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# New boundary conditions for the $c = -2$ ghost system

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We investigate a novel boundary condition for the $bc$ system with central charge $c = -2$. Its boundary state is constructed and tested in detail. It appears to give rise to the first example of a local logarithmic boundary sector within a bulk theory whose Virasoro zero modes are diagonalizable.

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## I. INTRODUCTION

The $bc$ system with Virasoro central charge $c = -2$, and the closely related symplectic fermion model, have been studied extensively in the past, both in the bulk and on the boundary (see e.g. [1–4] and references therein). Most of the past work was driven by formal questions in the context of logarithmic conformal field theory (logCFT), of which the symplectic fermions provide the simplest possible example. By definition, the operator products of a logCFT contain terms with logarithmic singularities such as e.g.

$$\Psi_1(z,\bar z)\Psi_2(0,0) \sim |z|^{2\Delta}(\Phi(0) + \log|z|\tilde\Phi(0))\ldots.$$

Whenever logarithmic terms appear, they imply that the generator $D = L_0 + \bar L_0$ of dilatations cannot be diagonalized. In our example above, for instance, the action of $D$ on the pair $(\Phi, \tilde\Phi)$ is easily found: It is represented by a $2 \times 2$ matrix with $2\Delta$ along the diagonal and $D_{12} = 1$, $D_{21} = 0$. Hence, $D$ can only be brought into Jordan normal form. We conclude that a nondiagonalizable generator $D$ is a characteristic feature of a logCFT. Models with this property are necessarily nonunitary. Nevertheless, there exist many such theories that are highly relevant for applications.

After this brief excursion into more general logCFTs let us come back to the $bc$ system. Recently, it was pointed out [5–7] that the $bc$ system at $c = -2$ plays a crucial role for the solution of WZNW models on supergroups. In fact, it enters through a Kac-Wakimoto type representation of such theories. The latter reduces the solution of the WZNW model on superspaces to that on the corresponding bosonic base. An extension of such a free fermion representation to the boundary sector requires one to impose boundary conditions on the $bc$ system. In the special case of trivial gluing conditions for WZNW currents the relevant boundary conditions for the $bc$ system have not been discussed in the existing literature. We shall fill this gap below.

As the name indicates, the $bc$ system involves two sets of chiral bulk fields $c$, $\bar c$ and $b$, $\bar b$ of conformal dimension $h_c = 0$ and $h_b = 1$, respectively. In the conventional setup, we would glue $c$ to $\bar c$ and $b$ to $\bar b$ along the boundary [8]. But for $c = -2$ there exists another possibility: namely, to glue $b$ to a derivative of $\bar c$ and vice versa. More precisely, we can demand that

$$b(z) = \mu\bar\partial\bar c(\bar z), \qquad \bar b(\bar z) = -\mu\partial c(z) \quad \text{for } z = \bar z. \tag{1}$$

These relations guarantee trivial gluing conditions for the energy momentum tensor $T = -b\partial c$. It is not difficult to check that the dynamics of the previous $bc$ system is described by the action

$$S = \frac{1}{2\pi}\int d^2z[b\bar\partial c + \bar b\partial\bar c] - \frac{\mu}{2\pi}\int du\, c\,\partial_u\bar c. \tag{2}$$

The fields are defined on the upper half plane and variations of $S$ vanish provided the ghosts have the usual holomorphicity properties in the bulk and satisfy in addition the gluing conditions (1) on the boundary. Our aim here is to solve the theory that is defined by the action (2) and the boundary condition (1). We shall set $\mu = 1$ throughout our discussion. Formulas for the general case are easily obtained from the ones we display below.

## II. SOLUTION OF THE BOUNDARY THEORY

In order to construct the state space and the fields explicitly, we introduce an algebra that is generated by the modes $c_n$, $b_n$ and two additional zero modes $\xi_0^b$, $\xi_0^c$ subject to the conditions

$$\{c_n, b_m\} = n\delta_{n,-m}, \tag{3}$$

$$\{\xi_0^c, b_0\} = 1, \qquad \{\xi_0^b, c_0\} = 1. \tag{4}$$

All other anticommutators in the theory are assumed to vanish. The state space of our boundary theory is generated from a ground state with the properties

$$c_n|0\rangle = b_n|0\rangle = 0 \quad \text{for } n \geq 0 \tag{5}$$

by application of "raising operators," including the zero modes $\xi_0^b$ and $\xi_0^c$. On this space we can introduce the local fields $c$, $\bar c$, $b$, $\bar b$ through the prescription

$$b(z) = \sum_{n\in Z} b_n z^{-n-1}, \tag{6}$$

 

$$c(z) = \sum_{n\neq 0} \frac{c_n}{n} z^{-n} + c_0 \ln z + \xi_0^c, \quad (7)$$

$$\bar{b}(\bar{z}) = \sum_{n\neq 0} c_n \bar{z}^{-n-1} - c_0 \bar{z}^{-1}, \quad (8)$$

$$\bar{c}(\bar{z}) = -\sum_{n\neq 0} \frac{b_n}{n} \bar{z}^{-n} + b_0 \ln\bar{z} - \xi_0^b. \quad (9)$$

It is not difficult to check with the help of Eqs. (3) and (4) that these fields satisfy the correct local anticommutation relations

$$\{b(z), c(w)\} = \delta(z-w), \qquad \{\bar{b}(\bar{z}), \bar{c}(\bar{w})\} = \delta(\bar{z}-\bar{w})$$

in the interior of the upper half plane. They also fulfill our boundary conditions (1) with $\mu = 1$.

For later use let us also spell out the construction of the Virasoro generators in terms of fermionic modes,

$$L_n = \sum_{m\neq 0} :b_{n-m}c_m: - b_n c_0.$$

It is important to stress that—due to the term $c_0 b_0$—the element $L_0$ satisfies $L_0 \xi_0^c \xi_0^b |0\rangle = |0\rangle$. Since $L_0$ vanishes on all other ground states, it is nondiagonalizable. In other words, our boundary theory is an example of a logarithmic conformal field theory. The logarithms in this model, however, are restricted to the boundary sector since the bulk Hamiltonian is diagonalizable (see below).

Computations of correlation functions in our boundary theory require one to introduce a dual vacuum with the properties

$$\langle 0|c_n = \langle 0|b_n = 0 \quad \text{for } n \leq 0, \quad (10)$$

$$\langle 0|\xi_0^c \xi_0^b|0\rangle = 1. \quad (11)$$

For the $c = -2$ ghost system the ground states $|0\rangle$ and $\langle 0|$ which are annihilated by the zero modes $b_0$ and $c_0$ are at the same time $SL(2, C)$ invariant vacua. Consistency with the commutation relations requires $\langle 0|0\rangle = \langle 0|\{b_0, \xi_0^c\}|0\rangle = 0$. Therefore, the simplest nonvanishing quantity is $\langle 0|\xi_0^c \xi_0^b|0\rangle$ for our boundary theory (see also [3] for a more detailed discussion).

Finally, we would like to display the boundary state $|N\rangle$ for our new boundary condition. Before we provide explicit formulas let us briefly recall that the bulk fields are obtained as

$$c(z) = \xi_0^c + \sum_{n\neq 0} \frac{c_n}{n} z^{-n}, \qquad b(z) = \sum_{n\in Z} b_n z^{-n-1}$$

and similarly for their antiholomorphic counterparts. Note that there are no modes $c_0$, $\bar{c}_0$ and $\xi_0^b$, $\bar{\xi}_0^b$ in the bulk of our $bc$ ghost system. This feature distinguishes the $c = -2$ ghosts from the closely related symplectic fermions. According to the standard rules, the boundary state for our boundary theory must satisfy the following Ishibashi conditions [9]:

$$(b_n - \bar{c}_{-n})|N\rangle = 0, \qquad (c_n + \bar{b}_{-n})|N\rangle = 0 \quad (12)$$

for $n \neq 0$ and $b_0|N\rangle = \bar{b}_0|N\rangle = 0$. As one may easily check, the unique solution to these conditions is given by

$$\frac{|N\rangle}{\sqrt{2\pi}} = \exp\left(-\sum_{m=1}^{\infty} \frac{1}{m}(c_{-m}\bar{c}_{-m} + b_{-m}\bar{b}_{-m})\right)|0\rangle \quad (13)$$

where $|0\rangle$ is a state in the bulk theory that satisfies conditions of the form (5) for both chiral and antichiral modes. There also exists a dual boundary state $\langle N|$, satisfying the conditions

$$\langle N|(b_n + \bar{c}_{-n}) = 0, \qquad \langle N|(c_n - \bar{b}_{-n}) = 0 \quad (14)$$

for $n \neq 0$ and $\langle N|b_0 = \langle N|\bar{b}_0 = 0$. These linear relations are related to Eqs. (12) by conjugation using that $c_n^* = -c_{-n}$ for $n \neq 0$ and $b_n^* = b_{-n}$ etc. The dual boundary state is given by the following explicit formula

$$\frac{\langle N|}{\sqrt{2\pi}} = \langle 0|\exp\left(\sum_{m=1}^{\infty} \frac{1}{m}(c_m\bar{c}_m + b_m\bar{b}_m)\right) \quad (15)$$

involving a dual closed string ground state $\langle 0|$ that obeys conditions of the form (10) for modes of chiral and antichiral fields and that is normalized by $\langle 0|\xi_0^c \bar{\xi}_0^c|0\rangle = 1$ [see comments after Eq. (11)].

## III. CARDY CONSISTENCY CONDITIONS

Having constructed our new boundary theory, and, in particular, its boundary state, we would now like to perform two Cardy-like consistency tests. To begin with, let us verify that $|N\rangle$ satisfies world-sheet duality. We stress that in this paper we consider a theory in which bulk and boundary theory consist of Ramond sectors only, a choice that we shall comment in more detail below. In such a model, world-sheet duality relates quantities that are periodic in both world-sheet space and time. The simplest such quantity in our boundary theory would be $\mathrm{tr}[q^{L_0+1/12}(-1)^F]$ which vanishes since bosonic and fermionic states come in pairs on each level of the state space. The same is certainly true for $\langle N|\tilde{q}^{L_0+1/12}(-1)^F|N\rangle$, in agreement with world-sheet duality. In order to probe finer details of the theory, we need to consider quantities with additional insertions of fields or zero modes. Here, we shall establish the relation

$$\mathrm{tr}\,(q^{H^o}(-1)^F c(z)\bar{c}(\bar{z})) = \langle N|\tilde{q}^{1/2H^c}(-1)^{1/2F^c} c(\xi)\bar{c}(\bar{\xi})|N\rangle, \quad (16)$$

where $H^o = L_0 + 1/12$, $q = \exp(2\pi i\tau)$, $\xi = \exp(-\frac{1}{\tau} \times \ln z)$, and $F^c = F + \bar{F}$, as usual. The closed string Hamiltonian is given by

$$H^c = \sum_{m\in Z} [:b_{-m}c_m: + :\bar{b}_{-m}\bar{c}_m:] + 1/6.$$

Validity of Eq. (16) is required by the definition of boundary states (see e.g. [10]). Starting with the left-hand side, it is rather easy to see that

$$\mathrm{tr}\,(q^{L_0+1/12}(-1)^F c(z)\bar{c}(\bar{z})) = -\mathrm{tr}(q^{L_0+1/12}(-1)^F \xi_0^c \xi_0^b) = 2\pi i\tau\eta(q)^2 = -2\pi\eta(\tilde{q})^2. \tag{17}$$

In the computation we split off the term $c_0 b_0$ from $H^o$ and use it to saturate the fermionic zero modes. The rest is then straightforward. We can reproduce the same result if we insert our explicit formulas for the boundary states $|N\rangle$ and $\langle N|$ into the right-hand side of Eq. (16).

It is possible to perform another similar test of our boundary theory using the usual trivial boundary conditions of the ghost system. In this case, the field $c(z)$ is identified with its own antiholomorphic partner $\bar{c}(\bar{z})$ along the boundary and likewise for the pair $b$ and $\bar{b}$. Let us recall that the boundary state $|\mathrm{id}\rangle$ and its dual $\langle\mathrm{id}|$ take the form [8]

$$|\mathrm{id}\rangle = \exp\left(\sum_{m=1}^{\infty}\frac{1}{m}(c_{-m}\bar{b}_{-m} + \bar{c}_{-m}b_{-m})\right)(\xi_0^c - \xi_0^{\bar{c}})|0\rangle$$

$$\langle\mathrm{id}| = i\langle 0|(\xi_0^c - \xi_0^{\bar{c}})\exp\left(\sum_{m=1}^{\infty}\frac{1}{m}(\bar{b}_m c_m + b_m\bar{c}_m)\right), \tag{18}$$

where we use the same notations as before. For the exchange of closed string modes between $|N\rangle$ and $\langle\mathrm{id}|$ the above formulas imply

$$\langle\mathrm{id}|\tilde{q}^{1/2H^c}(-1)^{1/2F^c}c(\xi)|N\rangle = \langle\mathrm{id}|\tilde{q}^{1/2H^c}(-1)^{1/2F^c}\xi_0^c|N\rangle = \sqrt{2\pi}\tilde{q}^{1/12}\prod_{n=1}^{\infty}(1+\tilde{q}^{2n}) = \sqrt{\frac{\pi\theta_2(2\tilde{\tau})}{\eta(2\tilde{\tau})}}. \tag{19}$$

Once more we had to insert the field $c(z)$ in order to get a nonvanishing result. For comparison with a world-sheet dual, we need to quantize the ghost system on the upper half plane with trivial boundary conditions on the positive real axis and our nontrivial ones on the other half. A moment of reflection reveals that the following combinations

$$\chi^+(z) = \frac{1}{\sqrt{2}}(b(z) + i\partial c(z)),$$

$$\chi^-(z) = \frac{1}{\sqrt{2}}(ib(z) + \partial c(z))$$

diagonalize the monodromy, i.e. they obey the following simple periodicity relations $\chi^{\pm}(e^{2\pi i}z) = \pm i\chi^{\pm}(z)$. Hence, they take the following form [1]:

$$\chi^{\pm}(z) = \sum_{r\in Z\mp\frac{1}{4}}\chi_r^{\pm}z^{-r-1}.$$

The modes $\chi_r^{\pm}$ satisfy the same canonical commutation relations, $\{\chi_r^+, \chi_s^-\} = r\delta_{r,-s}$, as before. Formulas for the Virasoro generators can easily be worked out. For us, it suffices to display the zero mode $\tilde{L}_0$,

$$\tilde{L}_0 = -\sum_{r\in Z-(1/4)} :\chi_r^+\chi_{-r}^-: - \frac{3}{32}. \tag{20}$$

The constant shift by 3/32 is needed in order to obtain standard Virasoro relations with the other generators (see also [11] for a closely related analysis of twisted sectors in the bulk theory). The state space of our boundary theory contains two ground states $|\Omega_{\pm}\rangle$ which are related to each other by the action of a zero mode $\xi^c$. On this space we can introduce the field $c$ through

$$c(z) = \sqrt{\pi}\xi^c + \frac{i}{\sqrt{2}}\sum_{r\in Z-(1/4)}\frac{\chi_r^+}{r}z^{-r} - \frac{1}{\sqrt{2}}\sum_{r\in Z+(1/4)}\frac{\chi_r^-}{r}z^{-r}.$$

From the construction of the state space and our formula for $\tilde{H}^o = \tilde{L}_0 + 1/12$ we infer the following expression for the mixed open string amplitude,

$$\mathrm{tr}\,(q^{\tilde{H}^o}(-1)^F c(z)) = \sqrt{\pi}q^{-(1/96)}\prod_{n=0}^{\infty}(1-q^{1/2(n+1/2)}) = \sqrt{\frac{\pi\theta_4(\tau/2)}{\eta(\tau/2)}},$$

which reproduces exactly the previous result (19) upon modular transformation and concludes our investigation of the new boundary theory. Let us remark that the same partition function was found recently in [12] with the help of boundary loop models.

## IV. CONCLUSIONS AND OUTLOOK

The choice of our new gluing condition for the $bc$ system was motivated by the interest in branes on supergroups. As we discuss in [13], generic maximally symmetric branes in a WZNW model on a supergroup turn out to satisfy Neumann-type boundary conditions in the fermionic coordinates. This implies that all fermionic zero modes must act nontrivially on the space of open string states. In our toy model, the role of the fermionic coordinates is played by $c$ and $\bar{c}$. Hence, we needed to find boundary conditions with a four-fold degeneracy of ground states. For the standard boundary conditions of the $bc$ system, $c = \bar{c}$ along the boundary and hence only one fermionic zero mode survives, giving rise to a 2-dimensional space of ground states. In this sense, the usual boundary conditions of the $bc$ systems are localized in one of the fermionic directions. Our boundary conditions come with two nonvanishing zero modes $\xi_0^b$ and $\xi_0^c$ (and their dual momenta $c_0$ and $b_0$). This property makes them a good model for maximally symmetric branes on supergroups.

There exist various extensions of our theory that we want to briefly comment about. In our analysis we focused on the Ramond-Ramond sector of the *bc* ghost system in the bulk. It is certainly straightforward to include a Neveu-Schwarz–Neveu-Schwarz sector in case this is required by the application. Furthermore, we can also replace the bulk theory by its logarithmic cousin, the symplectic fermion model. Since the formulas and results are very similar, we refrain from giving more details. Boundary theories for symplectic fermion theories have been studied extensively in the past (see e.g. [4,14–20]). We would like to stress, however, that our boundary condition seems to be new, also in the context of symplectic fermions.

Let us be a bit more specific and relate our constructions to the results in [4]. A comparison of the gluing conditions shows that our state $|\mathrm{id}\rangle$ is a close relative of the $(N, \pm)$ boundary condition of [4]. In fact, all boundary theories considered in [4] glue $\partial c$ to $\bar{\partial}\bar{c}$ and $b$ to $\bar{b}$, with different choices of signs. None of these models displays any enhancement of zero modes in the boundary spectrum. In this sense, we would prefer to consider them all as being of the same type (mixed Dirichlet-Neumann in the context of the *bc* system). Using the notations of [4], our new boundary theories arise when we glue $\chi^+$ to $\bar{\chi}^-$ and vice versa. Such a choice gives rise to a nontrivial gluing automorphism on the so-called triplet algebra and therefore it was excluded from the analysis in [4].

In the case of the *bc* ghost system, the boundary state $|N\rangle$ has a rather novel feature: it describes a logarithmic boundary theory in a nonlogarithmic bulk. Put differently, the *bc* ghost system possesses a diagonalizable bulk Hamiltonian $H^c$. Nevertheless, the Hamiltonian $H^o$ of our new boundary theory is nondiagonalizable. Hence, logarithmic singularities can appear, but *only* when two boundary fields approach each other. To the best of our knowledge, such a behavior has never been encountered before. It shows that conformal field theories may be logarithmic even if none of its correlators on the sphere contain logarithms.

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