Radion stabilization from the vacuum on flat extra dimensions

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Volume stabilization in models with flat extra dimensions could follow from vacuum energy residing in the bulk when translational invariance is spontaneously broken. We study a simple toy model that exemplifies this mechanism which considers a massive scalar field with nontrivial boundary conditions at the end points of the compact space, and includes contributions from brane and bulk cosmological constants. We perform our analysis in the conformal frame where the radion field, associated with volume variations, is defined, and present a general strategy for building stabilization potentials out of those ingredients. We also provide working examples for the interval and the T^n/Z_2 orbifold configuration.

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I. INTRODUCTION

Extra compact spatial dimensions are a well-known fundamental ingredient of string theory, which needs to be formulated in at least ten dimensions (or 11 for M theory) to be consistent. The space generated by the six (seven) spacelike extra dimensions may have a nontrivial configuration and topology, and be characterized by a variety of sizes, which, according to some speculations [1], may even be as large as a few micrometers, in contrast with the much smaller Planck length, $\ell_P \sim 10^{-33}$ cm. The idea seems to find some motivation from the study of the nonperturbative regime of the $E_8 \times E_8$ theory by Witten and Horava [2], where one of these extra dimensions appears to be larger than the naively expected Planck size for quantum gravity physics. The possibility that there could be such extra dimensions has renewed the interest in a class of models once inspired by the works of Kaluza and Klein [3] and lately suggested by several authors [4,5]. It has also motivated a large number of studies oriented to explore phenomenological uses of such new dimensions.

Of particular interest are the so-called brane models, in which our observable world is constrained to live on a four dimensional hypersurface (the brane) embedded in a flat, higher dimensional space (the bulk), such that the extra dimensions can only be tested by gravity, and perhaps standard model singlets, a setup that resembles D-brane theory constructions. Further modifications to this basic scenario have also considered the possibility that some or even all standard model fields may probe some of the extra dimensions. Nonetheless, most models are studied only in the effective field theory limit, valid below the fundamental string scale. These models have the extra feature that they may provide an understanding of the large difference among Planck (M_P) and electroweak ($m_{\rm ew}$) scales almost by construction, since now the Planck scale ceases to be fundamental. It is replaced by the truly fundamental gravity scale, M_*, associated with quantum gravity in the $4 + n$ dimensional theory. Both scales are then related by the volume of the compact manifold, vol_n, throughout the expression [1]

$$M_P^2 = M_*^{n+2}\mathrm{vol}_n, \tag{1.1}$$

which indicates that the so far unknown value for M_* could be anywhere within $m_{\rm ew}$ and M_P. If it happens to be in the TeV range there would be no big hierarchy, but a rather large volume is required. A number of possible theoretical uses of such extra dimensions has been explored, including new possible ways for the understanding of mass hierarchies [6], the origin of neutrino masses [7], the number of matter generations [8], baryon number violation [9], the origin of dark matter [10], new mechanisms for symmetry breaking [11], and model building [12], among many others. Experimental implications of some of those models have also been under the scope of many investigations (for references, see for instance [13,14]).

Most phenomenological models built on this scenario usually assume that the extra dimensions are stable, which typically becomes a fundamental requirement since most effects of extra dimensions on low energy physics depend either on the effective size of the compact space $b_0 \sim \mathrm{vol}_n^{1/n}$ or the effective Planck scale. However, if the compact space were dynamical, those quantities would become time dependent, against observations.

As it can be easily realized, during the early inflation period and the later evolution of the Universe, a static bulk appears to be hard to accept against an expanding 4D world. Indeed, there are indications that the contrary is rather more likely to happen [15]. Inflaton energy may also

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induce dynamical effects on the extra space, by driving the so-called radion field, associated with the overall extra volume variations, beyond its desired stable point. This may particularly affect the large extra dimensional case where the inflaton contribution might increase the effective volume by a factor of a few.

Understanding the stability of the compact space can be seen as finding the mechanism that provides the force which keeps the radion fixed at its zero value. Thus, in order to have a stable bulk volume, there has to be a potential which provides such a force. Some ideas on the possible origin of this potential can be found in the literature, ranging from pure quantum effects to string theory nontrivial flux constructions; see for instance [16–24]. In this paper we explore an idea first introduced in the context of warped extra dimensions [22], and later discussed for a single, flat extra dimension in Ref. [24], where vacuum energy is regarded as the one responsible for generating the stabilization potential. Although the use of this mechanism on flat backgrounds might look trivial at first sight, we believe an extended and careful analysis is worthwhile for two reasons. First of all, the mechanism on flat compact space mimics the stringy scenario where fluxes are used for stabilization, providing a bottom-up toy model where other problems, such as metric backreactions and quantum stability, could be tested. Second, as we shall observe, the definition of the radion field on the frame where gravity action becomes standard implies the introduction of conformal factors on matter actions, which have a nontrivial impact on the stabilization analysis. This is a feature that has usually been overlooked in previous works (see for instance [24]).

We will consider the generic model where a massive scalar bulk field develops a vacuum configuration that explicitly violates translational invariance along the extra space. Such a vacuum, in the effective four dimensions, appears as a potential energy that depends on the size of the extra dimension, and thus it is interpreted as a radion potential. We argue that these potentials can be built to have a minimum at a finite and nonzero value of the extra dimensional size, providing a successful stabilization at the tree level of the theory. Our analysis will concentrate mainly on the phenomenological modeling for the stabilization potential on flat backgrounds, which, given the number of particle physics models built on such an assumption [6–12], we believe have some interest on their own. Such an approximation, however, would lack the immediate link with the more fundamental string theory that motivates it. And, although the ingredients we shall consider are the minimum we expect to come from a real string theory construction, this is an issue we will not address here. Our main goal will be to demonstrate in a practical constructive way that vacuum energy could be enough to provide the required bulk stabilization, and to keep things simple, we will work in the assumption that backreactions are negligible.

The paper is organized as follows. First, we discuss the general aspects of radion stabilization by vacuum energy on flat extra dimensions. To clarify the basis of the mechanism, we start by briefly reviewing the definition of the radion field, conformally mapping the initial action to the physical Einstein frame, where the 4D gravity action is kept as usual and gravity coupling remains constant. We discuss the effect of such a metric conformal transformation on other Lagrangian terms on the action, particularly, on those that would later contribute to the stabilization potential. We show that, in general, when the radius is away from its stable value, some conformal factors remain on the potential energy. Such factors define the couplings of the radion to matter fields, both in the bulk and on the brane. They also affect the stabilization potential by introducing overall inverse volume factors. In Sec. III, we discuss the mechanism for radion stabilization based on vacuum energy. We first show that, surprisingly, stabilization can be accomplished with the sole introduction of cosmological constants, which exemplifies the nontrivial features of the mechanism. Next, we shall consider brane and bulk cosmological constants as well as bulk scalar vacuum contributions. We provide some general guidelines for building a successful radion potential, which, aside from having a nontrivial minimum, may also insure a zero 4D effective cosmological constant in the Einstein frame. Finally, in Sec. IV, we investigate the implementation of the present mechanism for the interval and for T^n/Z_2 orbifolds. For these examples, we show that brane and bulk cosmological constants play an important role to control the profile of the stabilization potentials. We end with some concluding remarks and observations.

II. THE RADION IN THE EINSTEIN FRAME

A. Dimensional reduction and the radion field

We start the discussion by assuming that Einstein gravity holds in the complete $(4 + n)$D theory, and proceed with dimensional reduction to introduce the definition of the radion field and its couplings. Thus we first write down the Einstein-Hilbert action

$$S = \frac{M_*^{2+n}}{2} \int d^4x d^n y \sqrt{|g_{(4+n)}|} R_{(4+n)} \qquad (2.1)$$

where $R_{(4+n)}$ stands for the $(4 + n)$ dimensional scalar curvature, and $|g_{(4+n)}|$ is the absolute determinant of the $(4 + n)$D metric. We then consider the background metric parametrization $ds^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu - h_{ab}dy^a dy^b$, which is conformally consistent with 4D Poincaré invariance and describes a compact and flat extra space. So we assume y^a as dimensionless coordinates on a unitary and closed manifold. Thus, h_{ab} has length dimension 2. Here we use, for the indices, the convention $A, B = \mu, a$ where $\mu = 0, \ldots, 3$ and $a = 5, \ldots, 4 + n$. Notice we are not considering the usual vectorlike A^a_μ connection pieces. This is because we want to concentrate only on the variations of the metric along the transverse directions for the rest of our discussion.

Upon dimensional reduction, one obtains at the zero mode level

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g_{(4)}|}\frac{\sqrt{|h|}}{\mathrm{vol}_n}\bigg\{R_{(4)} - \frac{1}{4}\partial_\mu h^{ab}\partial^\mu h_{ab} - \frac{1}{4}h^{ab}\partial_\mu h_{ab}\cdot h^{cd}\partial^\mu h_{cd}\bigg\}, \tag{2.2}$$

where vol_n stands for the volume of the extra space at the desired stable configuration, as defined above in Eq. (1.1). In this initial frame, gravity is not well defined. There is an extra factor which is, in general, different from unity when the compact volume differs from that of vol_n. In order to get a proper gravity action, one has to go to a different frame. Thus, we perform the conformal transformation

$$g_{\mu\nu} \to e^{2\varphi}g_{\mu\nu}, \tag{2.3}$$

with $e^{2\varphi} = \mathrm{vol}_n/\sqrt{|h|}$, to obtain the 4D gravity in canonical form,

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g_{(4)}|}\bigg\{R_{(4)} - \frac{1}{4}\partial_\mu h^{ab}\partial^\mu h_{ab} + \frac{1}{8}h^{ab}\partial_\mu h_{ab}\cdot h^{cd}\partial^\mu h_{cd}\bigg\}, \tag{2.4}$$

in what we shall refer to as the conformal (or Einstein) frame. Next, $g_{\mu\nu}$ can be assumed to be the standard metric for a Poincaré invariant brane Universe or the Friedmann-Robertson-Walker metric for cosmology. We will, however, keep $g_{\mu\nu}$ undefined as far as possible. Nevertheless, to simplify, we shall take $h_{ab} = b^2\delta_{ab}$, such that b represents the actual size of the compact space.

If the bulk had the desired stable configuration, the physical size of the extra dimension would be given by the identification $b = b_0$, such that $\mathrm{vol}_n = b_0^n$. However, on cosmological grounds at least, it is plausible that b would be a time dependent field; thus, the actual physical volume of the bulk would, rather, be given as $\mathrm{vol}_{\mathrm{phys}} = \sqrt{|h|} = b^n(t)$. A more general dependence $b(x)$ on the four space-time coordinates may also be possible. This would describe local variations on the bulk radius along the brane. Although we will not explicitly refer to this case here, we will kept most expressions as general as possible.

As it can be read from the action, in the conformal frame the effective Planck scale is well defined and constant. However, volume variation effects appear as the scalar field

$$\sigma(t) = M_P\sqrt{\frac{n(n+2)}{2}}\ln\bigg(\frac{b}{b_0}\bigg). \tag{2.5}$$

This field is usually called the radion, and it is defined in such a way that it sets to zero when the stabilized volume is reached. Indeed, with the use of this radion, the last effective 4D action becomes

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g_{(4)}|}R_{(4)} + \frac{1}{2}\int d^4x\sqrt{|g_{(4)}|}(\partial^\mu\sigma)(\partial_\mu\sigma), \tag{2.6}$$

where the last term corresponds to the action of a runaway scalar mode. Hence, without potential, the radion field can take any value. Furthermore, under any perturbation, the volume of the extra space is totally unstable. In general, an active radion means a variable bulk, and it could be seen as an unwanted and harmful scenario. As we will discuss below, this field couples to all other fields in the theory, affecting dispersion relations and the definition of coupling constants. Also, its couplings to the inflaton may introduce potential threats to standard cosmology (see for instance Ref. [15]). This can be disastrous, and thus, it is important to provide a radion potential capable of keeping the radion at its zero value.

Some comments are in order. The very definition of the radion depends on the background metric we have chosen. Different geometries would mean different mathematical forms for the radion field, but the latter would always be present. Flat backgrounds are the simplest examples where calculations can be worked out very clearly. Thus, hereafter we will assume the bulk to be flat. Nevertheless, one has to keep in mind that, in any realistic scenario, backreactions due to the energy that sources the stabilization potential may require refining the compactification analysis to take such effects into account. To keep our analysis simple, however, we will neglect such effects.

As already mentioned, in what follows, we shall consider two possible sources of energy contributing to stabilization: first, pure cosmological constants, which are usually seen as the zero level energy produced by the actual physics living on the space-time of the theory. The actual cosmological constant is rather small and one can safely take it to be zero for simplicity, but in a theory with extra dimensions what we see in four dimensions is just the result of all contributions that come from the various sectors of the theory. Thus, in a bulk-brane scenario, both bulk and brane cosmological constants could be expected. Branes, of course, should be located at the fixed points on the compact dimension. The next possible source comes from position dependent vacuum configurations on the bulk, which can be modeled using bulk scalar fields. These could actually come from many sectors of string theory, usually as extra degrees of freedom of vectorlike or tensorlike fields. However, we will not need to make any assumption on the origin of such fields, other than them having nontrivial bulk configurations. That will allow us to keep our analysis general, and to address the question of whether such ingredients could be enough to build appropriate stabilization potentials from a phenomenological perspective.

B. Radion couplings and effective potentials

Before entering into the discussion of the stabilization mechanism, we shall first make a note of the effect of the above introduced conformal transformation [Eq. (2.3)] on other physical actions besides that of gravity. Consider for instance a bulk scalar field, $\phi(x, y)$. The corresponding action, in the initial $4 + n$D frame, before performing the conformal transformation on the metric, goes as

$$S_\phi = \int d^4x d^n y \sqrt{|g_{(4)}|}\sqrt{|h|}\left[\frac{1}{2}G^{AB}\partial_A\phi\partial_B\phi - U(\phi)\right]. \tag{2.7}$$

Without loss of generality, we can always assume that ϕ has a proper Kaluza-Klein (KK) mode decomposition, which should be defined for each given topology of the compact space. Such modes are, in general, the orthogonal solutions to the free equation of motion, only considering up to mass terms in the above general action, with the proper boundary conditions. A typical expansion should have the formal expression

$$\phi(x, y) = \sum_{\vec{n}} \frac{\xi_{\vec{n}}(y)}{\sqrt{\mathrm{vol}_n}}\phi_{\vec{n}}(x), \tag{2.8}$$

where $\vec{n}$ stands for all the KK indices, and the KK modes, ξ, should obey the formal normalization condition

$$\int d^n y \xi_{\vec{n}}\xi_{\vec{n}'} = \delta_{\vec{n}\vec{n}'}. \tag{2.9}$$

By introducing this expression in the action, and including the conformal transformation, we get

$$S_\phi = \int d^4x\sqrt{|g_{(4)}|}\left[\sum_{\vec{n}}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi_{\vec{n}}\partial_\nu\phi_{\vec{n}}\right) - e^{-\alpha\sigma/M_P}U_{\mathrm{eff}}(\phi_{\vec{n}})\right], \tag{2.10}$$

where we have replaced the conformal factor terms in favor of the radion field, explicitly using the equivalent expressions

$$e^{2\varphi} = \left(\frac{\mathrm{vol}_n}{\sqrt{|h|}}\right) = \left(\frac{b_0}{b}\right)^n = e^{-\alpha\sigma/M_P} \tag{2.11}$$

with $\alpha = \sqrt{2n/(n+2)}$. Thus, we notice that in the conformal frame the radion couples exponentially to an effective potential, which is formally defined through the integral

$$U_{\mathrm{eff}} = \mathrm{vol}_n \cdot \int d^n y\left(\frac{1}{2}\frac{\vec{\nabla}_y\phi\cdot\vec{\nabla}_y\phi}{b^2} + U(\phi)\right), \tag{2.12}$$

with $\vec{\nabla}_y$ the gradient on the compact space coordinates. Note also that the last expression actually corresponds to the potential energy, U_{ini}, one calculates in the initial frame, but for the global factor vol_n instead of the physical volume $\sqrt{|h|}$. In fact, one can also write $U_{\mathrm{eff}} = e^{-\alpha\sigma/M_P}U_{\mathrm{ini}}$. The first term in the above equation would contribute to the whole potential in Eq. (2.10) with the KK mass term. The KK squared mass, as usual, appears proportional to the squared inverse physical radius, b^{-2}, up to an overall conformal factor $(b_0/b)^n$. So, in the Einstein frame, the effective mass of KK modes should follow the time dependence of radius variations with a power law modulation.

The overall conformal factor on potential terms is in fact a general feature for most actions. It also appears, for instance, in the case of a bulk cosmological constant, where the action $S_\Lambda = \int d^4x d^n y\sqrt{|g_{(4+n)}|}\Lambda$ is easily integrated over the extra dimensions, to become in the Einstein frame

$$S_\Lambda = \int d^4x\sqrt{|g_{(4)}|}\Lambda_n e^{-\alpha\sigma/M_P}, \tag{2.13}$$

with the effective cosmological constant $\Lambda_n = \mathrm{vol}_n \cdot \Lambda$. We must take into account these overall factors when discussing any bulk generated potential.

Similarly, for brane fields, although 4D actions do not involve a $\sqrt{|h|}$ factor but just $\sqrt{|g_{(4)}|}$ for the induced metric on the brane, the conformal transformation we used for the rest of the theory shall introduce a radion coupling at least for brane scalar and fermion fields as well as the brane cosmological constant. Indeed, for a scalar field, χ, one obtains

$$\int d^4x\sqrt{|g_{(4)}|}e^{-\alpha\sigma/M_P}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - e^{-\alpha\sigma/M_P}U(\chi)\right), \tag{2.14}$$

whereas one gets

$$\int d^4x\sqrt{|g_{(4)}|}e^{-(3/2)\alpha\sigma/M_P}g^{\mu\nu}e^a_\nu(i\bar{\psi}D_\mu\gamma_a\psi) \tag{2.15}$$

for a massless brane fermion, ψ. In the case of a 3-brane cosmological constant, λ, one has

$$\int d^4x\sqrt{|g_{(4)}|}e^{-2\alpha\sigma/M_P}\lambda. \tag{2.16}$$

There is no coupling of the radion field to massless gauge fields, though. The above couplings affect the definition of the canonical brane field through the modification of the kinetic terms, and hence the corresponding dispersion relations. They also transform cosmological constants into radius functions. Clearly, all these results reduce to the usual ones for a stable bulk, when $b = b_0$ ($\sigma = 0$), and certainly, one can use standard expressions at first order, when the radion is close to the minimum, such that its couplings can be treated perturbatively.

III. RADION STABILIZATION BY VACUUM ENERGY

As we already mentioned, some ideas on how to generate a stabilization potential for the radion can be found already in the literature [16–24]. In particular, for a single extra dimension, it has been pointed out [22,24] that a radion potential can be produced if translational invariance is broken in the bulk by the vacuum expectation value (vev) of a scalar field. Here, we will further explore this idea for flat extra dimensions. We shall perform our analysis in the conformal frame where the radion has been identified. The basics of the mechanism we are exploring are rather simple. Bulk field configurations may provide an effective 4D energy. Furthermore, if the bulk energy density breaks translational invariance along the bulk coordinates, one gets different amounts of energy for different volume sizes, thus generating a potential energy, which we shall find convenient to write in terms of the radius as $U_{\rm rad}(b)$. Of course, if there is a nontrivial minimum for $U_{\rm rad}(b)$, this would be identified as b_0.

A. Stabilization by cosmological constants

Let us first notice that the use of only a cosmological constant, either from 3D-branes located at fixed points or the bulk, does not provide a desirable scenario. For any individual case the radion potential is just an exponentially decaying function without a nontrivial minimum. However, the combination of both contributions may work. From Eqs. (2.13) and (2.16), the most general radion potential one can build in this case is

$$U^{\lambda}_{\rm rad}(\sigma) = e^{-\alpha\sigma/M_P}(\Lambda_n + e^{-\alpha\sigma/M_P}\lambda). \tag{3.1}$$

Clearly $U^{\lambda}_{\rm rad}(0) = \Lambda_n + \lambda$, whereas $U^{\lambda}_{\rm rad} \to 0$ for $\sigma \to \infty$. This potential has a minimum at $\sigma_0 = (M_P/\alpha) \times \ln(-2\lambda/\Lambda_n)$, provided $\lambda > 0$. The requirement that $\sigma_0 = 0$ be a minimum implies that $\Lambda_n + 2\lambda = 0$. At first sight this condition might be seen as a fine-tuning; nevertheless, this is actually what fixes the stable radius to $b_0 = (-2\lambda/\Lambda)^{1/n}$. Therefore, the only appropriate potential for this case goes as

$$U^{\lambda}_{\rm rad}(\sigma) = \lambda e^{-\alpha\sigma/M_P}(e^{-\alpha\sigma/M_P} - 2). \tag{3.2}$$

In Fig. 1 we have plotted the general profile for this potential. It is worth noticing that the potential diverges exponentially for negative values of σ, or radii smaller than b_0. This suggests that our configuration of bulk and brane cosmological constants works perfectly to keep the extra dimension from collapsing into itself. However, as it is clear, the depth of the potential is given by a single parameter which by construction can not be fixed to zero—the brane cosmological constant λ. This implies, at the stable volume configuration, a nonzero and negative effective 4D cosmological constant, $U^{\lambda}_{\rm rad}(0) = -\lambda$. This is troublesome since the observed cosmological constant is

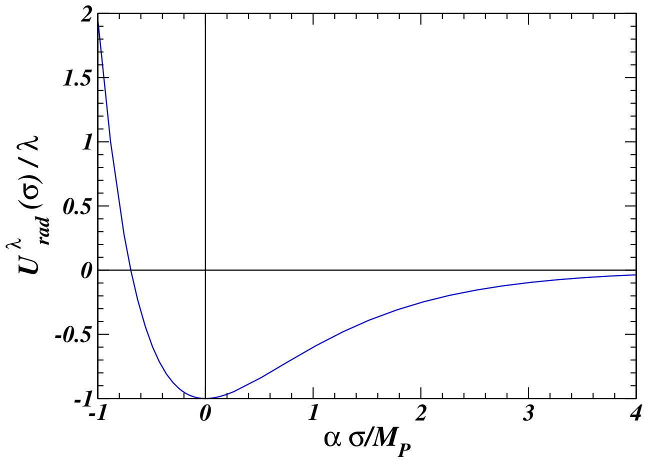

FIG. 1 (color online). Radion stabilization potential profile generated by the sole introduction of brane and bulk cosmological constants according to Eq. (3.2).

rather small and positive. Furthermore, from the potential one gets a Planck suppressed effective radion mass at the minimum,

$$m_\sigma = \alpha\sqrt{2\lambda}/M_P, \tag{3.3}$$

which may also imply a radion mass that is too light, against observational limits on gravity strength coupled scalars, that indicate $m_\sigma > 10^{-3}$ eV, which would require that $\lambda > \text{TeV}^4$. These features are indeed a potential problem, and thus, one is forced to depart from this simple model. We should notice that the same conclusions are reached when one does the analysis for the radius instead of the radion field, as expected.

It is worth stressing the fact that the potential naively calculated in the initial frame, that is, without properly including the conformal factors, is only a polynomial function of the radius, $b^n\Lambda + \lambda$, whose only minimum, at the best, resides at $b = 0$. This clearly shows the risk of getting misleading results when the analysis is not properly performed in the Einstein frame, and the conformal factors are taken into account.

An alternative for $n > 1$ could be to add brane tensions at the natural boundaries of the compact space, too. A realization of such a scenario from string theory may of course need the introduction of intersecting brane configurations. For instance, a $(n+2)$-brane tension, τ, would contribute to the effective action with the term

$$\int d^4x\sqrt{|g_{(4)}|}e^{-\beta\sigma/M_P}\tau_n, \tag{3.4}$$

where $\beta = (n+1)\alpha/n$ and $\tau_n = b_0^{n-1}\tau$. Now, $U^{\tau}_{\rm rad} = e^{-\alpha\sigma/M_P}(\Lambda_n + e^{-\alpha\sigma/M_P}\lambda) + e^{-\beta\sigma/M_P}\tau_n$ has a minimum for $\sigma = 0$, provided that $\alpha(\Lambda_n + 2\lambda) + \beta\tau_n = 0$. An additional condition can now be imposed by requiring that $U^{\tau}_{\rm rad}(\sigma = 0) = 0$, which gives $\Lambda_n + \lambda + \tau_n = 0$. This condition would always imply that at least one of the cosmological constants we are considering is negative, and conspire to (almost) cancel the effective cosmological constant. By combining these two equations we find the unique solution $\Lambda_n = (n-1)\lambda$, which fixes the radius at $b_0 = [(n-1)\lambda/\Lambda]^{1/n}$, and also $\tau_n = -n\lambda$. Thus, the potential becomes

$$U^\tau_{\rm rad} = \lambda e^{-\alpha\sigma/M_P}[n(1 - e^{-\alpha\sigma/nM_P}) + (e^{-\alpha\sigma/M_P} - 1)], \tag{3.5}$$

whereas the associated radion mass is now

$$m^2_\sigma = \left(\frac{n-1}{n}\right)\frac{\alpha^2}{M_P^2}\lambda. \tag{3.6}$$

Note that the last expression has a similar form as the result given above [Eq. (3.3)]. Again, it now implies that all cosmological constants in the model obey $\lambda, \sim\Lambda_n, \sim|\tau_n| > \mathrm{TeV}^4$. Notice that $U^\tau_{\rm rad}$ decays exponentially for large σ. Thus, the potential profile presents a potential barrier that isolates the local minimum $\sigma = 0$ from infinity, as it is depicted in Fig. 2. This is going to be a constant feature for the examples we shall discuss below. This, of course, may indicate the risk of a possible spontaneous decompactification by quantum tunneling. However, notice that the width is given in Planck mass units, above which we cannot really trust our effective analysis due to possible stringy (or quantum gravity) effects, that we do not have under control here, and that could substantially modify the potential for larger values of σ. This issue is out of the scope of the present paper, and therefore, we will not discuss it any further, nor will we do so for the cases presented below, when it appears.

B. Potential building

The next simplest example one can provide for bulk energy is a y-dependent vacuum. This arises in models where nontrivial boundary conditions are imposed on a bulk scalar field configuration. To elaborate, let us consider

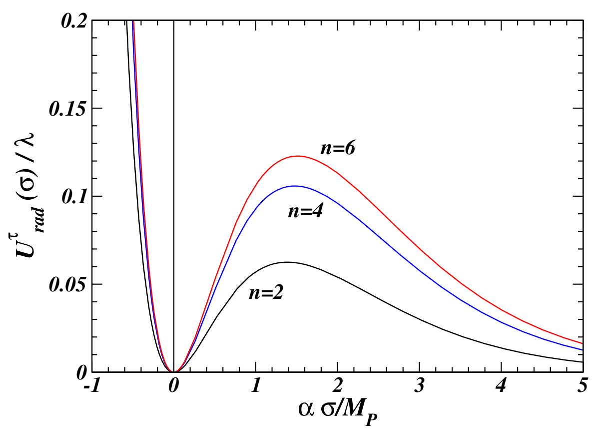

FIG. 2 (color online). Radion potential profile generated by cosmological constants according to Eq. (3.5), for n as indicated.

a massive scalar field, ϕ, described by the action given in Eq. (2.7) for $U(\phi) = \frac{1}{2}m^2\phi^2$. Therefore, the vacuum configuration in the initial frame, with a given volume of size b, should be a solution to the equation of motion

$$[-\nabla^2_y + \kappa^2]\langle\phi\rangle(y) = 0, \tag{3.7}$$

where $\kappa = mb$ and ∇^2_y is the Laplacian operator on the extra dimension coordinates. Notice also that we are considering only those vacuums which do not break translational invariance along the brane. The above equation should be complemented with the boundary conditions defined at the end points of the compact manifold. Without them, there would be no y-dependent vacuum energy in the minimal configuration. On the orbifold, for instance, these conditions are given on 3-branes located at the fixed points. They define localized sources for the bulk vacuum. These boundary conditions may be due to some other physics sited on the branes, which forces the bulk field to pick up a nontrivial vacuum expectation value. As an example, one can consider the coupling to some brane scalar field, χ, as $\chi\phi\delta(y-y_0)$, where y_0 is a fixed point where the brane is located. If χ develops a vev, this shall induce a vev on ϕ that varies along the bulk. This mechanism has been used in several models to explain small vevs in distant branes [25]. Another possibility may be the existence of localized potential terms for the bulk field on the branes, which favors a nontrivial localized vev. This happens, for instance, if one considers a Higgs-type localized potential $(\phi^2 - v)^2\delta(y-y_0)$.

Regardless of the mechanism that fixes the boundary conditions for $\langle\phi\rangle$, these induce a nontrivial profile for the vev along the bulk. Once such a vev is given, by setting it back into the Lagrangian, $\mathcal{L}$, at any given radius b, one formally gets in the Einstein frame the radion potential contribution:

$$U^\phi_{\rm rad}(b) = \left(\frac{b_0}{b}\right)^{2n} U_{\rm ini}(b), \tag{3.8}$$

here written in terms of the radius, and where we have conveniently used the potential as it is read in the initial frame (before conformal transformation),

$$U_{\rm ini}(b) = -b^n \cdot \int d^n y\, \mathcal{L}(\langle\phi\rangle). \tag{3.9}$$

By writing the potential this way, it becomes clear that, in general, a minimum for $U_{\rm ini}$ is not a minimum of $U^\phi_{\rm rad}$. The conformal factor deforms the potential, and may even compromise stabilization in some cases. Nevertheless, the function $U_{\rm ini}$ will prove to be a useful reference when analyzing the radion potential, as we shall see in the examples below.

On the other hand, even if one has a nontrivial minimum for the above potential, there is no guarantee that the potential would be zero at minimum. Such a case can, however, be controlled with the addition of cosmological constants. Therefore, the generic potential one can build goes as

$$U_{\rm rad}(b) = \left(\frac{b_0}{b}\right)^n\left[\left(\frac{b_0}{b}\right)^n[U_{\rm ini}(b) + \lambda] + \Lambda_n\right]. \qquad (3.10)$$

It is worth noticing that the explicit b_0 dependence on the above equation, although it may be annoying, is actually harmless. It is introduced by the conformal factors, and we keep it everywhere just for dimensional reasons. Nevertheless, it is actually ignored for the minimization of the potentials, since we can always factor it away using $\Lambda_n = b_0^n\Lambda$.

Before working out some examples, we notice that the radion mass provided by our mechanism has the general form

$$m_\sigma^2 = \left(\frac{b_0\alpha}{nM_P}\right)^2\left[\frac{d^2U_{\rm rad}(b)}{db^2}\right]_{b=b_0}. \qquad (3.11)$$

Therefore, it always comes with a Planck suppression, which may, however, be overcome provided the potential well is steep enough.

Among the many situations in which a minimum could appear for the radion potential in Eq. (3.10), two are of special interest for model building. Both are realized when $U_{\rm ini}(b)$ already has a nontrivial minimum, b_i.

(i) First, one can always guarantee that the actual minimum in the conformal frame remains the same, such that $b_0 = b_i$. This actually happens when the brane cosmological constant is used to shift the minimum of $U_{\rm ini}(b)$ to zero in the initial frame, by choosing $\lambda = -U_{\rm ini}(b_0)$, and one takes $\Lambda = 0$ to insure a zero effective cosmological constant in the conformal frame. In this case, the overall power law factor in Eq. (3.10) has no impact on the location of the minimum of the potential. As a matter of fact, it is easy to see that within these conditions $U_{\rm rad}(b_0) = 0$, whereas $U'_{\rm rad}(b_0) = U'_{\rm ini}(b_0) - \frac{2n}{b_0}[U_{\rm ini}(b_0) + \lambda] = 0$, and $U''_{\rm rad}(b_0) = U''_{\rm ini}(b_0)$, from a similar reasoning. The last equation also shows that the radion mass can be calculated directly from $U_{\rm ini}(b)$. Hence, it is usually enough to establish the existence for a nontrivial minimum on $U_{\rm ini}(b)$ to know that there is a working situation in the Einstein frame.

(ii) Second, one can take advantage of the interplay among the two cosmological constants to provide more control on the potential depth. The key observation is that, for any given function $f(b)$, its zeros are fixed points under the modulation by a $1/b^n$ factor, provided $b \neq 0$. This is actually the analytical reason why the minimum of $U_{\rm ini}(b)$ is kept in the previous situation despite the conformal factors. Also, since $1/b^n$ is always positive, it does not change the sign of any given value of $f(b)$. However, it suppresses the function for large b. Thus, one can subtract a large cosmological constant, $\lambda \ll -\| U_{\rm ini}(b_i) \|$, to $U_{\rm ini}(b)$, to shift the minimum towards negatives values, as to compensate for the $1/b^n$ modulation, and provide a deeper well for the effective potential:

$$U_{\rm eff}(b) = \left(\frac{b_0}{b}\right)^n[U_{\rm ini}(b) + \lambda]. \qquad (3.12)$$

Finally, a large positive $\Lambda_n = -U_{\rm eff}(b_0)$ should be chosen in order to cancel the radion potential at the minimum. It is not hard to see that in this scenario $b_0 \neq b_i$. As a matter of fact, the minimum of $U_{\rm eff}$ shall now also become the minimum of the above radion potential (3.10). Moreover, since $U''_{\rm rad}(b_0) = U''_{\rm eff}(b_0)$ also, the radion mass may, in this case, be calculated directly from the effective potential instead.

It is not difficult to understand what a mismatching $\delta\lambda = U_{\rm ini}(b_i) + \lambda \neq 0$ does for the deviation of the actual minimum, b_0, from b_i in previous scenarios. We can imagine a simple situation where $\delta\lambda$ is small, such that in the limit where it is neglected we start with the minimum at b_i, as described in the first item above. By switching on $\delta\lambda$ we shall be moving into the second scenario just described. So, the actual minimum should now be displaced from b_i by $\delta b = b_0 - b_i$. Being the minimum of $U_{\rm eff}$, b_0 fulfills the condition $b_0U'_{\rm ini}(b_0) - n\delta\lambda = 0$, which at first order gives

$$\delta b \approx \left(\frac{n}{b_iU''_{\rm ini}(b_i)}\right)\delta\lambda. \qquad (3.13)$$

As the coefficient within parentheses is positive by definition, we conclude that the minimum is shifted according to the sign of $\delta\lambda$, and clearly, we require $\Lambda_n \approx -\delta\lambda$. On the other hand, by looking at the second derivatives of the potentials, we find that at b_0 one gets $U''_{\rm eff} = U''_{\rm ini} + \delta U''$, where

$$\delta U'' \approx -\frac{n(n+1)}{b_i^2}\delta\lambda. \qquad (3.14)$$

Therefore, for $\delta\lambda < 0$, the potential around the minimum gets tightened and the radion mass is increased.

The case where the $U_{\rm ini}$ minimum is trivial, meaning $b_i = 0$ or infinity, is hard to handle, in general. However, as in the case of the sole cosmological constants, there may be some scenarios where $U_{\rm rad}$ do have a nontrivial minimum. Whether this is so would have to be studied for each particular case, though. We will illustrate this situation in the next section.

IV. RADION STABILIZATION ON ORBIFOLDS

A. The interval

To exemplify the mechanism let us elaborate on the simplest case of one single extra dimension where the coordinate y takes values in the interval [0,1]. The general solution to the equation for the vacuum state (3.7) is then

$$\phi(y) = Ae^{\kappa y} + Be^{-\kappa y}, \tag{4.1}$$

where the constants A and B are given in terms of the boundary conditions, which we assumed to be $\phi(0) = v_0$ and $\phi(1) = v_1$, where $v_{0,1}$ have mass dimension 3/2 by definition. Thus, one gets

$$A = \frac{v_1 - v_0 e^{-\kappa}}{e^{\kappa} - e^{-\kappa}} \quad \text{and} \quad B = \frac{v_0 e^{\kappa} - v_1}{e^{\kappa} - e^{-\kappa}}. \tag{4.2}$$

It is straightforward to calculate the potential in the initial frame according to Eq. (3.9), which goes as

$$U_{\rm ini}(b) = \frac{m}{2}\frac{(v_0^2 + v_1^2)\cosh\kappa - 2v_0 v_1}{\sinh\kappa}. \tag{4.3}$$

Note that the potential is invariant under the exchange $v_0 \leftrightarrow v_1$. This was expected because the physical situation we are describing within the interval (equivalent to the one dimensional orbifold S^1/Z_2) is invariant under exchange of the boundaries, which can be seen as an effect of parity symmetry. This potential has a sizable minimum at

$$mb_i = \text{arccosh}\left(\frac{v_0^2 + v_1^2}{2v_0 v_1}\right). \tag{4.4}$$

Hence, the stable radius is proportional to the inverse mass of the bulk scalar field by a factor fixed by the boundary conditions, which ranges from zero to infinity. This provides great freedom on the bulk scalar mass, and allows for a simple realization of large extra dimensions, at the price of moving the hierarchy to the boundary conditions. Particularly, for large v_0/v_1 ratios, one gets the approximate expression $mb_0 \approx [\ln(v_0/v_1)]^2$. At the minimum we get

$$U_{\rm ini}(b_i) = \frac{m}{2} \parallel v_0^2 - v_1^2 \parallel, \tag{4.5}$$

and so the potential is always positive. Notice also that the potential goes asymptotically to a constant: $U_{\rm ini}(b \to \infty) = m(v_0^2 + v_1^2)/2$, and for small b, behaves like $\sim(v_0 - v_1)^2/2b$, provided $b_0 \neq 0$.

Clearly, $v_0 = v_1$ is not a favored scenario. First of all, it implies $b_i = 0$, where the potential vanishes. Nevertheless, the $1/b^2$ squared modulation removes this minimum and kills the asymptotic behavior, such that the only possible minimum in the Einstein frame becomes $b \to \infty$. Furthermore, by including brane and bulk cosmological constants one gains new terms that go as $\Lambda/b + \lambda/b^2$. As we have shown, this piece of the potential has a nontrivial minimum by itself, provided $\lambda > 0$ and $\Lambda < 0$. The same situation holds for the whole radion potential. This can be easily seen as follows. First, consider that close to zero $U_{\rm rad}(b)$ diverges as λ/b^2. Thus $b \neq 0$ requires a positive λ. Next, we notice that Λ has to be negative to compensate the other monotonic and positive defined parts of the potential to provide a minimum. However, we now notice that the asymptotic form for the potential goes as $\sim\Lambda/b$, and thus $U_{\rm rad}(b)$ reaches zero asymptotically from below, which implies that $U_{\rm rad}(b_0)$ is strictly negative. Therefore, we are driven to this conclusion: one can find a way to provide a stabilization potential in this case, but one always ends with a nonzero cosmological constant, which is not very attractive. This suggests that asymmetric boundary conditions on both ends of the interval may be preferred. Notice, however, that for either v_0 or v_1 null, b_0 would go to infinity, and we will end in a similar situation.

Next, we proceed to study the radion potential in the Einstein frame by assuming that $v_0 > v_1$, for simplicity. The opposite case is actually equivalent due to the $v_0 \leftrightarrow v_1$ exchange symmetry. As the potential in the initial frame already has a minimum, the two scenarios for model building described in the previous section shall be useful.

As the first approximation, we add a brane cosmological constant $\lambda = -m(v_0^2 - v_1^2)/2$, and take $\Lambda = 0$. Thus, the resulting radion potential, $U_{\rm rad}(b) = (b_0/b)^2[U_{\rm ini}(b) + \lambda]$, keeps the minimum at $b_0 = b_i$, as defined by Eq. (4.4), and fixes $U_{\rm rad}(b_0)$ to zero. However, now $U_{\rm rad}(b)$ approaches zero asymptotically like $\sim mb_0^2 v_1^2/b^2$ for large b, and so an infinite b also appears as a possible stable configuration. Both minima are separated from each other by a potential barrier, and so there is the slight possibility of tunneling for the radion when perturbed. Of course, the height and the width of the potential barrier depend on the parameters of the theory, particularly on the size of the boundary conditions, and one may hope that some configurations with large values for mv_1^2 would ameliorate this possible problem. All these features can be observed in Fig. 3, where we have plotted this radion potential (continuous lines) in units of $mv_1^2/2$ to make it dimensionless, for different values of the v_0/v_1 ratio. Notice that, as expected, a larger v_0/v_1 ratio tends to increase the relative height and width of the potential barrier, making the potential well deeper and

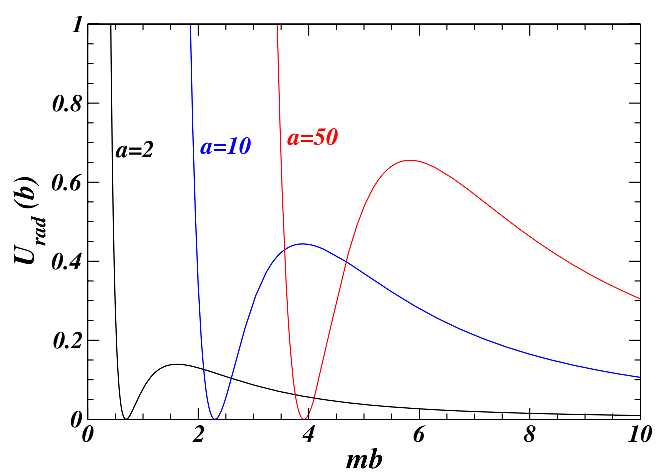

FIG. 3 (color online). Radion stabilization potential profiles generated by vacuum energy in the interval, in units of $mv_1^2/2$, for given values of $a = v_0/v_1$, as indicated.

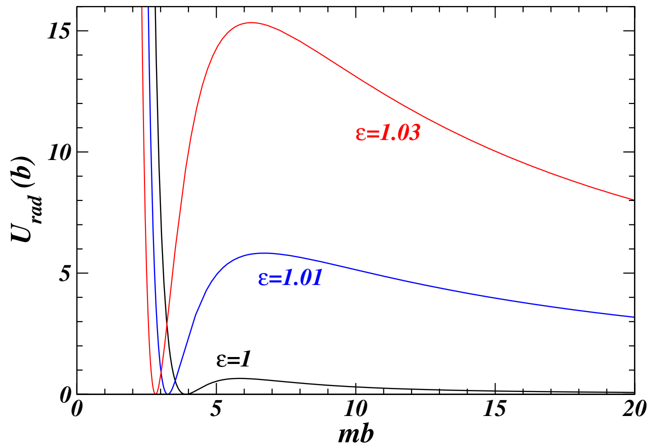

FIG. 4 (color online). Radion stabilization potentials for one extra dimension, with nonzero Λ. As before, potentials are plotted in units of $mv_1^2/2$, for the ratio $v_0/v_1 = 50$ and for slightly different values of $\lambda = -\varepsilon(v_0^2/v_1^2 - 1)$ in the same units as the potential, with ε as indicated.

narrower, and, at the same time, raising the hierarchy among b_0 and m.

Using Eq. (3.11) we calculate the radion mass for this case and obtain

$$m_\sigma^2 = \frac{4}{3}\left(\frac{m}{M_P}\right)^2 \frac{v_0^2 v_1^2}{m \parallel v_0^2 - v_1^2 \parallel}\left[\operatorname{arccosh}\left(\frac{v_0^2 + v_1^2}{2v_0 v_1}\right)\right]^2. \tag{4.6}$$

Thus, the radion mass is also sizable by adjusting the boundary conditions, just as is the size of the extra dimensions, according to Eq. (4.4). For a large v_0/v_1 ratio the above equation can be approximated as $m_\sigma^2 \approx (4/3) \times [\ln(v_0/v_1)]^2 m v_1^2/M_P^2$, which means that, if $\ln(v_0/v_1) \sim O(1)$, then $mv_1^2 > \text{TeV}^4$ to maintain $m_\sigma > 10^{-3}$ eV, but this also would indicate that b_0 cannot be too large. On the contrary, a larger hierarchy would easily provide a large radion mass, without implying a large m, thus allowing for a larger compactification radius.

In a second approach, one can use the cosmological constants to greatly improve on the potential depth, as described in the previous section. Notice, however, that this procedure will not substantially change the asymptotic behavior of the radion potential, because we shall only choose a different set of cosmological constants, keeping the functional form of the potential as given in Eq. (3.10) with $U_{\rm ini}$ replaced by Eq. (4.3). Yet, for large b we get $U_{\rm rad}(b) \sim b_0^2\Lambda/b$, where now the chosen $\Lambda = -U_{\rm eff}(b_0)$ could actually become quite large. Thus, the radion potential shall remain with two local minima, b_0 and infinity, but now with a wider and taller potential barrier in between. The corresponding effective potential does have a nontrivial minimum, as the nonzero solution for $\kappa = mb$ in the equation

$$4\lambda \sinh\kappa + m(v_0^2 + v_1^2)(\sinh 2\kappa + 2\kappa) = 4mv_0 v_1(\sinh\kappa + \kappa\cosh\kappa).$$

There is no analytical solution to the last expression, and thus, one has to proceed numerically in most cases, or at least perturbatively for small displacements. As discussed already, since we are now using $\delta\lambda < 0$, we can expect a minimum shifted to smaller values, and a tighter potential well for larger values of $|\lambda|$. All this is confirmed by the numerical analysis, as it can be checked in Fig. 4, where we have plotted the radion potential profile for the ratio $v_0/v_1 = 50$, and for some different values of λ, chosen as numerical multiples of $U_{\rm ini}(b \to \infty)$, for example.

B. The T^n/Z_2 orbifold

Let us now explore in some detail a more general example. Next, we shall consider a model where the bulk manifold is given by a T^n/Z_2 orbifold, where the Z_2 corresponds to the identification of points on the symmetric T^n torus with common radii b, according to the mapping $\vec{y} \to -\vec{y}$. To simplify matters, we will consider only the whole volume variations which do not alter this overall geometry, such that the metric on the compact space remains of the form $ds^2_{\rm compact} = b^2\delta_{ij}dy^i dy^j$, where the y_i coordinates on the torus have values in the interval $\mathcal{I} = [-1, 1]$. Of course, on the orbifold, physical compact space is smaller. It can be chosen to be represented by the reduced $[0, 1] \times \mathcal{I} \times \cdots \times \mathcal{I}$ space. In this orbifold, there are 2^n fixed points which correspond to the vertices of the unitary hypercube $\mathcal{I}_0^n = \mathcal{I}_0 \times \cdots \times \mathcal{I}_0$, where $\mathcal{I}_0 = [0, 1]$. This symmetric T^n/Z_2 orbifold has a residual discrete symmetry $\mathcal{R}^n_{\pi/2}$, given as the set of rotations by $\pi/2$ around any y_i coordinate axis. This symmetry transformation maps fixed points, located at the same distance from the origin, among themselves.

Since the potential we are to build is due to boundary conditions on the fixed points, the fact that all y directions should have the same size suggests a totally symmetric potential under the same $\mathcal{R}^n_{\pi/2}$ symmetry. Thus, in principle, only $n + 1$ boundary conditions on equal classes of fixed points can be allowed to be different, if this symmetry is to be unbroken. Moreover, we can work out our analysis considering only the contribution of the vacuum that resides on the hypercubic slice $\mathcal{I}_0^n$. The total potential energy on the orbifold shall be just a 2^{n-1} multiple of this.

The solution, ϕ, to the equation of motion (3.7) on the flat n-dimensional space we are considering can be factored as $\phi(\vec{y}) = \Pi_i^n \varphi_i(y_i)$, where each independent factor is a solution to the generic equation $\varphi_i'' - k^2\varphi_i = 0$, where $nk^2 = \kappa^2 = m^2b^2$, with the boundary conditions $\varphi_i(0) = v_{i0}$ and $\varphi_i(1) = v_{i1}$, such that the whole field configuration has boundary conditions given by products of v_i's. However, these $2n$ boundary conditions are not all independent. The $\mathcal{R}^n_{\pi/2}$ symmetry indicates that $v_{i1}v_{j0}$ is a constant for all $i \neq j$, and so both v_{i1} and v_{i0} are independent of the index. This way, only two independent boundary conditions are actually needed, which we now choose as $v_{0,1}$, and thus, all φ_i would be the same function already given in Eq. (4.1), but evaluated for the corresponding y_i coordinate: $\varphi_i(y_i) = \varphi(y_i) = Ae^{ky_i} + Be^{-ky_i}$, with the global constants $A = (v_1 - v_0 e^k)/\sinh k$ and $B = (v_0 e^{-k} - v_1)/\sinh k$. In this scenario different directions along any coordinate axis look alike for the scalar field. That is the reason why volume varies as a whole while the basic geometry stands still. We also note that $v^n_{0,1}$ should now have mass dimension $1 + n/2$ as the bulk scalar field ϕ.

The potential energy from this vacuum, as calculated in the initial frame, is given now by the general expression $U^n_{\rm ini}(b) = 2^{n-1} \times \frac{1}{2} b^{n-2} \int_0^1 dy[n(\varphi'(y))^2 + \kappa^2 \varphi^2(y)] \cdot [\int_0^1 dy \times \varphi^2(y)]^{n-1}$. After some algebra, one gets the rather complicated expression

$$U^n_{\rm ini}(b) = \frac{n^{n/2}}{2m^{n-2}} \left(\frac{(v_0^2 + v_1^2)\cosh k - 2v_0 v_1}{\sinh k} \right) \left(\frac{2v_0 v_1 (k \cosh k - \sinh k) + (v_0^2 + v_1^2)(\cosh k \sinh k - k)}{\sinh^2 k} \right)^{n-1}, \tag{4.7}$$

for which one cannot establish the existence for a minimum by exact analytical methods. A numerical analysis, however, shows that a minimum exists only for $n = 1$, which reduces to the case we discussed already in the previous section. One can get some understanding for the reasons of this fact by looking at the behavior at small and large b. For a small radius one gets $U^n_{\rm ini} \propto (1 - a^2)^2 (1 + a + a^2)^{n-1} b^{n-2}$, where $a = v_0/v_1$, such that for $n = 1$ it diverges linearly as we already know, whereas it goes to a constant for $n = 2$ and to zero with a power law for larger n. In contrast, for a large radius the potential goes exponentially to a constant value $\propto (1 + a^2)^n$. Interpolating between these two extreme values with exponentially dominating pieces, like those in the potential, leaves little room to develop any additional minimum.

As before, a minimum for the corresponding radion potential with $n > 1$ may exist for some added configuration of bulk and brane cosmological constants. Consider once more the radion potential in Eq. (3.10) with our present $U^n_{\rm ini}$. It is clear from the previous analysis that $U^n_{\rm rad} \propto {\rm const.}/b^{2+n} + \lambda/b^{2n}$, for small b, and thus, one would require $\lambda > 0$. On the other hand, at large b one gets $U^n_{\rm rad} \propto \Lambda/b^n$. This is altogether a similar behavior as the one already seen in the case of the interval for $v_0/v_1 = 1$ (the symmetric case). Nevertheless, here the conclusion arises regardless of the value of the v_0/v_1 ratio. As before, a negative Λ would be enough to get a nontrivial minimum, but at the unwanted cost of a strictly negative value for $U^n_{\rm rad}(b_0)$.

A more appealing scenario emerges if instead of λ we assume that the boundaries of the hypercube contribute to the potential energy with some surface energy, fed by $(n-2)$-brane tensions. Thus, we add a potential term similar to the one provided in Eq. (3.4). Next we consider the effective potential written as

$$U^n_{\rm eff} = \left(\frac{b_0}{b}\right) \left[\left(\frac{b_0}{b}\right)^{n-1} U^n_{\rm ini}(b) + \tau_n \right]. \tag{4.8}$$

The term between squared parentheses in the above equation still has no local minimum by itself, but now we can choose τ_n to insure that $U^n_{\rm eff}$ will have one, by using a variation of the second strategy discussed at the end of Sec. III. First, notice that $U^n_{\rm ini}/b^{n-1}$ goes as $\sim{\rm const.}/b$ for small b, so it is linearly divergent at zero. Second, the same term vanishes asymptotically as $\sim (v_0^2 + v_1^2)^n / b^{n-1}$. Hence, when shifting $U^n_{\rm ini}/b^{n-1}$ by adding a negative τ, we still get a function with no local minimum, which now, however, crosses to negative values at some point, and approaches τ for large b. Hence, the observation we made in the previous section will apply: the crossing is a fixed point under the modulation by the overall $1/b$ factor, yet to be included in order to build $U^n_{\rm eff}$. As a matter of fact, the multiplication by $1/b$ also changes the asymptotic form, and now $U^n_{\rm eff}$ shall reach zero at infinity from below, as $\sim \tau_n/b$. Therefore, a local minimum, b_0, must now emerge within the region beyond the crossing point, where $U^n_{\rm eff}$ is negative. Finally, we shall consider a positive $\Lambda = -U^n_{\rm eff}(b_0)$, to shift the minimum value of

$$U^n_{\rm rad} = \left(\frac{b_0}{b}\right)^n \left[\left(\frac{b_0}{b}\right) \left[\left(\frac{b_0}{b}\right)^{n-1} U^n_{\rm ini}(b) + \tau_n \right] + \Lambda_n \right] \tag{4.9}$$

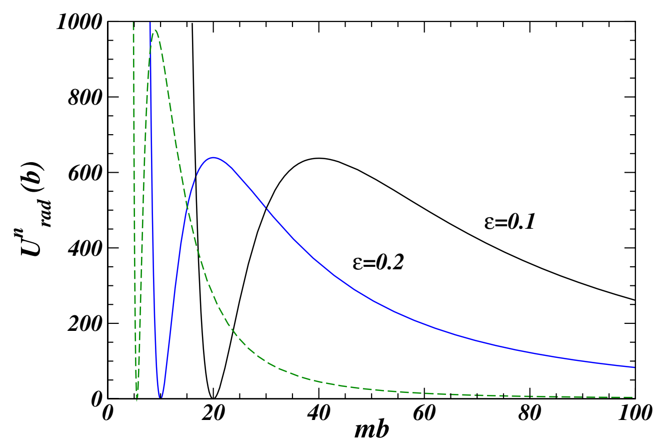

FIG. 5 (color online). Radion stabilization potentials generated for the T^n/Z_2 orbifold, in units of $n^{n/2} v_1^{2n}/2m^{n-2}$ and for $v_0/v_1 = 10$. Continuous lines plot the profile for $n = 2$ and $\tau = -\varepsilon(1 + v_0^2/v_1^2)^n$ in the same units as the potential, with ε as indicated. We also depict the profile for $n = 3$ and $\varepsilon = 0.1$ (dashed line) with the potential conveniently scaled by a factor of $1/100$.

to zero. A further contribution of λ is not required now, although it may be included, too.

As an example we have chosen $\tau_n = -\varepsilon(1 + v_0^2/v_1^2)^n \times (n^{n/2} v_1^{2n}/2m^{n-2})$ and made plots for the potential profile for $v_0/v_1 = 10$ and for $n = 2, 3$, using values of ε as shown in Fig. 5. Notice again the characteristic form of these profiles, which interpose a potential barrier between the local minimum and infinity, and whose width is actually sizable. Note also that the narrower and taller barrier in the $n = 3$ case (shown in the figure with an appropriate scaling factor to fit it within the used scale) is actually an apparent effect due to the use of a numerically larger value of τ, although we are using the same value for ε. This is also the reason why we now look at larger mb_0 values, when compared to previous figures.

V. CONCLUSIONS

Summarizing, our present work pinpoints a clear conclusion: the combination of bulk and brane cosmological constants and bulk vacuum energy from scalar fields does provide successful and manageable scenarios for the understanding of the stabilization of the radion field, within the context of the four dimensional effective theory, in flat extra dimension models. We have developed some basic strategies to handle and build radion potentials, with local minima and a zero effective cosmological constant, out of the two above-mentioned minimal ingredients.

Our analysis has been properly done in the Einstein frame, where the radion is defined as a scalar field associated with volume variations, and gravity is written in the standard form. We properly included the volumetric suppressions introduced by conformal factors in all the different contributions to the radion potential we considered. We have shown that, due to these factors, the use of a bulk cosmological constant and brane tension configurations may be enough to provide stabilization for the radion. However, for the one extra dimension case, an effective four dimensional negative cosmological constant arises.

The further addition of a nontrivial y-dependent vacuum energy introduces the required freedom to obtain working scenarios for the stabilization of the radion. These scenarios are good toy models where other common problems of dynamical stabilization could be consistently analyzed, as other moduli stabilization or metric backreactions, that we have not discussed in here. For example, a generalization of the present ideas to the stabilization of other moduli fields is, in principle, possible. A trivial extension for the T^n/Z_2 orbifold may consider a separate stabilization of each bulk direction, using scalar fields located at the different boundaries on the orbifold, such that the problem would get reduced to one dimensional cases. Other configurations may also be possible. Backreactions, on the other hand, are less trivial to analyze and still require some study.

Our results are an indication that it is well possible to build phenomenological stabilization potentials out of the most common ingredients that any bulk-brane theory could have: brane and bulk cosmological constants, and bulk scalar degrees of freedom with nontrivial bulk configurations. We made no claims on the possible size of the extra compact space, but, rather, emphasized the fact that, even though the size always appears related to the scalar mass, in our constructions there are many possible situations where the hierarchy on those parameters is conveniently sizable. Nevertheless, such freedom usually means moving such hierarchy to the boundary conditions on the scalar vacuum.

On the other hand, and mostly due to the conformal factors, all examples we have provided suffer from the same potential illness: a decompactified extra dimensional volume also appears as a plausible scenario. We have not consider, however, any string correction or quantum gravity effect in our analysis. This is due to the very nature of our effective low energy (4D) approach. We believe this problem might be ameliorated in a real quantum gravity theory calculation, and it probably should not be a matter of concern here. Moreover, close to the minimum and due to the Planck suppressions, our model provides a workable scenario on which an effective theory approach should properly describe radion physics. In particular, the approach may supply the physical radion mass, characteristic of each particular model, and certainly the profile of the radion potential close to the minimum, too.

As a final note, let us mention that, because cosmological constants contribute nontrivially to the radion potential, any redefinition of these, either introduced by hand or due to quantum contributions, may alter the stabilization of the volume in two possible ways. It may shift the minimum of the potential and introduce a nontrivial contribution to the effective 4D cosmological constant. Intriguingly, this seems to establish a connection of the cosmological constant hierarchy problem with the volume stabilization which may deserve further study.

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