\perp}$	$-0.11 \pm 0.12$	$0.02 \pm 0.03$	$0.00 \pm 0.00$
	$0.22 \pm 0.25$	$0.05 \pm 0.07$	$0.00 \pm 0.00$
$\phi_{\parallel}$	$2.37^{+0.14}_{-0.13}$	$2.34 \pm 0.09$	$2.53 \pm 0.02$
	$2.34 \pm 0.17$	$2.33 \pm 0.09$	$2.52 \pm 0.02$
$\phi_{\perp}$	$2.36 \pm 0.14$	$2.49 \pm 0.06$	$2.56 \pm 0.03$
	$2.58 \pm 0.17$	$2.48 \pm 0.06$	$2.55 \pm 0.03$
$\Delta\phi_{\parallel}$	$0.10 \pm 0.14$	$0.02 \pm 0.06$	$0.00 \pm 0.00$
	$0.07 \pm 0.21$	$0.03 \pm 0.08$	$-0.01 \pm 0.00$
$\Delta\phi_{\perp}$	$0.04 \pm 0.14$	$0.03 \pm 0.07$	$0.00 \pm 0.00$
	$0.19 \pm 0.21$	$0.04 \pm 0.09$	$-0.01 \pm 0.00$

fitted results for  $B \rightarrow K\eta^{(\prime)}$  and  $B \rightarrow \phi K^*$  are collected in Tables II and III.

The results obtained in [17], where the weak annihilation effects are not included, are given by<sup>8</sup>

$$\begin{aligned}
 |\tilde{a}_{23,\text{DY}}| &= 4.36^{+0.28}_{-0.18} \times 10^{-3}, \\
 |\tilde{a}_{25,\text{DY}}| &= 5.38^{+0.49}_{-0.31} \times 10^{-3},
 \end{aligned} \quad (50)$$

to be compared with our present upper bounds,

$$|\tilde{a}_{23}| \leq 7.1 \times 10^{-4}, \quad |\tilde{a}_{25}| \leq 6.1 \times 10^{-4}, \quad (51)$$

which are extracted from Fig. 2 and (46), and are much smaller than the values in (50). This indicates that the contributions of tensor operators induced from the scalar/pseudoscalar operators in the MSSM-Higgs are too small to explain the polarization puzzle, while the puzzle can be accommodated by the weak annihilations effects.

#### D. Consistency with SM and $B_s \rightarrow \mu^+ \mu^-$

The current upper bound for the branching fraction of  $B_s \rightarrow \mu^+ \mu^-$  [37,48] at 90% confidence level (CL) is

<sup>8</sup>Here  $\tilde{a}_{23}$  and  $\tilde{a}_{25}$  are defined by  $\tilde{a}_{23} \equiv a_{23} + \frac{1}{2}a_{24}$  and  $\tilde{a}_{25} \equiv a_{25} + \frac{1}{2}a_{26}$ , respectively [17]. We have factored out the CKM factor.

TABLE IV. Best-fit parameters obtained in NP-(A) and NP-(B) considering  $B \rightarrow \phi K^*$  annihilations. Numbers with (\*) indicate that they reach the upper or lower bound in the parameter space. Best-fit values of  $B \rightarrow K\eta^{(\prime)}$  and  $B \rightarrow \phi K^*$  are shown in Tables II and III, respectively. Weak phased NP parameters ( $DB\xi$ ,  $DB\xi'$ ) are plotted in Fig. 2.

	Combined fit with $\phi K^*$ annihilation effects	
	Scenario (A)	Scenario (B)
$\chi^2_{\text{min}}/\text{d.o.f.}$	9.8/17	15.5/17
$(B-A)/B$	$0.55 \pm 0.76$ (*)	$0.43 \pm 0.19$ (*)
$\delta, \delta'$	$\delta = +30 \pm 85^\circ$	$\delta' = -38 \pm 70^\circ$
$\rho_A[K\eta^{(\prime)}]$	$1.00 \pm 0.33$ (*)	$0.51 \pm 0.41$ (*)
$\phi_A[K\eta^{(\prime)}]$	$117 \pm 17^\circ$	$53 \pm 47^\circ$
$\rho_H[K\eta^{(\prime)}]$	$0.11 \pm 0.67$ (*)	$1.00 \pm 0.59$ (*)
$\phi_H[K\eta^{(\prime)}]$	$69 \pm 156^\circ$	$-167 \pm 25^\circ$
$\rho_A[\phi K^*]$	$0.57 \pm 0.04$	$0.55 \pm 0.02$
$\phi_A[\phi K^*]$	$-96 \pm 3^\circ$	$-85 \pm 3^\circ$

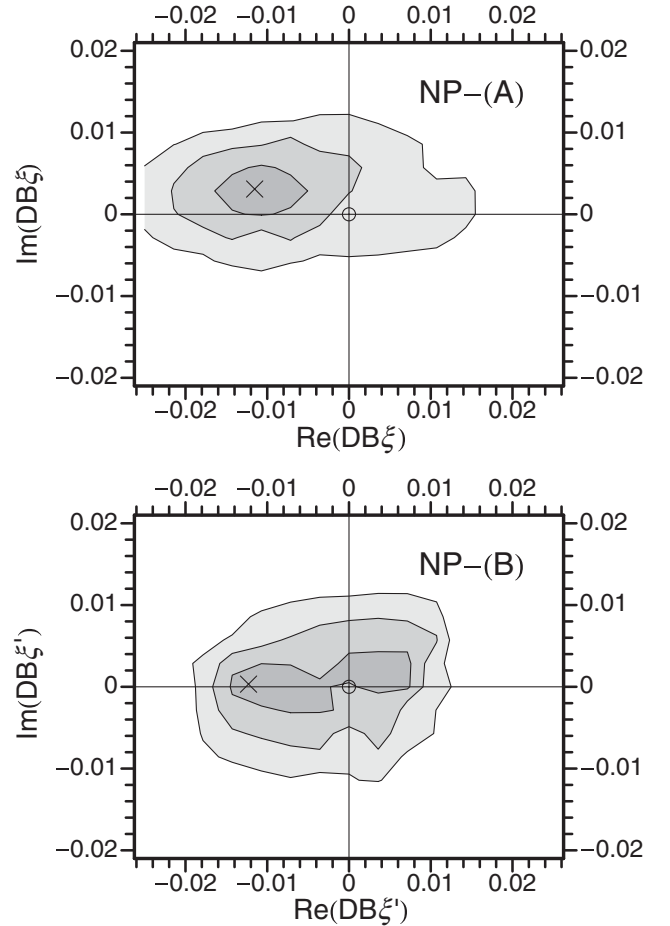


FIG. 2. Contour plots for  $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\text{min}}$  in  $\text{Re}(DB\xi)$  vs  $\text{Im}(DB\xi)$  [or  $\text{Re}(DB\xi')$  vs  $\text{Im}(DB\xi')$ ] for the NP scenario-A [or NP scenario-B]. Allowed regions of  $\Delta\chi^2 < 1$ ,  $1 < \Delta\chi^2 < 4$ , and  $4 < \Delta\chi^2 < 9$  are shown by dark, medium-dark, and light-gray regions, respectively. The “×” symbol indicates the location of the global minimum,  $\chi^2_{\text{min}}$ . The origin corresponds to the SM. The circle at the origin indicates the allowed upper limit from the  $B_s \rightarrow \mu^+ \mu^-$  data.

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \leq 7.5 \times 10^{-8}. \quad (52)$$

The branching fraction of  $b \rightarrow s \bar{\ell} \ell$  due to operators  $O_i^{(\ell)}$  (with  $i = 7, 9, 15, 17, 19, 21$ ) for  $\ell = e, \mu, \tau$ <sup>9</sup> is given by

$$\begin{aligned} \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) &= \tau_{B_s} \frac{G_F^2 m_{B_s}^3}{16\pi} f_{B_s}^2 \left( \frac{m_{B_s}}{m_b + m_s} \right)^2 |V_{tb} V_{ts}^*|^2 \\ &\times \sqrt{1 - 4\hat{m}^2} \{ (1 - 4\hat{m}^2) \\ &\times |c_{15}^{(\ell)} - c_{17}^{(\ell)} + c_{19}^{(\ell)} - c_{21}^{(\ell)}|^2 + |c_{15}^{(\ell)} \\ &+ c_{17}^{(\ell)} - c_{19}^{(\ell)} - c_{21}^{(\ell)} + 2\hat{m}(c_7^{(\ell)} - c_9^{(\ell)})|^2 \}, \end{aligned} \quad (53)$$

where  $\hat{m} \equiv m_\mu/m_{B_s}$ ,  $c_i^{(\ell)}$  are the Wilson coefficients of  $O_i^{(\ell)}$  at  $\mu = m_b$ , and we have used

$$\langle 0 | \bar{s} \gamma_5 b | \bar{B}_s \rangle = -i f_{B_s} \frac{m_{B_s}^2}{m_b + m_s}. \quad (54)$$

Here we note that if RGE effects are not large,  $c_i^{(\ell)} \sim (m_\ell/m_s) c_i$  and

$$\begin{aligned} (c_{15} - c_{17} + c_{19} - c_{21}) &= 2DA(\xi - \xi') \\ &= 2(1 - (B - A)/B)DB(\xi - \xi'), \\ (c_{15} + c_{17} - c_{19} - c_{21}) &= -2DB(\xi + \xi'). \end{aligned} \quad (55)$$

Using  $B_s$ 's lifetime  $\tau_{B_s} = 1.437$  ps, mass  $m_{B_s} = 5.366$  GeV, decay constant  $f_{B_s} = 215 \pm 25$  MeV, quark masses  $m_b = 4.9 \pm 0.1$  GeV, and  $m_s = 145 \pm 25$  MeV, we obtain upper bounds of  $DB\xi^{(\ell)}$  as

$$|DB\xi^{(\ell)}| \leq 9.2 \times 10^{-4}. \quad (56)$$

In Fig. 2 we have shown the SM ( $DB\xi = DB\xi' = 0$ ) and the upper bound of the NP effect constrained by the  $B_s \rightarrow \mu^+ \mu^-$  decay as the origin and a small circle at the origin, respectively. In the figures we have also drawn the contours of  $\Delta\chi^2 = 1, 4, \text{ and } 9$ , where  $\Delta\chi^2 \equiv \chi^2 - \chi_{\min}^2$ . The  $B_s \rightarrow \mu^+ \mu^-$  allowed region shares partly the  $\Delta\chi^2 \leq 1$  ( $1\sigma$ ) region in NP-(B), and is just outside of the  $\Delta\chi^2 \leq 4$  ( $2\sigma$ ) region in the scenario NP-(A). Since in both cases the  $B_s \rightarrow \mu^+ \mu^-$  data and the SM are located within contours where  $\chi^2/\text{d.o.f.}$  is sufficiently small, we can safely conclude that our two scenarios are consistent with the data for  $B_s \rightarrow \mu^+ \mu^-$  decay and with the SM.

In [23], the authors discuss the scalar/pseudoscalar operators induced by  $R$ -parity violating interactions in the supersymmetric standard models. Because they did not take into account the weak annihilation effects and possible constraints from  $B \rightarrow K\eta^{(\prime)}$ , large contributions due to

tensor operators to explain the  $B \rightarrow \phi K^*$  polarization puzzle are required and therefore the estimated magnitudes of the effects of scalar/pseudoscalar operators are much larger than the upper bound of  $B_s \rightarrow \mu^+ \mu^-$ .

#### IV. SUMMARY

We have studied the scalar/pseudoscalar operators, and tensor operators where the latter are obtained from scalar/pseudoscalar operators by the Fierz transformation, in  $B \rightarrow \phi K^*$  and  $B \rightarrow K\eta^{(\prime)}$  decays. We have considered the scalar/pseudoscalar operators induced by penguin diagrams of MSSM neutral-Higgs bosons.

Without the weak annihilations in  $B \rightarrow \phi K^*$ , we cannot obtain any reasonable solution to explain both  $B \rightarrow \phi K^*$  and  $B \rightarrow K\eta^{(\prime)}$  decays simultaneously in the NP region ( $-0.1 \leq (B - A)/B \leq 1$ ) of the MSSM induced by the neutral-Higgs bosons. Taking into account weak annihilation effects in  $B \rightarrow \phi K^*$ , we obtain best-fit results in good agreement with the  $B \rightarrow K\eta^{(\prime)}$  and  $B \rightarrow \phi K^*$  data. From the fitted parameters we estimate the magnitudes of the contributions due to NP tensor operators. They are, however, much smaller than the results of [17], which are introduced to explain the  $B \rightarrow \phi K^*$  polarization puzzle. The polarization puzzle can be explained mostly by a weak annihilation effect, as pointed out in [13–15]. The contributions of NP operators are constrained mainly by the fit of  $B \rightarrow K\eta^{(\prime)}$  data. While our results may allow nonvanishing NP effects, the data for the decays  $B \rightarrow K\eta^{(\prime)}$ ,  $B \rightarrow \phi K^*$  are consistent with the  $B_s \rightarrow \mu^+ \mu^-$  data as well as the SM prediction.

Finally, we remark on the recently observed large longitudinal polarization fraction  $f_L$  in  $B \rightarrow \phi K_2^*(1430)$  [1]. If tensor operators play a significant role in  $B \rightarrow VT$  (where  $T$  denotes a tensor meson) decays,  $f_L$  may significantly deviate from unity. The current  $B \rightarrow \phi K_2^*(1430)$  experiment seems to be consistent with our conclusion since in our analysis the effect due to tensor operators is found to be very small. However, in the present study we cannot exclude the possibility that sizable NP effects contribute directly to tensor operators, instead of scalar/pseudoscalar operators, and, moreover, a cancellation may take place between weak annihilations and contributions due to NP tensor operators in the  $B \rightarrow \phi K_2^*(1430)$  decay. For the point of view of the new physics,  $B \rightarrow \phi K_2^*(1430)$  may be sensitive to the  $B \rightarrow K_2^*$  tensor form factor which can be further explored from the  $B \rightarrow K_2^*(1430)\gamma$  decay.

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<sup>9</sup>We have defined  $O_i^{(\ell)}$  as  $O_{7(9)}^{(\ell)} = \bar{s}(1 - \gamma_5)b\bar{\ell}(1 \pm \gamma_5)\ell$ , and  $O_i^{(\ell)}$  ( $i = 15, \dots, 21$ ) with replacements:  $\bar{s}(1 \pm \gamma_5)s \rightarrow \bar{\ell}(1 \pm \gamma_5)\ell$  and  $\bar{s}\gamma^\mu(1 + \gamma_5)s \rightarrow \bar{\ell}\gamma^\mu(1 + \gamma_5)s$ .

### APPENDIX A: DECAY CONSTANTS AND FORM FACTORS FOR $\eta$ AND $\eta'$ MESONS

The  $|\eta\rangle$  and  $|\eta'\rangle$  meson states are defined as the mixed states of  $|\eta_q\rangle$  and  $|\eta_s\rangle$ , as stated in (20) [32]. In this section we summarize the notations in [32]. Decay constants  $f_{\eta^{(l)}}^{q,s}$  are given by

$$\begin{aligned} f_{\eta}^q &= f_q \cos\phi_{\eta}, & f_{\eta}^s &= -f_s \sin\phi_{\eta}, \\ f_{\eta'}^q &= f_q \sin\phi_{\eta}, & f_{\eta'}^s &= f_s \cos\phi_{\eta}, \end{aligned} \quad (\text{A1})$$

and in the same way, pseudoscalar densities  $h_{\eta^{(l)}}^{q,s}$  are defined as

$$\begin{aligned} h_{\eta}^q &= h_q \cos\phi_{\eta}, & h_{\eta}^s &= -h_s \sin\phi_{\eta}, \\ h_{\eta'}^q &= h_q \sin\phi_{\eta}, & h_{\eta'}^s &= h_s \cos\phi_{\eta}, \end{aligned} \quad (\text{A2})$$

where  $h_{q,s}$  are defined by

$$\begin{aligned} h_q &= f_q(m_{\eta}^2 \cos^2\phi_{\eta} + m_{\eta'}^2 \sin^2\phi_{\eta}) \\ &\quad - \sqrt{2}f_s(m_{\eta'}^2 - m_{\eta}^2) \sin\phi_{\eta} \cos\phi_{\eta}, \\ h_s &= f_s(m_{\eta}^2 \cos^2\phi_{\eta} + m_{\eta'}^2 \sin^2\phi_{\eta}) \\ &\quad - \frac{1}{\sqrt{2}}f_q(m_{\eta'}^2 - m_{\eta}^2) \sin\phi_{\eta} \cos\phi_{\eta}. \end{aligned} \quad (\text{A3})$$

$B \rightarrow \eta^{(l)}$  form factors are defined as

$$F^{B \rightarrow \eta^{(l)}} = F_1 \frac{f_{\eta^{(l)}}^q}{f_{\pi}} + F_2 \frac{\sqrt{2}f_{\eta^{(l)}}^q + f_{\eta^{(l)}}^s}{\sqrt{3}f_{\pi}}, \quad (\text{A4})$$

and in the present paper we take  $F_1 = F_0^{B \rightarrow \pi}(0)$  and  $F_2 = 0$ .  $f_{\pi}$  is the decay constant of the pion.

### APPENDIX B: DECAY CONSTANTS AND FORM FACTORS IN $B \rightarrow VV$ DECAYS

We have used

$$\langle \phi(q, \varepsilon_{\phi}) | V^{\mu} | 0 \rangle = f_{\phi} m_{\phi} \varepsilon_{\phi}^{\mu*}, \quad (\text{B1})$$

$$\begin{aligned} \langle \bar{K}^*(p_{K^*}, \varepsilon_{K^*}) | V_{\mu} | \bar{B}(p_B) \rangle \\ = \frac{2}{m_B + m_{K^*}} \epsilon_{\mu\nu\alpha\beta} \varepsilon_{K^*}^{\nu*} p_B^{\alpha} p_{K^*}^{\beta} V(q^2), \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \langle \bar{K}^*(p_{K^*}, \varepsilon_{K^*}) | A_{\mu} | \bar{B}(p_B) \rangle \\ = i \left[ (m_B + m_{K^*}) \varepsilon_{K^*}^{\mu*} A_1(q^2) - (\varepsilon_{K^*}^* \cdot p_B) \right. \\ \left. \times (p_B + p_{K^*})_{\mu} \frac{A_2(q^2)}{m_B + m_{K^*}} \right] \\ - 2im_{K^*} \frac{\varepsilon_{K^*}^* \cdot p_B}{q^2} q_{\mu} [A_3(q^2) - A_0(q^2)], \end{aligned} \quad (\text{B3})$$

for current operators, and

$$\langle \phi(q, \varepsilon_1) | \bar{s} \sigma^{\mu\nu} s | 0 \rangle = -if_{\phi}^T (\varepsilon_1^{\mu*} q^{\nu} - \varepsilon_1^{\nu*} q^{\mu}), \quad (\text{B4})$$

$$\begin{aligned} \langle \bar{K}^*(p', \varepsilon_2) | \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b | \bar{B}(p) \rangle \\ = i \epsilon_{\mu\nu\rho\sigma} \varepsilon_2^{\nu*} p^{\alpha} p'^{\beta} 2T_1(q^2) + \{ \varepsilon_2^{\mu*} (m_B^2 - m_{K^*}^2) \\ - (\varepsilon_2^* \cdot p)(p + p')_{\mu} \} T_2(q^2) \\ + (\varepsilon_2^* \cdot p_B) \left[ q_{\mu} - \frac{q^2}{m_B^2 m_{K^*}^2} (p + p')_{\mu} \right] T_3(q^2), \end{aligned} \quad (\text{B5})$$

for tensor operators. In (B3) and (B5),  $A_3(0) = A_0(0)$ ,  $T_1(0) = T_2(0)$ , and

$$A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2). \quad (\text{B6})$$

### APPENDIX C: THE COEFFICIENTS $a_i^{p,h}$ CORRESPONDING TO RIGHT-HANDED FOUR-QUARK OPERATORS

In (27), the expressions for effective parameters  $a_{11(12)}^{p,h}$  corresponding to right-handed four-quark operators are

$$\begin{aligned} a_i^{p,h}(V_1 V_2) = \left[ \left( c_i + \frac{c_{i\pm 1}}{N_c} \right) N_i(V_2) + \frac{c_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \left( V_i^h(V_2) \right. \right. \\ \left. \left. + \frac{4\pi^2}{N_c} H_i^h(V_1 V_2) \right) + P_i^{p,h}(V_2) \right], \end{aligned} \quad (\text{C1})$$

with  $N_i(V_2) = 1$  for  $i = 11, 12$ . For  $a_{13(14)}^{p,h}$ , one should replace  $c_i$  by  $\bar{c}_i$ , and have  $N_{13}(V_2) = 1$ ,  $N_{14}(V_2) = 0$ .  $V_i^h(V_2)$  account for vertex corrections,  $H_i^h(V_1 V_2)$  for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the  $B$  meson, and  $P_i(V_2)$  for penguin contractions. The vertex corrections read

$$V_i^0(V_2) = \begin{cases} \int_0^1 dx \Phi_{V_2}(x) [12 \ln \frac{m_b}{\mu} - 18 + g_T(x)], & (i = 11, 12), \\ \int_0^1 dx \Phi_{V_2}(x) [-12 \ln \frac{m_b}{\mu} + 6 - g(1-x)], & (i = 13), \\ \int_0^1 dx \Phi_{v_2}(x) [-6 + h(x)], & (i = 14), \end{cases} \quad (\text{C2})$$

and

$$V_i^+(V_2) = \begin{cases} \int_0^1 dx \Phi_{b_2}(x) [12 \ln \frac{m_b}{\mu} - 18 + g_T(x)], & (i = 11, 12), \\ \int_0^1 dx \Phi_{a_2}(x) [-12 \ln \frac{m_b}{\mu} + 6 - g_T(1-x)], & (i = 13), \\ 0, & (i = 14), \end{cases} \quad (C3)$$

where  $\Phi_V(x)$ ,  $\Phi_v(x)$ ,  $\Phi_a(x)$ ,  $\Phi_b(x)$ ,  $g(x)$ ,  $h(x)$ , and  $g_T(x)$  are defined in [15,31].

$H_i^h(V_1 V_2)$  have the expressions:

$$H_{11}^0(V_1 V_2) = H_{12}^0(V_1 V_2) = \frac{f_B f_{V_1} f_{V_2}}{X_0^{(\bar{B}V_1, V_2)}} \int_0^1 d\rho \frac{\Phi_1^B(\rho)}{\rho} \int_0^1 dv \int_0^1 du \left( \frac{\Phi_{V_1}(v) \Phi_{V_2}(u)}{\bar{u} \bar{v}} + r_{\chi'}^{V_1} \frac{\Phi_{v_1}(v) \Phi_{V_2}(u)}{u \bar{v}} \right), \quad (C4)$$

$$H_{13}^0(V_1 V_2) = -\frac{f_B f_{V_1} f_{V_2}}{X_0^{(\bar{B}V_1, V_2)}} \int_0^1 d\rho \frac{\Phi_1^B(\rho)}{\rho} \int_0^1 dv \int_0^1 du \left( \frac{\Phi_{V_1}(v) \Phi_{V_2}(u)}{u \bar{v}} + r_{\chi'}^{V_1} \frac{\Phi_{v_1}(v) \Phi_{V_2}(u)}{\bar{u} \bar{v}} \right), \quad (C5)$$

$H_{14}^0(V_1 V_2) = 0$ , and

$$\begin{aligned} H_{11}^+(V_1 V_2) &= H_{12}^+(V_1 V_2) = -\frac{f_B f_{V_1}^{\perp} f_{V_2}}{X_+^{(\bar{B}V_1, V_2)}} \int_0^1 d\rho \frac{\Phi_1^B(\rho)}{\rho} \int_0^1 dv \int_0^1 du \frac{\Phi_{V_1}^{\perp}(v) \Phi_{b_2}(u)}{u \bar{v}^2}, \\ H_{13}^+(V_1 V_2) &= \frac{f_B f_{V_1}^{\perp} f_{V_2}}{X_+^{(\bar{B}V_1, V_2)}} \int_0^1 d\rho \frac{\Phi_1^B(\rho)}{\rho} \int_0^1 dv \int_0^1 du \frac{\Phi_{V_1}^{\perp}(v) \Phi_{a_2}(u)}{\bar{u} \bar{v}^2}, \\ H_{14}^+(V_1 V_2) &= -\frac{f_B f_{V_1} f_{V_2}}{2X_+^{(\bar{B}V_1, V_2)}} \int_0^1 d\rho \frac{\Phi_1^B(\rho)}{\rho} \int_0^1 dv \int_0^1 du \frac{\Phi_{a_2}(v) \Phi_{V_2}^{\perp}(u)}{u \bar{u} \bar{v}}, \end{aligned} \quad (C6)$$

where

$$\begin{aligned} X_0^{(\bar{B}V_1, V_2)} &= \frac{f_{V_2}}{2m_{V_1}} \left[ (m_B^2 - m_{V_1}^2 - m_{V_2}^2)(m_B + m_{V_1}) A_1^{BV_1}(m_{V_2}^2) - \frac{4m_B^2 p_c^2}{m_B + m_{V_1}} A_2^{BV_1}(m_{V_2}^2) \right], \\ X_+^{(\bar{B}V_1, V_2)} &= -f_{V_2} m_{V_2} \left[ (m_B + m_{V_1}) A_1^{BV_1}(m_{V_2}^2) \mp \frac{2m_B p_c}{m_B + m_{V_1}} V^{BV_1}(m_{V_2}^2) \right], \end{aligned} \quad (C7)$$

with  $q = p_B - p_{V_1} \equiv p_{V_2}$ . Here  $\Phi_1^B(\rho)$  is one of the two light-cone distribution amplitudes of the  $\bar{B}$  meson [49].  $P_i^{h,p}$  are strong penguin contractions. We obtain

$$\begin{aligned} P_{12}^{0,p}(V_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ (c_{12} + c_{14}) \sum_{i=u}^b \left[ \frac{4n_f}{3} \ln \frac{m_b}{\mu} - (n_f - 2) G_{V_2}(0) - G_{V_2}(s_c) - G_{V_2}(1) - \frac{8}{3} \right] \right. \\ &\quad \left. + c_{11} \sum_{i=u}^b \left[ \frac{8}{3} \ln \frac{m_b}{\mu} - G_{V_2}(0) - G_{V_2}(1) + \frac{4}{3} \right] \right\}, \end{aligned} \quad (C8)$$

$$P_{14}^{0,p}(V_2) = -\frac{C_F \alpha_s}{4\pi N_c} \left\{ (c_{12} + c_{14}) \sum_{i=u}^b [(n_f - 2) \hat{G}_{V_2}(0) + \hat{G}_{V_2}(s_c) + \hat{G}_{V_2}(1)] + c_{11} \sum_{i=u}^b [\hat{G}_{V_2}(0) + \hat{G}_{V_2}(1)] \right\}, \quad (C9)$$

$P_{11}^{h,p} = P_{13}^{h,p} = P_{12}^{+,p} = P_{14}^{+,p}$ , where  $s_i = m_i^2/m_b^2$  and the functions  $G_{M_2}(s)$  and  $\hat{G}_{M_2}(s)$  are given by

$$G_{M_2}(s) = -4 \int_0^1 du \Phi_{V_2}(u) \left[ \int_0^1 dx x \bar{x} \ln(s - \bar{u} x \bar{x} - i\epsilon) \right], \quad \hat{G}_{V_2}(s) = -4 \int_0^1 du \Phi_{v_2}(u) \left[ \int_0^1 dx x \bar{x} \ln(s - \bar{u} x \bar{x} - i\epsilon) \right]. \quad (C10)$$

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