

Circular polarization of primordial gravitational waves in string-inspired inflationary cosmologyMasaki Satoh,^{1,*} Sugumi Kanno,^{2,†} and Jiro Soda^{1,‡}¹*Department of Physics, Kyoto University, Kyoto 606-8501, Japan*²*Department of Physics, McGill University, Montréal, QC H3A 2T8, Canada*

(Received 3 July 2007; published 23 January 2008)

We study a mechanism to produce the circular polarization of primordial gravitational waves. The circular polarization is generated during the superinflation driven by the Gauss-Bonnet term in the string-inspired cosmology. The instability in the tensor mode caused by the Gauss-Bonnet term and the parity violation due to the gravitational Chern-Simons term are the essential ingredients of the mechanism. We also discuss detectability of the produced circular polarization of gravitational waves. It turns out that the simple model of single-field inflation contradicts cosmic microwave background (CMB) observations. To circumvent this difficulty, we propose a two-field inflation model. In this two-field model, the circular polarization of gravitational waves is created in the frequency range designed by the big-bang observer (BBO) or the deci-hertz gravitational-wave observatory (DECIGO).

DOI: [10.1103/PhysRevD.77.023526](https://doi.org/10.1103/PhysRevD.77.023526)

PACS numbers: 98.80.Cq

I. INTRODUCTION

It is widely believed that the most promising candidate for a unified theory including quantum gravity is superstring theory. It is therefore interesting to prove or disprove superstring theory from an observational point of view. To this end, primordial gravitational waves have been considered as the most efficient probe, since the gravitational waves can carry the information of the Universe at the Planck time.

In this paper, we focus on a robust prediction of superstring theory, namely, the parity violation due to the gravitational Chern-Simons term. In fact, the Chern-Simons term appears in the Green-Schwarz mechanism which is necessary to cancel the anomaly in the theory [1]. It also arises as a string correction [2]. Intriguingly, the existence of the Chern-Simons term can be probed by the primordial gravitational waves. This is because the Chern-Simons term can generate the circular polarization of gravitational waves through the parity violation, while it is difficult to find other effects to produce the circular polarization of primordial gravitational waves. Hence, if we detect the circular polarization in the primordial gravitational waves, it would be a strong indication of existence of the Chern-Simons term in the early universe. Thus, it can be interpreted as an evidence of superstring theory. In addition to this fundamental motivation, there is a phenomenological interest in Chern-Simons gravity as an alternative theory of gravity [3]. Even from this phenomenological point of view, the circular polarization of gravitational waves deserves detailed investigation.

Concerning the issue if the Chern-Simons term can produce the primordial gravitational waves with circular polarization, there has been already several works [4–6]. It

turned out that, however, there exists no observable amount of circular polarization of gravitational waves [7,8], if the background spacetime undergoes the standard slow-roll inflation. The purpose of this paper is to point out that this disappointing result is reversed if we properly take into account another stringy effect. The point is that the Chern-Simons term is not the only term that could be induced by the stringy corrections. There is another term, the so-called Gauss-Bonnet term. If we consider the Chern-Simons term, it would be mandatory to incorporate the Gauss-Bonnet term. Hence, we consider both terms, the Gauss-Bonnet and Chern-Simons terms, and study a possibility to have the circular polarization of primordial gravitational waves. The reason we expect a different result from the one obtained from the standard slow-roll inflation is the existence of the superinflationary phase induced by the Gauss-Bonnet term [9–13]. And more importantly, as was first discovered in [14–16], there exists an instability in gravitational-wave modes during the superinflationary stage. It is this instability that generates the circular polarization of primordial gravitational waves. In fact, we show the primordial gravitational waves are fully polarized due to the Gauss-Bonnet term. We also discuss the detectability of the polarization of gravitational waves. The detectability of circular polarization of gravitational waves depends on when the superinflation occurs. Since the curvature perturbation has a blue spectrum in the superinflationary regime, we need to assume the superinflation occurs in the late stage of the slow-roll inflation. As a realization, we propose a two-field inflation model where the circular polarization of gravitational waves could be produced in the BBO [17] or DECIGO [18] frequency range.

The organization of this paper is as follows. In Sec. II, we present basic equations. First, we study the evolution of the background spacetime in the Gauss-Bonnet–Chern-Simons gravity. For the background evolution, the Gauss-Bonnet term is crucial, while the Chern-Simons term plays no role. We show there are two types of inflation, namely,

*satoht@tap.scphys.kyoto-u.ac.jp

†sugumi@hep.physics.mcgill.ca

‡jiro@tap.scphys.kyoto-u.ac.jp

the standard slow-roll inflation and the superinflation. Next, we derive equations for gravitational waves. Here, the Chern-Simons term plays an important role. In Sec. III, we explain the essential mechanism to produce the circular polarization of gravitational waves. We quantize the gravitational waves in the superinflationary background and calculate the degree of the circular polarization of gravitational waves. There, we will show that the significant circular polarization is created due to the instability caused by the Gauss-Bonnet term. We note that the model presented in this section is just for illustration. More realistic modes are considered in the next section. In Sec. IV, we discuss the detectability of the circular polarization generated during the superinflationary regime. We point out the defect of the single-field inflation model. Then, as a model consistent with current observations, we propose a two-field inflation model where the Gauss-Bonnet and Chern-Simons term couple with the inflaton only in the second stage of the inflation. The final section is devoted to the conclusion.

II. BASIC EQUATIONS IN GAUSS-BONNET-CHERN-SIMONS GRAVITY

Before discussing primordial gravitational waves from the quantum fluctuation during the inflation, we need to reconsider the chaotic inflationary scenario driven by an inflaton ϕ with a potential $V(\phi)$. Although the Chern-Simons term does not alter the background geometry, the Gauss-Bonnet term changes the inflationary scenario at the early stage of the inflation. The change would be important for the primordial gravitational waves.

We start with the action motivated from string theory given by [9]

$$S = \frac{1}{2} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left[\frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi + V(\phi) \right] - \frac{1}{16} \int d^4x \sqrt{-g} \xi(\phi) R_{\text{GB}}^2 + \frac{1}{16} \int d^4x \sqrt{-g} \omega(\phi) R \tilde{R}, \quad (1)$$

where the first term of action is the Einstein-Hilbert term and g is the determinant of the metric $g_{\mu\nu}$. We have set the unit $8\pi G = 1$. In the above action (1), we have taken into account the Gauss-Bonnet term

$$R_{\text{GB}}^2 = R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} - 4R^{\alpha\beta} R_{\alpha\beta} + R^2 \quad (2)$$

and the Chern-Simons term

$$R \tilde{R} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta}{}^{\rho\sigma}, \quad (3)$$

where $\epsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita tensor density. We have also allowed the coupling of the inflaton field both to the Gauss-Bonnet $\xi(\phi)$ and Chern-Simons terms $\omega(\phi)$. Otherwise, these topological terms vanish identically. It

should be noted that, as is well known, the Chern-Simons term does not contribute to the dynamics of the isotropic and homogeneous universe.

From now on, for simplicity, we consider a typical potential, $V = 1/2m^2\phi^2$. We also specify the coupling function $\xi(\phi) = \omega(\phi) = 16\phi^4$ for concreteness, although we keep them in the formula as far as we can. For other coupling functions, the analysis is similar and easy to perform.

A. Background spacetime

Let us consider the background spacetime with spatial isotropy and homogeneity. Then, the metric is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) [-d\eta^2 + \delta_{ij} dx^i dx^j]. \quad (4)$$

Here, we have also assumed the flat space and used the conformal time η . Taking the variations of the action (1), we have equations

$$3\mathcal{H}^2 = \frac{1}{2} \phi'^2 + \frac{1}{2} m^2 a^2 \phi^2 + \frac{3}{2a^2} \mathcal{H}^3 \xi', \quad (5)$$

$$(2\mathcal{H}' + 3\mathcal{H}^2) \left(1 - \frac{1}{2a^2} \mathcal{H} \xi' \right) + \mathcal{H}^2 \left(1 + \frac{\mathcal{H}}{2a^2} \xi' - \frac{1}{2a^2} \xi'' \right) - m^2 a^2 \phi^2 = 0, \quad (6)$$

$$\phi'' + 2\mathcal{H} \phi' + \frac{3}{2a^2} \mathcal{H}^2 \mathcal{H}' \xi_{,\phi} + m^2 a^2 \phi = 0, \quad (7)$$

where we have defined $\mathcal{H} = a'/a$. Here, the prime denotes the derivative with respect to the conformal time η .

In the conventional case, namely, without the Gauss-Bonnet term, the Hubble friction in the equation of motion for the scalar field makes the rolling of the scalar field slow. Then, the slow-roll inflation is soon realized for the initial condition with a large Hubble. However, in the presence of the Gauss-Bonnet term, the force due to the Gauss-Bonnet term becomes dominant for a large value of ϕ . If the scalar field starts with a negative value, the force term accelerates the scalar field and makes the kinetic term dominant. Hence, the situation, $\mathcal{H}^2 \ll \phi'^2$, $m^2 a^2 \phi^2 \ll \phi'^2$, is realized. Thus, Eqs. (5) and (6) become

$$a^2 \phi'^2 + 3\mathcal{H}^3 \xi' = 0, \quad (8)$$

$$(2\mathcal{H}' + 3\mathcal{H}^2) \xi' + \mathcal{H}(\xi'' - \mathcal{H} \xi') = 0. \quad (9)$$

The scalar field ϕ rolls down from the negative side towards zero according to the above Eqs. (8) and (9). Now, it is easy to solve Eqs. (8) and (9) as

$$\phi = -\sqrt{15/16}(-\eta)^{5/6}, \quad a(\eta) = (-\eta)^{-1/6}, \quad (10)$$

$$\mathcal{H} = \frac{1}{(-6\eta)}, \quad \eta < 0.$$

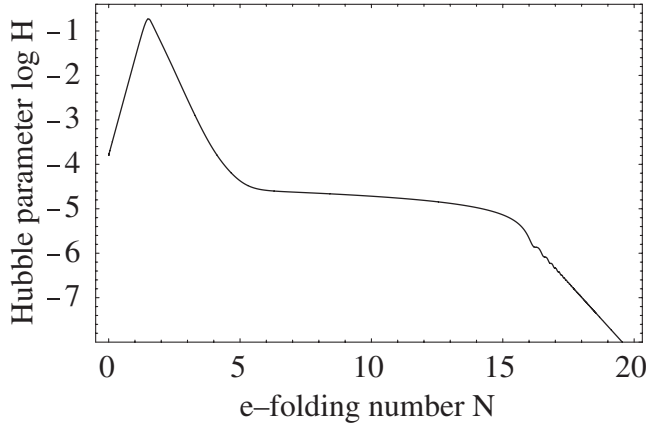


FIG. 1. A typical evolution of the background spacetime is numerically calculated and displayed. A short period of the superinflationary phase is followed by a long period of the slow-roll inflationary phase.

We note that the superinflation $H' > 0$ is realized in this phase. Here, $H = \mathcal{H}/a$ is the Hubble parameter. As the scalar field rolls down, the Gauss-Bonnet term decreases. Eventually, the conventional Hubble friction overcomes the Gauss-Bonnet effect and the slow-roll inflation commences. A typical evolution of the spacetime is shown in Fig. 1. In the superinflationary phase $H' > 0$, the weak energy condition is violated. Hence, the system may show the instability. Of course, as you can see in Fig. 1, this instability is a transient one. To make this point clear, we rewrite the equations of motion (7) as the autonomous system for ϕ and H and plot the phase flow diagram in Fig. 2. This diagram shows that the superinflationary phase will be followed by the standard phase where the Hubble parameter is decreasing.

In the next section, we will show that the transient superinflation phase gives rise to the circular polarization of gravitational waves.

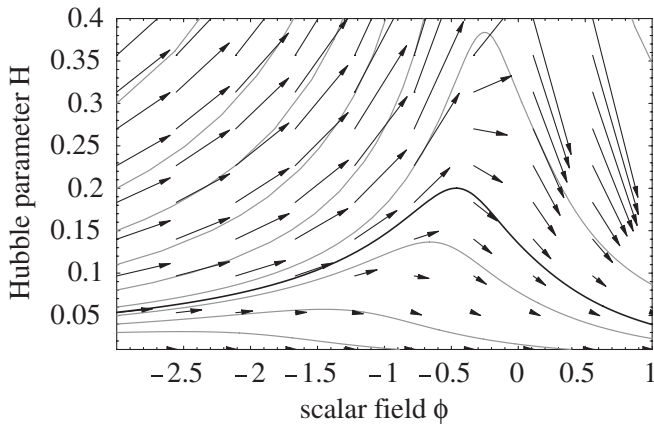


FIG. 2. Phase diagram around $\phi \simeq 0$ is drawn. The thick line in the diagram denotes the trajectory corresponding to Fig. 1. This tells that the asymptotic solution breaks down before reaching $\phi = 0$, and thus the singularity is avoided.

B. Action for gravitational waves

In this subsection, we will deduce the quadratic action for the gravitational waves from the action (1). Let us consider the tensor perturbation

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta)[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \quad (11)$$

where h_{ij} satisfies the transverse-traceless conditions $h_{ij,j} = h_{ii} = 0$. Substituting the metric into the Einstein action with the Gauss-Bonnet term, we obtain the quadratic action [15,19,20]

$$\begin{aligned} & \frac{1}{2} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left[\frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi + V(\phi) \right] \\ & - \frac{1}{16} \int d^4x \sqrt{-g} \xi(\phi) R_{\text{GB}}^2 \\ & = \frac{1}{8} \int d^4x a^2 \left[\left(1 - \frac{\mathcal{H} \xi'}{2a^2} \right) h'^{ij} h'_{ij} \right. \\ & \quad \left. - \left(1 + \frac{\mathcal{H} \xi'}{2a^2} - \frac{\xi''}{2a^2} \right) h^{ijkl} h_{ijkl} \right]. \end{aligned} \quad (12)$$

In addition to these terms, the parity violating Chern-Simons term gives

$$\begin{aligned} & \frac{1}{16} \int d^4x \sqrt{-g} \omega(\phi) R \tilde{R} \\ & = -\frac{1}{16} \int d^4x \omega' \epsilon^{ijk} (h'_{ai} h'^a{}_{kl} - h_{ai|b} h^a{}_{|b}{}^l{}_j), \end{aligned} \quad (13)$$

where the stem $|$ denotes a derivative with respect to the spatial coordinates. Here, we have used the convention $\epsilon^{0ijk} = -\epsilon^{ijk}$ with $\epsilon^{123} \equiv 1$. It is convenient to expand h_{ij} by plane waves

$$\frac{h_{ij}(\eta, \mathbf{x})}{\sqrt{2}} = \sum_{A=+, \times} \int \frac{d^3k}{(2\pi)^3} \psi_{\mathbf{k}}^A(\eta) e^{i\mathbf{k} \cdot \mathbf{x}} p_{ij}^A, \quad (14)$$

where we have introduced the circular polarization tensor p_{ij}^A ($A = R, L$) defined by

$$p_{ij}^R \equiv \frac{1}{\sqrt{2}}(p_{ij}^+ + ip_{ij}^\times), \quad p_{ij}^L \equiv \frac{1}{\sqrt{2}}(p_{ij}^+ - ip_{ij}^\times). \quad (15)$$

Here, the polarization tensors p_{ij}^+ and p_{ij}^\times are the plus and cross modes, respectively. The polarization tensors are normalized as $p_{ij}^{*A} p_{ij}^B = 2\delta^{AB}$. The circular polarization has the property

$$\frac{k_s}{k} \epsilon^{rsj} p_{ij}^A = -i\lambda^A (p^r{}_i)^A, \quad A = R, L, \quad (16)$$

where $\lambda^R \equiv +1$ and $\lambda^L \equiv -1$ represent the right and left-handed circular polarizations, respectively. This makes the action of the Chern-Simons term diagonal.

Putting above results together and defining a new variable $\mu_{\mathbf{k}}^A \equiv z_{\mathbf{k}}^A \psi_{\mathbf{k}}^A$, we obtain

$$S^{\text{GW}} = \sum_{A=\text{R,L}} \frac{1}{2} \int d\eta \frac{d^3k}{(2\pi)^3} \left[|\mu_{\mathbf{k}}^A|^2 - \left(1 + \frac{\mathcal{H} \xi'}{z_A^2} - \frac{\xi''}{2z_A^2} \right) k^2 |\mu_{\mathbf{k}}^A|^2 + \frac{z_A''}{z_A} |\mu_{\mathbf{k}}^A|^2 \right], \quad (17)$$

where we have defined

$$z_A = a(\eta) \sqrt{1 - \frac{\mathcal{H} \xi'}{2a^2} - \lambda^A k \frac{\omega'}{2a^2}}, \quad \lambda^{\text{R}} = 1, \quad (18)$$

$$\lambda^{\text{L}} = -1.$$

Thus, the equation of motion becomes

$$(\mu_{\mathbf{k}}^A)'' + \left[\left(1 + \frac{\mathcal{H} \xi'}{z_A^2} - \frac{\xi''}{2z_A^2} \right) k^2 - \frac{z_A''}{z_A} \right] \mu_{\mathbf{k}}^A = 0. \quad (19)$$

The term z_A''/z_A can be interpreted as the effective potential. The coefficient of the wave number k may be interpreted as the square of the speed of sound waves.

In the long wavelength limit, we have the solution

$$\mu_{\mathbf{k}}^A = G z_{\mathbf{k}}^A + D z_{\mathbf{k}}^A \int \frac{d\eta}{z_{\mathbf{k}}^{A2}}, \quad (20)$$

where G, D are the constants of integration. In the conventional inflationary scenario, the first term corresponds to the growing mode which is actually constant when we translated back to the metric perturbations. Hence, the primordial gravitational waves are frozen on superhorizon scales. However, in the presence of Gauss-Bonnet and Chern-Simons terms, the role of the growing and decaying modes are interchanged. This gives rise to an interesting effect on the polarization.

III. A MECHANISM TO PRODUCE CIRCULAR POLARIZATION

In this section, we present a mechanism to produce the circular polarization of primordial gravitational waves. The interplay between the instability induced by the Gauss-Bonnet term and the parity violation due to the Chern-Simons term is essential for the mechanism. The mechanism is efficient and generic in the sense that both terms are ubiquitous in string theory.

Now, let us quantize the gravitational waves and calculate the degree of polarization. What we should do is promote $\mu_{\mathbf{k}}^A$ to the operator and expand by mode functions $u_{\mathbf{k}}^A(\eta)$ as

$$\mu_{\mathbf{k}}^A = a_{\mathbf{k}}^A u_{\mathbf{k}}^A + a_{\mathbf{k}}^{\dagger A} u_{\mathbf{k}}^{*A}, \quad (21)$$

where $u_{\mathbf{k}}^{*A}$ is the complex conjugate of $u_{\mathbf{k}}^A$ and $a_{\mathbf{k}}^A$ and $a_{\mathbf{k}}^{\dagger A}$ denote the annihilation and creation operators. The mode functions $u_{\mathbf{k}}^A$ satisfy the same equation for $\mu_{\mathbf{k}}^A$. Once the positive frequency mode $u_{\mathbf{k}}^A$ is specified, the vacuum is defined by

$$a_{\mathbf{k}}^A |0\rangle = 0. \quad (22)$$

Then, we can calculate the vacuum fluctuations

$$\langle 0 | \mu_{\mathbf{k}}^A |^2 | 0 \rangle = |u_{\mathbf{k}}^A|^2. \quad (23)$$

Here, we focus only on the gravitational waves which leaves the horizon during the superinflationary phase, because the gravitational waves still under the horizon when superinflation ends are the same as those of the slow-roll inflation.

During the stage prior to the slow-roll inflation, z_A can be approximated as $z_A = \sqrt{-\mathcal{H} \xi'/2 - \lambda^A k \omega'/2}$. In spite of this simplification, it is still difficult to solve the equation of motion analytically. Hence, we solve the equation at superhorizon scale and subhorizon scale, separately. After that, we connect these solutions smoothly at the horizon crossing.

First, let us consider the subhorizon scale, $\mathcal{H} \ll k$ or $-k\eta \gg 1/6$. Leaving the terms up to the order of $1/(-k\eta)$, we obtain the equation

$$(u_{\mathbf{k}}^A)'' + k^2 \left(1 - \lambda^A \frac{8}{3} \frac{1}{-k\eta} \right) u_{\mathbf{k}}^A = 0. \quad (24)$$

This can be solved analytically by using confluent hypergeometric functions. Instead of doing that, we estimate the degree of the polarization approximately. The initial condition can be set by imposing the Bunch-Davies vacuum. More precisely, we choose the positive frequency mode

$$u_{\mathbf{k}}^A = \frac{1}{\sqrt{2k}} e^{-ik\eta} \quad (25)$$

in the asymptotic past. Namely, deep inside the Hubble horizon, both the left and right polarization modes are simply oscillating. At the time $-k\eta = 8/3$, the asymmetry shows up. After that time, $-k\eta < 8/3$, we can approximate the solutions as

$$u_{\mathbf{k}}^A = A_1 \left(\frac{3\lambda^A (-k\eta)}{8k^2} \right)^{1/4} \exp\left(2\sqrt{\frac{8\lambda^A}{3}} (-k\eta) \right) + A_2 \left(\frac{3\lambda^A (-k\eta)}{8k^2} \right)^{1/4} \exp\left(-2\sqrt{\frac{8\lambda^A}{3}} (-k\eta) \right), \quad (26)$$

where we have used WKB approximation which is valid for the period we are considering. To determine the constants of integration A_1 and A_2 , we have to match solutions smoothly at $-k\eta \sim 8/3$ as

$$u_{\mathbf{k}}^A(\eta)|_{\eta < 8/3} = u_{\mathbf{k}}^A(\eta)|_{\eta > 8/3}, \quad (27)$$

$$(u_{\mathbf{k}}^A)'(\eta)|_{\eta < 8/3} = (u_{\mathbf{k}}^A)'(\eta)|_{\eta > 8/3}.$$

Apparently, both A_1 and A_2 are not zero. Hence, for the left-handed circular polarization modes $\lambda^{\text{L}} = -1$, the solution shows damping oscillation. While, for the right-handed circular polarization mode $\lambda^{\text{R}} = 1$, the solution grows rapidly. It is this instability due to the Gauss-

Bonnet term that produces the difference between the left and right-handed circular polarization modes. The ratio can be quantified by

$$\frac{|u_R|^2}{|u_L|^2} = \frac{\exp(-8/3)}{\exp(-32/3)} \sim 2980. \quad (28)$$

Thus, we can expect the fully polarized gravitational waves.

To make the analysis more accurate, we need the solutions to continue to the superhorizon scale, $\mathcal{H} \gg k$ or $-k\eta \ll 1/6$. On the superhorizon scales, the solution can be obtained as

$$(u_{\mathbf{k}}^A)'' + \frac{2}{9}k^2 \frac{1}{(-k\eta)^2} u_{\mathbf{k}}^A = 0. \quad (29)$$

This can be solved easily as

$$u_{\mathbf{k}}^A = B_1(-k\eta)^{1/3} + B_2(-k\eta)^{2/3}, \quad (30)$$

where B_1 and B_2 are constants of integration. To determine the constants B_1 and B_2 , we have to match solutions smoothly at $-\eta_{\text{cross}} = 1/(6k)$ as

$$\begin{aligned} u_{\mathbf{k}}^A(\eta_{\text{cross}})|_{\mathcal{H} \ll k} &= u_{\mathbf{k}}^A(\eta_{\text{cross}})|_{\mathcal{H} \gg k}, \\ (u_{\mathbf{k}}^A)'(\eta_{\text{cross}})|_{\mathcal{H} \ll k} &= (u_{\mathbf{k}}^A)'(\eta_{\text{cross}})|_{\mathcal{H} \gg k}. \end{aligned} \quad (31)$$

What we want to calculate is the degree of the circular polarization defined as the difference between the power of right and left-handed circularly polarized gravitational waves at the end of the superinflation phase:

$$\Pi(k) \equiv \frac{|u_{\mathbf{k}}^R(\eta_{\text{end}})|^2 - |u_{\mathbf{k}}^L(\eta_{\text{end}})|^2}{|u_{\mathbf{k}}^R(\eta_{\text{end}})|^2 + |u_{\mathbf{k}}^L(\eta_{\text{end}})|^2}, \quad (32)$$

where η_{end} represents the time when the superinflation ends. We have performed a matching analysis and displayed a numerical result in Fig. 3. Because of the behavior just before the horizon crossing, namely, the right-handed circular polarization mode is growing and the left-handed

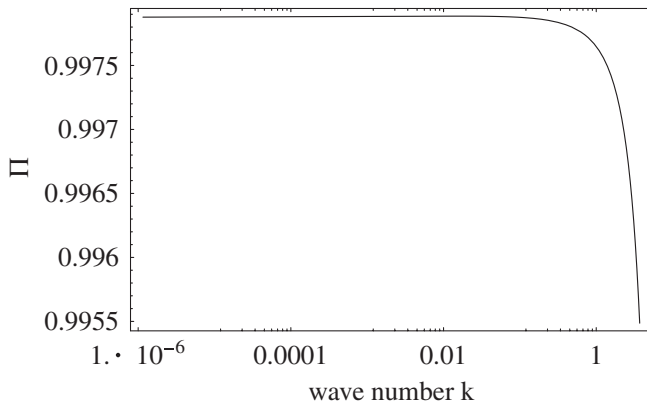


FIG. 3. The degree of the polarization $\Pi(k)$ as a function of wave numbers k is shown. As expected, the gravitational waves are almost 100% circularly polarized.

circular polarization mode is decaying, consequently the net polarization is produced. The resultant polarization is sufficiently large and hence detectable by BBO or DECIGO.

To calculate the spectrum of the polarization of gravitational waves which is directly observed today, we have to connect solutions from the superinflation phase to the slow-roll inflation, radiation dominant, and matter dominant phase. This part of calculations is standard and does not alter the polarization.

Finally, we should mention a subtle point in the calculation. From the definition of z_A (18), it is obvious that there is a point η_{sing} where $z_A = 0$. That occurs close to the time when the left-handed circular polarization mode crosses the horizon. There, the solution behaves as

$$u_{\mathbf{k}}^L = C_1 \sqrt{|\eta - \eta_{\text{sing}}|} + C_2 \sqrt{|\eta - \eta_{\text{sing}}|} \ln(|\eta - \eta_{\text{sing}}|), \quad (33)$$

where C_1 and C_2 are constants of integration. We can see that $u_{\mathbf{k}}^L \rightarrow 0$, in the limit $\eta \rightarrow \eta_{\text{sing}}$, so it causes no problem to solve Eq. (19). However, the relevant quantity is the physical amplitude $\psi_{\mathbf{k}}^L$, rather than $u_{\mathbf{k}}^L$, namely,

$$\psi_{\mathbf{k}}^L = \frac{u_{\mathbf{k}}^L}{z_L} \sim C_1 + C_2 \ln(|\eta - \eta_{\text{sing}}|). \quad (34)$$

Apparently, the physical amplitude diverges at $\eta \rightarrow \eta_{\text{sing}}$. In the previous work [7,8], this issue was serious, because this divergence means the breakdown of the linear analysis. It suggests that we should take into account the nonlinear effect which is not available. In our case, the relevant mode is the right polarization mode which has no divergence problem. As the breakdown of the linear analysis occurs just at the horizon crossing, the nonlinear effects work almost instantaneously. Hence, it is reasonable to assume that the nonlinear effect for the left-handed circular polarization mode hardly affects the evolution of the right-handed circular polarization mode. Therefore, we believe the consequence about the degree of polarization does not change.

IV. DETECTABILITY OF CIRCULAR POLARIZATION OF GRAVITATIONAL WAVES

First of all, we should emphasize that the analysis done in the previous section is explanatory. The frequency range where the circular polarization is created will change depending on when the superinflation occurs. In this section, we will discuss the indirect detection through cosmic microwave background (CMB) and the direct detections by the interferometer.

As to the indirect detection of circular polarization through the CMB radiation, the required degree of circular polarization has been obtained as $|\Pi| \geq 0.35(r/0.05)^{-0.6}$ [21], where r is the tensor-to-scalar ratio. The relevant frequency in this case is around $f \sim 10^{-17}$ Hz. As to the

direct detection of circular polarization, the required degree of circular polarization has been estimated as $\Pi \sim 0.08(\Omega_{\text{GW}}/10^{-15})^{-1}(\text{SNR}/5)$ around the frequency $f \sim 1$ Hz [22,23], where Ω_{GW} is the density parameter of the stochastic gravitational waves and SNR is the signal to the noise ratio. Here, 10 years observational time is assumed. Of course, for the isotropic component of circular polarization of stochastic gravitational waves, one needs to break the symmetry of the detector configuration.

Thus, in order to know if we can detect polarized gravitational waves, we need the degree of the polarization, the intensity of the gravitational waves Ω_{GW} or the tensor-to-scalar ratio r , and also the spectrum around the relevant frequency ranges. As we have shown already, we can get the polarization $\Pi \sim 1$ easily. Moreover, it is not difficult to get the required Ω_{GW} or r provided standard model parameters. We can calculate the spectrum numerically assuming the Bunch-Davis vacuum. We have presented a numerical result in Fig. 4. This result can be understood analytically. In fact, the initial mode function behaves $\propto 1/\sqrt{k}$ inside the horizon which should match to the growing mode $B_1(-k\eta)^{1/3}$ on the superhorizon scales. Matching the condition at $-k\eta \sim 1$ gives the k dependence of the coefficient B_1 as $B_1 \propto 1/\sqrt{k}$. Thus, we have $u_k \propto k^{1/3-1/2} \sim k^{-1/6}$. Consequently, the power spectrum for the gravitational waves becomes $k^3|u_k|^2 \propto k^{8/3}$. Since the spectrum has the blue index $n \sim 2.66$, the polarization would be detectable even in the low intensity case. Now, the point is the frequency range, namely, the time when the polarization is created. In the following discussion, we will mainly focus on this issue.

In the case of the single inflaton model we have discussed, the frequency where the circular polarization is significant lies around the CMB scale $f \sim 10^{-17}$ Hz. Hence, there is a possibility for the circular polarization to be indirectly detected through the CMB observation,

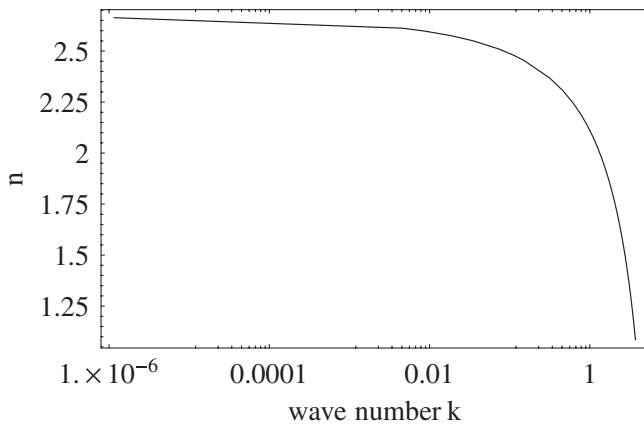


FIG. 4. The numerical result of the spectral index n for the power spectrum of primordial gravitational waves as a function of wave numbers k is shown. In the relevant frequency range, we have obtained the spectral index $n = 2.66$ numerically.

more precisely, through the temperature and B mode or E mode and B mode correlations [4,21,24]. In a recent paper [21], it is argued that a high degree of polarization is necessary for the polarization to be detected in the future observations. The high degree of polarization can be attainable by the mechanism we have found in this paper. However, in this simple model, we have to worry about the curvature perturbations which is also expected to have the blue spectrum during the superinflation. One possible way out of this problem is to assume the curvature perturbation created by the inflaton is negligible and to resort to the curvaton scenario [25–27] as is common to the pre-big-bang type models [28–31]. However, it is not clear if the concrete realization of this kind of model is possible. Moreover, one may feel that introducing the curvaton mechanism into the scenario is not appealing even if it exists. Fortunately, there exists another interesting possibility. We will devote the remaining part of this section to the model which is consistent with observational constraints.

To circumvent the problem of the curvature perturbations, we propose a two-field inflation model defined by the action

$$\begin{aligned}
 S = & \frac{1}{2} \int d^4x \sqrt{-g} R \\
 & - \int d^4x \sqrt{-g} \left[\frac{1}{2} \nabla^\mu \chi \nabla_\mu \chi + \frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi + V(\phi, \chi) \right] \\
 & - \frac{1}{16} \int d^4x \sqrt{-g} \xi(\phi) R_{\text{GB}}^2 + \frac{1}{16} \int d^4x \sqrt{-g} \omega(\phi) R \tilde{R},
 \end{aligned} \tag{35}$$

where the potential $V(\phi, \chi)$ is assumed to take a hybrid type. The main difference between our model and the conventional inflation model is simply the Gauss-Bonnet and Chern-Simons term. In the first stage, the field χ drives the slow-roll inflation. During this stage, the field ϕ takes a constant large field value. Hence, both the Gauss-Bonnet and the Chern-Simons terms are decoupled from the system. During this conventional inflationary period, the temperature fluctuations of CMB are created. And the spectrum of the fluctuations is almost flat. Then, at some point, the motion in the direction of ϕ is triggered and the Gauss-Bonnet and Chern-Simons terms take part in the dynamics. In this second stage, the ϕ field rolls down the hill rapidly and the Gauss-Bonnet term induces a superinflationary phase. During this stage, the circular polarization of the primordial gravitational waves is produced through the parity violation of the Chern-Simons term. Subsequently, the second standard slow-roll inflation driven by the same field ϕ takes over.

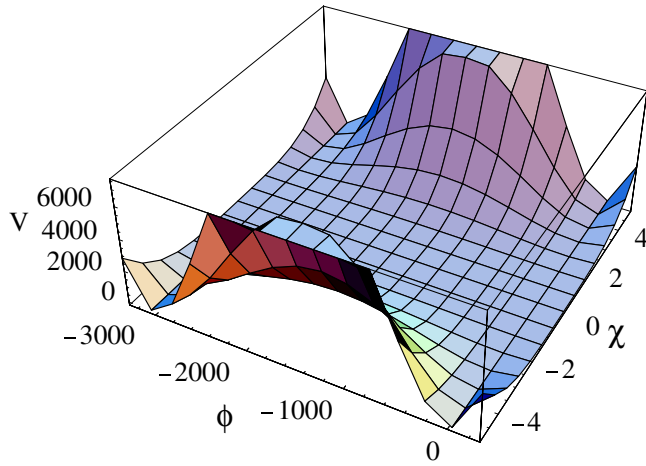


FIG. 5 (color online). Potential function for the two-field model is depicted. Initially, the scalar field ϕ is stuck to the point $\phi = -3000$ and the other scalar field χ slowly rolls down to give rise to the inflation. At some point, ϕ starts to roll down toward $\phi = 0$. Then, the coupling to the Gauss-Bonnet term induces the superinflation where the circular polarization of gravitational waves is created. Subsequently, the standard slow-roll inflation follows.

Here, we present an example. The potential function is

$$V(\phi, \chi) = V_p(\lambda_\chi \chi^4 + 1) \left\{ (2/\phi_m)^2 [(\phi + \phi_m/2)^2 - (\phi_m/2)^2] \right\}^2 + \frac{1}{2} m_\chi^2 \chi^2 + \left(\frac{\phi}{\phi_m} \right)^2 V_m, \quad (36)$$

where the parameters are set as

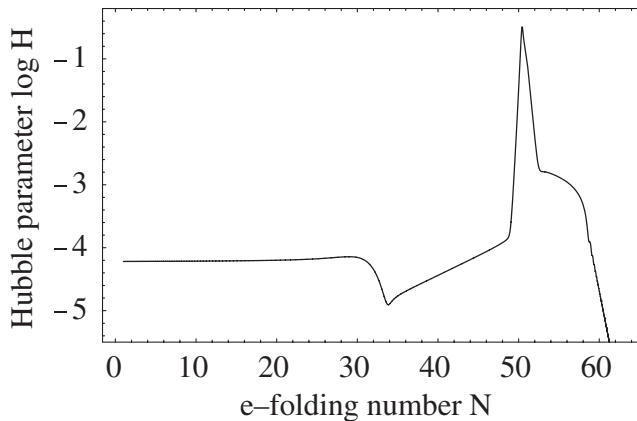


FIG. 6. The time evolution of the Hubble parameter is numerically calculated. The curvature perturbations on the CMB scales are produced during the slow-roll phase. Hence, this model is consistent with CMB observations. The Gauss-Bonnet term induces the superinflation around $N \approx 50$ where the circular polarization would be created. It could be observed directly through the interferometer detector.

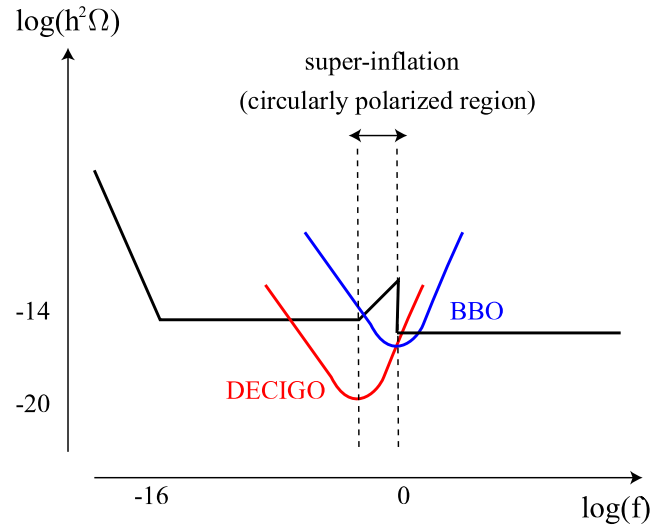


FIG. 7 (color online). A schematic picture of the expected spectrum is depicted. The frequency range where we can observe the circular polarization depends on the model parameters. In that range, the spectrum is blue. In other parts, the spectrum is almost flat. The expected sensitivities of BBO and DECIGO are also plotted.

$$\begin{aligned} \phi_m = 3000, \quad V_m = 10^{-8}, \quad V_p = 0.235, \\ \lambda_\chi = 10^2, \quad m_\chi = 10^{-6}. \end{aligned} \quad (37)$$

This potential is shown in Fig. 5. Coupling ξ and ω are the same as those of the single-field model, namely $\xi = \omega = 16\phi^4$. We have calculated the evolution of the Hubble parameter numerically (see Fig. 6).

For this model, the duration of the first and second slow-roll inflation is tunable. Hence, it is possible to have the gravitational waves with the significant circular polarization in the BBO or DECIGO frequency range (see Fig. 7). Since the spectrum is blue, even the ground based detectors may find the circular polarization as is pointed out recently [32].

To make a more precise prediction of the polarization of gravitational waves, we need a more complete analysis. In particular, we should calculate the spectrum of the curvature perturbations of this model. We leave these details for future publications [33].

V. CONCLUSION

We have studied a mechanism to produce the circular polarization of gravitational waves in the string-inspired cosmology. It turned out that the circularly polarized gravitational waves are ubiquitous in string cosmology. The point we have observed is that there are two key terms in string theory, namely, the Chern-Simons term and the Gauss-Bonnet term. The Chern-Simons term violates the parity invariance, therefore it makes a room for the circularly polarized gravitational waves to be produced.

However, in the previous works [4,7,8], it had been shown that there is no significant circular polarization of gravitational waves within the conventional inflationary scenario. In this paper, we have shown that the Gauss-Bonnet term reversed the previous conclusion. The Gauss-Bonnet term has changed the background evolution in such a way that the superinflationary epoch appears during the conventional inflationary stage. During the superinflation, there exists an instability in the tensor modes. It is the instability that produces a significant circular polarization $\Pi \sim 1$. We have also discussed the detectability of the polarization. We have shown that the detectability depends on the specific scenario of inflation. In the single-field inflation model, the circular polarization would be observable at the CMB scale. The consistency with the observation of the temperature fluctuations requires the curvaton scenario in this case. To circumvent the situation, we have also proposed a two-field inflation model. In the two-field model, the frequency range where the circular polarization is produced depends on the parameter. It would be possible to make a model to produce gravitational waves with circular polarization in the BBO or DECIGO frequency range. In this case, curvature fluctuations on large scales are produced by the standard slow-roll inflation. Hence, we do not need the curvaton scenario any more.

There are several directions to be explored. We should investigate the two-field inflation model in detail [33]. In particular, it is interesting to seek a concrete realization of the model in the string landscape. It is also intriguing to search other mechanisms to produce a circular polarization of gravitational waves. One simple possibility is to consider the nonvacuum initial state in the Lorentz violating inflationary scenario [34]. The coupling to the Lorentz violating sector could violate the parity invariance. Hence, it is possible to take the asymmetric vacuum for right and left-handed circular polarizations. In fact, the Chern-Simons term already allows this possibility. There may be other interesting mechanisms with noninflationary origin, such as the turbulence [35] or the helical magnetic fields [24,36]. The Chern-Simons term at present may become an interesting subject in the light of the dark energy [37–41].

ACKNOWLEDGMENTS

J.S. is supported by the Japan-U.K. Research Cooperative Program, the Japan-France Research Cooperative Program, Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Science and Culture of Japan No. 18540262 and No. 17340075. S.K. is supported by JSPS Grant-in-Aid for Research Abroad.

-
- [1] M.B. Green and J.H. Schwarz, Phys. Lett. B **149**, 117 (1984).
 - [2] I. Antoniadis, E. Gava, and K.S. Narain, Phys. Lett. B **283**, 209 (1992).
 - [3] R. Jackiw and S.Y. Pi, Phys. Rev. D **68**, 104012 (2003).
 - [4] A. Lue, L.M. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999).
 - [5] K. Choi, J.c. Hwang, and K.W. Hwang, Phys. Rev. D **61**, 084026 (2000).
 - [6] S. Alexander, M.E. Peskin, and M.M. Sheikh-Jabbari, Phys. Rev. Lett. **96**, 081301 (2006).
 - [7] S. Alexander and J. Martin, Phys. Rev. D **71**, 063526 (2005).
 - [8] D.H. Lyth, C. Quimbay, and Y. Rodriguez, J. High Energy Phys. 03 (2005) 016.
 - [9] I. Antoniadis, J. Rizos, and K. Tamvakis, Nucl. Phys. **B415**, 497 (1994).
 - [10] S. Kawai and J. Soda, Phys. Rev. D **59**, 063506 (1999).
 - [11] S. Kawai and J. Soda, arXiv:gr-qc/9906046.
 - [12] S. Kawai and J. Soda, Phys. Lett. B **460**, 41 (1999).
 - [13] C. Cartier, J.c. Hwang, and E.J. Copeland, Phys. Rev. D **64**, 103504 (2001).
 - [14] S. Kawai, M.a. Sakagami, and J. Soda, Phys. Lett. B **437**, 284 (1998).
 - [15] J. Soda, M.a. Sakagami, and S. Kawai, arXiv:gr-qc/9807056.
 - [16] S. Kawai, M.a. Sakagami, and J. Soda, arXiv:gr-qc/9901065.
 - [17] E.S. Phinney *et al.*, The Big Bang Observer, NASA Mission Concept Study (2003).
 - [18] N. Seto, S. Kawamura, and T. Nakamura, Phys. Rev. Lett. **87**, 221103 (2001).
 - [19] M. Gasperini, Phys. Rev. D **56**, 4815 (1997).
 - [20] C. Cartier, E.J. Copeland, and M. Gasperini, Nucl. Phys. **B607**, 406 (2001).
 - [21] S. Saito, K. Ichiki, and A. Taruya, J. Cosmol. Astropart. Phys. 09 (2007) 002.
 - [22] N. Seto, Phys. Rev. Lett. **97**, 151101 (2006).
 - [23] N. Seto, Phys. Rev. D **75**, 061302 (2007).
 - [24] C. Caprini, R. Durrer, and T. Kahniashvili, Phys. Rev. D **69**, 063006 (2004).
 - [25] K. Enqvist and M.S. Sloth, Nucl. Phys. **B626**, 395 (2002).
 - [26] D.H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002).
 - [27] T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001); **539**, 303(E) (2002).
 - [28] J.E. Lidsey, D. Wands, and E.J. Copeland, Phys. Rep. **337**, 343 (2000).
 - [29] M. Gasperini and G. Veneziano, Phys. Rep. **373**, 1 (2003).
 - [30] S. Kanno, M. Sasaki, and J. Soda, Prog. Theor. Phys. **109**, 357 (2003).
 - [31] S. Kanno, J. Soda, and D. Wands, J. Cosmol. Astropart. Phys. 08 (2005) 002.
 - [32] N. Seto and A. Taruya, Phys. Rev. Lett. **99**, 121101 (2007).

- [33] M. Satoh, S. Kanno, and J. Soda (work in progress).
- [34] S. Kanno and J. Soda, Phys. Rev. D **74**, 063505 (2006).
- [35] T. Kahniashvili, G. Gogoberidze, and B. Ratra, Phys. Rev. Lett. **95**, 151301 (2005).
- [36] T. Kahniashvili, New Astron. Rev. **50**, 1015 (2006).
- [37] G. Calcagni, S. Tsujikawa, and M. Sami, Classical Quantum Gravity **22**, 3977 (2005).
- [38] T. Koivisto and D.F. Mota, Phys. Rev. D **75**, 023518 (2007).
- [39] B.M. Leith and I.P. Neupane, J. Cosmol. Astropart. Phys. 05 (2007) 019.
- [40] S. Nojiri, S.D. Odintsov, and P.V. Tretyakov, Phys. Lett. B **651**, 224 (2007).
- [41] E. Elizalde, S. Jhingan, S. Nojiri, S.D. Odintsov, M. Sami, and I. Thongkool, arXiv:0705.1211.