Azimuthal anisotropy of direct photons

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The electromagnetic bremsstrahlung produced by a quark interacting with nucleons or nuclei is azimuthally asymmetric. In the light-cone dipole approach this effect is related to the orientation dependent dipole cross section. Such a radiation anisotropy is expected to contribute to the azimuthal asymmetry of direct photons in *pA* and *AA* collisions, as well as in deep-inelastic scattering and in the production of dileptons.

I. INTRODUCTION

Direct photons, i.e. photons not from hadronic decay, are of particular interest, since they do not participate in the strong interaction and therefore carry undisturbed information about the dynamics of the primary hard collision.

Here we present the basic color-dipole formalism for calculating the azimuthal distribution of direct photons radiated by a quark interacting either with a nucleon or nuclear targets. For this purpose we further develop the dipole approach proposed in [1,2] for electromagnetic bremsstrahlung by a quark interacting with nucleons and nuclei. This technique can be applied to dilepton [3–5] and prompt photon [6,7] production in *pp*, *pA* and heavy ion collisions. It can be also used for calculating the azimuthal angle dependence in the radiation of dileptons, or in deep-inelastic scattering.

An azimuthal asymmetry appears due to dependence of the interaction of a dipole on its orientation. Indeed, a colorless $\bar{q}q$ dipole is able to interact only due to the difference between the impact parameters of q and $\bar{q}$ relative to the scattering center. If $\vec{b}$ is the impact parameter of the center of gravity of the dipole, and $\vec{r}$ is the transverse separation of the q and $\bar{q}$, then the dipole interaction should vanish if $\vec{r} \perp \vec{b}$, but should be maximal if $\vec{r} \parallel \vec{b}$. One can see this on a simple example of a dipole interacting with a quark in Born approximation. The partial elastic amplitude reads

$$\mathrm{Im}f_{\bar{q}q}^q(\vec{b}, \vec{r}) = \frac{2}{9\pi^2} \int \frac{d^2q d^2q' \alpha_s(q^2)\alpha_s(q'^2)}{(q^2 + \mu^2)(q'^2 + \mu^2)}$$
$$\times [e^{i\vec{q}\cdot(\vec{b}+\vec{r}/2)} - e^{i\vec{q}\cdot(\vec{b}-\vec{r}/2)}]$$
$$\times [e^{i\vec{q}'\cdot(\vec{b}+\vec{r}/2)} - e^{i\vec{q}'\cdot(\vec{b}-\vec{r}/2)}]. \quad (1)$$

Here we assume for the sake of simplicity that q and $\bar{q}$ have equal longitudinal momenta, i.e. they are equally distant from the dipole center of gravity. The general case of unequal sharing of the dipole momentum is considered later in (23). We introduced in (1) an effective gluon mass μ which takes into account confinement and other possible nonperturbative effects.

Integrating in (1) with a fixed α_s we arrive at

$$\mathrm{Im}f_{\bar{q}q}^q(\vec{b}, \vec{r}) = \frac{8\alpha_s^2}{9}\left[K_0\left(\mu \left| \vec{b} + \frac{\vec{r}}{2} \right| \right)\right.$$
$$\left. - K_0\left(\mu \left| \vec{b} - \frac{\vec{r}}{2} \right| \right)\right]^2, \quad (2)$$

where $K_0(x)$ is the modified Bessel function. This expression explicitly exposes a correlation between $\vec{r}$ and $\vec{b}$: the two terms cancel each other if $\vec{b} \cdot \vec{r} = 0$.

II. DIRECT PHOTONS: DIPOLE REPRESENTATION

The radiation of direct photons, which in the parton model looks like a Compton process $gq \rightarrow \gamma q$, in the target rest frame should be treated as electromagnetic bremsstrahlung by a quark interacting with the target. In the light-cone dipole approach the transverse momentum distribution of photon bremsstrahlung by a quark propagating interacting with a target t (nucleon, $t = N$, or nucleus, $t = A$) at impact parameter $\vec{b}$, can be written in the factorized form [2]

$$\frac{d\sigma^{qT \rightarrow \gamma X}(b, p, \alpha)}{d(\ln\alpha)d^2p d^2b} = \frac{1}{(2\pi)^2} \sum_{in,f} \int d^2r_1 d^2r_2 e^{i\vec{p}\cdot(\vec{r}_1 - \vec{r}_2)}$$
$$\times \phi_{\gamma q}^\star(\alpha, \vec{r}_1)\phi_{\gamma q}(\alpha, \vec{r}_2)$$
$$\times F_t(\vec{b}, \alpha\vec{r}_1, \alpha\vec{r}_2, x). \quad (3)$$

Here $\vec{p}$ and $\alpha = p_\gamma^+/p_q^+$ are the transverse and fractional light-cone momenta of the radiated photon, $\phi_{\gamma q}(\alpha, \vec{r})$ is the light-cone distribution amplitude for the $q\gamma$ Fock component with transverse separation $\vec{r}$, and $F_t(\vec{b}, \alpha\vec{r}_1, \alpha\vec{r}_2, x)$ is an effective partial amplitude to be discussed below. The product of the distribution amplitudes, summed in (3) over initial and final polarizations of the quark and photon, reads [2]

$$\sum_{in,f} \phi_{\gamma q}^{\star}(\alpha, \vec{r}_1)\phi_{\gamma q}(\alpha, \vec{r}_2) = \frac{\alpha_{em}}{2\pi^2} m_q^2 \alpha^2 \left\{ \alpha^2 K_0(\alpha m_q r_1) K_0(\alpha m_q r_2) + [1 + (1 - \alpha)^2] \frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2} K_1(\alpha m_q r_1) K_1(\alpha m_q r_2) \right\}. \quad (4)$$

Here m_q is the effective quark mass, which is in fact an infrared cutoff parameter, and can be adjusted to photoproduction data [8], or shadowing [9], and whose value is $m_q \approx 0.2$ GeV.

In Eq. (3) the effective partial amplitude $F_t(\vec{b}, \alpha\vec{r}_1, \alpha\vec{r}_2, x)$ is a linear combination of $\bar{q}q$ dipole partial amplitudes at impact parameter b,

$$F_t(\vec{b}, \alpha\vec{r}_1, \alpha\vec{r}_2, x) = \text{Im}[f_{q\bar{q}}^t(\vec{b}, \alpha\vec{r}_1, x) + f_{q\bar{q}}^t(\vec{b}, \alpha\vec{r}_2, x)$$
$$- f_{q\bar{q}}^t(\vec{b}, \alpha(\vec{r}_1 - \vec{r}_2), x)], \quad (5)$$

where x is the Bjorken variable of the target gluons.

III. AZIMUTHAL ASYMMETRY IN QUARK-NUCLEON COLLISIONS

In the case of a nucleon target ($t = N$), the partial elastic amplitude $f_{\bar{q}q}^N(\vec{b}, \vec{r})$ of interaction of the $\bar{q}q$ dipole with a proton at impact parameter $\vec{b}$ is related to the dipole cross section as

$$\sigma_{\bar{q}q}^N(r) = 2 \int d^2b \, \text{Im} f_{\bar{q}q}^N(\vec{b}, \vec{r}), \quad (6)$$

where $\sigma_{q\bar{q}}^N(r)$ is the total cross section of a $\bar{q}q$-proton collision. Here and further on, unless specified otherwise, the dipole cross section and partial amplitudes implicitly depend on the Bjorken variable x of the target gluons.

The cross section $\sigma_{\bar{q}q}^N(r)$ has been rather well determined by data on deep-inelastic scattering [10]. With this input, and using Eq. (3), one can calculate the inclusive differential cross section of direct photon emission. This was done in [7] for pp collisions, with results in good agreement with data.

Using the partial elastic amplitude $f_{\bar{q}q}^N(\vec{b}, \vec{r})$ one can also calculate the differential elastic cross section of dipole-nucleon scattering. Neglecting the real part, the amplitude reads

$$\frac{d\sigma_{el}^{(\bar{q}q)N}(r)}{dq_T^2} = \frac{1}{4\pi} \left| \int d^2b e^{i\vec{q}_T \cdot \vec{b}} \, \text{Im} f_{\bar{q}q}^N(\vec{b}, \vec{r}) \right|^2$$
$$\approx \frac{[\sigma_{\bar{q}q}^N(r)]^2}{16\pi} \exp[-B_{el}^{(\bar{q}q)N}(r)q_T^2]. \quad (7)$$

In the second line of this equation we rely on the small-q_T approximation. This defines the forward slope of the differential cross section, which can be calculated as

$$B_{el}^{(\bar{q}q)N}(r) = \frac{1}{2}\langle s^2 \rangle = \frac{1}{\sigma_{\bar{q}q}^N(r)} \int d^2s s^2 \, \text{Im} f_{\bar{q}q}^N(\vec{s}, \vec{r}). \quad (8)$$

The slope for small-dipole-proton elastic scattering was measured in diffractive electroproduction of ρ-mesons at high Q^2 at HERA [11]. The measured slope, $B_{el}^{(\bar{q}q)N}(r) \approx 5$ GeV^{-2}, agrees with the expected value $B_{el}^{(\bar{q}q)N}(r) \approx B_{el}^{pp}/2$.

The objective of this paper is the azimuthal asymmetry of photon radiation. First of all, we calculate the asymmetry of the cross section Eq. (3) for quark-nucleon collisions. The only vector available for such asymmetry is the impact parameter $\vec{b}$, and therefore we should trace a correlation between the vectors $\vec{p}$ and $\vec{b}$. The popular correlation function is defined as

$$v_2^{qN}(b, p, \alpha) = \langle \hat{v}_2 \rangle_{\phi_p} = 2\left\langle \left(\frac{\vec{p} \cdot \vec{b}}{pb}\right)^2 \right\rangle_{\phi_p} - 1$$
$$= \frac{\int_0^{2\pi} d\phi_p \hat{v}_2 \frac{d\sigma^{qT \to \gamma X}(b, p, \alpha)}{d(\ln\alpha)d^2 p d^2 b}}{\int_0^{2\pi} d\phi_p \frac{d\sigma^{qT \to \gamma X}(b, p, \alpha)}{d(\ln\alpha)d^2 p d^2 b}}, \quad (9)$$

where the averaging is performed integrating in (3) over the azimuthal angle ϕ_p of the transverse momentum $\vec{p}$.

IV. RADIATION PRODUCED BY A QUARK PROPAGATING THROUGH A NUCLEUS

In this case the partial amplitude to be used in (5), for a $\bar{q}q$ dipole colliding with a nucleus at impact parameter b, reads,

$$\text{Im} f_{q\bar{q}}^A(\vec{b}, \vec{r}) = 1 - \left[1 - \frac{1}{2A}\sigma_{q\bar{q}}^N(r)\tilde{T}_A(\vec{b}, \vec{r})\right]^A$$
$$\approx 1 - \exp\left[-\frac{1}{2}\sigma_{q\bar{q}}^N(r)\tilde{T}_A(\vec{b}, \vec{r})\right]. \quad (10)$$

The effective nuclear thickness $\tilde{T}_A$ is defined as [12]

$$\tilde{T}_A(\vec{b}, \vec{r}) = \frac{2}{\sigma_{\bar{q}q}^N(r)} \int d^2s \, \text{Im} f_{\bar{q}q}^N(\vec{s}, \vec{r}) T_A(\vec{b} + \vec{s}), \quad (11)$$

where the nuclear thickness function is defined as an integral of the nuclear density along the particle trajectory, $T_A(b) = \int_{-\infty}^{\infty} dz\rho_A(b, z)$.

Calculating $v_2^{qA}(b, p, \alpha)$, we can average over ϕ_p,

$$\left\langle \left(\frac{\vec{p} \cdot \vec{b}}{pb}\right)^2 \right\rangle_{\phi_p} \propto \int_0^{2\pi} d\phi_p \left(\frac{\vec{p} \cdot \vec{b}}{pb}\right)^2 \frac{d\sigma^{qA \to \gamma X}(b, p, \alpha)}{d(\ln\alpha)d^2 p d^2 b}, \quad (12)$$

analytically. Instead of integration over the direction of $\vec{p}$ at fixed $\vec{b}$, one can integrate over the direction of $\vec{b}$ at fixed $\vec{p}$. The advantage of such a replacement is obvious: all the b-dependence in (3) is located in the effective amplitude F^A and it has an explicit and simple form.

Indeed, the mean value of s^2 is according to (8) $\langle s^2 \rangle \approx 0.4$ fm^2, which is much smaller than the heavy nucleus radius squared, R_A^2. Therefore we can expand $T_A(\vec{b} + \vec{s})$ as

$$T_A(\vec{b} + \vec{s}) = T_A(b) + \frac{\vec{s} \cdot \vec{b}}{b} T_A'(b) + \frac{1}{2}\left(\frac{\vec{s} \cdot \vec{b}}{b}\right)^2 T_A''(b) + \dots. \tag{13}$$

Correspondingly, the partial amplitude (10) can be expanded as

$$\mathrm{Im} f_{q\bar{q}}^A(\vec{b}, \vec{r}) \approx 1 - \exp\left[-\frac{1}{2}\sigma_{q\bar{q}}^N(r)T_A(b)\right]$$
$$\times \left\{ 1 - \frac{1}{b}T_A'(b)\gamma_1(\vec{b}, \vec{r}) \right.$$
$$\left. - \frac{1}{2b^2}\left[T_A''(b)\gamma_2(\vec{b}, \vec{r}) - T_A'^2(b)\gamma_1^2(\vec{b}, \vec{r})\right]\right\}, \tag{14}$$

where

$$\gamma_n(\vec{b}, \vec{r}) = \int d^2 s\, \mathrm{Im} f_{q\bar{q}}^N(\vec{s}, \vec{r})(\vec{s} \cdot \vec{b})^n. \tag{15}$$

Integrating the amplitude (14) together with $\hat{v}_2$ over ϕ_b we find that the first two terms in the curly brackets in (14) give zero, and the rest is

$$\mathrm{Im} \tilde{f}_{q\bar{q}}^A(b, \vec{r}) \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi_b\, \mathrm{Im} f_{q\bar{q}}^A(\vec{b}, \vec{r})\hat{v}_2(\phi_b)$$
$$= e^{-(1/2)\sigma_{q\bar{q}}^N(r)T_A(b)}\frac{1}{4}\left[T_A''(b)g(r) - T_A'^2(b)h(r)\right]. \tag{16}$$

Here

$$g(r) = \int d^2 s\, \mathrm{Im} f_{\bar{q}q}^N(\vec{s}, \vec{r})\left[2\frac{(\vec{p} \cdot \vec{s})^2}{p^2} - s^2\right]; \tag{17}$$

$$h(r) = \int d^2 s_1 d^2 s_2\, \mathrm{Im} f_{\bar{q}q}^N(\vec{s}_1, \vec{r})\, \mathrm{Im} f_{\bar{q}q}^N(\vec{s}_2, \vec{r})$$
$$\times \left[2\frac{(\vec{s}_1 \cdot \vec{p})(\vec{s}_2 \cdot \vec{p})}{p^2} - (\vec{s}_1 \cdot \vec{s}_2)\right]. \tag{18}$$

Equation (16) shows that the azimuthal asymmetry is strongly enhanced at the nuclear periphery. Indeed, at small impact parameters the amplitude Eq. (16) is suppressed by the factor $\exp[-\frac{1}{2}\sigma_{q\bar{q}}^N(r)T_A(b)]$, and, moreover, $T_A'(b) \approx -2\rho_0 b/\sqrt{R_A^2 - b^2}$ and $T_A''(b) \approx 2\rho_0 R_A^2/(R_A^2 - b^2)^{3/2}$ are vanishingly small and peak at the periphery ($\rho_0 \approx 0.16$ fm^{-3} is the central nuclear density). The smallness of both the amplitude and azimuthal asymmetry justifies also the expansion made in (14).

As far as the partial amplitudes $f_{\bar{q}q}^N(\vec{s}, \vec{r})$ and their asymmetric part Eq. (16) are known, one can calculate the azimuthal asymmetry of photons radiated in quark-nucleus

collisions,

$$v_2^{qA}(b, p, \alpha)\frac{d\sigma^{qA \to \gamma X}(b, p, \alpha)}{d(\ln\alpha)d^2 p d^2 b}$$
$$= \sum_{in,f}\int d^2 r_1 d^2 r_2 e^{i\vec{p} \cdot (\vec{r}_1 - \vec{r}_2)}\phi_{\gamma q}^\star(\alpha, \vec{r}_1)\phi_{\gamma q}(\alpha, \vec{r}_2)$$
$$\times \tilde{F}^A(b, \alpha\vec{r}_1, \alpha\vec{r}_2), \tag{19}$$

where

$$\tilde{F}^A(b, \alpha\vec{r}_1, \alpha\vec{r}_2) = \mathrm{Im} \tilde{f}_{q\bar{q}}^A(b, \alpha r_1) + \mathrm{Im} \tilde{f}_{q\bar{q}}^A(b, \alpha r_2)$$
$$- \mathrm{Im} \tilde{f}_{q\bar{q}}^A(b, \alpha|\vec{r}_1 - \vec{r}_2|). \tag{20}$$

Notice that the cross section in the left-hand side of Eq. (19) can also be calculated without using the expansion (13), relying on the eikonal approximation, $\tilde{T}_A(\vec{b}, \vec{r}) \approx T_A(b)$, which is known to be quite accurate for heavy nuclei.

V. PARTIAL DIPOLE AMPLITUDE $f_{\bar{q}q}^N(\vec{b}, \vec{r})$

The next step is to model the partial dipole amplitude $f_{\bar{q}q}^N(\vec{b}, \vec{r})$. An azimuthal asymmetry can only emerge if the amplitude $f_{\bar{q}q}^N(\vec{b}, \vec{r})$ contains a correlation between the vectors $\vec{b}$ and $\vec{r}$. If such a correlation is lacking, the function Eqs. (17) and (18) are equal to zero. A model for $f_{\bar{q}q}^N(\vec{b}, \vec{r})$ having no $\vec{b} - \vec{r}$ correlation was proposed in [13].

It is rather straightforward to calculate the partial amplitude within the two-gluon exchange model [14],

$$\mathrm{Im} f_{\bar{q}q}^N(\vec{b}, \vec{r}) = \frac{2}{3\pi^2}\int\frac{d^2 q d^2 q' \alpha_s(q^2)\alpha_s(q'^2)}{(q^2 + \mu^2)(q'^2 + \mu^2)}$$
$$\times e^{i\vec{b} \cdot (\vec{q} - \vec{q}')}(1 - e^{i\vec{q} \cdot \vec{r}})(1 - e^{-i\vec{q}' \cdot \vec{r}})$$
$$\times [F_N(\vec{q} - \vec{q}') - F_N^{(2q)}(\vec{q}, \vec{q}')], \tag{21}$$

where $F_N(k) = \langle\Psi_N|\exp(i\vec{k} \cdot \vec{\rho}_1)|\Psi_N\rangle$ is the nucleon form factor, and $F_N^{(2q)}(\vec{q}, \vec{q}') = \langle\Psi_N|\exp[i\vec{q} \cdot \vec{\rho}_1 - i\vec{q}' \cdot \vec{\rho}_2]|\Psi_N\rangle$ is the so-called two-quark-nucleon form factor. Both can be calculated using the three-valence quark-nucleon wave function $\Psi_N(\vec{\rho}_1, \vec{\rho}_2, \vec{\rho}_3)$.

An effective gluon mass μ is introduced in (21) in order to imitate confinement. We fix its value at $\mu = m_\pi$ in order to reproduce the large hadronic cross sections.

The Born amplitude is unrealistic since it leads to an energy independent dipole cross section $\sigma_{\bar{q}q}(r, x)$. This dipole cross section has been well probed by measurements of the proton structure function at small Bjorken x at HERA, and was found to rise towards small x, with an x dependent steepness. In fact, it can be expressed via the unintegrated gluon density $\mathcal{F}(x, q^2)$,

$$\sigma(r, x) = \frac{4\pi}{3} \int \frac{d^2q}{q^4} (1 - e^{-i\vec{q}\cdot\vec{r}}) \alpha_s(q^2) \mathcal{F}(x, q^2). \quad (22)$$

Analogously, the partial amplitude for dipole-nucleon elastic scattering at impact parameter $\vec{b}$ between the centers of gravity of the dipole and nucleon reads

$$\begin{aligned}
\text{Im} f_{\bar{q}q}^N(\vec{b}, \vec{r}, \beta) &= \frac{1}{12\pi} \int \frac{d^2q d^2q'}{q^2 q'^2} \alpha_s \mathcal{F}(x, \vec{q}, \vec{q}') \\
&\times e^{i\vec{b}\cdot(\vec{q}-\vec{q}')} (e^{-i\vec{q}\cdot\vec{r}\beta} - e^{i\vec{q}\cdot\vec{r}(1-\beta)}) \\
&\times (e^{i\vec{q}'\cdot\vec{r}\beta} - e^{-i\vec{q}'\cdot\vec{r}(1-\beta)}). \quad (23)
\end{aligned}$$

Here the dipole has transverse separation $\vec{r}$, fractional light-cone momenta of the quark and antiquark, $1 - \beta$ and β respectively. Since the radiated photon takes away fraction α of the quark momentum, the corresponding dipole has $\beta = 1/(2 - \alpha)$. The impact parameter $\vec{b}$ of the dipole is the transverse distance from the target to the dipole center of gravity, which is shifted towards the fastest q or $\bar{q}$ in accordance with (23).

In (23) $\alpha_s = \sqrt{\alpha_s(q^2)\alpha_s(q'^2)}$, and we introduced the off-diagonal unintegrated gluon density $\mathcal{F}(x, \vec{q}, \vec{q}')$, which in the Born approximation limit takes the form

$$\begin{aligned}
\mathcal{F}(x, \vec{q}, \vec{q}') &\Rightarrow \mathcal{F}_{\text{Born}}(\vec{q}, \vec{q}') \\
&= \frac{4\alpha_s}{\pi} [F_N(\vec{q} - \vec{q}') - F_N^{(2q)}(\vec{q}, \vec{q}')]. \quad (24)
\end{aligned}$$

Besides, the partial elastic amplitude Eq. (23) should satisfy the conditions Eqs. (6) and (8). For the dipole cross section we rely on the popular saturated shape [10] fitted to HERA data for $F_2^p(x, Q^2)$, and we choose the following form of $\mathcal{F}(x, \vec{q}, \vec{q}')$,

$$\begin{aligned}
\mathcal{F}(x, \vec{q}, \vec{q}') &= \frac{3\sigma_0}{16\pi^2\alpha_s} q^2 q'^2 R_0^2(x) \\
&\times \exp\left[-\frac{1}{8} R_0^2(x)(q^2 + q'^2)\right] \\
&\times \exp[-R_N^2(\vec{q} - \vec{q}')^2/2], \quad (25)
\end{aligned}$$

where $\sigma_0 = 23.03$ mb, $R_0(x) = 0.4$ fm $\times (x/x_0)^{0.144}$ with $x_0 = 3.04 \times 10^{-4}$ [10] and $x = p/\sqrt{s}$ [7]. We assume here that the Pomeron-proton form factor has the Gaussian form, $F_{\text{IP}}^p(k_T^2) = \exp(-k_T^2 R_N^2/2)$, so the slope of the pp elastic differential cross section is $B_{\text{el}}^{pp} = 2R_N^2 + 2\alpha_{\text{IP}}' \ln(s/s_0)$, where $\alpha_{\text{IP}}' \approx 0.25$ GeV^{-2} is the slope of the Pomeron trajectory, $s_0 = 1$ GeV^2. $R_N^2 \approx \langle r_{\text{ch}}^2 \rangle/3$ is the part of the slope of the elastic cross section related to the Pomeron-proton form factor.

With this unintegrated gluon density the partial amplitude Eq. (23) can be calculated explicitly,

$$\begin{aligned}
\text{Im} f_{\bar{q}q}^N(\vec{b}, \vec{r}, x, \beta) &= \frac{\sigma_0}{8\pi B_{\text{el}}} \left\{ \exp\left[-\frac{[\vec{b} + \vec{r}(1 - \beta)]^2}{2B_{\text{el}}}\right] \right. \\
&+ \exp\left[-\frac{(\vec{b} - \vec{r}\beta)^2}{2B_{\text{el}}}\right] \\
&\left. - 2\exp\left[-\frac{r^2}{R_0^2} - \frac{[\vec{b} + (1/2 - \beta)\vec{r}]^2}{2B_{\text{el}}}\right] \right\},
\end{aligned}$$
(26)

where $B_{\text{el}}(x) = R_N^2 + R_0^2(x)/8$. This amplitude satisfies the conditions Eqs. (6) and (8). This expression also goes beyond the usual assumption that the dipole cross section is independent of the light-cone momentum sharing β. The partial amplitude Eq. (26) does depend on β, but this dependence disappears after integration over impact parameter $\vec{b}$.

VI. NUMERICAL RESULTS

Now we are in a position to calculate $v_2^{qN}(b, p, \alpha)$. Examples of quark-nucleon collisions radiating a photon, with $\alpha = 1$ and at different impact parameters and energies, are depicted in Fig. 1. The results show that the anisotropy of the dipole interaction rises with impact parameter, reaching rather large values. As a function of the transverse momentum of the radiated photons, $v_2^{qN}(b, p, \alpha)$ vanishes at large p_T. Such a behavior could be anticipated, since the interaction of vanishingly small dipoles responsible for large p is not sensitive to the dipole orientation.
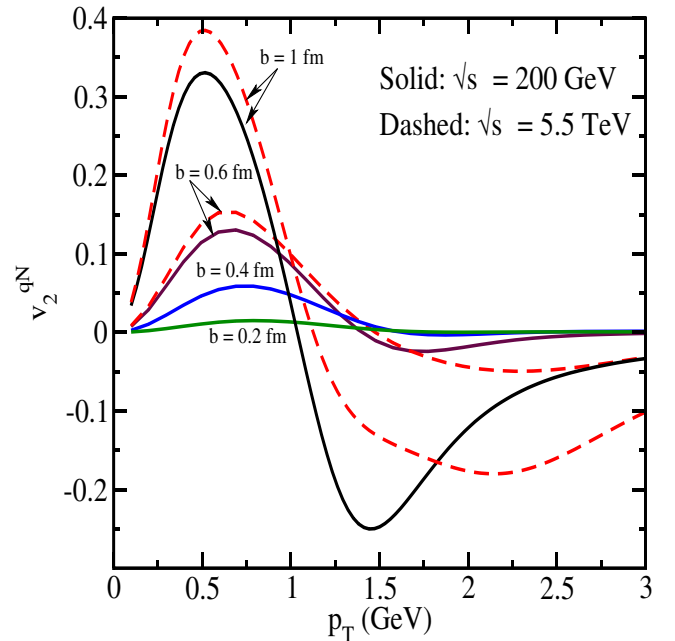

FIG. 1 (color online). The anisotropy parameter $v_2^{qN}(b, p, \alpha)$ as a function of p calculated at $\alpha = 1$ for different impact parameters b and energies: $\sqrt{s} = 200$ GeV (solid, $b = 0.2$, 0.4, 0.6, 1 fm), and $\sqrt{s} = 5500$ GeV (dashed, $b = 0.6$, 1 fm).

The next step is calculating the azimuthal asymmetry in quark-nucleus collisions. The results are plotted in Fig. 2 as a function of transverse momentum, at different impact parameters and at the energies of the Relativistic Heavy Ion Collider (RHIC) and LHC.

The first observation is the smallness of v_2^{qA}, which is suppressed an order of magnitude compared to v_2^{qN}. At first glance this might look strange, since the quark interacts with nucleons anyway. However, a quark propagating through a nucleus interacts with different nucleons located at different azimuthal angles relative to the quark trajectory. Their contributions to v_2^{qA} tend to cancel each other, restoring the azimuthal symmetry. Such cancellation would be exact if the nuclear profile function $T_A(b)$ were constant. We have a nonzero, but small v_2^{qA} only due to the variation of T_A with b, i.e. the presence of finite first and second derivatives, as was derived in Eq. (16).

The results of a numerical integration (without expansion (14)), depicted in Figs. 1 and 2, also confirm the anticipation based on Eq. (16) that the azimuthal asymmetry is enhanced on the nuclear periphery.

We used the Woods-Saxon parametrization for nuclear density [15]. The anisotropy of electromagnetic radiation appears only on the nuclear periphery and according to (16) is extremely sensitive to the behavior of the nuclear thickness function at the very edge of the nucleus. Electron scattering data, which is the main source of information about the electric charge distribution in nuclei, is not

FIG. 3 (color online). Azimuthal anisotropy of direct photons with $\alpha = 1$ from quark-lead collisions at $b = 5$ and 7 fm. Solid and dashed curves are calculated with Woods-Saxon (WS) and hard sphere (HS) parametrizations of nuclear density, respectively.

sensitive to the neutron distribution, which is known to be enlarged on the periphery. Therefore the details of the shape of the density distribution on the nuclear surface are poorly known. As a simple estimate of the theoretical uncertainty related to this problem one can use an alternative parametrization of the nuclear density, such as the simple and popular hard sphere form, $\rho(r) = \rho_0 \Theta(R_A - r)$. We compare in Fig. 3 the anisotropy parameters $v_2^{qA}(p, b, \alpha)$ calculated with hard sphere (dashed) and Woods-Saxon (solid) parametrizations. As one could expect, the hard sphere density leads to a quite larger anisotropy, since the derivatives of the nuclear profile function are much sharper.

VII. SUMMARY

Summarizing, we extended the dipole description of electromagnetic radiation [1,2] in quark-nucleon and nucleus collisions to calculation of the azimuthal angle distribution. This problem involves more detailed features of the dipole amplitude, namely, its dependence on dipole size and impact parameter, as well as on their correlation. We propose a simple model generalizing the unintegrated gluon density fitted to HERA data for the proton structure function to an off-diagonal gluon distribution. The latter satisfies all the imposed boundary conditions.

The developed theoretical tools can be applied to the calculation of the azimuthal asymmetry in deep-inelastic
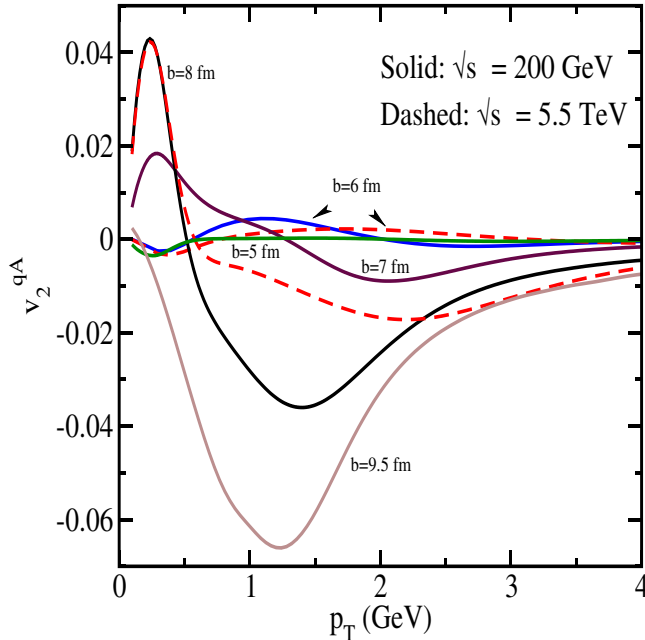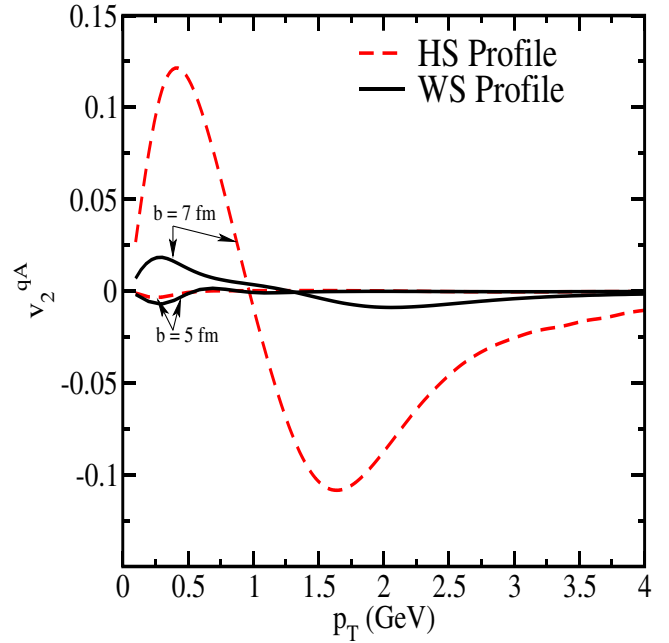

FIG. 2 (color online). Azimuthal anisotropy of direct photons with $\alpha = 1$ from quark-lead collisions at different impact parameters as is labeled in the plot. Solid and dashed curves correspond to the energies of RHIC ($b = 5, 6, 7, 8, 9.5$ fm), and LHC ($b = 6, 8$ fm), respectively.

scattering and in Drell-Yan reactions on a proton, as well as to the production of direct photons and Drell-Yan pairs in proton-nucleus and heavy ion collisions.

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