Harmonic generation of gravitational wave induced Alfvén waves

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Here we consider the nonlinear evolution of Alfvén waves that have been excited by gravitational waves from merging binary pulsars. We derive a wave equation for strongly nonlinear and dispersive Alfvén waves. Because of the weak dispersion of the Alfvén waves, significant wave steepening can occur, which in turn implies strong harmonic generation. We find that the harmonic generation is saturated due to dispersive effects, and use this to estimate the resulting spectrum. Finally we discuss the possibility of observing the above process.

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I. INTRODUCTION

The launch of large projects for the detection of gravitational waves such as LIGO (Laser Interferometer Gravitational Wave Observatory) together with ambitious projects under development such as LISA (Laser Interferometer Space Antenna) [1] has increased the hope for successful detection of gravitational waves (GW's) during the next few decades, and stimulated much work (see e.g. Refs. [1,2]). Naturally the research devoted to detection concerns low amplitude GW's. In an astrophysical context close to the GW source, the gravitational waves can propagate in a plasma medium, and the amplitudes are larger. In Refs. [3–10] the authors have studied nonlinear responses to the gravitational wave by the plasma medium, although the backreaction has been neglected. The nonlinear response gives rise to effects such as parametric instabilities [5–8], large density fluctuations [3,9], photon acceleration [3], and wave collapse [10]. The application of gravitational wave processes to astrophysics has been discussed by, for example, Refs. [11–15], and to cosmology by Refs. [16–19]. A number of works studying nonlinear propagation of gravitational waves including the backreaction from the plasma have also been written, see e.g. Refs. [7,9,20].

In Ref. [10] the coupled evolution of GW's and (compressional) Alfvén waves were considered, and a condition for nonlinear wave collapse was derived. Here we develop that work, taking into account the dispersive properties of the Alfvén waves. For conditions, when wave collapse does *not* occur, we here show that the nonlinear evolution leads to significant wave steepening, and associated high harmonic generation. Furthermore, we find that the harmonic generation is saturated due to the dispersive effects associated with the Hall current. As a result, we are able to estimate the resulting wave spectrum. Because of the strong harmonic generation, it turns out that electromagnetic wave frequencies several orders of magnitudes larger than the original GW frequency can be generated. Finally we discuss the possibility that such radiation might be observable with satellite based radio arrays [21,22].

II. COUPLED ALFVÉN AND GRAVITATIONAL WAVES

The metric of a linearized gravitational wave propagating in the z-direction can be written as [23]

$$\begin{aligned} \mathrm{d}s^2 = -\mathrm{d}t^2 + [1 + h(z - ct)]\mathrm{d}x^2 \\ + [1 - h(z - ct)]\mathrm{d}y^2 + \mathrm{d}z^2, \end{aligned} \tag{1}$$

where we have assumed linear polarization, as will be justified below. For an observer comoving with the time coordinate, the natural frame for measurements is given by

$$\begin{aligned} e_0 = \partial_t, \qquad e_1 = (1 - \tfrac{1}{2}h)\partial_x, \\ e_2 = (1 + \tfrac{1}{2}h)\partial_y, \qquad e_3 = \partial_z. \end{aligned} \tag{2}$$

We now introduce the 3-vector notation such that $\boldsymbol{\nabla} \equiv (e_1, e_2, e_3)$, $\boldsymbol{E} \equiv (E^1, E^2, E^3)$ is the electric field and $\boldsymbol{B} \equiv (B^1, B^2, B^3)$ is the magnetic field. It can be shown [11] that in the frame (2) Maxwell's equations can be written

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}, \tag{3}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{4}$$

$$\partial_t \boldsymbol{E} - c^2 \boldsymbol{\nabla} \times \boldsymbol{B} = -\frac{\boldsymbol{j}_E}{\varepsilon_0} - \frac{\boldsymbol{j}}{\varepsilon_0}, \tag{5}$$

$$\partial_t \boldsymbol{B} + \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\boldsymbol{j}_B}{c\varepsilon_0}. \tag{6}$$

Here $\rho \equiv \sum_s q\gamma n$ is the charge density, q and n denote the charge and the proper particle density for a particle of species s, and $\boldsymbol{j}_E$ and $\boldsymbol{j}_B$ are the effective gravitational current densities, defined by

$$j_E^1 = j_B^2 \equiv \frac{c\varepsilon_0}{2}(E^1 - cB^2)\partial_z h, \tag{7}$$

$$j_E^2 = -j_B^1 \equiv -\frac{c\varepsilon_0}{2}(E^2 + cB^1)\partial_z h. \tag{8}$$

Next we assume the presence of a background magnetic field, $\boldsymbol{B}_0 = B_0\mathbf{e}_1$, and introduce perturbations such that $n = n_0 + \delta n$, $\boldsymbol{B} = (B_0 + B_x)\mathbf{e}_1$, $\boldsymbol{E} = E_y\mathbf{e}_2$, and $\mathbf{v} = v_y\mathbf{e}_2 + v_z\mathbf{e}_3$, corresponding to compressional Alfvén waves excited by the GW's. We also note from Ref. [3] that in the case of gravitational waves propagating in a magnetized plasma, with the magnetic field perpendicular to the direction of propagation, only the linear component of the GW polarization considered in Eq. (1) couples effectively to the electromagnetic wave. Furthermore $v_y \ll c$ (as $v_y/c \sim h$), and we therefore neglect terms of the type v_y^2 and $v_y h$, but allow for $v_z \sim c$. We will also consider slow variations such that $\partial_t \ll \omega_c \equiv qB_0/m$ for each plasma species.

Following Ref. [10], letting the variables depend on z and t, we find that the Maxwell and fluid equations can be reduced to

$$\partial_t E_y - c^2\partial_z B_x + \sum_s \frac{q}{\varepsilon_0}\gamma(n_0 + \delta n)v_y = -\frac{1}{2}E_y\partial_t h + \frac{c^2}{2}(B_0 + B_x)\partial_z h, \tag{9}$$

$$\partial_t B_x - \partial_z E_y = -\tfrac{1}{2}E_y\partial_z h + \tfrac{1}{2}(B_0 + B_x)\partial_t h, \tag{10}$$

$$\partial_t(\gamma(n_0 + \delta n)) = -\partial_z(\gamma(n_0 + \delta n)v_z), \tag{11}$$

$$\partial_t(\gamma v_y) + v_z\partial_z(\gamma v_y) = \frac{q}{m}(E_y + v_z(B_0 + B_x)), \tag{12}$$

$$\partial_t(\gamma v_z) + v_z\partial_z(\gamma v_z) = -\frac{q}{m}v_y(B_0 + B_x), \tag{13}$$

where Eqs. (11)–(13) holds for each particle species. However, for notational convenience we omit the index denoting species, as we at this stage want to cover both the case of an electron-ion plasma as well as an electron-positron plasma. The above system of Eqs. (9)–(13) should be complemented by the nontrivial part of the linearized Einstein field equations [3,20,24]

$$(c^{-2}\partial_t^2 - \partial_z^2)h = \kappa(\delta T_{11} - \delta T_{22}), \tag{14}$$

where $\kappa \equiv 8\pi G/c^4$, G is Newton's gravitational constant, and δT_{ij} are the perturbed components of the energy momentum tensor. Provided the coupled Alfvén waves and GW's move together close to the speed of light, the vacuum expressions for the effective currents (7) and (8) holds approximately. The above system of equations has previously been studied by Refs. [3,10]. In particular Ref. [10] derived a system of two coupled equations of the form

$$(\partial_t + \mathcal{U}(B_x)\partial_z)B_x = \tfrac{1}{2}B_0\partial_t h, \tag{15}$$

$$\left(\frac{1}{c^2}\partial_t^2 - \partial_z^2\right)h = -2\kappa\frac{B_0 B_x}{\mu_0}, \tag{16}$$

where $\mathcal{U}(B_x) = c - (c^3/2C_A^2)(B_0/(B_0 + 2B_x))^{3/2}$, $C_A \equiv c(\sum_s \omega_p^2/\omega_c^2)^{-1/2}$ is the nonrelativistic Alfvén velocity, and $\omega_p = (n_0 q^2/\epsilon_0 m)^{1/2}$ is the plasma frequency for each species. In contrast to Refs. [3,10] we will here concentrate on the evolution of the Alfvén waves at a distance further from the gravitational source, where the Alfvén-GW coupling is of less importance, and the GW source terms for the Alfvén waves can be neglected. Thus from now on we will assume the perturbations to be of the form $\psi \approx \psi(z - V_A t)$, where $V_A = C_A/\sqrt{1 + C_A^2/c^2}$ is the (roughly constant) relativistic Alfvén velocity, which allows us to use $\partial_t \approx -V_A\partial_z$. With these approximations we can reduce the system (9)–(13) to

$$(\partial_t^2 - c^2\partial_z^2)B_x + \sum_s \frac{mn_0}{\varepsilon_0}V_A\partial_z\left[\frac{\partial_z(\gamma v_z)}{B_0 + B_x}\right] = 0, \tag{17}$$

$$(v_z - V_A)\partial_z(\gamma v_y) - \frac{q}{m}[v_z(B_0 + B_x) - V_A B_x] = 0, \tag{18}$$

$$(v_z - V_A)\partial_z(\gamma v_z) + \frac{q}{m}v_y(B_0 + B_x) = 0. \tag{19}$$

Since $\omega_c^{-1}\partial_t \ll 1$ and $\gamma v_y/c \ll 1$, Eq. (18) can be used to obtain an approximate expression of v_z,

$$v_z = V_A\frac{B_x}{B_0 + B_x}, \tag{20}$$

which in turn can be used together with (17) to express the wave equation as

$$(\partial_t^2 - c^2\partial_z^2)B_x + \sum_s \frac{mn_0}{\varepsilon_0}V_A^2\partial_z \times\left[\frac{B_0\partial_z B_x}{[B_0^2 + 2B_0B_x + (1 - V_A^2/c^2)B_x^2]^{3/2}}\right] = 0. \tag{21}$$

By using $\partial_t^2 - c^2\partial_z^2 \approx -(c + V_A)(\partial_t + c\partial_z)\partial_z$ we obtain

$$\partial_t B_x + \mathcal{V}(B_x)\partial_z B_x = 0, \tag{22}$$

where

$$\mathcal{V}(B_x) = \left\{c - (c - V_A)\left[1 + 2\frac{B_x}{B_0} + \left(1 - \frac{V_A^2}{c^2}\right)\frac{B_x^2}{B_0^2}\right]^{-3/2}\right\}. \tag{23}$$

We note here that the velocity $\mathcal{V}(B_x)$ agrees with the velocity $\mathcal{U}(B_x)$ in Eq. (15) (used previously by Ref. [10]) to first order in c/C_A. Expression (22) is slightly more accurate, however, as $c/C_A \ll 1$ has not been used in the later derivation.

Making the transformation $t = \tau$, $z = \zeta + \int_0^\tau \mathcal{V}(B(\zeta, \tau'))d\tau'$ we have $\partial_t \to \partial_\tau - (\mathcal{V}(B_x)/R)\partial_\zeta$ and $\partial_z \to (1/R)\partial_\zeta$, where $R \equiv 1 + \int_0^\tau(\partial\mathcal{V}(B_x)/\partial\zeta)d\tau'$. As a consequence the general solution of Eq. (22) can be written $B_x(z, t) = B_x(\zeta)$, which formally shows that wave steepening continues until wave breaking occurs, such that $R \to 0$

and $\partial_z \to \infty$ at some point for a finite time. Thus, in order to include dispersive effects that will prevent wave breaking we will need a more accurate expression than Eq. (20) for v_z. Dispersive effects are then included by keeping terms of a higher order in an $\omega_c^{-1}\partial_t$ expansion. How to implement this is to some extent dependent on whether we study an electron-ion plasma (in which case it is the higher order expression for the ion polarization drift that first leads to wave dispersion), or whether we consider an electron-positron plasma (in which case electron and positron motion contributes to wave dispersion simultaneously). For the astrophysical applications to be considered below, an electron-ion plasma is more appropriate, and thus we will focus on this case in the remainder of the article, using index i to denote ion quantities below. The only consequence for the calculations that has been made above is that we can simplify the expression for the Alfvén velocity slightly, such that $C_A = c(\sum_s \omega_p^2/\omega_c^2)^{-1/2} = (B_0^2/\mu_0 m_i n_0)^{1/2}$. Next, using Eq. (20) as an approximation of v_{zi} in Eq. (19) gives

$$v_{yi} = \frac{m_i}{q_i}\left(\frac{B_0^2 V_A^2}{B_0 + B_x}\right)\frac{\partial_z B_x}{[B_0^2 + 2B_0 B_x + (1 - \frac{V_A^2}{c^2})B_x^2]^{3/2}}, \tag{24}$$

which can be inserted into Eq. (18), resulting in a more accurate expression for v_{zi},

$$v_{zi} = \frac{V_A B_x}{B_0 + B_x} - \frac{V_A^3 B_0^3}{(B_0 + B_x)^2}\left(\frac{m_i}{q_i}\right)^2 \times \left\{\frac{\partial_z^2 B_x}{[B_0^2 + 2B_0 B_x + (1 - \frac{V_A^2}{c^2})B_x^2]^2} - \frac{4[B_0 + (1 - \frac{V_A^2}{c^2})B_x]\{\partial_z B_x\}^2}{[B_0^2 + 2B_0 B_x + (1 - \frac{V_A^2}{c^2})B_x^2]^3}\right\}. \tag{25}$$

This corrected expression of v_{zi} combined with Eq. (17) results in a wave equation of the form

$$\partial_t B_x + \mathcal{V}(B_x)\partial_z B_x + \frac{c - V_A}{B_0 + B_x}\left(\frac{V_A}{\omega_{ci}}\right)^2 \partial_z \times \left\{\frac{B_0^7}{(B_0 + B_x)}\left[\frac{\partial_z^2 B_x}{g^5(B_x)} - \frac{4[B_0 + (1 - \frac{V_A^2}{c^2})B_x](\partial_z B_x)^2}{g^7(B_x)}\right]\right\} = 0, \tag{26}$$

where $\mathcal{V}(B_x)$ is given by (23) and the auxiliary function $g(B_x)$ is defined by

$$g(B_x) \equiv \left[B_0^2 + 2B_0 B_x + \left(1 - \frac{V_A^2}{c^2}\right)B_x^2\right]^{1/2}. \tag{27}$$

As we can see, Eq. (26) now describes the fully nonlinear evolution combined with dispersive effects. In the weakly nonlinear limit this reduces to the celebrated Korteweg de Vries (KdV) equation,

$$\partial_t B_x + V_A \partial_z B_x + 3(c - V_A)\frac{B_x}{B_0}\partial_z B_x + (c - V_A)\frac{V_A^2}{\omega_{ci}^2}\partial_z^3 B_x = 0. \tag{28}$$

As is well known [25], for vanishing boundary conditions the general solution to the KdV equation involves a train of solitons, where the steepening effects due to nonlinearity are balanced by the dispersive effects due to the last term of Eq. (28). Our main interest here is the steepening of an initially sinusoidal wave profile. A study of the KdV equation for such initial conditions shows that the wave steepening induced by the nonlinearity leads to a harmonic content. However, as we are interested in generation of *large* harmonic content, with wave steepening occurring on a comparatively fast time scale, we need to study the fully nonlinear dispersive Eq. (26) rather than Eq. (28).

An initially sinusoidal wave described by Eq. (26) can at first be approximated with Eq. (22) if the initial wave frequency ω fulfills $\omega \ll \omega_{ci}$. As a consequence the sinusoidal profile will undergo wave steepening until dispersive effects become important leading to a saturation of the steepening and the approach to a steady state profile, as can be described by Eq. (26). Thus we are looking for solutions to Eq. (26) that are periodic and static in a frame moving with the wave. The general methods for finding solutions of this type is described in some detail in Ref. [26]. First we let $\partial_t \to -V_{An}\partial_z$, where V_{An} is a constant representing the nonlinearly modified Alfvén velocity. Integrating the resulting equation with respect to z, we get an equation that is analogous to a particle moving in a potential [26]. For appropriate choices of integration constants, the motion corresponds to a particle moving in a potential well, in which case we obtain solutions of the desired type, i.e. $B_x = B_x(z - V_{An}t)$, where B_x is a periodic function of the argument. The resulting differential equation for B_x has to be solved numerically. The degree of wave steepening in the resulting wave profile depends on the amplitude of the oscillations as well as on ω/ω_{ci}, parameters which are related to the integration constants. For given integration constants, the corresponding physical parameters can be calculated straightforwardly, and thereby solutions with parameters of physical interest can be found by some trial and error. Solutions of astrophysical relevance showing significant wave steepening corresponding to large harmonic content of the wave profile will be presented in the preceding section.

III. ASTROPHYSICAL EXAMPLE

In this section we are going to investigate the nonlinearly modified Alfvén waves and to what extent these can be observed. Based on the fact that electromagnetic signals are easier to detect than gravitational ones of the same power, we will be looking for a spectral signature of GW-generated Alfvén waves. As is well known, the gravita-

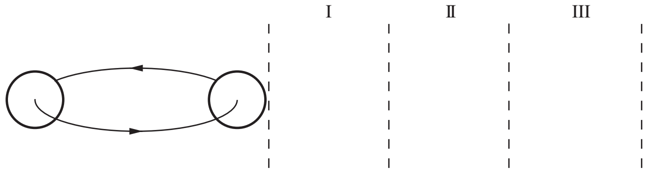

FIG. 1. The neighborhood of the binary system is divided into three regions: region I ($10R_S - 30R_S$), the energy conversion zone; region II ($30R_S - 3500R_S$), the nonlinearity enhancement zone; and region III ($3500R_S - 7000R_S$), the wave steepening zone.

tional sources have high powers only up to frequencies of the order ~1 kHz, which is much below the lowest observable radio frequencies [21,22]. Thus for GW-induced radio wave detection to be possible, we will need a mechanism to convert some fraction of the electromagnetic energy to higher frequencies. As we will demonstrate below, the processes studied in Sec. II can provide the basis for such mechanisms, and thus lead to a large increase of the frequency for a significant fraction of the electromagnetic spectrum.

Below we will study a concrete example. As a source of gravitational radiation we consider a binary system. At least one of the objects should have a strong magnetic field (in order to make the Alfvén phase velocity close to c), and the objects should be compact (as to make the gravitational wave frequency and amplitude before merging reasonably large). For definiteness we study a system consisting of two neutron stars of equal mass $M_\odot$, separated by a distance of $20R_S$, where $R_S = 2GM_\odot/c^2 \approx 3$ km. Furthermore, the surface magnetic field of each neutron star is assumed to be 4×10^6 T. The surroundings of the binary system can loosely be divided into three regions (Fig. 1), depending on which physical mechanism is dominating.

A. Energy conversion zone

The interval $10R_S - 30R_S$ from the center of mass roughly constitutes region I, which is the region where most of the gravitational energy is gained by the Alfvén wave. Using a Newtonian approximation, with $d = \alpha R_S$ and $r = \beta R_S$, it is straightforward to show that $|h| \sim (2\alpha\beta)^{-1}$, where d is the separation distance between the binary objects and r is the observation distance from the center of mass of the system. In order to obtain an estimate of the amplitude of the generated EM wave, we note that in the near zone (i.e. where the magnetic pulsar magnetic field decays as a dipole) the plasma density is low, and we expect that for our purposes here, we can approximate the medium as vacuum. A calculation of the electromagnetic amplitude generated by the GW during such circumstances has been done by Ref. [3]. Combining the above expression for the gravitational wave amplitude at given distances with Eq. (14) of Ref. [3], the GW-induced magnetic field amplitude δB can thus be estimated. The result is

$$\frac{\delta B}{B_0} \sim 1.8 \times 10^{-4}, \tag{29}$$

at the of end region I.

B. Nonlinearity enhancement zone

In region II (approximate interval $30R_S - 3500R_S$ from the center of mass) the ratio determining the degree of nonlinearity, $\delta B/B_0$, is increasing quadratically. The reason is that δB suffers spherical attenuation (due to the high frequency) whereas B_0 decays as a dipole field until the light cylinder is reached. The outer bound of region II, with a light cylinder at $3500R_S$, is somewhat artificially chosen, and corresponds to a pulsar period of 35 ms. Note, however, that the nonlinearity enhancement mechanism is sufficient to reach the strongly nonlinear regime (i.e. $\delta B/B_0 \sim 1$) within region II even for faster pulsars.

C. Wave steepening zone

In region III (approximate interval $3500R_S - 7000R_S$), the amplitude of the Alfvén waves is strongly nonlinear, and thus pronounced wave steepening will occur here, as described by Eq. (28). The characteristic propagation length L_p for large steepening to occur is of the order

$$L_p \sim \frac{\lambda}{2}\frac{V_A}{\delta V_A}, \tag{30}$$

where δV_A is the velocity difference between the maximum and minimum wave velocity within the amplitude wave profile with dispersive effects neglected. At the beginning of region III we have $B_0 \simeq 3 \times 10^{-3}$ T, assuming a moderate plasma density, of the order $n_0 \simeq 10^{10}$ m^{-3}, gives an Alfvén velocity $V_A \simeq 0.9c$. Furthermore, a relatively strong nonlinearity with $\delta B/B_0 = 0.25$, consistent with the estimate made in Sec. III B, corresponds to $\delta V_A \simeq 0.2c$, which combined with a wavelength $\lambda \simeq 1.7 \times 10^6$ m results in a steepening distance located reasonably well within region III.

Because of the nonlinear velocity dependence of the amplitude, wave steepening continues until the dispersive effects become important. As the initial GW-generated angular frequency is $\omega \approx 1.1 \times 10^3$ s^{-1}, whereas $\omega_{ci} \approx 2.7 \times 10^5$ s^{-1} in our case, and the amplitude is strongly nonlinear, the steady state profiles are dramatically changed as compared to the initially sinusoidal wave profiles. The steady state profile derived numerically from Eq. (28) corresponding to $\omega/\omega_{ci} = 1/240$ and $\delta B/B_0 \approx 0.25$ is shown in Fig. 2. Since, the temporal and spatial derivatives are orders of magnitudes larger than the corresponding values for the initial profile, the spectral content has changed dramatically from the initially quasimonochromatic wave. The spectrum corresponding to the same data as in Fig. 2 is shown in Fig. 3. There it is seen that the highest harmonic generation of Alfvén waves may approach the 100 kHz range. Unfortunately, this is still not

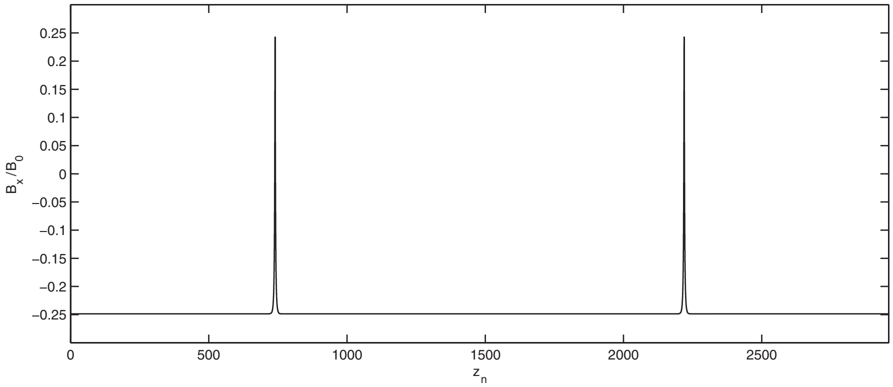

FIG. 2. The wave profile for a moderately nonlinear amplitude ($\delta B/B_0 \approx 0.25$) as a function of the normalized distance $z_n = z\omega_{ci}/V_A$. The wavelength $\lambda_n \approx 1500$ corresponds to a ratio $\omega/\omega_{ci} \approx 1/240$.

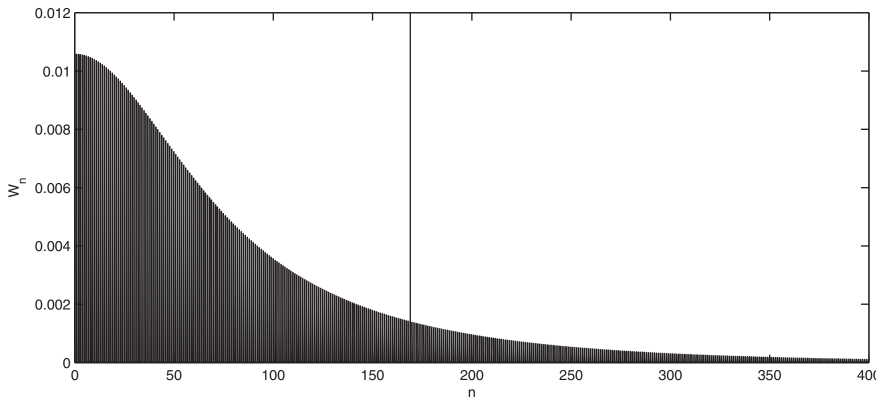

FIG. 3. The normalized energy density (total energy density = 1) content W_n of the wave profile as a function of the harmonic number n corresponding to the wave profile in Fig. 2. The vertical line around $n \approx 170$ separates the harmonics exceeding the lower sensitivity bound of SIRA from the frequencies that are too low to be detected. For our given example, the amount of energy that can be detected is of the order of 10% of the total wave energy.

enough to reach the radio window for earth based antennas, where the lower cutoff lies around 20 MHz. However, proposals such as the Astronomical Low Frequency Array (ALFA) [21] and more recently the Solar Imaging Radio Array (SIRA) [22] show that satellite based antennas can provide an opening for observation, since such arrays are planned for a sensitivity down to frequencies of the order 30 kHz. As shown in Fig. 3, the spectral content exceeding this frequency is of the order 10% of the total wave energy density. Thus the astrophysical processes outlined here in Sec. III opens for the possibility to correlate and interpret GW observations made by LIGO or LISA with radio wave observations made with SIRA.

IV. SUMMARY AND DISCUSSION

We have considered the coupling of GW's and Alfvén waves in a magnetized plasma. The dependence of the Alfvén velocity on the magnetic field amplitude makes the evolution equation for the Alfvén waves strongly nonlinear. As described in Sec. II, GW-generated Alfvén waves are subject to nonlinear wave steepening, which is directly associated with harmonic generation. In order to find a saturation mechanism for the steepening process, we have included dispersive effects in our model by solving the momentum equation to third order in an ∂_t/ω_c expansion. As demonstrated in Sec. III C, the high amplitude steady state solutions for the Alfvén waves show strong harmonic generation, which for sufficient amplitude does not saturate until the frequencies of the harmonics approach the cyclotron frequency of the ions.

As a specific example we have considered a case where the GW's associated with a neutron star-pulsar merging generate strongly nonlinear Alfvén waves, resulting in high harmonic generation of the Alfvén waves. As shown, the original GW-frequencies of the order ~200 Hz may generate signals with harmonic number $n \sim 200$ or even larger. Unfortunately this is not sufficient to reach the radio window for earth based detection, but proposals for satellite based radio observations [22] that can detect radio signals down to ~30 kHz could make observations possible. This opens for the future possibility to correlate GW observations made by, for example, LISA [1] with radio wave observations made with SIRA [22], to make detailed comparison with theories for the coupled electromagnetic-gravitational evolution. The process described here can be even more effective than in our example, if pulsars with stronger magnetic fields are considered.

Finally we should point out that for the Alfvén waves generated in our example to travel through interstellar distances, they must be more or less decoupled from the medium to become ordinary radio waves. For this to happen, the density should fall off with distance sufficiently fast after the wave steepening region in the previous example, such that the plasma frequency falls beyond the highest parts of the wave frequency before the cyclotron frequency does so. In our example this is fulfilled, since the highest harmonic frequencies can exceed the plasma frequency already within the wave steepening region. However, for an accretion disc that is still thick at a distance when the magnetic field falls below the wave frequency, cyclotron damping could limit the possibilities for radio wave detection to a significant extent.

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[1] M. Maggiore, Phys. Rep. **331**, 283 (2000); see also http://www.ligo.caltech.edu/ and http://lisa.jpl.nasa.gov/.

[2] B. F. Schutz, Classical Quantum Gravity **16**, A131 (1999).

[3] G. Brodin, M. Marklund, and M. Servin, Phys. Rev. D **63**, 124003 (2001).

[4] G. Brodin and M. Marklund, Phys. Rev. Lett. **82**, 3012 (1999).

[5] D. Papadopoulos, N. Stergioulas, L. Vlahos, and J. Kuijpers, Astron. Astrophys. **377**, 701 (2001).

[6] G. Brodin, M. Marklund, and P. K. S. Dunsby, Phys. Rev. D **62**, 104008 (2000).

[7] A. B. Balakin, V. R. Kurbanova, and W. Zimdahl, J. Math. Phys. (N.Y.) **44**, 5120 (2003).

[8] M. Servin, G. Brodin, M. Bradley, and M. Marklund, Phys. Rev. E **62**, 8493 (2000).

[9] Yu G. Ignat'ev, Phys. Lett. A **230**, 171 (1997).

[10] A. Källberg, G. Brodin, and M. Bradley, Phys. Rev. D **70**, 044014 (2004).

[11] M. Marklund, G. Brodin, and P. K. S. Dunsby, Astrophys. J. **536**, 875 (2000).

[12] J. Moortgat and J. Kuijpers, Astron. Astrophys. **402**, 905 (2003).

[13] H. J. M. Cuesta, Phys. Rev. D **65**, 064009 (2002).

[14] H. Isliker, I. Sandberg, and L. Vlahos, Phys. Rev. D **74**, 104009 (2006).

[15] J. Moortgat and J. Kuijpers, Mon. Not. R. Astron. Soc. **368**, 1110 (2006).

[16] D. Papadopoulos, Classical Quantum Gravity **19**, 2939 (2002).

[17] M. Marklund, P. K. S. Dunsby, and G. Brodin, Phys. Rev. D **62**, 101501 (2000).

[18] P. A. Hogan and E. M. O'Shea, Phys. Rev. D **65**, 124017 (2002).

[19] A. Kuiroukidis, K. Kleidis, D. B. Papadopoulos, and L. Vlahos, Astron. Astrophys. **471**, 409 (2007).

[20] M. Servin and G. Brodin, Phys. Rev. D **68**, 044017 (2003).

[21] D. L. Jonews *et al.*, Adv. Space Res. **26**, 743 (2000).

[22] R. J. MacDowall *et al.*, Proc. SPIE-Int. Soc. Opt. Eng. **5659**, 284 (2005).

[23] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1975).

[24] L. P. Grishchuk and A. G. Polnarev, *General Relativity and Gravitation*, edited by A. Held (Plenum, New York, 1980), Vol. 2, pp 416–430.

[25] R. K. Dodd, J. C. Eilbeck, J. D. Gibbon, and H. C. Morris, *Solitons and Nonlinear Wave Equations* (Academic Press, London, 1982).

[26] P. K. Shukla, *Nonlinear Waves*, edited by L. Debnath (Cambridge University Press, New York, 1983), Chap. 11.