

# Leptogenesis within a generalized quark-lepton symmetry

F. Buccella

*Dipartimento di Scienze Fisiche, Università di Napoli, Via Cintia, Napoli, Italy,  
and INFN, Sezione di Napoli, Italy*

D. Falcone

*Dipartimento di Scienze Fisiche, Università di Napoli, Via Cintia, Napoli, Italy*

L. Oliver

*Laboratoire de Physique Theorique, Université de Paris XI, Batiment 210, Orsay Cedex, France*

(Received 29 October 2007; published 14 February 2008)

We reexamine the question of baryogenesis via leptogenesis in schemes of the seesaw mechanism with quark-lepton symmetry. Within the phenomenological approach of textures, we propose to relax this strict symmetry and propose weaker conditions, namely, models of the neutrino Dirac mass matrix  $M_D$  which have the same hierarchy as the matrix elements of  $M_u$ . We call this guideline generalized *hierarchical* quark-lepton symmetry. We consider in detail particular cases in which the moduli of the matrix elements of  $M_D$  are equal to those of  $M_u$ . We try for the heavy Majorana mass matrix diagonal and off-diagonal forms. We find that an ansatz for  $M_D$  preserving the hierarchy, together with an off-diagonal model for the heavy Majorana neutrino mass, is consistent with neutrino masses, neutrino mixing, and baryogenesis via leptogenesis for an intermediate mass scale  $m_R \sim 10^{12}$  GeV. The preservation of the hierarchical structure could come from a possible symmetry scheme.

DOI: [10.1103/PhysRevD.77.033002](https://doi.org/10.1103/PhysRevD.77.033002)

PACS numbers: 14.60.Pq, 12.15.Ff, 98.80.Cq

## I. INTRODUCTION

The seesaw mechanism [1] can explain the smallness of neutrino masses and is consistent with large lepton mixing [2]. In a nutshell, it is based on the existence of very heavy right-handed Majorana neutrinos. A cosmological consequence is the generation of a baryon asymmetry in the universe by means of the out-of-equilibrium decays of the heavy right-handed neutrinos to leptons and  $SU(2)_L$  Higgs bosons, which create a lepton asymmetry, partially converted to a baryon asymmetry by electroweak sphalerons, a mechanism known as baryogenesis via leptogenesis [3].

Recently, much progress has been done in the study of leptogenesis, especially in the exploration of flavor effects [4]. In the present paper we turn again to the link between leptogenesis and fermion mass matrices. We are interested in the compatibility of quark-lepton symmetry with leptogenesis. In previous papers this compatibility has been strongly questioned [5]. In Refs. [6,7] it was achieved only in the case of some degeneracy in the right-handed neutrino masses. Often an inverse seesaw formula was used.

In order to further explore this subject, we adopt here a typical form of the quark mass matrices [8], together with minimal models for the Majorana mass matrix [9], and the direct seesaw formula.

In a second step, in the exploration of models that could give a more natural heavy Majorana neutrino mass spectrum together with a right order of magnitude for the leptogenesis, we relax the strict quark-lepton symmetry. We adopt a weaker hypothesis, namely, keeping the hierarchy of the Dirac mass matrix elements, with the moduli

of the elements of  $M_D$  being equal to the moduli of the elements of  $M_u$ . Of course, this investigation does not imply that it could not be possible that other classes of mass matrices might yield a reasonable lepton asymmetry even for strict quark-lepton symmetry.

## II. MASS MATRICES

According to the seesaw mechanism, the effective mass matrix of neutrinos is given by the formula

$$M_\nu \simeq M_D M_R^{-1} M_D, \quad (1)$$

where  $M_D$  is the Dirac mass matrix and  $M_R$  the Majorana mass matrix. For  $M_R \gg M_D$ , we have  $M_\nu \ll M_D$ .

Our starting point for fermion mass matrices is the following symmetric forms of quark mass matrices [8,10], for which we simply give an order of magnitude of the matrix elements,

$$M_u \simeq \begin{pmatrix} 0 & i\epsilon_u^3 & \epsilon_u^4 \\ i\epsilon_u^3 & \epsilon_u^2 & \epsilon_u^2 \\ \epsilon_u^4 & \epsilon_u^2 & 1 \end{pmatrix} m_t, \quad (2)$$

$$M_d \simeq \begin{pmatrix} 0 & \epsilon_d^3 & \epsilon_d^4 \\ \epsilon_d^3 & \epsilon_d^2 & \epsilon_d^2 \\ \epsilon_d^4 & \epsilon_d^2 & 1 \end{pmatrix} m_b, \quad (3)$$

with  $\epsilon_u^2 = m_c/m_t$ ,  $\epsilon_d^2 = m_s/m_b$ , i.e.  $\epsilon_u \simeq 0.05$  and  $\epsilon_d \simeq 0.15$ , which agree with the mass spectrum of the quarks and the Cabibbo-Kobayashi-Maskawa mixing matrix. Then, simple quark-lepton symmetry leads to the relations

$$M_D = M_u, \quad M_e = M_d, \quad (4)$$

for example, in SO(10) with Higgses transforming as **10** representations. Indeed, SO(10) is a favored scenario for neutrino mass and leptogenesis, since one has right-handed neutrinos with heavy Majorana masses and  $B - L$  spontaneous symmetry breaking. As it is well known, the relation  $M_e = M_d$  can be naturally modified in SO(10) with a **126** representation in order to have  $-3$  Clebsch-Gordan coefficient, coming from color, in the (2,2) entry for  $M_e$  relatively to  $M_d$ , which yields the better relation  $m_s = m_\mu/3$  at the unification scale [11].

For the right-handed neutrinos we take minimal mass matrices, namely:

(i) the diagonal,

$$M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R, \quad (5)$$

and (ii) the off-diagonal

$$M_R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & c & 0 \\ 1 & 0 & 0 \end{pmatrix} m_R. \quad (6)$$

Moreover, in the diagonal model we will choose  $b \simeq \epsilon^4$ ,  $a \simeq \epsilon^5$ , and in the off-diagonal model  $c \simeq \epsilon^2$ , as explained below. We also examine other cases of rank-3 matrices in the Appendix.

In the two cases (5) and (6), application of the seesaw formula gives the phenomenological viable form [12,13]

$$M_\nu \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} m_\nu, \quad (7)$$

where, from now on, we denote

$$\epsilon = \epsilon_u \simeq 0.05. \quad (8)$$

This neutrino mass matrix corresponds to maximal mixing and a normal hierarchy with the overall scale  $m_\nu \simeq 0.05$  eV, fixed by the atmospheric and accelerator neutrino oscillations, but with two different mass scales  $m_R$ .

The matrix (7) gives only a qualitative account for the experimental situation of neutrino masses and mixing, since it yields the square mass differences,  $m_3^2 - m_2^2 \sim (0.1 \text{ eV})^2$ ,  $m_2^2 - m_1^2 \sim 0$  and, after diagonalization of  $M_\nu$  and  $M_e$ , a large lepton mixing.

It must be emphasized that our purpose in this paper is only an order-of-magnitude analysis, namely, to examine the consistency between a neutrino spectrum and a lepton mixing matrix with approximately maximal mixing, and the amount of needed leptogenesis to explain the baryon asymmetry of the universe.

### III. LEPTOGENESIS

Since we are interested in an order-of-magnitude calculation, we consider leptogenesis formulas in the single-flavor approximation. The importance of flavor effects has been recently underlined [4].

The calculation must be done in the basis where the right-handed mass matrix is diagonal (with eigenvalues  $M_1, M_2, M_3$ ). The baryon asymmetry, baryon to entropy fraction, is given by

$$Y_B \simeq \frac{1}{2} Y_L \quad (9)$$

and the lepton asymmetry by

$$Y_L \simeq 0.3 \frac{\epsilon_1}{g_*} \left( \frac{0.55 \times 10^{-3} \text{ eV}}{\tilde{m}_1} \right)^{1.16} \quad (10)$$

in the strong washout regime, and

$$Y_L \simeq 0.3 \frac{\epsilon_1}{g_*} \left( \frac{\tilde{m}_1}{3.3 \times 10^{-3} \text{ eV}} \right) \quad (11)$$

in the opposite weak washout regime. The parameter  $g_*$  is the number of light degrees of freedom, of the order  $g_* \simeq 100$  in the standard case. Strong washout is realized for  $\tilde{m}_1 \gg 3 \times 10^{-3}$ , where  $\tilde{m}_1 = (M_D^\dagger M_D)_{11}/M_1$ .

Notice that  $Y_B$  is smaller than the baryon to photon ratio  $\eta$  by roughly a factor 7. The experimental value of the baryon asymmetry is (see [14])

$$(Y_B)_{\text{exp}} \simeq 9 \times 10^{-11}. \quad (12)$$

The  $CP$ -violating asymmetry  $\epsilon_1$ , related to the decay of the lightest right-handed neutrino, is given here below case by case. We now consider the two different textures for the right-handed neutrino mass matrices proposed above.

### IV. THE DIAGONAL MODEL

In the diagonal model (i), application of the seesaw formula gives the effective neutrino mass matrix

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/b + \epsilon^8 & i\epsilon^5/b + \epsilon^6 & i\epsilon^5/b + \epsilon^4 \\ * & -\epsilon^6/a + \epsilon^4/b + \epsilon^4 & i\epsilon^7/a + \epsilon^4/b + \epsilon^2 \\ * & * & \epsilon^8/a + \epsilon^4/b + 1 \end{pmatrix} \frac{m_1^2}{m_R}. \quad (13)$$

A structure similar to (7) is achieved for  $b \sim \epsilon^4$  and  $a \sim \epsilon^5$ .

Since we hopefully expect

$$\frac{m_l^2}{m_R} \simeq 0.05 \text{ eV} \quad (14)$$

to describe the neutrino spectrum and lepton mixing, from (7) we get

$$m_R \sim 10^{15} \text{ GeV}, \quad (15)$$

near a unification scale. In this case

$$\epsilon_1 \simeq \frac{3}{16\pi v^2} \left( \frac{\text{Im}(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_2} + \frac{\text{Im}(M_D^\dagger M_D)_{13}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_3} \right), \quad (16)$$

where  $v$  is the vacuum expectation value of the Higgs field, and we get

$$\epsilon_1 \simeq 2 \times 10^{-9}. \quad (17)$$

We have also

$$\tilde{m}_1 \simeq \epsilon m_\nu, \quad (18)$$

that lies in the weak washout regime, so that

$$Y_B \simeq 2 \times 10^{-12}. \quad (19)$$

Note that  $a \sim \epsilon^4$  is also possible. In such a case there is degeneracy in the lightest right-handed neutrinos and the lepton asymmetry is enhanced [15]. Note also that, although we do not consider flavor effects, they can in principle preserve the asymmetry related to the decay of the second neutrino [16]; here  $\epsilon_2 \sim 10^{-13}$ .

## V. THE OFF-DIAGONAL MODEL

In the off-diagonal model (ii), the seesaw formula gives

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/c & i\epsilon^7 + i\epsilon^5/c & \epsilon^8 + i\epsilon^5/c \\ * & i\epsilon^5 + \epsilon^4/c + i\epsilon^5 & \epsilon^6 + \epsilon^4/c + i\epsilon^3 \\ * & * & \epsilon^4 + \epsilon^4/c + \epsilon^4 \end{pmatrix} \times \frac{m_l^2}{m_R}. \quad (20)$$

A structure similar to (7) is now achieved for  $\epsilon^4 \leq c \leq \epsilon^2$ . For  $c \simeq \epsilon^4$  one has  $m_\nu \simeq m_l^2/m_R$ , while in the more interesting case  $c \simeq \epsilon^2$  one has

$$m_\nu \simeq \epsilon^2 \frac{m_l^2}{m_R} \simeq 0.05 \text{ eV}, \quad (21)$$

and hence, from  $\epsilon \simeq 0.05$ , the intermediate scale

$$m_R \sim 10^{12} \text{ GeV}. \quad (22)$$

In this last case

$$\epsilon_1 \simeq \frac{3}{16\pi v^2} \left( \frac{\text{Im}(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_1} + \frac{\text{Im}(M_D^\dagger M_D)_{23}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_3} \right). \quad (23)$$

After a right-handed rotation in the 1-3 sector, we get for  $c \simeq \epsilon^2$ ,

$$\epsilon_1 \simeq 5 \times 10^{-11}. \quad (24)$$

With smaller values of the parameter  $c$ , we obtain a larger scale  $m_R$  and a smaller amount of the  $CP$  asymmetry.

Notice one point. The simple ansatz (6) implies two degenerate very heavy Majorana neutrinos  $M_1 = M_3 = m_R$  and a lighter one  $M_2 \simeq \epsilon^2 m_R$ . This is a quite different situation than the one considered in [6,7], which needed a quasidegeneracy of the lightest heavy neutrinos decaying out of equilibrium. Of course, the proposal (6) can be easily modified to have three heavy Majorana neutrinos of quite different masses.

## VI. RELAXING QUARK-LEPTON SYMMETRY PRESERVING HIERARCHY OF MATRIX ELEMENTS

From the results of the preceding sections, we realize that, keeping strict quark-lepton symmetry (4) with  $M_u$  given by (2), both the diagonal and the off-diagonal models for the heavy Majorana right-handed neutrinos provide a too small baryon asymmetry.

Keeping the two models for the heavy Majorana right-handed neutrinos, we will now try to modify the quark-lepton symmetry relation (4), while preserving for the  $M_D$  matrix elements the same order of magnitude in powers of  $\epsilon$ , i.e. we relax the quark-lepton symmetry relation while keeping the same hierarchy. Instead of  $M_D = M_u$  with  $M_u$  given by (2), we propose then a Dirac neutrino mass matrix of the form

$$M_D \simeq \begin{pmatrix} 0 & O(\epsilon^3) & O(\epsilon^4) \\ O(\epsilon^3) & O(\epsilon^2) & O(\epsilon^2) \\ O(\epsilon^4) & O(\epsilon^2) & O(1) \end{pmatrix} m_l. \quad (25)$$

The interest of such an ansatz is that possible symmetries could link this matrix to  $M_u$ .

To have a guide about which a scheme of this kind may give the right baryon asymmetry, we perform the following exercise. We modify the Dirac mass matrix  $M_D$  given by  $M_D = M_u$  by putting  $i$  factors in several matrix elements. We make the trial

$$M_D \simeq \begin{pmatrix} 0 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} m_l, \quad (26)$$

i.e. we drop the  $i$  factors in (2). Of course, this matrix is real and cannot lead to a  $CP$  asymmetry. However, this form can give us a hint of the possible interesting cases. With this last form of  $M_D$  we begin by computing the *real*

quantities

$$\eta_1 \simeq \frac{3}{16\pi v^2} \left( \frac{(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_2} + \frac{(M_D^\dagger M_D)_{13}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_3} \right), \quad (27)$$

for the diagonal case (i), and

$$\eta_1 \simeq \frac{3}{16\pi v^2} \left( \frac{(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_1} + \frac{(M_D^\dagger M_D)_{23}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_3} \right) \quad (28)$$

for the off-diagonal case (ii).

For the diagonal model (i) we find  $\eta_1 \simeq 10^{-8}$ , while in the off-diagonal model (ii) this increases enormously, to  $\eta_1 \simeq 10^{-4}$ . This suggests that potentially the off-diagonal model is able to provide a sufficient amount of asymmetry, looking for appropriate complex models of  $M_D$ .

Following this guideline, we do consider a model of the Dirac mass matrix with  $i$  factors in several matrix elements. Of course, we first have to check that the effective neutrino mass matrix is in agreement with the phenomenologically successful neutrino mass matrix (7), and then calculate the baryon asymmetry. We find that adding  $i$  factors in positions 2-2, 2-3, and 3-2 this is viable. Therefore, our ansatz for the Dirac neutrino mass matrix is

$$M_D \simeq \begin{pmatrix} 0 & i\epsilon^3 & \epsilon^4 \\ i\epsilon^3 & i\epsilon^2 & i\epsilon^2 \\ \epsilon^4 & i\epsilon^2 & 1 \end{pmatrix} m_t. \quad (29)$$

We turn now to the schemes (i) and (ii) for the heavy right-handed neutrino masses. Let us consider first the diagonal model (i). We find the neutrino mass matrix

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/b + \epsilon^8 & -\epsilon^5/b + i\epsilon^6 & -\epsilon^5/b + \epsilon^4 \\ * & -\epsilon^6/a - \epsilon^4/b - \epsilon^4 & i\epsilon^7/a - \epsilon^4/b + i\epsilon^2 \\ * & * & \epsilon^8/a - \epsilon^4/b + 1 \end{pmatrix} \frac{m_t^2}{m_R}. \quad (30)$$

Notice that using the values  $b \sim \epsilon^4$  and  $a \sim \epsilon^5$  proposed in Sec. IV to get a structure similar to (7), we have lost this structure adopting now the new form for  $M_D$  (27), since the (3,3) entry in (28) is small, of the order  $\epsilon^3$ . Therefore, maximal mixing can only be achieved in this case with some fine-tuning. Moreover, one finds in this case a too small value of  $CP$  violation and baryon asymmetry, very close to (17) and (19).

Let us turn now to the more interesting off-diagonal model (ii). We find the neutrino mass matrix

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/c & i\epsilon^7 - \epsilon^5/c & \epsilon^8 - \epsilon^5/c \\ * & -\epsilon^5 - \epsilon^4/c - \epsilon^5 & i\epsilon^6 - \epsilon^4/c + i\epsilon^3 \\ * & * & \epsilon^4 - \epsilon^4/c + \epsilon^4 \end{pmatrix} \times \frac{m_t^2}{m_R}. \quad (31)$$

With  $c \sim \epsilon^2$ , as proposed in Sec. V, one gets at leading order in  $\epsilon$  exactly the desired form (7) with  $m_\nu \sim \epsilon^2(m_t^2/m_R)$ , that suggests the intermediate scale (22). Moreover, one finds a larger amount of  $CP$  violation

$$\epsilon_1 \simeq 4 \times 10^{-7}. \quad (32)$$

In this last case we have

$$\tilde{m}_2 = (M_D^\dagger M_D)_{22}/M_2 \simeq m_\nu, \quad (33)$$

a value that lies in the strong washout regime, so that

$$Y_B \sim 5 \times 10^{-12} \quad (34)$$

that is somewhat short of the order of magnitude (12). Remember that in the off-diagonal case the lightest right-handed neutrino corresponds to the second one in flavor.

However, we stress here that for

$$c \sim \epsilon^{3/2} \quad (35)$$

which is admissible, agreement with leptogenesis is improved, since now we have  $m_\nu \sim \epsilon^{5/2}(m_t^2/m_R) \sim 0.05$  and we get the scale

$$m_R \sim 3 \times 10^{11} \text{ GeV}, \quad (36)$$

the  $CP$  asymmetry

$$\epsilon_1 \simeq 2 \times 10^{-6}, \quad (37)$$

and therefore

$$Y_B \sim 1 \times 10^{-10} \quad (38)$$

that is of the right order of magnitude (12).

We have diagonalized the neutrino mass matrix in this latter case (31), and have checked that, to a good approximation, it gives the spectrum and mixing that follow from the simple ansatz (7).

To summarize, in the present context the off-diagonal model at the intermediate scale is preferred by leptogenesis with respect to the diagonal model at the unification scale.

## VII. CONCLUSION

The present work relies on the phenomenological approach of textures. It is relevant to recall here that, for example, within  $SO(10)$ , it is possible to obtain any relation between quark and lepton mass matrices by using general Higgs representations, i.e. several **10s** and **126s**. However, we stress that the preservation of the hierarchical structure of mass matrices seems to point towards horizontal symmetries. Then it is also possible to generate relations between mass matrices [17].

In conclusion, we propose to relax the simple quark-lepton symmetry for the Dirac neutrino mass matrix  $M_D = M_u$  and consider a kind of generalized quark-lepton symmetry that keeps the hierarchy of the matrix elements in terms of powers of  $\epsilon$ . We could call this phenomenological approach hierarchical quark-lepton symmetry. In particular, we have considered in detail a case in which the moduli of the matrix elements of  $M_D$  are equal to those of  $M_u$ .

Out of the several models, we have found one scheme consistent with the neutrino mass spectrum, lepton mixing, and leptogenesis: the off-diagonal matrix (6) with one lightest right-handed neutrino corresponding to the second flavor, together with a Dirac mass matrix (29) with imaginary second row and column. The diagonal form is known to be viable only for degenerate lightest masses of the Majorana heavy neutrinos.

### ACKNOWLEDGMENTS

One of us (L. O.) acknowledges partial support by the EU Contract No. MRTN-CT-2006-035482 (FLAVIANET) and advice from Jean-Claude Raynal.

### APPENDIX

We consider here two other interesting possibilities of rank-3 matrices for the heavy Majorana neutrinos :

$$M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_R, \quad (\text{A1})$$

$$M_R = \begin{pmatrix} 0 & e & 0 \\ e & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R. \quad (\text{A2})$$

For these two models, the effective neutrino mass matrix is not in agreement with the structure (7). However, considering the real case, since the structure in sector 2-3 of the effective matrix is achieved and the smallness of the first row and column could be due to running effects [18], we perform again the calculation. In model (A1) we get  $d \sim \epsilon^4$  and  $m_R \sim 10^{13}$  GeV and, moreover,  $Y_L \sim 10^{-13}$ . In model (A2) we obtain  $e \sim \epsilon^5$  and  $m_R \sim 10^{16}$  GeV and the lepton asymmetry is enhanced due to degeneracy in the two lightest right-handed neutrinos [15].

- 
- [1] P. Minkowski, Phys. Lett. **67B**, 421 (1977); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
  - [2] A. Yu. Smirnov, Phys. Rev. D **48**, 3264 (1993); D. Falcone, Int. J. Mod. Phys. A **21**, 3015 (2006).
  - [3] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986); M. Luty, Phys. Rev. D **45**, 455 (1992); L. Covi, E. Roulet, and F. Vissani, Phys. Lett. B **384**, 169 (1996); R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradis, Nucl. Phys. **B575**, 61 (2000).
  - [4] A. Abada, S. Davidson, A. Ibarra, F.-X. Josse-Michaux, M. Losada, and A. Riotto, J. High Energy Phys. 09 (2006) 010.
  - [5] D. Falcone and F. Tramontano, Phys. Rev. D **63**, 073007 (2001); E. Nezri and J. Orloff, J. High Energy Phys. 04 (2003) 020; D. Falcone, Phys. Rev. D **68**, 033002 (2003).
  - [6] F. Buccella, D. Falcone, and F. Tramontano, Phys. Lett. B **524**, 241 (2002).
  - [7] E. K. Akhmedov, M. Frigerio, and A. Yu. Smirnov, J. High Energy Phys. 09 (2003) 021.
  - [8] S. Antusch, S. F. King, and R. N. Mohapatra, Phys. Lett. B **618**, 150 (2005).
  - [9] D. Falcone, Phys. Lett. B **572**, 50 (2003).
  - [10] G.C. Branco, D. Emmanuel-Costa, and R. Gonzalez Felipe, Phys. Lett. B **483**, 87 (2000); R.G. Roberts, A. Romanino, G.G. Ross, and L. Velasco-Sevilla, Nucl. Phys. **B615**, 358 (2001).
  - [11] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979); H. Georgi and D.V. Nanopoulos, Nucl. Phys. **B159**, 16 (1979).
  - [12] F. Vissani, J. High Energy Phys. 11 (1998) 025.
  - [13] R.N. Mohapatra and W. Rodejohann, Phys. Rev. D **72**, 053001 (2005).
  - [14] D.N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **170**, 377 (2007).
  - [15] C.H. Albright and S.M. Barr, Phys. Rev. D **70**, 033013 (2004).
  - [16] O. Vives, Phys. Rev. D **73**, 073006 (2006); P. Di Bari, Nucl. Phys. **B727**, 318 (2005).
  - [17] D. Falcone, Phys. Rev. D **64**, 117302 (2001).
  - [18] M. Frigerio and A. Yu. Smirnov, J. High Energy Phys. 02 (2003) 004.