

# Estimate of the partial width for $X(3872)$ into $p\bar{p}$

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We present an estimate of the partial width of  $X(3872)$  into  $p\bar{p}$  under the assumption that it is a weakly bound hadronic molecule whose constituents are a superposition of the charm mesons  $D^{*0}\bar{D}^0$  and  $D^0\bar{D}^{*0}$ . The  $p\bar{p}$  partial width of  $X$  is therefore related to the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  near the threshold. That cross section at an energy well above the threshold is estimated by scaling the measured cross section for  $p\bar{p} \rightarrow K^{*0}K^0$ . It is extrapolated to the  $D^{*0}\bar{D}^0$  threshold by taking into account the threshold resonance in the  $1^{++}$  channel. The resulting prediction for the  $p\bar{p}$  partial width of  $X(3872)$  is proportional to the square root of its binding energy. For the current central value of the binding energy, the estimated partial width into  $p\bar{p}$  is comparable to that of the P-wave charmonium state  $\chi_{c1}$ .

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## I. INTRODUCTION

Since the surprising discovery of the  $X(3872)$  in 2003 [1], there has been a steadily growing list of new  $c\bar{c}$  mesons discovered at the  $B$  factories [2–9]. These discoveries have revealed that the spectrum of  $c\bar{c}$  mesons is richer than the charmonium states predicted by quark potential models. The candidates for some of the new  $c\bar{c}$  mesons include charm meson molecules, tetraquark states, and charmonium hybrid states [10]. An ideal experiment for studying some of the new  $c\bar{c}$  mesons would be  $p\bar{p}$  collisions at resonance. The effectiveness of resonant  $p\bar{p}$  collisions for studying conventional charmonium states was demonstrated by the E760 and E835 experiments at Fermilab [11,12]. The “ $c\bar{c}$  Mesons” section of the Review of Particle Physics [13] was largely rewritten by these experiments. There will be future opportunities to study  $c\bar{c}$  mesons through resonant  $p\bar{p}$  collisions. The Panda experiment at GSI is expected to begin taking data on  $p\bar{p}$  collisions at energies in the charmonium region around 2014 [14]. A recently proposed  $p\bar{p}$  resonance experiment at Fermilab could begin even earlier [15].

The rate at which a resonance is produced in  $p\bar{p}$  collisions is proportional to its partial width into  $p\bar{p}$ . Thus a resonant  $p\bar{p}$  annihilation experiment would be useful for studying the new  $c\bar{c}$  mesons provided their  $p\bar{p}$  partial widths are sufficiently large. To estimate the  $p\bar{p}$  partial widths of conventional charmonium states, one can take advantage of the measured  $p\bar{p}$  partial widths of the  $\eta_c$ ,  $J/\psi$ ,  $\chi_{c0}$ ,  $\chi_{c1}$ ,  $\chi_{c2}$ , and  $\psi(2S)$ . It is difficult to estimate the  $p\bar{p}$  partial widths for most of the more exotic candidates for the new  $c\bar{c}$  mesons. A weakly bound S-wave charm meson molecule is an exception. Its  $p\bar{p}$  partial width is related in a simple way to the cross section near threshold for  $p\bar{p}$  annihilation into the charm mesons that are its constituents. The energy dependence of that cross section near threshold is determined by the mass and width of the resonance. If the S-wave contribution to the cross section for  $p\bar{p}$  annihilation into the charm mesons well above the threshold was known, a reasonable extrapolation to the

threshold could be made in terms of the position and width of the resonance. Cross sections for  $p\bar{p}$  annihilation into pairs of charm mesons have not been measured. However at energies well above threshold, those cross sections can be estimated by scaling the measured cross sections for  $p\bar{p}$  annihilation into the corresponding strange mesons. In this paper, we will use this strategy to estimate the partial width of the  $X(3872)$  into  $p\bar{p}$ .

We will assume in this paper that the  $X(3872)$  is a weakly bound hadronic molecule whose constituents are a superposition of  $D^{*0}\bar{D}^0$  and  $D^0\bar{D}^{*0}$ . We first summarize the evidence for this identification. Measurements of the mass of the  $X(3872)$  by the Belle, CDF, *BABAR*, and D0 Collaborations [1,16–18], combined with a new measurement of the  $D^0$  mass by the CLEO Collaboration [19], imply that the mass is extremely close to the  $D^{*0}\bar{D}^0$  threshold:

$$M_X - (m_{D^*} + m_{\bar{D}^0}) = -0.6 \pm 0.6 \text{ MeV}. \quad (1)$$

The observation of the decay of  $X$  into  $J/\psi\gamma$  [20] implies that the charge conjugation quantum number is  $C = +1$ . Studies of the decays of  $X$  into  $J/\psi\pi^+\pi^-$  [21,22] strongly favor the spin and parity  $J^P = 1^+$ , although  $2^-$  is not excluded. The observation of decays into  $D^0\bar{D}^0\pi^0$  [23,24] excludes  $J \geq 2$ , because the available phase space is so small. Thus the quantum numbers of the  $X(3872)$  have been determined to be  $J^{PC} = 1^{++}$ . These quantum numbers imply that the  $X$  has an S-wave coupling to the charm meson channel  $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$ . The mass measurement in Eq. (1) implies that it is a resonant coupling. This is sufficient to conclude that the  $X(3872)$  is either a charm meson molecule with particle content  $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$  or it is a virtual state of the charm mesons. The reason such an unambiguous statement can be made is that nonrelativistic systems with S-wave threshold resonances have universal properties that depend on the large scattering length of the constituents but are otherwise insensitive to their interactions at shorter distances [25]. These universal properties have been exploited to describe the decays of  $X$  into

$D^0\bar{D}^0\pi^0$  and  $D^0\bar{D}^0\gamma$  [26], the production process  $B \rightarrow KX$  [27,28], the line shapes of the  $X$  [29], and decays of  $X$  into  $J/\psi$  and pions [30]. These applications rely on factorization formulas that separate the length scale  $a$  from all the shorter distance scales of QCD [29,31]. Hanhart *et al.* have analyzed the data from the Belle and BABAR Collaborations on the decays  $B^+ \rightarrow K^+ + X$  in the decay channels  $J/\psi\pi^+\pi^-$  and  $D^0\bar{D}^0\pi^0$  and concluded that the  $X$  must be a virtual state of charm meson [32]. A more recent analysis that took into account consistently the effects of the  $D^{*0}$  width concluded that a charm meson molecule was preferred by the data, although a virtual state could not be excluded [33].

In this paper, we exploit the identification of the  $X(3872)$  as a weakly bound charm meson molecule to give an order-of-magnitude estimate of its partial width into  $p\bar{p}$ . In Sec. II, we estimate the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  well above the threshold by scaling measured cross sections for  $p\bar{p} \rightarrow K^{*-}K^+$ . In Sec. III, we extrapolate the cross sections for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  to the threshold under the assumption that there is an S-wave threshold resonance in the  $1^{++}$  channel. In Sec. IV, we deduce the partial width for  $X$  into  $p\bar{p}$  from the extrapolated cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$ . We discuss the results in Sec. V.

## II. CROSS SECTION FOR $p\bar{p} \rightarrow D^{*0}\bar{D}^0$ FAR ABOVE THRESHOLD

Given that  $X(3872)$  is a weakly bound S-wave hadronic molecule composed of  $D^{*0}\bar{D}^0$  or  $D^0\bar{D}^{*0}$ , its partial width into  $p\bar{p}$  is proportional to the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  near the  $D^{*0}\bar{D}^0$  threshold. In this section, we estimate that cross section at energies well above the threshold by scaling the measured cross section for  $p\bar{p} \rightarrow K^{*-}K^+$ .

At the quark level, the process  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  is  $uud + \bar{u}\bar{u}\bar{d} \rightarrow c\bar{u} + \bar{c}u$ . This process can proceed through the short-distance annihilation process  $ud + \bar{u}\bar{d} \rightarrow c + \bar{c}$  followed by a long-distance process in which the remaining constituents  $\bar{u}$  and  $u$  of the  $\bar{p}$  and  $p$  bind to the  $c$  and  $\bar{c}$  to form the charm mesons. The  $\bar{u}$  and  $u$  carry only a small fraction of the momenta of the outgoing charm mesons. This process can proceed most easily via the Feynman process in which the  $u$  and  $\bar{u}$  that do not annihilate carry only a small fraction of the momenta of the colliding  $p$  and  $\bar{p}$ . The momenta transferred to the  $u$  and to the  $\bar{u}$  are therefore small. We assume that the cross section for this process can be factored into the rate for the short-distance annihilation process  $ud + \bar{u}\bar{d} \rightarrow c + \bar{c}$  and a long-distance probability factor associated with the  $u$  and  $\bar{u}$  that do not annihilate.

We first consider the short-distance factor. The short-distance process involves the annihilation of  $ud + \bar{u}\bar{d}$  into two virtual gluons which then create a  $c\bar{c}$  pair. Since it involves 6 external partons, the dimensional counting rules for this process imply that its rate scales with the center-of-mass energy  $\sqrt{s}$  like  $1/s^3$  [34,35]. This should be con-

trasted with the rate for  $p\bar{p}$  annihilation at resonance into charmonium. In this case, the short-distance process is  $uud + \bar{u}\bar{u}\bar{d} \rightarrow c\bar{c}$ . The dimensional counting rules imply that its rate scales like  $1/M^8$ , where  $M$  is the charmonium mass [36]. In the case of  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$ , the rate for the short-distance subprocess scales with two fewer powers of the center-of-mass energy, because one of the three quarks in the proton is not required to annihilate.

We now consider the long-distance factor. This factor is the probability for the surviving  $u$  and  $\bar{u}$  from the colliding  $p$  and  $\bar{p}$  to become constituents of the outgoing charm mesons. In the initial state, the  $ud$  and the  $\bar{u}\bar{d}$  that annihilate act as lightlike colored sources for the remaining low-momentum  $u$  and  $\bar{u}$ . In the final state, the  $c$  and  $\bar{c}$  act as colored sources for the low-momentum  $\bar{u}$  and  $u$ . In the  $p\bar{p}$  rest frame, these sources have equal and opposite velocities  $\beta = [(s - 4m_c^2)/s]^{1/2}$ . The effect of the short-distance process is to suddenly replace the lightlike colored sources  $ud$  and  $\bar{u}\bar{d}$  by colored sources  $c$  and  $\bar{c}$  with equal and opposite velocities  $\beta$ . The long-distance factor  $P(\beta)$  is the square of the amplitude for the low-momentum  $u$  and  $\bar{u}$  to evolve from constituents of the  $p$  and  $\bar{p}$  into constituents of the  $\bar{D}^0$  and  $D^{*0}$  after the sudden change in color sources.

The long-distance factor  $P(\beta)$  depends on the velocity  $\beta$  of the colored sources in the final state, but it does not depend on the flavor of the particles that serve as the colored sources. The amplitude would be the same if the charm quarks  $c$  and  $\bar{c}$  were replaced by strange quarks  $s$  and  $\bar{s}$  with the same velocities  $\beta$ . Thus the rate for producing the charm mesons  $D^{*0} + \bar{D}^0$ , whose quark content is  $c\bar{u} + \bar{c}u$ , can be related to the rate for producing strange mesons  $K^{*-} + K^+$ , whose quark content is  $s\bar{u} + \bar{s}u$ . If we take into account the masses of the mesons, a  $K^{*-}K^+$  pair with center-of-mass energy  $s_K^{1/2}$  has the same relative velocity  $2\beta$  in the center-of-mass frame as a  $D^{*0}\bar{D}^0$  pair with center-of-mass energy  $s_D^{1/2}$  if the center-of-mass energies satisfy

$$\frac{s_K \lambda^{1/2}(s_K^{1/2}, m_{K^*}, m_K)}{s_K^2 - (m_{K^*}^2 - m_K^2)^2} = \beta = \frac{s_D \lambda^{1/2}(s_D^{1/2}, m_{D^*}, m_D)}{s_D^2 - (m_{D^*}^2 - m_D^2)^2}, \quad (2)$$

where  $\lambda(x, y, z) = x^4 + y^4 + z^4 - 2(x^2y^2 + y^2z^2 + z^2x^2)$ .

The cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  can be expressed as the product of a flux factor  $[s(s - 4m_p^2)]^{-1/2}$ , a phase space factor  $\lambda^{1/2}(s^{1/2}, m_{D^*}, m_D)/(8\pi s)$ , and a matrix element factor. Under our factorization assumption, the matrix element factor is the product of a short-distance factor that scales asymptotically like  $s^{-2}$  and a long-distance factor  $P(\beta)$  that is a function of the velocities  $\beta$  of the  $c$  and  $\bar{c}$  created by the short-distance process. If we use the asymptotic scaling behavior of the short-distance factor in the matrix element, the energy dependence of the cross section is given by

$$\sigma[p\bar{p} \rightarrow D^{*0}\bar{D}^0; s] \sim \frac{1}{[s(s-4m_p^2)]^{1/2}} \frac{P(\beta)}{s^2} \times \frac{\lambda^{1/2}(s^{1/2}, m_{D^*}, m_D)}{s}. \quad (3)$$

In the limit  $s \rightarrow \infty$ , the right side of Eq. (3) approaches  $P(1)/s^3$  in accord with the dimensional counting rules. There is an expression analogous to Eq. (3) for the cross section for  $p\bar{p} \rightarrow K^{*-}K^+$  with the same long-distance factor  $P(\beta)$ , but with  $m_K$  and  $m_{K^*}$  replaced by  $m_D$  and  $m_{D^*}$ . Using Eq. (2) to eliminate that factor, we obtain a scaling relation between the two cross sections:

$$\sigma[p\bar{p} \rightarrow D^{*0}\bar{D}^0; s_D] \approx \sigma[p\bar{p} \rightarrow K^{*-}K^+; s_K] \times \frac{[s_K(s_K-4m_p^2)]^{1/2}}{[s_D(s_D-4m_p^2)]^{1/2}} \left(\frac{s_K}{s_D}\right)^3 \times \frac{\lambda^{1/2}(s_D^{1/2}, m_{D^*}, m_D)}{\lambda^{1/2}(s_K^{1/2}, m_{K^*}, m_K)}, \quad (4)$$

where  $s_K$  is the function of  $s_D$  obtained by solving Eq. (2).

The cross sections for  $p\bar{p} \rightarrow K^{*-}K^+$  have been measured for antiproton momenta in the lab frame ranging from 702 to 1642 MeV [37–39]. This corresponds to center-of-mass energies  $\sqrt{s}$  ranging from 1990 to 2304 MeV. The only recent measurements were from the Crystal Barrel Collaboration [39]. The cross section was measured to be  $(460 \pm 50) \mu\text{b}$  at  $\sqrt{s} = 2050$  MeV and  $(147 \pm 22) \mu\text{b}$  at 2304 MeV [39]. The relative velocities  $\beta$  given by Eq. (2) are 0.74 and 0.80, respectively. The data points are shown in Fig. 1 as a function of the center-of-mass energy. Also shown in the figure are curves that pass through the Crystal Barrel data points and whose dependence on the energy is given by the analog of Eq. (3) for  $K^{*-}K^+$  with the probability factor  $P(\beta)$  approximated by a constant. Notice that the two curves do not differ dramatically. This suggests that at these energies, the matrix element factor in the  $K^{*-}K^+$  cross section may already be close to its asymptotic behavior proportional to  $P(1)/s^3$ .

To estimate the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$ , we insert the Crystal Barrel data points for the  $K^{*-}K^+$  cross section into Eq. (4), where  $s_K$  is the function of  $s = s_D$  obtained by solving Eq. (2). The center-of-mass energies  $s_K^{1/2}$  for the two Crystal Barrel data points are 2050 and 2304 MeV. The corresponding center-of-mass energies  $s_D^{1/2}$  for the process  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  are 5714 and 6391 MeV. If we use the higher-energy Crystal Barrel data point as the input, our estimate for the cross section for  $D^{*0}\bar{D}^0$  is

$$\sigma[p\bar{p} \rightarrow D^{*0}\bar{D}^0; s] \approx \left(\frac{m_{D^*} + m_D}{\sqrt{s}}\right)^6 \frac{\lambda^{1/2}(s^{1/2}, M_{D^*}, M_D)}{[s(s-4m_p^2)]^{1/2}} \times (4800 \text{ nb}). \quad (5)$$

If we use the lower-energy Crystal Barrel data point as the input, the factor of 4800 nb is replaced by 5600 nb. The

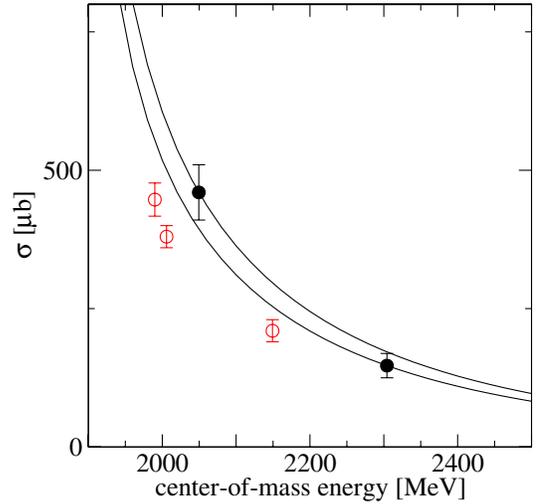


FIG. 1 (color online). Measured cross sections for  $p\bar{p} \rightarrow K^{*-}K^+$  as a function of the center-of-mass energy. The open dots are the data from Refs. [37,38]. The two solid dots are data from the Crystal Barrel Collaboration [39]. The curves passing through the Crystal Barrel data points are extrapolations whose energy dependence is given by the analog of Eq. (3) for  $K^{*-}K^+$  with the probability factor  $P(\beta)$  approximated by a constant.

relatively small difference implies that the probability factor  $P(\beta)$  in Eq. (3) does not depend dramatically on  $\beta$  in the region  $0.74 < \beta < 0.80$ . If we were to change the assumed scaling behavior of the short-distance factor in the square of the matrix element from  $(\sqrt{s})^{-6}$  to  $(\sqrt{s})^{-6 \pm 1}$ , the prediction in Eq. (5) would change by a multiplicative factor of  $2.8^{\pm 1}$ .

### III. CROSS SECTION FOR $p\bar{p} \rightarrow D^{*0}\bar{D}^0$ NEAR THRESHOLD

Given the estimate in Eq. (5) of the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  at energies far above threshold, we would like to obtain an estimate of the cross section near threshold. We cannot simply use Eq. (5) to extrapolate to the threshold region, because this expression ignores the suppression of higher partial waves near the threshold, which can be taken into account through the probability factor  $P(\beta)$  in Eq. (3), and because it also ignores the threshold resonance associated with the  $X(3872)$ . We will assume that the threshold resonance is in the S-wave  $C = +1$  channel, and that this is the only effect that gives dramatic dependence of the S-wave phase shift on the center-of-mass energy  $\sqrt{s}$ .

As the center-of-mass energy approaches the threshold at  $\sqrt{s} = M_{D^{*0}} + M_{D^0}$ , the terms in the matrix element with nonzero orbital angular momentum quantum number  $L$  are suppressed by a factor of  $(\sqrt{s} - M_{D^{*0}} - M_{D^0})^{L/2}$ . Only the S-wave  $L = 0$  term survives in the limit. If the S-wave threshold resonance could be ignored, a crude extrapolation of the cross section for  $D^{*0}\bar{D}^0$  in Eq. (5) towards the

threshold could be obtained simply by multiplying it by the fraction  $f_{L=0}$  of the cross section that comes from S-wave scattering. The determination of the S-wave fraction requires measurements of the angular distribution. There is some information on the angular distribution for the related process  $p\bar{p} \rightarrow K^{*-}K^+$ . In the Crystal Barrel experiment [39], it was estimated that angular momenta up to  $L = 5$  contribute to the cross section at center-of-mass energy 2304 MeV. In Ref. [38], the angular distribution for  $p\bar{p} \rightarrow K^{*-}K^+$  was measured at center-of-mass energy 2006 MeV. The ratios  $a_L/a_0$  of the coefficients of the Legendre polynomials in the partial wave expansion were given for  $L$  up to 5. From those results, we can infer that the fraction of the cross section at 2006 MeV that comes from S-wave scattering is about 9.3%. Since no experimental information is available on the S-wave fraction  $f_{L=0}$  for  $D^{*0}\bar{D}^0$ , we will use the S-wave fraction for  $K^{*-}K^+$  at 2006 MeV as an estimate:  $f_{L=0} \approx 9.3\%$ .

If the threshold resonance could be ignored, we could estimate the cross section for  $D^{*0}\bar{D}^0$  near threshold obtained simply by extrapolating Eq. (5) to the threshold and multiplying it by the fraction  $f_{L=0}$ . However the existence of the  $X(3872)$  with quantum numbers  $1^{++}$  implies that there is a resonant enhancement of the cross section for  $D^{*0}\bar{D}^0$  near the threshold due to scattering in the S-wave  $C = +1$  channel. The enhancement applies only to the fraction  $f_{C=+1}$  of the cross section that comes from  $p\bar{p}$  scattering into channels that are even under charge conjugation. Experiments on  $p\bar{p}$  annihilation at very low  $\bar{p}$  momentum have revealed a suppression of the production of final states that are even under charge conjugation. In Ref. [38], the suppression of  $C = +1$  final states was also observed in final states containing pairs of strange mesons at the center-of-mass energy 2006 MeV. The suppression factor in the case of  $K^{*-}K^+$  can be deduced from measurements of  $K^{*0}K^0$ , which is related by isospin symmetry. The cross section for  $p\bar{p} \rightarrow K^{*0}K_S^0$  followed by the decay  $K^{*0} \rightarrow K_S^0\pi^0$ , which gives a final state with  $C = +1$ , is only  $17 \pm 4 \mu\text{b}$ . The cross section for  $p\bar{p} \rightarrow K^{*0}K_S^0$  with no restriction on the decays of  $K^{*0}$  is  $130 \pm 10 \mu\text{b}$ . The ratio of these two cross sections gives an even-charge-conjugation fraction of about 13.1%. Since no experimental information is available on the even-charge-conjugation fraction  $f_{C=+1}$  for  $D^{*0}\bar{D}^0$ , we will use the even-charge-conjugation fraction for  $K^{*0}K_S^0$  at 2006 MeV as an estimate:  $f_{C=+1} \approx 13.1\%$ .

To extrapolate the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  to the threshold region, we will exploit the universal features of S-wave threshold resonances in nonrelativistic 2-body systems [25]. These universal features can be conveniently expressed in terms of the scattering length  $a$  in the resonant S-wave channel. If the resonance is sufficiently close to the threshold,  $|a|$  will be much larger than all other length scales associated with the structure of the particles and their interactions. The scattering amplitude in the resonant

channel has the universal form

$$f(E) = \frac{1}{-1/a + \sqrt{-2ME - i\epsilon}}, \quad (6)$$

where  $E$  is the energy relative to the 2-body threshold and  $M$  is the reduced mass. If  $a$  is real and positive, the amplitude in Eq. (6) has a pole at  $E = -E_X$ , where

$$E_X = \frac{1}{2Ma^2}. \quad (7)$$

The pole is associated with a bound state with binding energy  $E_X$ . If  $a$  is real and negative, there is no pole on the physical sheet of the complex energy  $E$ , so there is no bound state. There is however a pole on the unphysical sheet of  $E$  and it is conventionally referred to as a ‘‘virtual state.’’ The scattering length  $a$  is complex if the 2-body system has inelastic scattering channels. If  $a$  is complex, its real part can still be used to distinguish between a bound state ( $\text{Re}a > 0$ ) and a virtual state ( $\text{Re}a < 0$ ). In the case  $\text{Re}a > 0$ , the bound state has a nonzero width determined by the imaginary part of  $a$ .

The existence of the  $X(3872)$  with quantum numbers  $1^{++}$  and mass given by Eq. (1) implies that there is an S-wave threshold resonance in the channel

$$(D^*\bar{D})_+^0 = \frac{1}{\sqrt{2}}(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}). \quad (8)$$

The scattering amplitude in this channel is given by the universal expression in Eq. (6), where  $E = \sqrt{s} - m_{D^{*0}} - m_{D^0}$  and  $M$  is the reduced mass  $M_{D^*\bar{D}} = m_{D^{*0}}m_{D^0}/(m_{D^{*0}} + m_{D^0})$ . Since  $X(3872)$  decays, its constituents have inelastic scattering channels, so the scattering length  $a$  must be complex. In Ref. [32], the authors analyzed the data from the Belle and BABAR Collaborations on the  $X(3872)$  resonance produced via  $B^+ \rightarrow K^+ + X$  and decaying through the channels  $J/\psi\pi^+\pi^-$  and  $D^0\bar{D}^0\pi^0$ . They concluded that  $X(3872)$  must be a virtual state with  $\text{Re}a < 0$ . In a subsequent analysis of the same data, the nonzero width of the constituent  $D^{*0}$  or  $\bar{D}^{*0}$  of the  $X$  was taken into account [33]. The conclusion of that analysis was that the data preferred a bound state corresponding to  $\text{Re}a > 0$ , although a virtual state was not excluded. We will assume that the  $X(3872)$  is indeed a bound state. We also assume for simplicity that the imaginary part of  $a$  is small compared to its real part, so the binding energy of the  $X$  can be approximated by the simple expression in Eq. (7) with  $M$  replaced by  $M_{D^*\bar{D}}$ .

We proceed to use the universality of S-wave threshold resonances to extrapolate the estimated cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  to the threshold region. Matrix elements corresponding to the projection of the charm mesons onto the channel  $(D^*\bar{D})_+^0$  defined in Eq. (8) will be enhanced at energies near the  $D^{*0}\bar{D}^0$  threshold by the resonance associated with the  $X(3872)$ . For the process  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$ , the resonance enhances the matrix element

near the threshold by a dimensionless factor

$$(2/\pi)\Lambda f(E) = \frac{(2/\pi)\Lambda}{-1/a - ik_{\text{cm}}}, \quad (9)$$

where  $k_{\text{cm}} = (2M_{D^*\bar{D}}E)^{1/2}$  and  $\Lambda$  is the momentum scale above which the effects of the nonzero range of the interaction become important. If the S-wave effective range  $r_s$  for the scattering of the charm mesons was known, we could use  $\Lambda \approx 2/r_s$  as a quantitative estimate. A natural order-of-magnitude estimate for  $\Lambda$  is  $m_\pi$ , which is the next smallest relevant momentum scale after  $1/|a|$ . This is a relevant momentum scale, because the low-energy scattering of  $D^{*0}\bar{D}^0$  is dominated by pion exchange. Our estimate of the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  near the threshold is obtained by extrapolating the estimated cross section in Eq. (5) to the threshold region and multiplying it by the S-wave fraction  $f_{L=0} \approx 9.3\%$ , the even-charge-conjugation fraction  $f_{C=+1} \approx 13.1\%$ , and the resonance factor  $(4/\pi^2)\Lambda^2|f(E)|^2$ :

$$\sigma[p\bar{p} \rightarrow D^{*0}\bar{D}^0; E] \approx (1.9 \text{ nb}) \frac{\Lambda^2 k_{\text{cm}}}{m_\pi(1/a^2 + k_{\text{cm}}^2)}. \quad (10)$$

The factor of  $m_\pi$  in the denominator in Eq. (10) was introduced to make the last factor dimensionless. If the lower-energy Crystal Barrel data point was used as the input for Eq. (5), the prefactor in Eq. (10) would have been 5.5 nb.

#### IV. PARTIAL WIDTH INTO $p\bar{p}$

To relate the cross sections for  $p\bar{p}$  annihilation into  $D^{*0}\bar{D}^0$  and  $X(3872)$ , we begin by considering the forward scattering amplitude for  $p\bar{p} \rightarrow p\bar{p}$  near the  $D^{*0}\bar{D}^0$  threshold. This amplitude can be expressed as the product of three factors: a short-distance factor for the transition from  $p\bar{p}$  to the resonant channel  $(D^*\bar{D})_+^0$ , a long-distance resonance factor  $f(E)$  given by Eq. (6), and a short-distance factor for the transition from  $(D^*\bar{D})_+^0$  to  $p\bar{p}$ . By the optical theorem, the total  $p\bar{p}$  cross section is proportional to the imaginary part of the forward scattering amplitude. The resonant contribution to the imaginary part associated with  $D^{*0}\bar{D}^0$  or  $D^0\bar{D}^{*0}$  in the final state comes from the imaginary part of the resonance factor:

$$\text{Im} f(E) = \frac{k_{\text{cm}}}{1/a^2 + k_{\text{cm}}^2} \theta(E) + \frac{\pi}{M_{D^*\bar{D}}a} \delta(E + E_X), \quad (11)$$

where  $k_{\text{cm}} = (2M_{D^*\bar{D}}E)^{1/2}$  and  $E_X = 1/(2M_{D^*\bar{D}}a^2)$  is the binding energy of the  $X$  given in Eq. (7). The first term on the right side corresponds to the production of  $D^{*0}\bar{D}^0$  and  $D^0\bar{D}^{*0}$  above the threshold. The second term corresponds to the production of the  $X(3872)$  resonance. The cross sections for producing  $D^{*0}\bar{D}^0$  near threshold,  $D^0\bar{D}^{*0}$  near threshold, and  $X(3872)$  are

$$\sigma[p\bar{p} \rightarrow D^{*0}\bar{D}^0; E] = \frac{1}{2} \sigma_{\text{SD}} \frac{m_\pi k_{\text{cm}}}{1/a^2 + k_{\text{cm}}^2} \theta(E), \quad (12a)$$

$$\sigma[p\bar{p} \rightarrow D^0\bar{D}^{*0}; E] = \frac{1}{2} \sigma_{\text{SD}} \frac{m_\pi k_{\text{cm}}}{1/a^2 + k_{\text{cm}}^2} \theta(E), \quad (12b)$$

$$\sigma[p\bar{p} \rightarrow X; E] = \sigma_{\text{SD}} \frac{\pi m_\pi (2M_{D^*\bar{D}}E_X)^{1/2}}{M_{D^*\bar{D}}} \delta(E + E_X), \quad (12c)$$

where  $\sigma_{\text{SD}}$  is a short-distance factor. In Eq. (12c), we have used Eq. (7) to eliminate the scattering length  $a$  in favor of the binding energy  $E_X$ . A factor of  $m_\pi$  has been inserted into the long-distance factor to ensure that  $\sigma_{\text{SD}}$  has the dimensions of a cross section. The delta function of the energy in Eq. (12c) arises from neglecting the width of the  $X(3872)$  resonance. An estimate for the short-distance factor  $\sigma_{\text{SD}}$  can be obtained by comparing the cross section in Eq. (12a) with the estimated cross section in Eq. (10):

$$\sigma_{\text{SD}} = \frac{2\Lambda^2}{m_\pi^2} (1.9 \text{ nb}). \quad (13)$$

The cross section for  $p\bar{p} \rightarrow X$  in the narrow resonance limit is related to the partial width for  $X \rightarrow p\bar{p}$  by simple kinematics. If we denote the T-matrix element for  $X \rightarrow p\bar{p}$  by  $\mathcal{M}$ , the partial width into  $p\bar{p}$  is

$$\Gamma[X \rightarrow p\bar{p}] = \frac{1}{2M_X} \frac{1}{3} \sum_{\text{spins}} |\mathcal{M}|^2 \frac{(M_X^2 - 4m_p^2)^{1/2}}{8\pi M_X}. \quad (14)$$

The cross section for  $p\bar{p} \rightarrow X$  in the narrow resonance limit averaged over the spins of the proton and antiproton is

$$\sigma[p\bar{p} \rightarrow X; E] = \frac{1}{2M_X(M_X^2 - 4m_p^2)^{1/2}} \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 \times \frac{\pi}{M_X} \delta(E + E_X). \quad (15)$$

Eliminating the T-matrix element from Eqs. (14) and (15), we obtain

$$\sigma[p\bar{p} \rightarrow X; E] = \frac{6\pi^2 \Gamma[X \rightarrow p\bar{p}]}{M_X^2 - 4m_p^2} \delta(E + E_X). \quad (16)$$

Comparing with Eq. (12c) and using the estimate of the short-distance factor  $\sigma_{\text{SD}}$  in Eq. (13), we obtain an estimate of the partial width of  $X$  into  $p\bar{p}$ :

$$\Gamma[X \rightarrow p\bar{p}] = \frac{(M_X^2 - 4m_p^2)\Lambda^2(2M_{D^*\bar{D}}E_X)^{1/2}}{3\pi m_\pi M_{D^*\bar{D}}} (1.9 \text{ nb}). \quad (17)$$

The partial width is proportional to the square root of the binding energy  $E_X$ . The current measurement of the binding energy is given by Eq. (1):  $E_X = 0.6 \pm 0.6$  MeV. The expression in Eq. (17) can be reduced to

$$\Gamma[X \rightarrow p\bar{p}] = \left(\frac{\Lambda}{m_\pi}\right)^2 \left(\frac{E_X}{0.6 \text{ MeV}}\right)^{1/2} (28 \text{ eV}). \quad (18)$$

The total width of the  $X$  must be greater than the width of its constituent  $D^{*0}$ , which is about 70 keV. Thus the estimate in Eq. (18) implies that the branching fraction for  $X \rightarrow p\bar{p}$  is less than about  $4 \times 10^{-4}$ .

## V. DISCUSSION

We have estimated the partial width of  $X(3872)$  into  $p\bar{p}$ . The estimate was obtained by a sequence of well-motivated steps. The first step was estimating the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  well above the threshold by scaling measured cross sections for  $p\bar{p} \rightarrow K^{*+}K^+$ . The second step was estimating the contribution from the S-wave  $1^{++}$  channel using suppression factors  $f_{L=0}$  and  $f_{C=+1}$  for the  $K^{*+}K^+$  process. The third step was extrapolating the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  to the  $D^{*0}\bar{D}^0$  threshold under the assumption that there is an S-wave threshold resonance in the  $1^{++}$  channel. The cross section for  $p\bar{p} \rightarrow X$  was obtained from the cross section for  $p\bar{p} \rightarrow D^{*0}\bar{D}^0$  near threshold by using the universal properties of an S-wave threshold resonance. The partial width for  $X \rightarrow p\bar{p}$  is related to the cross section for  $p\bar{p} \rightarrow X$  by simple kinematic factors. Our estimate for the partial width is given in Eq. (18).

It is widely believed that the production of a loosely bound molecular state in a process involving energies much greater than the binding energy should be strongly suppressed. There is some suppression, but it is not as strong as one might expect. In the estimate in Eq. (18), the suppression is taken into account by the factor proportional to  $E_X^{1/2}$ . This factor goes to 0 as  $E_X \rightarrow 0$ , suggesting that there would be complete suppression in this limit. However the suppression factor has this simple form proportional to  $E_X^{1/2}$  only if  $E_X$  is greater than  $\Gamma_X/2$ , where  $\Gamma_X$  is the total width of the  $X$ . There is no further suppression once  $E_X$  becomes comparable to  $\Gamma_X/2$ . A lower bound on  $\Gamma_X$  is provided by the width of the constituent  $D^{*0}$  or  $\bar{D}^{*0}$ :  $\Gamma[D^{*0}] = 66 \pm 15 \text{ keV}$ . Thus even if the binding energy is much smaller than 0.6 MeV, the associated suppression factor cannot decrease the estimate in Eq. (18) by more than about a factor of 0.2.

The estimate for the partial width for  $X(3872) \rightarrow p\bar{p}$  in Eq. (18) is proportional to  $\Lambda^2$ , where  $\Lambda$  is the momentum scale above which the effects of the nonzero range of the interaction between the charm mesons becomes important. Thus the rate is very sensitive to  $\Lambda$ . The scale  $\Lambda$  can be identified with  $2/r_s$ , where  $r_s$  is the effective range for the scattering of the charm mesons, which is not known. We have suggested that a natural scale for  $\Lambda$  is  $m_\pi$ , since low-energy scattering of charm mesons is dominated by pion exchange. The estimates of the rate for  $B \rightarrow K + X$  in Refs. [27,28] were also proportional to  $\Lambda^2$ . In this case,

the choice  $\Lambda = m_\pi$  may underestimate the rate by about an order of magnitude. Thus setting  $\Lambda = m_\pi$  in Eq. (18) may also underestimate the partial width of  $X \rightarrow p\bar{p}$  by an order of magnitude.

Since the P-wave charmonium state  $\chi_{c1}$  has the same quantum numbers  $1^{++}$  as the  $X(3872)$ , it is useful to compare the  $p\bar{p}$  partial widths of the  $X(3872)$  with that of  $\chi_{c1}$ , which is  $60 \pm 5 \text{ eV}$ . If we set  $\Lambda = m_\pi$  and  $E_X = 0.6 \text{ MeV}$  in Eq. (18), our estimate of the  $p\bar{p}$  partial width of  $X(3872)$  is within a factor of 2 of that of  $\chi_{c1}$ . The E760 experiment on charmonium production at resonance in  $p\bar{p}$  collisions measured the total width of the  $\chi_{c1}$  with an uncertainty of 0.14 MeV [40]. Our estimate for the partial width of  $X(3872)$  suggests that a new resonant  $p\bar{p}$  experiment at GSI [14] or at Fermilab [15] should be able to measure the properties of the  $X(3872)$  with comparable or higher accuracy than those of the  $\chi_{c1}$ .

It is also useful to compare the production rates of  $X(3872)$  and  $\chi_{c1}$  in other processes. The production rate of  $X$  by the exclusive decay  $B^+ \rightarrow K^+ + X$  has been measured by the Belle and BABAR Collaborations [1,18]. Dividing the measured product of the branching fractions for  $B^+ \rightarrow K^+ + X$  and  $X \rightarrow J/\psi\pi^+\pi^-$  by the branching fraction for  $B^+ \rightarrow K^+ + \chi_{c1}$  [13], we obtain

$$\frac{\Gamma[B^+ \rightarrow K^+ + X]}{\Gamma[B^+ \rightarrow K^+ + \chi_{c1}]} = \frac{0.023 \pm 0.005}{Br[X \rightarrow J/\psi\pi^+\pi^-]}. \quad (19)$$

Some information on the branching fraction for  $X$  into  $J/\psi\pi^+\pi^-$  is provided by a measurement of the relative production rates for  $J/\psi\pi^+\pi^-$  and  $D^0\bar{D}^0\pi^0$  in the  $X(3872)$  resonance region by the Belle Collaboration [23]:

$$\frac{Br[X \rightarrow D^0\bar{D}^0\pi^0]}{Br[X \rightarrow J/\psi\pi^+\pi^-]} = 8.8_{-3.6}^{+3.1}. \quad (20)$$

This measurement suggests that the branching fraction for  $X$  into  $J/\psi\pi^+\pi^-$  is less than about 1/8.8, which implies that the production ratio in Eq. (19) is greater than about 0.2.

The production rates of  $X(3872)$  and  $\chi_{c1}$  are subject to various suppression factors. In the case of  $X(3872)$ , they include the factor  $E_X^{1/2}$  associated with the large mean separation of its constituents. In the case of  $\chi_{c1}$ , they include a factor of  $v^5$  associated with the small relative momentum  $v$  of the  $c$  and  $\bar{c}$ . These suppression factors apply equally well to all production processes. However there is one suppression factor in resonant  $p\bar{p}$  production that does not apply to exclusive  $B$  meson decay. This is the suppression factor associated with the annihilation of all three quarks in the proton in the case of  $\chi_{c1}$  and the annihilation of only two of the quarks in the case of  $X(3872)$ . Because of this suppression factor, the production ratio for  $X(3872)$  and  $\chi_{c1}$  in resonant  $p\bar{p}$  collisions should be larger than the production ratio in Eq. (19). This is

consistent with the estimated production ratio in resonant  $p\bar{p}$  collisions obtained by dividing Eq. (18) by the measured partial width for  $\chi_{c1} \rightarrow p\bar{p}$ .

In summary, our estimate of the partial width for  $X(3872)$  into  $p\bar{p}$  in Eq. (18) indicates that it should be comparable to and perhaps even larger than that of the  $\chi_{c1}$ . This implies that a future experiment on  $p\bar{p}$  collisions at

the  $X(3872)$  resonance will allow the properties of this remarkable meson to be studied in great detail.

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