Is $Z^+(4430)$ a loosely bound molecular state?

Xiang Liu,[1,*] Yan-Rui Liu,[2,†] Wei-Zhen Deng,[1,‡] and Shi-Lin Zhu[1,§]

[1] *Department of Physics, Peking University, Beijing 100871, China*

[2] *Institute of High Energy Physics, P.O. Box 918-4, Beijing 100049, China*

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Since $Z^+(4430)$ lies very close to the threshold of $D^*\bar{D}_1$, we investigate whether $Z^+(4430)$ could be a loosely bound S-wave state of $D^*\bar{D}_1$ or $D^*\bar{D}_1'$ with $J^P = 0^-, 1^-, 2^-$, i.e., a molecular state arising from the one-pion-exchange potential. The potential from the crossed diagram is much larger than that from the diagonal scattering diagram. With various trial wave functions, we notice that the attraction from the one-pion-exchange potential alone is not strong enough to form a bound state with realistic pionic coupling constants deduced from the decay widths of D_1 and D_1'.

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I. INTRODUCTION

Recently Belle Collaboration observed a sharp peak in the $\pi^+\psi'$ invariant mass spectrum in the exclusive $B \to K\pi^+\psi'$ decays with a statistical significance of 7σ [1]. This resonancelike structure is named as $Z^+(4430)$. The fit with a Breit-Wigner form yields its mass $m = 4433 \pm 4(\text{stat}) \pm 1(\text{syst})$ MeV and a narrow width $\Gamma = 44^{+17}_{-13}(\text{stat})^{+30}_{-11}(\text{syst})$ MeV. It is very interesting to note that the width of $Z^+(4430)$ is roughly the same as that of D_1.

The product branching fraction is measured to be $\mathcal{B}(B \to KZ^+(4430)) \cdot \mathcal{B}(Z^+(4430) \to \pi^+\psi') = (4.1 \pm 1.0(\text{stat}) \pm 1.3(\text{syst})) \times 10^{-5}$ [1]. For comparison, we list the production rate of $X(3872)$ and $Y(4260)$ in B decays. From Ref. [2] we have

$$\mathcal{B}(B^- \to K^-X(3872)) \cdot \mathcal{B}(X(3872) \to J/\Psi\pi^+\pi^-) = (1.28 \pm 0.41) \times 10^{-5},$$

and from Ref. [3]

$$\mathcal{B}(B^- \to K^-X(3872)) \cdot \mathcal{B}(X(3872) \to J/\Psi\pi^+\pi^-) = (10.1 \pm 2.5 \pm 1.0) \times 10^{-6}.$$

For $Y(4260)$, *BABAR* Collaboration gave the upper limit of the branching fraction [3]

$$\mathcal{B}(B^- \to K^-Y(4260)) \cdot \mathcal{B}(Y(4260) \to J/\Psi\pi^+\pi^-) < 2.9 \times 10^{-5}.$$

It is plausible that (1) $\mathcal{B}(B \to KZ^+(4430))$ is comparable to both $\mathcal{B}(B^- \to K^-X(3872))$ and $\mathcal{B}(B^- \to K^-Y(4260))$; (2) $\pi^+\psi'$ is one of the main decay modes of $Z^+(4430)$ if it is a resonance.

The peak $Z^+(4430)$ inspired several theoretical speculations of its underlying structure. Rosner suggested that $Z^+(4430)$ is a S-wave threshold effect because $Z^+(4430)$ lies close to the $D^*(2010)\bar{D}_1(2420)$ threshold [4]. The production mechanism was speculated as follows. The b quark first decays into a strange quark and a pair of $c\bar{c}$ while a pair of light quarks are created from the vacuum. In other words, B meson first decays into a K and a pair of D mesons. Then the D meson pair rescatters into $\pi^+\psi'$. He also suggested other possible charged states.

Maiani, Polosa and Riquer identified this signal as the first radial excitation of the tetraquark supermultiplet to which $X(3872)$ and $X(3876)$ belong [5]. With their assignment the quantum number of $Z^+(4430)$ is $J^{PC} = 1^{+-}$ where the C-parity is for the neutral member within the same multiplet. $Z^+(4430)$ decays into $\pi^+\psi'$ via S-wave. Moreover their scheme requires the ground state with $J^{PC} = 1^{+-}$ around 3880 MeV which decays into $\pi^+\psi$ and $\eta_c\rho^+$.

With a QCD-string model, Gershtein, Likhoded, and Pronko argued that both $X(3872)$ and $Z^+(4430)$ are tetraquark states [6]. They speculate that the two quarks and two antiquarks sit on the four corners of a square while any $q\bar{q}$ pair is a color-octet state. The decays of tetraquarks involve the reconnection of the color string.

Cheung, Keung, and Yuan discussed the bottom analog of $Z^+(4430)$ assuming it is a tetraquark bound state [7]. According to their estimate, the doubly-charged Z_{bc} state lies around 7.6 GeV while the bottomonium analog Z_{bb} of $Z^+(4433)$ is about 10.7 GeV.

Qiao suggested $Z^+(4433)$ be the first radial excitation of $\Lambda_c - \Sigma_c^0$ bound state [8]. Within Qiao's scheme, all of the recently observed states $Y(4260)$, $Y(4361)$, $Z^+(4430)$ and $Y(4664)$ are accommodated in the extended heavy baryonium framework.

Lee, Mihara, Navarra, and Nielsen calculated the mass of $Z^+(4430)$ $m_Z = (4.40 \pm 0.10)$ GeV in the framework of QCD sum rules, assuming it is a 0^- molecular state of $D^*(2010)\bar{D}_1(2420)$ [9]. They predicted the analogous mesons Z_s at $m_{Z_s} = (4.70 \pm 0.06)$ GeV, which is above the $D_s^*D_1$ threshold and Z_{bb} around $m_{Z_{bb}} = (10.74 \pm 0.12)$ GeV.

*xiangliu@pku.edu.cn
†yrliu@ihep.ac.cn
‡dwz@th.phy.pku.edu.cn
§zhusl@phy.pku.edu.cn

Bugg proposed that $Z^+(4430)$ is a threshold cusp arising from the deexcitation of the $D^*(2010)D_1(2420)$ pair into lower mass D states [10]. The imaginary part $\mathrm{Im}f(s)$ of the elastic S-wave amplitude $f(s)$ is a step function near the threshold. From the dispersion relation, the real part of the amplitude near the threshold looks like $\mathrm{Re}f(s)\sim \int_{s_0} \frac{\theta(s)}{s'-s} ds' \sim \ln(s-s_0)$. Therefore $|f(s)|^2$ contains a sharp cusp near threshold.

However, none of the above schemes explains why $Z^+(4430)$ does *not* decay into $\pi^+\psi$. One notes that the momentum of D and $\bar{D}$ is small and close to each other in the rest frame of the parent B meson, especially when one (or two) of the D meson pair is an excited state. Therefore, there is plenty of time for the D meson pair to move together and rescatter into $\pi^+\psi'$. However, the most puzzling issue of the rescattering mechanism is the absence of any signal in the π^+J/ψ channel. One may wonder whether the mismatch of the Q-values of the initial and final states plays an important role. If so, one should also expect a signal in the $\pi^+\psi(3S)$ channel since there is nearly no mismatch of the Q-value now. Another potential scapegoat is the specific nodal structure in the wave functions of the final states. Detailed calculations along the above two directions are highly desirable to investigate the origin of the nonobservation of $Z^+(4430)$ in the $\pi^+\psi$ mode.

In the heavy quark limit, the angular momentum of the light quark $j_l = \vec{l} + \vec{S}_q$ is a good quantum number where l is the orbital angular momentum and S_q is the light quark spin. For the P-wave heavy mesons, $j_l = \frac{3}{2}$ or $\frac{1}{2}$, which correspond to the two doublets with $J^P = (1^+, 2^+)$ and $(0^+, 1^+)$, respectively. The ground states D and D^* belong to the $J^P = (0^-, 1^-)$ doublet with $j_l = \frac{1}{2}$. Since the 1^+ state in the $(1^+, 2^+)$ doublet decays into $D^*\pi$ via D-wave, it is very narrow and denoted as $D_1(2420)$ [11]. The 1^+ state in the $(0^+, 1^+)$ doublet decays into $D^*\pi$ via S-wave. Hence it is very broad and denoted as $D_1'(2430)$ [11].

Assuming $Z^+(4430)$ is a D_1D^* (or $D_1'D^*$) S-wave resonance, Meng and Chao found that the open-charm decay mode $D^*D^*\pi$ is dominant and the rescattering effects are significant in D_1D^* channel but not in $D_1'D^*$ channel since D_1' is very broad [12]. For the $J^P = 1^-$ candidate, the ratio $\Gamma(Z^+ \to \psi'\pi^+)/\Gamma(Z^+ \to J/\psi\pi^+)$ may reach 5.3 with a special set of parameters, which partly accounts for why the $Z^+(4430)$ is difficult to be found in $J/\psi\pi^+$.

Despite so many theoretical speculations proposed above, a dynamical study of the $Z^+(4430)$ signal is still missing. In this work we will explore whether $Z^+(4430)$ could be a S-wave molecular state of D^* and $\bar{D}_1'$ (or $\bar{D}_1$), which is loosely bound by the long-range pion exchange potential. We want to find out whether there exists the attractive force between D^* and $\bar{D}_1'$ ($\bar{D}_1$) in different channels.

This paper is organized as follows. We discuss the possible quantum numbers of $Z^+(4430)$ and its possible partner states in Sec. II. We collect the effective Lagrangians and various coupling constants in Sec. III. We derive the one-pion-exchange potential (OPEP) in Sec. IV. Then we present our numerical result and a short discussion in Sec. V.

II. QUANTUM NUMBER OF $Z^+(4430)$ AND OTHER POSSIBLE STATES

Natively, the smaller its angular momentum, the lower the mass of $Z^+(4430)$. Since $Z^+(4430)$ lies very close to the $D^*(2010)\bar{D}_1(2420)$ threshold, we consider only the possibility of $Z^+(4430)$ being the loosely bound S-wave state of D^* and $\bar{D}_1'$ (or $\bar{D}_1$). Therefore, its possible angular momentum and parity are $J^P = 0^-$, 1^-, 2^-. Moreover, $Z^+(4430)$ was observed in the $\psi'\pi^+$ channel. So it is an isovector state with positive G-parity, i.e., $I^G = 1^+$.

For a charged member within a molecular isovector multiplet, one can construct its flavor wave function with definite G parity in the following way. Suppose $|A\rangle$ is one component of its flavor wave function. Then the $G = +$ state reads:

$$|+\rangle = \frac{1}{\sqrt{2}}(|A\rangle + \hat{G}|A\rangle) \tag{1}$$

where $\hat{G} = e^{iI_y\pi}\hat{C}$ is the G-parity operator. Similarly, the $G = -$ state reads:

$$|-\rangle = \frac{1}{\sqrt{2}}(|A\rangle - \hat{G}|A\rangle). \tag{2}$$

In the present case, the flavor wave function of $Z^+(4430)$ is

$$|Z^+\rangle = \frac{1}{\sqrt{2}}[|A'\rangle + |B'\rangle] \tag{3}$$

with $|A'\rangle = |\bar{D}_1'^0 D^{*+}\rangle$ and $|B'\rangle = |D_1'^+ \bar{D}^{*0}\rangle$, where $D_1'^0$ and $D_1'^+$ belong to the $(0^+, 1^+)$ doublet in the heavy quark effective field theory. Or

$$|Z^+\rangle = \frac{1}{\sqrt{2}}[|A\rangle + |B\rangle] \tag{4}$$

with $|A\rangle = |\bar{D}_1^0 D^{*+}\rangle$ and $|B\rangle = |D_1^+ \bar{D}^{*0}\rangle$, where D_1^0 and D_1^+ belong to the $(1^+, 2^+)$ doublet.

The flavor wave functions of the partner states of Z^+ can be derived in the following way. Charmed mesons belong to the fundamental representation of flavor $SU(3)$. Therefore, the system with a charmed meson and an anticharmed meson belongs to $\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{8} + \mathbf{1}$. We list the wave functions of these hidden charm states, whose names are listed in Fig. 1. Here we use the system of $D^*_{(s)}$ and $D_{(s)1}$ as an illustration.

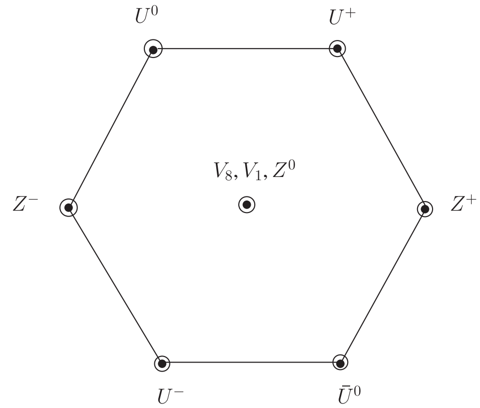

FIG. 1. The multiplets composed with charmed mesons and anticharmed mesons.

The flavor wave functions of these states are

$$\begin{aligned}
|Z^+\rangle &= \frac{1}{\sqrt{2}}(|\bar{D}_1^0 D^{*+}\rangle - c|\bar{D}^{*0} D_1^+\rangle),\\
|Z^0\rangle &= \frac{1}{2}[(|D_1^- D^{*+}\rangle - c|D^{*-} D_1^+\rangle) - (|\bar{D}_1^0 D^{*0}\rangle\\
&\quad - c|\bar{D}^{*0} D_1^0\rangle)],\\
|Z^-\rangle &= -\frac{1}{\sqrt{2}}(|D_1^- D^{*0}\rangle - c|D^{*-} D_1^0\rangle),\\
|U^+\rangle &= -\frac{1}{\sqrt{2}}(|\bar{D}_1^0 D_s^{*+} - c|\bar{D}^{*0} D_{s1}^+\rangle\rangle),\\
|U^0\rangle &= -\frac{1}{\sqrt{2}}(|D_1^- D_s^{*+}\rangle - c|D^{*-} D_{s1}^+\rangle),\\
|U^-\rangle &= -\frac{1}{\sqrt{2}}(|D_{s1}^- D^{*0}\rangle - c|D_s^{*-} D_1^0\rangle),\\
|\bar{U}^0\rangle &= \frac{1}{\sqrt{2}}(|D_{s1}^- D^{*+}\rangle - c|D_s^{*-} D_1^+\rangle),\\
|V_8\rangle &= \frac{1}{2\sqrt{3}}[(|D_1^- D^{*+}\rangle - c|D^{*-} D_1^+\rangle) + (|\bar{D}_1^0 D^{*0}\rangle\\
&\quad - c|\bar{D}^{*0} D_1^0\rangle) - 2(|D_{s1}^- D_s^{*+}\rangle - c|D_s^{*-} D_{s1}^+\rangle)],\\
|V_1\rangle &= \frac{1}{\sqrt{6}}[(|D_1^- D^{*+}\rangle - c|D^{*-} D_1^+\rangle) + (|\bar{D}_1^0 D^{*0}\rangle\\
&\quad - c|\bar{D}^{*0} D_1^0\rangle) + (|D_{s1}^- D_s^{*+}\rangle - c|D_s^{*-} D_{s1}^+\rangle)].
\end{aligned}$$

We use Z^0 in the $J = 0$ case as an example to illustrate how to determine c. At the quark level, this molecular state may be written as

$$J_{Z^0} = \tfrac{1}{2}[J_1 - cJ_2 - (J_3 - cJ_4)] \tag{5}$$

where

$$\begin{aligned}
J_1 &= (\bar{c}^a \gamma_\mu \gamma_5 d^a)(\bar{d}^e \gamma^\mu c^e),\\
J_2 &= (\bar{d}^a \gamma_\mu \gamma_5 c^a)(\bar{c}^e \gamma^\mu d^e),\\
J_3 &= (\bar{c}^a \gamma_\mu \gamma_5 u^a)(\bar{u}^e \gamma^\mu c^e),\\
J_4 &= (\bar{u}^a \gamma_\mu \gamma_5 c^a)(\bar{c}^e \gamma^\mu u^e).
\end{aligned}$$

In the above equation, a and e are the color indices. Under charge conjugate transformation, we have

$$\begin{aligned}
\hat{C} J_1 \hat{C}^{-1} &= -(\bar{d}^a \gamma_\mu \gamma_5 c^a)(\bar{c}^e \gamma^\mu d^e) = -J_2,\\
\hat{C} J_2 \hat{C}^{-1} &= -(\bar{c}^a \gamma_\mu \gamma_5 d^a)(\bar{d}^e \gamma^\mu c^e) = -J_1,\\
\hat{C} J_3 \hat{C}^{-1} &= -(\bar{u}^a \gamma_\mu \gamma_5 c^a)(\bar{c}^e \gamma^\mu u^e) = -J_4,\\
\hat{C} J_4 \hat{C}^{-1} &= -(\bar{c}^a \gamma_\mu \gamma_5 u^a)(\bar{u}^e \gamma^\mu c^e) = -J_3.
\end{aligned}$$

Therefore, we get

$$\hat{C} J_{Z^0} \hat{C}^{-1} = \tfrac{1}{2}[-J_2 + cJ_1 - (-J_4 + cJ_3)]. \tag{6}$$

In other words, the C-parity of Z^0 is $C = \pm$ for $c = \pm 1$. Since $Z^+(4430)$ is a state with isospin 1 and $G = +$, one requires $c = -1$ for these states. The quantum numbers of Z^0 are $I^G(J^{PC}) = 1^+(0, 1, 2)^{--}$.

We want to emphasize that the presence of both the charm and anticharm quark (and the light quark and antiquark) in the expression of J_i ensures there is no arbitrary phase factor under charge conjugate transformation.

There is one intuitive and natural way to interpret the above results if we consider the flavor $SU(4)$ symmetry, which is of course broken badly in reality. If we naively assume the flavor $SU(4)$ symmetry, then states within the same multiplet should carry the same coefficient under charge conjugate transformation. For example, $\hat{C}|D^{*-}\rangle = C(D^{*-})|D^{*+}\rangle$ where the coefficient $C(D^{*-})$ takes the same value as either $C(\rho^0)$ or $C(J/\psi)$, i.e., $C(D^{*-}) = -1$. Similarly, $C(D_1^-) = +1$. In other words, the $c = -1$ case leads to the Z^0 state with negative C-parity.

In addition, one obtains the flavor wave functions of these states with opposite G-parity if we take $c = +$ in the above equations. For example, we will also discuss whether $\tilde{Z}^+$ could be a molecular state:

$$|\tilde{Z}^+\rangle = \frac{1}{\sqrt{2}}(|\bar{D}_1^0 D^{*+}\rangle - |\bar{D}^{*0} D_1^+\rangle).$$

If $\tilde{Z}^+$ exists, this state may be discovered in either $J/\psi \pi^+ \pi^0$ or $\psi' \pi^+ \pi^0$ channel.

If we replace one of the c (or $\bar{c}$) by b (or $\bar{b}$), we can get molecular states such as $(b\bar{q}) - (\bar{c}q)$. With the dual replacement $c \to b$, $\bar{c} \to \bar{b}$, we get the hidden bottom molecular states $(b\bar{q}) - (\bar{b}q)$.

III. EFFECTIVE LAGRANGIANS AND COUPLING CONSTANTS

We collect the effective chiral Lagrangian used in the derivation of the OPEP in this section. In the chiral and heavy quark dual limits, the Lagrangian relevant to our calculation reads [13,14]

$$\begin{aligned}\mathcal{L} = & ig\,\mathrm{Tr}[H_b \not{A}_{ba}\gamma_5\bar{H}_a] + ig'\,\mathrm{Tr}[S_b\not{A}_{ba}\gamma_5\bar{S}_a] \\ & + ig''\,\mathrm{Tr}[T_{\mu b}\not{A}_{ba}\gamma_5\bar{T}^{\mu}_a] + [ih\,\mathrm{Tr}[S_b A_{ba}\gamma_5\bar{H}_a] \\ & + \mathrm{H.c.}] + \left\{i\frac{h_1}{\Lambda_\chi}\mathrm{Tr}[T^{\mu}_b(D_\mu \not{A})_{ba}\gamma_5\bar{H}_a] + \mathrm{H.c.}\right\} \\ & + \left\{i\frac{h_2}{\Lambda_\chi}\mathrm{Tr}[T^{\mu}_b(\not{D}A_\mu)_{ba}\gamma_5\bar{H}_a] + \mathrm{H.c.}\right\},\end{aligned} \tag{7}$$

where

$$H_a = \frac{1+\not{v}}{2}[P^{*\mu}_a\gamma_\mu - P_a\gamma_5], \tag{8}$$

$$S_a = \frac{1+\not{v}}{2}[P'^{\mu}_{1a}\gamma_\mu\gamma_5 - P^*_{0a}], \tag{9}$$

$$\begin{aligned}T^{\mu}_a = & \frac{1+\not{v}}{2}\left\{P^{*\mu\nu}_{2a}\gamma_\nu - \sqrt{\frac{3}{2}}P^{\nu}_{1a}\gamma_5 \right. \\ & \left. \times\left[g^{\mu}_{\nu} - \frac{1}{3}\gamma_\nu(\gamma^\mu - v^\mu)\right]\right\}\end{aligned} \tag{10}$$

and the axial-vector field A^{μ}_{ab} is defined as

$$A^{\mu}_{ab} = \frac{1}{2}(\xi^{\dagger}\partial^{\mu}\xi - \xi\partial^{\mu}\xi^{\dagger})_{ab} = \frac{i}{f_\pi}\partial^{\mu}\mathcal{M}_{ab} + \cdots$$

with $\xi = \exp(i\mathcal{M}/f_\pi)$, $f_\pi = 132$ MeV and

$$\mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{11}$$

After expanding Eq. (7) to the leading order of the pion field, we further obtain

$$\begin{aligned}\mathcal{L}_{D^{*+}D^{*+}\pi^0} = & g_{D^{*+}D^{*+}\pi^0}\epsilon^{\alpha\beta\mu\nu}D^{*+}_{\alpha}(\partial_\mu\pi^0)(\partial_\nu D^{*-}_{\beta}) \\ & + \mathrm{H.c.},\end{aligned} \tag{12}$$

$$\mathcal{L}_{D'^0_1 D'^0_1\pi^0} = g_{D'^0_1 D'^0_1\pi^0}\epsilon^{\alpha\beta\mu\nu}D'^0_{1\alpha}(\partial_\mu\pi^0)(\partial_\nu\bar{D}'^0_{1\beta}) + \mathrm{H.c.}, \tag{13}$$

$$\mathcal{L}_{D^0_1 D^0_1\pi^0} = g_{D^0_1 D^0_1\pi^0}\epsilon^{\alpha\beta\mu\nu}(D^0_{1\alpha})(\partial_\mu\pi^0)(\partial_\nu\bar{D}^0_{1\beta}) + \mathrm{H.c.}, \tag{14}$$

$$\begin{aligned}\mathcal{L}_{D^* D'_1\pi^0} = & g_{D^*D'_1\pi^0}[-(\partial^\alpha D^*_\beta)(\partial^\beta\pi^0)D'_{1\alpha} + D^*_\beta(\partial^\alpha\pi^0) \\ & \times(\partial^\beta D'_{1\alpha}) + (\partial^\mu D^*_\alpha)(\partial_\mu\pi^0)D'^{\alpha}_1] + \mathrm{H.c.}\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{L}_{D^* D_1\pi^0} = & g_{D^*D_1\pi^0}\Big[D^*_\beta D_{1\nu}g^{\lambda\nu}(\partial^\beta\partial_\lambda\pi^0) \\ & - D^*_\beta D_{1\nu}g^{\beta\nu}(\partial^\lambda\partial_\lambda\pi^0) + 2D^*_\beta D_{1\nu}g^{\beta\lambda}(\partial^\nu\partial_\lambda\pi^0) \\ & + \frac{1}{m_{D^*}m_{D_1}}(\partial^\lambda D^{*\nu})(\partial_\alpha\partial_\lambda\pi^0)(\partial^\alpha D_{1\nu})\Big],\end{aligned} \tag{16}$$

where D'_1 denotes the P-wave axial-vector state in the $(0^+, 1^+)$ doublet while D_1 is the 1^+ state in the $(1^+, 2^+)$ doublet. The coupling constants $g_{D^{*+}D^{*+}\pi^0}$, $g_{D'^0_1 D'^0_1\pi^0}$, $g_{D^0_1 D^0_1\pi^0}$ and $g_{D^* D^{(\prime)}_1\pi^0}$ are

$$\begin{aligned} g_{D^{*+}D^{*+}\pi^0} &= -\frac{\sqrt{2}g}{f_\pi}, \\ g_{D'^0_1 D'^0_1\pi^0} &= \frac{\sqrt{2}g'}{f_\pi}, \\ g_{D^0_1 D^0_1\pi^0} &= -\frac{5g''}{3\sqrt{2}f_\pi}, \\ g_{D^{*+}D'^+_1\pi^0} = g_{D^{*0}D'^0_1\pi^0} &= -\frac{i\sqrt{2}h}{f_\pi}, \\ g_{D^{*0}D^0_1\pi^0} = -g_{D^{*+}D^+_1\pi^0} &= -\frac{\sqrt{m_{D^*}m_{D_1}}}{\sqrt{3}f_\pi\Lambda_\chi}(h_1 + h_2). \end{aligned}$$

The coupling constant g was studied in many theoretical approaches such as QCD sum rules [15–18] and quark model [13]. In this work, we use the value $g = 0.59 \pm 0.07 \pm 0.01$ extracted by fitting the experimental width of D^* [19]. Falk and Luke obtained an approximate relation $|g'| = |g|/3$ and $|g''| = |g|$ in quark model [13]. However the phase between g' and g'' is not fixed. With the available experimental information, Casalbuoni and collaborators extracted $h = -0.56 \pm 0.28$ and $h' = (h_1 + h_2)/\Lambda_\chi = 0.55\ \mathrm{GeV}^{-1}$ [14]. If we replace the meson field in the heavy quark limit in the above equations by the fields in full QCD and scale the coupling constants by a factor $\sqrt{m_{D^*}}$ etc., we get the effective Lagrangian in full QCD, which is used below in the derivation of the potential.

IV. DERIVATION OF THE ONE-PION-EXCHANGE POTENTIAL

Study of the possible molecular states, especially the system of a pair of heavy mesons, started more than three decades ago. The presence of the heavy quarks lowers the kinetic energy while the interaction between two light quarks could still provide strong attraction. Okun and Voloshin proposed possibilities of the molecular states involving charmed mesons [20]. Rujula, Georgi and Glashow once suggested $\psi(4040)$ as a $D^*\bar{D}^*$ molecular state [21]. Törnqvist studied possible deuteronlike two-meson bound states such as $D\bar{D}^*$ and $D^*\bar{D}^*$ using a quark-pion interaction model [22]. Dubynskiy and Voloshin proposed that there exists a possible new resonance at the $D^*\bar{D}^*$ threshold [23,24].

Several groups suggested $X(3872)$ could be a good molecular candidate [25–29]. However, Suzuki argued that $X(3872)$ is not a molecule state of $D^0\bar{D}^{*0} + \bar{D}^0 D^{*0}$ [30]. Instead, $X(3872)$ may have a dominant $c\bar{c}$ component with some admixture of $D^0\bar{D}^{*0} + \bar{D}^0 D^{*0}$ [30–32].

In this work we will explore whether $Z^+(4430)$ could be a S-wave molecular state of D^* and $\bar{D}_1'$ (or $\bar{D}_1$), which is loosely bound by the long-range pion exchange potential. We first derive the scattering matrix elements between the pair of D mesons as shown in Fig. 2. With the Breit approximation, we can get the one-pion-exchange potentials. Since the flavor wave function of Z^+ contains two components, we have to consider both the direct scattering diagram Fig. 2(a) and the crossed diagram Fig. 2(b). Note only the crossed diagram contributes to OPEP in the case of $X(3872)$.

Recall the flavor wave function of $Z^+(4430)$ reads

$$|Z^+\rangle = \frac{1}{\sqrt{2}}[|A'\rangle + |B'\rangle] \tag{17}$$

with $|A'\rangle = |\bar{D}_1'^0 D^{*+}\rangle$ and $|B'\rangle = |D_1'^+ \bar{D}^{*0}\rangle$ or

$$|Z^+\rangle = \frac{1}{\sqrt{2}}[|A\rangle + |B\rangle] \tag{18}$$

with $|A\rangle = |\bar{D}_1^0 D^{*+}\rangle$ and $|B\rangle = |D_1^+ \bar{D}^{*0}\rangle$. The mass of $Z^+(4430)$ is expressed as

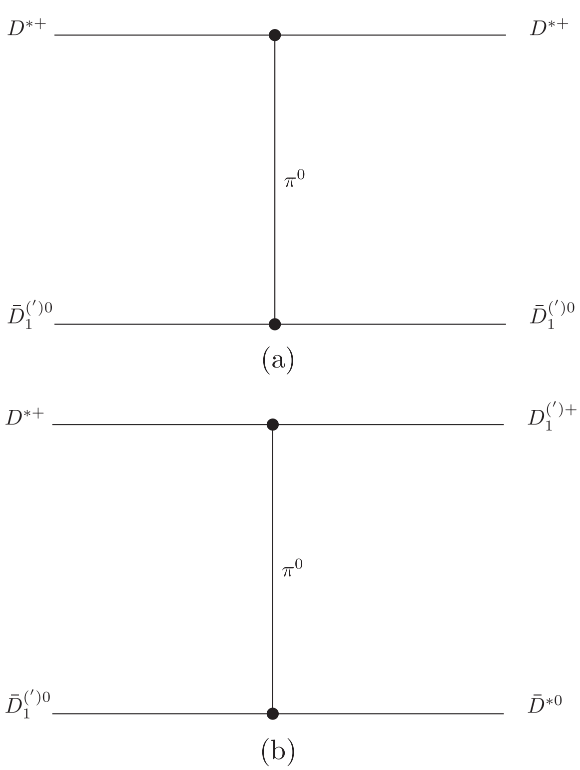

FIG. 2. (a) The single pion exchange in the direct scattering process $D^{*+}\bar{D}_1^{(\prime)0} \to D^{*+}\bar{D}_1^{(\prime)0}$. (b) The crossed process $D^{*+}\bar{D}_1^{(\prime)0} \to D_1^{(\prime)+}\bar{D}^{*0}$.

$$M_{Z^+} = m_{D^*} + m_{D_1^{(\prime)}} + T + E + \delta, \tag{19}$$

where T is the kinetic energy in the center of mass frame, $E = \langle D^{*+}\bar{D}_1^{(\prime)0}|\mathcal{H}_1|D^{*+}\bar{D}_1^{(\prime)0}\rangle$ and $\delta = \langle D^{*+}\bar{D}_1^{(\prime)0}|\mathcal{H}_2|\times D_1'^+\bar{D}^{*0}\rangle$. $\mathcal{H}_1$ and $\mathcal{H}_2$ correspond to the interaction in Fig. 2(a) and 2(b)respectively. For the possible $\tilde{Z}^+$ state with negative G-parity, we can get its mass through the replacement $+\delta \to -\delta$ in Eq. (19).

A. Scattering amplitudes

We collect the scattering amplitudes in the different channels below. For the process $D^{*+}(p_1, \epsilon_1)\bar{D}_1'^0(p_2, \epsilon_2) \to D^{*+}(p_3, \epsilon_3)\bar{D}_1'^0(p_4, \epsilon_4)$, the amplitude is

$$\begin{aligned} i\mathcal{M}[D^{*+}(p_1, \epsilon_1)\bar{D}_1'^0(p_2, \epsilon_2) &\to D^{*+}(p_3, \epsilon_3)\bar{D}_1'^0(p_4, \epsilon_4)] \\ &= \frac{2igg'}{f_\pi^2}\frac{1}{q^2 - m_\pi^2}\varepsilon^{\alpha\beta\mu\nu}\varepsilon^{\alpha'\beta'\mu'\nu'} \\ &\quad \times q_\mu q_{\mu'} p_{1\nu} p_{1\nu'}(\epsilon_{1\alpha}^{\lambda_1}\epsilon_{2\alpha'}^{\lambda_2})(\epsilon_{3\beta}^{\lambda_1'}\epsilon_{4\beta'}^{\lambda_2'}). \end{aligned} \tag{20}$$

For $D^{*+}(p_1, \epsilon_1)\bar{D}_1^0(p_2, \epsilon_2) \to D^{*+}(p_3, \epsilon_3)\bar{D}_1^0(p_4, \epsilon_4)$, one gets

$$\begin{aligned} i\mathcal{M}[D^{*+}(p_1, \epsilon_1)\bar{D}_1^0(p_2, \epsilon_2) &\to D^{*+}(p_3, \epsilon_3)\bar{D}_1^0(p_4, \epsilon_4)] \\ &= -\frac{5igg''}{3f_\pi^2}\frac{1}{q^2 - m_\pi^2}\varepsilon^{\alpha\beta\mu\nu}\varepsilon^{\alpha'\beta'\mu'\nu'} \\ &\quad \times q_\mu q_{\mu'} p_{1\nu} p_{1\nu'}(\epsilon_{1\alpha}^{\lambda_1}\epsilon_{2\alpha'}^{\lambda_2})(\epsilon_{3\beta}^{\lambda_1'}\epsilon_{4\beta'}^{\lambda_2'}). \end{aligned} \tag{21}$$

For $D^{*+}(p_1, \epsilon_1)\bar{D}_1'^0(p_2, \epsilon_2) \to D_1'^+(p_3, \epsilon_3)\bar{D}^{*0}(p_4, \epsilon_4)$, the amplitude is

$$\begin{aligned} i\mathcal{M}[D^{*+}(p_1, \epsilon_1)\bar{D}_1'^0(p_2, \epsilon_2) &\to D_1'^+(p_3, \epsilon_3)\bar{D}^{*0}(p_4, \epsilon_4)] \\ &= \frac{2ih^2}{f_\pi^2}\frac{1}{q^2 - m_\pi^2}[-p_1^\alpha q^\beta - q^\alpha p_3^\beta + (p_1 \cdot q)g^{\alpha\beta}] \\ &\quad \times [-p_4^{\alpha'} q^{\beta'} - q^{\alpha'} p_2^{\beta'} + (p_4 \cdot q)g^{\alpha'\beta'}] \\ &\quad \times (\epsilon_{1\beta}^{\lambda_1}\epsilon_{2\alpha'}^{\lambda_2})(\epsilon_{3\alpha}^{\lambda_1'}\epsilon_{4\beta'}^{\lambda_2'}). \end{aligned} \tag{22}$$

The amplitude of the process $D^{*+}(p_1, \epsilon_1)\bar{D}_1^0(p_2, \epsilon_2) \to D_1^+(p_3, \epsilon_3)\bar{D}^{*0}(p_4, \epsilon_4)$ is

$$\begin{aligned} i\mathcal{M}[D^{*+}(p_1, \epsilon_1)\bar{D}_1^0(p_2, \epsilon_2) &\to D_1^+(p_3, \epsilon_3)\bar{D}^{*0}(p_4, \epsilon_4)] \\ &= \frac{-im_{D^*}m_{D_1}(h_1 + h_2)^2}{3f_\pi^2\Lambda_\chi^2}\frac{1}{q^2 - m_\pi^2}\bigg[3q^\nu q^\beta - g^{\beta\nu}q^2 \\ &\quad + \frac{g^{\beta\nu}}{m_{D^*}m_{D_1}}(p_1 \cdot q)(q \cdot p_3)\bigg]\bigg[3q^{\nu'} q^{\beta'} - g^{\beta'\nu'}q^2 \\ &\quad + \frac{g^{\beta'\nu'}}{m_{D^*}m_{D_1}}(p_2 \cdot q)(q \cdot p_4)\bigg](\epsilon_{1\beta}^{\lambda_1}\epsilon_{2\nu'}^{\lambda_2})(\epsilon_{3\nu}^{\lambda_1'}\epsilon_{4\beta'}^{\lambda_2'}). \end{aligned} \tag{23}$$

Here the polarization vector is defined as $\epsilon^{\pm 1} = \frac{1}{\sqrt{2}} \times (0, \pm 1, i, 0)$ and $\epsilon^0 = (0, 0, 0, -1)$.

B. The one-pion-exchange potential

We impose the constraint on the scattering amplitudes that initial states and final states should have the same angular momentum. The molecular state $|J, J_z\rangle$ composed of the 1^- and 1^+ charm meson pair can be constructed as

$$|J, J_z\rangle = \sum_{\lambda_1,\lambda_2} \langle 1, \lambda_1; 1, \lambda_2 | J, J_z\rangle | p_1, \epsilon_1; p_2, \epsilon_2\rangle \tag{24}$$

where $\langle 1, \lambda_1; 1, \lambda_2 | J, J_z\rangle$ is the Clebsch-Gordan coefficient. Combining the equation with the scattering amplitudes, one gets the matrix element $i\mathcal{M}(J, Jz)$.

With the Breit approximation, the interaction potential in the momentum space is related to $i\mathcal{M}(J, Jz)$

$$V(q) = -\frac{1}{\sqrt{\prod_i 2m_i \prod_f 2m_f}} \mathcal{M}(J, Jz) \tag{25}$$

where m_i and m_f denote the masses of the initial and final states, respectively. We collect the expressions of the potential in Tables I and II. We have explicitly shown the resulting potentials are the same for the different J_z component.

1. OPEP from the direct scattering diagram

From Tables I and II, it is very interesting to note that the OPEP from the direct scattering diagram in the $J^P = 0^-$ and 1^- channel always has the same sign while it is opposite to that in the $J^P = 2^-$ channel. In other words, there must be attraction in one of three channels no matter what sign gg' takes.

After making the Fourier transformation, we get the one-pion-exchange potential in the configuration space. The final potentials are obtained with the following replacements: $x^2 \to r^2/3$, $x^4 \to r^4/5$, $x^2y^2 \to r^4/15$ etc. since we consider only the S-wave system. Alternatively, one may average the potential in the momentum space first.

In the derivation of the OPEP, we let $q^0 \approx 0$ as usually done for the direct scattering diagram in Fig. 2(a). The resulting potential from the direct scattering diagram is the familiar Yukawa potential plus a δ function, similar to OPEP in the nucleon-nucleon potential.

TABLE I. The one-pion-exchange potential between D^* ($\bar{D}^*$) and $\bar{D}_1'$ (D_1'). Here the expressions are for the $J_z = 0$ components. $A' = \bar{D}_1'^0 D^{*+}$ and $B' = D_1'^+ \bar{D}^{*0}$.

State	$A'(B') \to A'(B')$	$A'(B') \to B'(A')$
0^-	$\frac{gg'}{3f_\pi^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2}$	$-\frac{h^2}{2f_\pi^2}\frac{(q^0)^2}{(q^0)^2-\mathbf{q}^2-m_\pi^2}$
1^-	$\frac{gg'}{2f_\pi^2}\frac{q_z^2}{\mathbf{q}^2+m_\pi^2}$	$-\frac{h^2}{2f_\pi^2}\frac{(q^0)^2}{(q^0)^2-\mathbf{q}^2-m_\pi^2}$
2^-	$-\frac{gg'}{6f_\pi^2}\frac{2\mathbf{q}^2-3q_z^2}{\mathbf{q}^2+m_\pi^2}$	$-\frac{h^2}{2f_\pi^2}\frac{(q^0)^2}{(q^0)^2-\mathbf{q}^2-m_\pi^2}$

TABLE II. The one-pion-exchange potential between D^* ($\bar{D}^*$) and $\bar{D}_1$ (D_1). In this table, $A = \bar{D}_1^0 D^{*+}$ and $B = D_1^+ \bar{D}^{*0}$.

State	$A(B) \to A(B)$	$A(B) \to B(A)$
0^-	$-\frac{5gg''}{18f_\pi^2}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2}$	$\frac{h'^2}{6f_\pi^2}\frac{(\mathbf{q}^2)^2}{(q^0)^2-\mathbf{q}^2-m_\pi^2}$
1^-	$-\frac{5gg''}{12f_\pi^2}\frac{q_z^2}{\mathbf{q}^2+m_\pi^2}$	$-\frac{h'^2}{12f_\pi^2}\frac{2(\mathbf{q}^2)^2-3q_z^2\mathbf{q}^2}{(q^0)^2-\mathbf{q}^2-m_\pi^2}$
2^-	$\frac{5gg''}{36f_\pi^2}\frac{2\mathbf{q}^2-3q_z^2}{\mathbf{q}^2+m_\pi^2}$	$\frac{h'^2}{8f_\pi^2}\frac{(\mathbf{q}^2)^2-8q_z^2\mathbf{q}^2+9q_z^4}{(q^0)^2-\mathbf{q}^2-m_\pi^2}$

2. OPEP from the crossed diagram

First we note from Table II that the OPEP from the crossed diagram contains terms such as $\mathbf{q}^4$ in the numerator, which reflects the fact that D_1 decays into $D^*\pi$ via D-wave. But the approximation $q^0 \approx 0$ is not reasonable for the crossed diagram in Fig. 2(b) because $q^0 \approx M_{D_1} - M_{D^*} \approx 410$ MeV $\approx 3m_\pi$. In the following, we take $J^P = 0^-$ as an example to illustrate how to deal with the crossed diagram. Since the D_1' meson decays into $D^*\pi$, the crossed diagram contains a small imaginary part, which is roughly of the order $\Gamma_{D_1'}$. However only the real part of the scattering amplitude contributes to the potential. Hence the principal integration is always assumed throughout the Fourier transformation for the crossed diagram.

$$\begin{aligned} V(\mathbf{r}) &= \int \hat{\mathcal{P}}\Big[-\frac{h^2}{2f_\pi^2}\frac{(q^0)^2}{(q^0)^2 - \mathbf{q}^2 - m_\pi^2 + i\epsilon} e^{i\mathbf{q}\cdot\mathbf{r}}\Big]\frac{d\mathbf{q}}{(2\pi)^3} \\ &= -\frac{h^2}{2f_\pi^2}\int \hat{\mathcal{P}}\Big[\frac{(q^0)^2}{\mu^2 - \mathbf{q}^2 + i\epsilon} e^{i\mathbf{q}\cdot\mathbf{r}}\Big]\frac{d\mathbf{q}}{(2\pi)^3} \\ &= \frac{h^2(q^0)^2}{8\pi f_\pi^2}\frac{\cos(\mu r)}{r} \end{aligned}$$

where $\mu^2 = (q^0)^2 - m_\pi^2$ and $q^0 \approx M_{D_1} - M_{D^*}$. The resulting potential is of very long range and oscillating. We list the expressions of the potential in the coordinate space in Tables III and IV.

For the $\bar{D}^* D_1'$ system, it is clear from Table III that the OPEP from the crossed diagram has the same overall sign in all three channels. Moreover, the sign is positive. Because of the factor $q_0^2 \sim 9m_\pi^2$, the OPEP from the crossed diagram is numerically much larger than that from the direct scattering diagram. For the $\bar{D}^* D_1$ case,

TABLE III. The one-pion-exchange potential in the coordinate space with $A' = \bar{D}_1'^0 D^{*+}$ and $B' = D_1'^+ \bar{D}^{*0}$.

State	$A'(B') \to A'(B')$	$A'(B') \to B'(A')$
0^-	$\frac{gg'}{3f_\pi^2}[\delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r}e^{-m_\pi r}]$	$\frac{h^2(q^0)^2}{8\pi f_\pi^2}\frac{\cos(\mu r)}{r}$
1^-	$\frac{gg'}{6f_\pi^2}[\delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r}e^{-m_\pi r}]$	$\frac{h^2(q^0)^2}{8\pi f_\pi^2}\frac{\cos(\mu r)}{r}$
2^-	$-\frac{gg'}{6f_\pi^2}[\delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r}e^{-m_\pi r}]$	$\frac{h^2(q^0)^2}{8\pi f_\pi^2}\frac{\cos(\mu r)}{r}$

TABLE IV. The one-pion-exchange potential in the coordinate space with $A = \bar{D}_1^0 D^{*+}$ and $B = D_1^+ \bar{D}^{*0}$.

State	$A(B) \to A(B)$	$A(B) \to B(A)$
0^-	$-\frac{5gg''}{18f_\pi^2}[\delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r} e^{-m_\pi r}]$	$\frac{h'^2}{6f_\pi^2}[\nabla^2\delta(\mathbf{r}) - \mu^2\delta(\mathbf{r}) - \frac{\mu^4}{4\pi}\frac{\cos\mu r}{r}]$
1^-	$-\frac{5gg''}{36f_\pi^2}[\delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r} e^{-m_\pi r}]$	$-\frac{h'^2}{12f_\pi^2}[\nabla^2\delta(\mathbf{r}) - \mu^2\delta(\mathbf{r}) - \frac{\mu^4}{4\pi}\frac{\cos\mu r}{r}]$
2^-	$\frac{5gg''}{36f_\pi^2}[\delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r} e^{-m_\pi r}]$	$\frac{h'^2}{60f_\pi^2}[\nabla^2\delta(\mathbf{r}) - \mu^2\delta(\mathbf{r}) - \frac{\mu^4}{4\pi}\frac{\cos\mu r}{r}]$

we note from Table IV that the overall sign in the 1^- channel is different from that in both 0^- and 2^- channels.

With the coupling constants $g = 0.59$, $g' = g/3$ and $g'' = g$, the variation of the potential listed in Tables III and IV with r (in unit of GeV^{-1}) is shown in Figs. 3 and 4.

We also note that the OPEP does not depend on the mass of the charmed mesons. In other words, the same potential may be used in the discussion of the hidden bottom molecular states $(b\bar{q}) - (\bar{b}q)$.

V. RESULTS AND DISCUSSIONS

Besides those coupling constants in Sec. III, we also need the following parameters in our numerical analysis: $m_{D^*} = 2007$ MeV, $m_{D_1'} = 2430$ MeV, $m_{D_1} = 2420$ MeV, $m_{B^*} = 5325$ MeV, $m_{B_1'} = 5732$ MeV, $f_\pi = 132$ MeV, $m_\pi = 135$ MeV [11], $m_{B_1} = 5725$ MeV [33].

With the potentials derived above, we use the variational method to investigate whether there exists a loosely bound state. Our criteria of the formation of a possible loosely bound molecular state is (1) the radial wave function

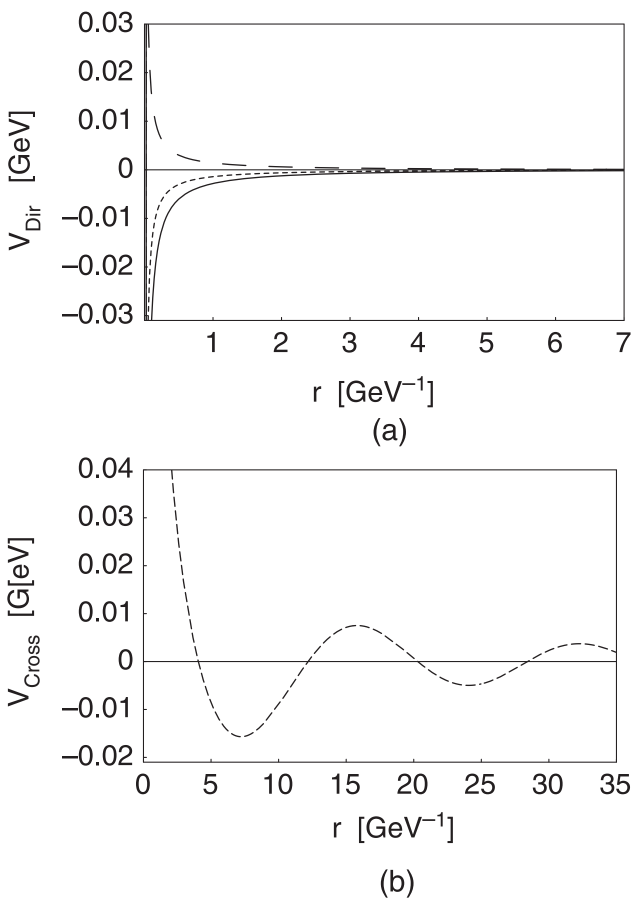

FIG. 3. (a) The solid, dotted, and dashed lines correspond to the potentials from the direct scattering diagram $A(B) \to A(B)$ in the $J^P = 0^-, 1^-, 2^-$ channels, respectively, where $g = 0.59$, $g' = g/3$, and $g'' = g$. (b) The potential from the crossed diagram $A(B) \to B(A)$.

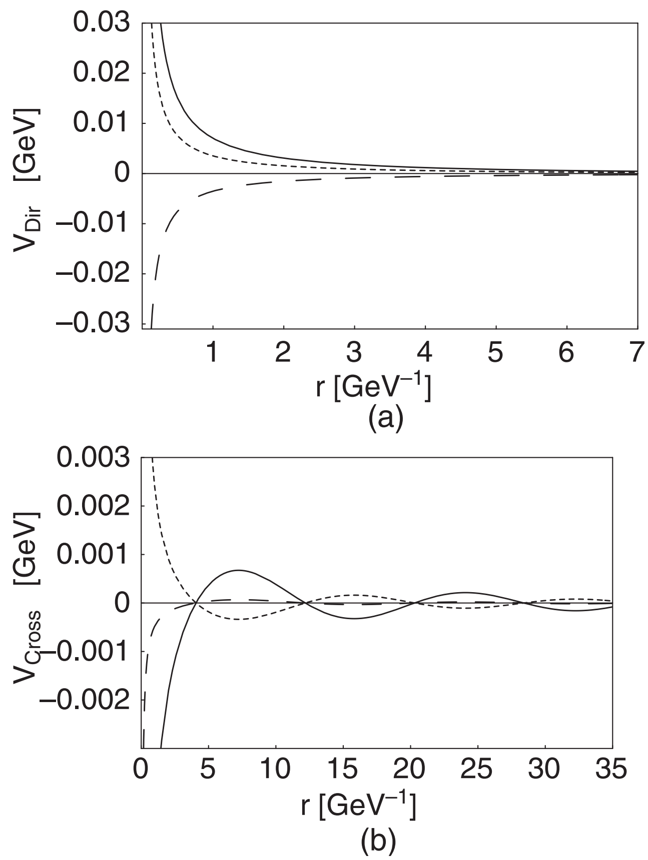

FIG. 4. (a) The solid, dotted, and dashed lines correspond to the potentials from the direct scattering diagram $A'(B') \to A(B')$ in the $J^P = 0^-, 1^-, 2^-$ channels, respectively, where $g = 0.59$, $g' = g/3$, and $g'' = g$. (b) The potential from the crossed diagram $A'(B') \to B'(A')$.

extend to 1 fm or beyond and (2) the minimum energy of the system is negative. Our trial wave functions include (a) $\psi(r) = (1 + \alpha r)e^{-\beta r}$; (b) $\psi(r) = (1 + \alpha r^2)e^{-\beta r^2}$; (c) $\psi(r) = r^2(1 + \alpha r)e^{-\beta r}$.

Unfortunately a solution to satisfy the above criteria does not exist for the system of $D_1' - D^*$ or $D_1 - D^*$ in all $J^P = 0^-, 1^-, 2^-$ channels with the realistic coupling constants $g = 0.59$, $g' = g/3$ and $g'' = g$ deduced from the width of D^*, D_1 and D_1'. Such a solution also does not exist if we switch the sign of gg' or enlarge the absolute value of gg' by a factor 3. The same conclusion holds for the system of $B' - B^*$, $B - B^*$ and $\tilde{Z}^+$ with negative G-parity.

In short summary, we have performed a dynamical study of the $Z^+(4430)$ signal to see whether it is a loosely bound molecular state of $D_1 - D^*$ or $D_1' - D^*$. We find that the interaction from the one-pion-exchange potential alone is not strong enough to bind the pair of charmed mesons with realistic coupling constants. Other dynamics is necessary if $Z^+(4430)$ is further established as a molecular state by the future experiments.

It is interesting to note that the one-pion-exchange potential alone does not bind the deuteron in nuclear physics either. In fact, the strong attractive force in the intermediate range is introduced in order to bind the deuteron, which is sometimes modeled by the sigma meson exchange. One may wonder whether the similar mechanism plays a role in the case of $Z^+(4430)$ and $X(3872)$. Further work along this direction is in progress.

If the $Z^+(4430)$ is really a $J^P = 1^-$ molecular state, the quantum number of its neutral partner state $Z^0(4430)$ is $J^{PC} = 1^{--}$. Such a state can be searched for in the e^+e^- annihilation processes. *BABAR* and Belle collaborations have observed several new charmonium (or charmonium-like) states with $J^{PC} = 1^{--}$ around this mass range including $Y(4260)$, $Y(4320)$, and $Y(4664)$ with the initial state radiation (ISR) technique, although these states do not appear as a peak in the R distribution. We strongly urge Belle and *BABAR* collaborations to search for the $Z^0(4430)$ state in the $\pi^0\psi'$ channel using the ISR technique. The absence of a signal will be an indication that the J^P of $Z^+(4430)$ is not 1^-.

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