

Coupling constant constraints in a nonminimally coupled phantom cosmology

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In the paper we investigate observational constraints on coupling to gravity constant parameter ξ using distant supernovae SNIa data, baryon oscillation peak (BOP), the cosmic microwave background radiation (CMBR) shift parameter, and the $H(z)$ data set. We estimate the value of this parameter to constrain the extended quintessence models with nonminimally coupled to gravity phantom scalar field. The combined analysis of observational data favors a value of ξ which lies in close neighborhood of the conformal coupling. While our estimations are model dependent they give rise to an indirect bound on the equivalence principle.

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At the present, scalar fields play the crucial role in modern cosmology. In the inflationary scenario they generate an exponential rate of evolution of the Universe as well as density fluctuations due to vacuum energy.

Observations of distant supernovae support the cosmological constant term. But two problems emerge in this context. Namely, the fine-tuning and cosmic coincidence problems. The lack of some fundamental mechanism which sets the cosmological constant almost zero is called the cosmological constant problem. The second problem called “cosmic conundrum” is the question of why the energy densities of both dark energy and dark matter are nearly equal at the present epoch. One of the solutions to this problem offers the idea of quintessence [1,2] which is a version of the time varying cosmological constant conception.

All these models base on the assumption that there is a minimal coupling of scalar field to gravity ($\xi = 0$). This *a priori* assumption requires some justification. There are many theoretical arguments suggesting that the nonminimal coupling (NC for short) should be considered. The nonzero ξ comes from quantum corrections [3], renormalization of classical theory that shifts it to one with nonzero ξ [4], in relativity the value of $\xi = 1/6$ (conformal coupling) is distinguished. Only in this case the Einstein equivalence principle is not violated (for details see [5]).

It is expected also that the value of coupling constant ξ should be fixed by the physics of the problem and there are no free parameters, and then we obtain the answer to the

question “what is the value of ξ ?” [6]. However, the answer to this question differs according to the theory of the scalar field employed. For example, in Einstein gravity with the polynomial quartic potential function and back-reaction, the value of NC was found [7] or a Higgs field in the standard model gives the value of NC nonpositive or larger (or equal) than $1/6$ [8].

The main goal of this paper is a statistical estimation of the coupling parameter from the astronomical observations. For this aim we consider the spatially flat Friedmann-Robertson-Walker (FRW) model where the source of gravity is a noninteracting mixture of dust matter and a nonminimally coupled phantom scalar field.

We treat ξ as a free parameter in the model and we are looking for constraints on its value from observational cosmology. For simplicity a simple and natural form of the potential function of the scalar field is assumed. For the vanishing coupling constant (minimal coupling) this potential corresponds to chaotic inflation [9]. This paradigm of inflation can be extended by including NC [6]. Also the scalar fields nonminimally coupled to gravity are simple, nonexotic models of phantoms and give rise to superacceleration [10].

Tsujikawa and Gumjudpai [11] also investigated constraints on the NC parameter from the CMB observations. Recently Jankiewicz and Kephart [12] applied the method of WKB approximation to study the NC in the FRW universe with the scalar field and found the best-fit value of the NC parameter very close to $1/6$. Futamase and Maeda [13], in the case of the quadratic potential of the inflaton field, found that modulus of NC less than 10^{-3} leads to sufficient inflation (see also [14] for other constraints).

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In order to determine the NC parameter we use a recent expansion history of the Universe rather than early stages of its evolution (inflation). In estimation of the parameter ξ we do not assume any arbitrary parametrization for the equation of state parameter $w(z)$, but we derive its exact form directly from the model (dynamics). While this equation has in general a very complex form, in practice its simple approximations are used. If we expand the $w(z)$ relation in the Taylor series with the respect to the redshift z then we recover the well-known parametrization in the linear form of the redshift or the scale factor. All the coefficients in the series depend on the parameter ξ and initial conditions. Then after constraining initial conditions from the observational data we can obtain corresponding limits on ξ .

We assume the flat model with the FRW geometry with the signature $(-, +, +, +)$ and a source of gravity is the phantom scalar field ψ with an arbitrary coupling constant ξ . The dynamics is governed by the action

$$S = \frac{1}{2} \int d^4x \sqrt{-g} (m_p^2 R + (g^{\mu\nu} \psi_{,\mu} \psi_{,\nu} + \xi R \psi^2 - m^2 \psi^2)), \quad (1)$$

where $m_p^2 = (8\pi G)^{-1}$; for simplicity and without loss of generality we assume $4\pi G/3 = 1$ and $m^2 = 1$.

In our paper [15] we studied generic features of the evolutionary paths of the flat FRW model with phantom scalar field nonminimally coupled to gravity. We reduced dynamics of the model to the simple case of autonomous dynamical system on the invariant submanifold (ψ, ψ') (a prime denotes differentiation with respect to the natural logarithm of the scale factor) and we founded that principally there is one asymptotic state, which corresponds to the critical point in the phase space $\psi_0 = \pm 1/\sqrt{6\xi}$ and $\psi'_0 = 0$. This critical point is also the de Sitter state ($w = -1$). There are two types of scenarios leading to this Lambda state (depending on the value of ξ), through

- (1) the monotonic evolution toward the critical point of a node type for $0 < \xi \leq 3/25$;
- (2) the damping oscillations around the critical point of a focus type for $3/25 < \xi < 1/3$.

In both evolutionary scenarios we obtained linearized solutions of the dynamical system in the vicinity of the critical point. Using the relation of the equation of state parameter on the dynamical variables ψ and ψ' we were able to derive exact forms of $w(z)$ relations in both scenarios (see [15] for details).

Now, we present a method of estimation model parameters from the combined analysis of the parameters w_0 , w_1 , and w_2 from the combined astronomical data for three parametrizations of $w(z)$ (see Table I).

To constrain the unknown values of model parameters we used the set of $N_1 = 192$ SNIa data, CMBR shift parameter, measurement of the parameter A coming from the SDSS luminous red galaxies, and $N_2 = 9$ observational $H(z)$ data. The likelihood function for such data sets has the following form:

$$\mathcal{L}(\bar{\theta}) \propto \exp[-\frac{1}{2}\chi^2(\bar{\theta})], \quad (2)$$

where $\bar{\theta} = (H_0, \Omega_{m,0}, w_0, w_1, w_2)$ and $\chi^2(\bar{\theta}) = \chi_{\text{SN}}^2(\bar{\theta}) + \chi_R^2(\bar{\theta}) + \chi_A^2(\bar{\theta}) + \chi_H^2(\bar{\theta})$.

Let us consider quantities from the definition of function χ^2 . Using the SNIa data [16–18] we base on the standard relation between the apparent magnitude m and luminosity distance d_L : $m - M = 5\log_{10} D_L + \mathcal{M}$, where M is the absolute magnitude of SNIa, $\mathcal{M} = -5\log_{10} H_0 + 25$, and $D_L = H_0 d_L$. The luminosity distance depends on the cosmological model considered and with the assumption that $k = 0$ is given by $d_L = (1+z)c \int_0^z \frac{dz'}{H(z')}$. The χ_{SN}^2 function has the following form:

$$\chi_{\text{SN}}^2(\bar{\theta}) = \sum_{i=1}^{N_1} \left(\frac{\mu_i^{\text{obs}} - \mu_i^{\text{th}}}{\sigma_i} \right)^2, \quad (3)$$

where $\mu_i^{\text{obs}} = m_i - M$, $\mu_i^{\text{th}} = 5\log_{10} D_{Li} + \mathcal{M}$.

As we have written before we also used constraints which come from the so-called CMBR shift parameter which is defined as $R^{\text{th}} = \sqrt{\Omega_{m,0}} \int_0^{z_{\text{dec}}} \frac{H_0}{H(z)} dz$. The χ_R^2 has the following form:

$$\chi_R^2(\bar{\theta}) = \left(\frac{R^{\text{obs}} - R^{\text{th}}}{\sigma_R} \right)^2, \quad (4)$$

where $R^{\text{obs}} = 1.70 \pm 0.03$ for $z_{\text{dec}} = 1089$ [19,20].

We also used constraints coming from the SDSS luminous red galaxies measurement of A parameter ($A^{\text{obs}} = 0.469 \pm 0.017$ for $z_A = 0.35$) [21], which is related to the baryon acoustic oscillation peak and defined in the following way: $A^{\text{th}} = \sqrt{\Omega_{m,0}} \left(\frac{H(z_A)}{H_0} \right)^{-(1/3)} \left[\frac{1}{z_A} \int_0^{z_A} \frac{H_0}{H(z)} dz \right]^{2/3}$. The χ_A^2 function has the following form:

$$\chi_A^2(\bar{\theta}) = \left(\frac{A^{\text{th}} - A^{\text{obs}}}{\sigma_A} \right)^2. \quad (5)$$

TABLE I. Estimations of expansion coefficients for three different parametrizations of $w(z)$.

Case	Parametrization	w_0	w_1	w_2
(1)	$w(z) = w_0 + w_1 z + w_2 z^2$	-1.29 ± 0.18	1.50 ± 1.03	-0.57 ± 0.75
(2)	$w(z) = w_0 + w_1 \frac{z}{1+z} + w_2 \left(\frac{z}{1+z} \right)^2$	-1.38 ± 0.13	2.75 ± 0.87	-2.13 ± 1.33
(3)	$w(z) = w_0 + w_1 \ln(1+z) + w_2 (\ln(1+z))^2$	-1.34 ± 0.24	2.16 ± 1.71	-1.25 ± 1.73

TABLE II. Solutions to the system of equations w_X^0 , w_X^1 , and w_X^2 with values of w_0 , w_1 , and w_2 from Table I.

Case	x_0	y_0	y_0/x_0	ξ
(1)	7.45 ± 7.05	-8.55 ± 7.77	-1.15 ± 2.13	0.14 ± 0.03
(2)	6.44 ± 2.75	-8.22 ± 3.11	-1.28 ± 1.03	0.15 ± 0.02
(3)	6.53 ± 8.77	-8.09 ± 10.12	-1.24 ± 3.21	0.15 ± 0.05

Finally we add constraints coming from observational $H(z)$ data ($N = 9$) [22–24]. This data is based on the differential ages (dt/dz) of the passively evolving galaxies which allow to estimate the relation $H(z) \equiv \dot{a}/a = -(1+z)^{-1}dz/dt$. The function χ_H^2 has the following form:

$$\chi_H^2(\bar{\theta}) = \sum_{i=1}^{N_2} \left(\frac{H(z_i) - H_i(z_i)}{\sigma_i^2} \right)^2. \quad (6)$$

After the marginalization likelihood function over the parameter H_0 in the range $(60, 80)$ the values of model parameters ($\Omega_{m,0}$, w_0 , w_1 , w_2) were obtained via the χ^2 minimization procedure and are gathered in Table I. There are also the 1σ uncertainties which correspond to the parameter value where the likelihood function differs from the best-fit likelihood by a factor of $e^{1/2}$.

For every fitting parametrization we obtain a set of three numbers (w_0 , w_1 , w_2) (see Table I). Then basing on expansion in the Taylor series of the exact forms of equations of state (EoS) functions up to second order term (w_X^0 , w_X^1 , w_X^2), we can solve the system of three equations for three values ξ , x_0 , and y_0 . The errors of Δx_0 , Δy_0 , and $\Delta \xi$ are calculated in a standard way. From Table II we can see that the parameter ξ is closed to its conformal value $1/6$. Therefore the combined analysis of observational data favors a value of ξ which lies in close neighborhood of the conformal coupling. The second conclusion is that the oscillatory scenario is favored. In the case of parametrization (2) we can exclude the monotonic scenario on the 1σ confidence level.

Finally we consider parameters w_0 , w_1 , w_2 as functions of parameters ξ , x_0 , y_0 . The values of model parameters $\Omega_{m,0}$, ξ , x_0 , y_0 obtained via the χ^2 minimization procedure together with 1σ errors are gathered in Table III.

The initial condition (x_0, y_0) defines the initial rate of change of scalar field $(\ln(\psi_i - \psi_0))' = \psi_i'/(\psi_i - \psi_0) = y_0/x_0 = (\ln x_0)'$. This value is negative and is presented

TABLE IV. The values of the parameter of the nonminimal coupling ξ estimated via a direct method together with 1σ , 2σ , and 3σ confidence levels. In the cases (1) and (3) we can exclude the monotonic scenario at a 2σ and in the case (2) at a 3σ level.

Case	ξ	1σ	2σ	3σ
(1)	0.14	0.01	0.02	0.04
(2)	0.15	0.01	0.02	0.03
(3)	0.15	0.01	0.02	0.05

in the third column of Table II and in the fourth column of Table III.

We can see that in this case the values of parameters do not differ from the values obtained by an indirect method via the estimation of w_0 , w_1 , w_2 but here the uncertainties of the parameter values are smaller. In Table IV we gathered the values of the parameter of nonminimal coupling ξ estimated via a direct method together with 1σ , 2σ , and 3σ confidence levels. The value of $\xi = 1/6$ is consistent with our results at a 3σ confidence level for parametrization (1) and at a 2σ confidence level for parametrization (2) and (3). As one can conclude the oscillatory scenario is favored, moreover, we can exclude the monotonic scenario at a 2σ confidence level for the parametrization (1) and (3) and at a 3σ confidence level for parametrization (2).

While the standard quintessence idea is the most popular one, scalar field cosmology with a minimally coupled scalar field requires the existence of special conditions for a tracker solution and potential functions of the scalar field [25]. The standard quintessence evolutionary scenario has been extended by the introduction of the nonminimal coupling between gravitation and scalar field. Hence the cosmology with such a field contains an additional parameter which should be determined from observational data. We demonstrate that astronomical data and cosmography may be very useful in estimation of this free parameter of a model.

In our approach the effects of a nonvanishing coupling constant are dynamically equivalent to the effects of substantial dark energy characterized by the coefficient of the equation of state. The dynamics with positive value of the NC parameter admits two types of evolutionary scenarios: the monotonic evolution and damping oscillations around the stationary de Sitter state. Given the estimated value of the parameter ξ we found that the oscillating scenario is favored.

TABLE III. The values of directly estimated parameters $\Omega_{m,0}$, ξ , x_0 , y_0 after substituting w_X^0 , w_X^1 , w_X^2 into the $H(z)$ formula for three different parametrizations of $w(z)$.

Case	$\Omega_{m,0}$	x_0	y_0	y_0/x_0	ξ
(1)	0.27 ± 0.02	7.33 ± 0.90	-8.43 ± 0.75	-1.15 ± 0.24	0.14 ± 0.01
(2)	0.27 ± 0.02	6.41 ± 0.32	-8.18 ± 0.32	-1.28 ± 0.11	0.15 ± 0.01
(3)	0.27 ± 0.02	6.60 ± 0.72	-8.16 ± 0.81	-1.24 ± 0.26	0.15 ± 0.01

We expanded the exact form of the EoS parameter up to second order term and then constrained the effective form of the EoS which determines directly the value of the NC parameter. As a result performed combined analysis we obtain that value of ξ from the neighborhood of conformal coupling is favored by observational astronomical data, i.e. SN Ia data, baryon oscillation peak, CMBR shift, and $H(z)$ data. It was demonstrated that the value of estimated NC parameter does not depend on a used expansion formula up to the second order although errors can be different.

The general relativity is based on the Einstein equivalence principle, which includes weak equivalence principle, local lorentz invariance, and local position invariance. Therefore if this universality condition is valid then gravity can be described in terms of Riemannian geometry. Our estimation of the coupling constant is based on the FRW model filled by a nonminimally coupled phantom scalar field but one should also note the existence of the direct constraints on weak equivalence principle (WEP). Then deviation value of the coupling constant from the conformal coupling can measure the deviation from the WEP and

the general relativity and gravitation (GRG) in the Einstein formulation. Our estimation, based on a model with the nonminimally coupled phantom scalar field, gives the deviation from the WEP $\Delta\xi/\xi \approx 10^{-1}$. On the other hand, the string theory suggests the existence of strength scalar field (dilations) whose couplings to matter can violate the WEP [26]. This provides a new motivation for high-precision experiments verifying the universality of the free fall to the 10^{-12} level [27–29]. The lunar laser ranging experiment has also verified that the moon and the earth fall with the same acceleration toward the sun to better than one part in 10^{12} precision [30]. Also observed limits on the evolution of the binary pulsar BO655 + 64 orbit provide new bounds on the violation of SEP [31] (for the description of current and future projects for the improvement of the accuracy of the experiments as compared to experiments on ground, see [32]).

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