Constraining neutron-star tidal Love numbers with gravitational-wave detectors

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(Received 12 September 2007; published 22 January 2008)

Ground-based gravitational wave detectors may be able to constrain the nuclear equation of state using the early, low frequency portion of the signal of detected neutron star–neutron star inspirals. In this early adiabatic regime, the influence of a neutron star's internal structure on the phase of the waveform depends only on a single parameter λ of the star related to its tidal Love number, namely, the ratio of the induced quadrupole moment to the perturbing tidal gravitational field. We analyze the information obtainable from gravitational wave frequencies smaller than a cutoff frequency of 400 Hz, where corrections to the internal-structure signal are less than 10%. For an inspiral of two nonspinning $1.4M_\odot$ neutron stars at a distance of 50 Megaparsecs, LIGO II detectors will be able to constrain λ to $\lambda \leq 2.0 \times 10^{37}$ g cm^2 s^2 with 90% confidence. Fully relativistic stellar models show that the corresponding constraint on radius R for $1.4M_\odot$ neutron stars would be $R \leq 13.6$ km (15.3 km) for a $n = 0.5$ ($n = 1.0$) polytrope with equation of state $p \propto \rho^{1+1/n}$.

I. BACKGROUND AND MOTIVATION

Coalescing binary neutron stars are one of the most important sources for gravitational wave (GW) detectors [1]. LIGO I observations have established upper limits on the event rate [2], and at design sensitivity LIGO II is expected to detect inspirals at a rate of ~ 2/day [3].

One of the key scientific goals of detecting neutron star (NS) binaries is to obtain information about the nuclear equation of state (EoS), which is at present fairly unconstrained in the relevant density range $\rho \sim 2 - 8 \times 10^{14}$ g cm^{-3} [4]. The conventional view has been that for most of the inspiral, finite-size effects have a negligible influence on the GW signal, and that only during the last several orbits and merger at GW frequencies $f \gtrsim 500$ Hz can the effect of the internal structure be seen.

There have been many investigations of how well the EoS can be constrained using these last several orbits and merger, including constraints from the GW energy spectrum [5], and, for black hole/NS inspirals, from the NS tidal disruption signal [6]. Several numerical simulations have studied the dependence of the GW spectrum on the radius [7]. However, there are a number of difficulties associated with trying to extract equation of state information from this late time regime: (i) The highly complex behavior requires solving the full nonlinear equations of general relativity together with relativistic hydrodynamics. (ii) The signal depends on unknown quantities such as the spins of the stars. (iii) The signals from the hydrodynamic merger (at frequencies $\gtrsim 1000$ Hz) are outside of LIGO's most sensitive band.

The purpose of this paper is to demonstrate the potential feasibility of instead obtaining EoS information from the early, low frequency part of the signal. Here, the influence of tidal effects is a small correction to the waveform's phase, but it is very clean and depends only on one parameter of the NS—its Love number [8].

II. TIDAL INTERACTIONS IN COMPACT BINARIES

The influence of tidal interactions on the waveform's phase has been studied previously using various approaches [8–13]. We extend those studies by (i) computing the effect of the tidal interactions for fully relativistic neutron stars, i.e. to all orders in the strength of internal gravity in each star, (ii) computing the phase shift analytically without the assumption that the mode frequency is much larger that the orbital frequency, and (iii) performing a computation of how accurately the Love number can be measured.

The basic physical effect is the following: the $l = 2$ fundamental f-modes of the star can be treated as forced, damped harmonic oscillators driven by the tidal field of the companion at frequencies below their resonant frequencies. Assuming circular orbits they obey equations of motion of the form [14]

$$\ddot{q} + \gamma\dot{q} + \omega_0^2 q = A(t)\cos[m\Phi(t)], \quad (1)$$

where $q(t)$ is the mode amplitude, ω_0 the mode frequency, γ a damping constant, m is the mode azimuthal quantum number, $\Phi(t)$ is the orbital phase of the binary, and $A(t)$ is a slowly varying amplitude. The orbital frequency $\omega(t) = \dot{\Phi}$ and $A(t)$ evolve on the radiation reaction time scale which is much longer than $1/\omega_0$. In this limit the oscillator evolves adiabatically, always tracking the minimum of its time-dependent potential. The energy absorbed by the oscillator up to time t is

$$E(t) = \frac{\omega_0^2 A(t)^2}{2(\omega_0^2 - m^2\omega^2)^2} + \gamma \int_{-\infty}^{t} dt' \frac{m^2\omega(t')^2 A(t')^2}{\omega_0^4 + m^2\omega(t')^2\gamma^2}.$$
$$(2)$$

The second term here describes a cumulative, dissipative effect which dominates over the first term for tidal interactions of main sequence stars. For NS-NS binaries, how-

ever, this term is unimportant due to the small viscosity [11], and the first, instantaneous term dominates.

The instantaneous effect is somewhat larger than often estimated for several reasons: (i) The GWs from the time varying stellar quadrupole are phase coherent with the orbital GWs, and thus there is a contribution to the energy flux that is linear in the mode amplitude. This affects the rate of inspiral and gives a correction of the same order as the energy absorbed by the mode [10]. (ii) Some papers [9,11,12] compute the orbital phase error as a function of orbital radius r. This is insufficient as one has to express it in the end as a function of the observable frequency, and there is a correction to the radius-frequency relation which comes in at the same order. (iii) The effect scales as the fifth power of neutron star radius R, and most previous estimates took $R = 10$ km. Larger NS models with e.g. $R = 16$ km give an effect that is larger by a factor of ~ 10.

III. TIDAL LOVE NUMBER

Consider a static, spherically symmetric star of mass m placed in a time-independent external quadrupolar tidal field $\mathcal{E}_{ij}$. The star will develop in response a quadrupole moment Q_{ij}. In the star's local asymptotic rest frame (asymptotically mass centered Cartesian coordinates) at large r the metric coefficient g_{tt} is given by (in units with $G = c = 1$) [15]:

$$\frac{(1 - g_{tt})}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{\delta^{ij}}{3}\right) + \frac{\mathcal{E}_{ij}}{2}x^i x^j + \dots \tag{3}$$

where $n^i = x^i/r$; this expansion defines the traceless tensors $\mathcal{E}_{ij}$ and Q_{ij}. To linear order, the induced quadrupole will be of the form

$$Q_{ij} = -\lambda \mathcal{E}_{ij}. \tag{4}$$

Here λ is a constant which we will call the tidal Love number (although that name is usually reserved for the dimensionless quantity $k_2 = \frac{3}{2}G\lambda R^{-5}$). The relation (4) between Q_{ij} and $\mathcal{E}_{ij}$ defines the Love number λ for both Newtonian and relativistic stars. For a Newtonian star, $(1 - g_{tt})/2$ is the Newtonian potential, and Q_{ij} is related to the density perturbation $\delta\rho$ by $Q_{ij} = \int d^3x \delta\rho(x_i x_j - r^2 \delta_{ij}/3)$.

We have calculated the Love numbers for a variety of fully relativistic NS models with a polytropic pressure-density relation $P = K\rho^{1+1/n}$. Most realistic EoS's resemble a polytrope with effective index in the range $n \simeq 0.5$–1.0 [16]. The equilibrium stellar model is obtained by numerical integration of the Tolman-Oppenheimer-Volkhov equations. We calculate the linear $l = 2$ static perturbations to the Schwarzschild spacetime following the method of [17]. The perturbed Einstein equations $\delta G^\nu_\mu = 8\pi \delta T^\nu_\mu$ can be combined into a second order differential equation for the perturbation to g_{tt}. Matching

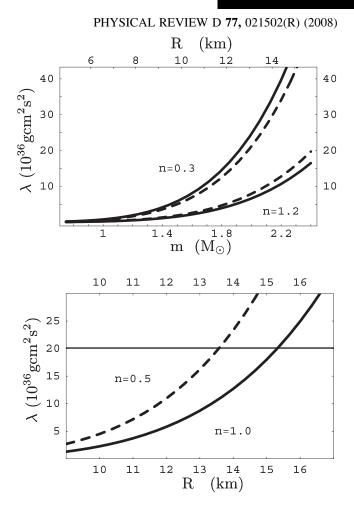FIG. 1. [Top] The solid lines bracket the range of Love numbers λ for fully relativistic polytropic neutron -star models of mass m with surface redshift $z = 0.35$, assuming a range of $0.3 \leq n \leq 1.2$ for the adiabatic index n. The top scale gives the radius R for these relativistic models. The dashed lines are corresponding Newtonian values for stars of radius R. [Bottom] Upper bound (horizontal line) on the weighted average $\tilde{\lambda}$ of the two Love numbers obtainable with LIGO II for a binary inspiral signal at distance of 50 Mpc, for two nonspinning, $1.4M_\odot$ neutron stars, using only data in the frequency band $f < 400$ Hz. The curved lines are the actual values of λ for relativistic polytropes with $n = 0.5$ (dashed line) and $n = 1.0$ (solid line).

the exterior solution and its derivative to the asymptotic expansion (3) gives the Love number. For $m/R \sim 10^{-5}$ our results agree well with the Newtonian results of Refs. [9,18]. Figure 1 shows the range of Love numbers for $m/R = 0.2256$, corresponding to the surface redshift $z = 0.35$ that has been measured for EXO0748-676 [19]. Details of this computation will be published elsewhere.

IV. EFFECT ON GRAVITATIONAL WAVE SIGNAL

Consider a binary with masses m_1, m_2 and Love numbers λ_1, λ_2. For simplicity, we compute only the excitation of star 1; the signals from the two stars simply add in the

phase. Let ω_n, $\lambda_{1,n}$ and Q_{ij}^n be the frequency, the contribution to λ_1 and the contribution to Q_{ij} of modes of the star with $l = 2$ and with n radial nodes, so that $\lambda_1 = \Sigma_n \lambda_{1,n}$ and $Q_{ij} = \Sigma_n Q_{ij}^n$. Writing the relative displacement as $\mathbf{x} = (r\cos\Phi, r\sin\Phi, 0)$, the action for the system is

$$S = \int dt \left[\frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\mu r^2\dot{\Phi}^2 + \frac{M\mu}{r} \right] - \frac{1}{2}\int dt Q_{ij}\mathcal{E}_{ij}$$
$$+ \sum_n \int dt \frac{1}{4\lambda_{1,n}\omega_n^2}\left[\dot{Q}_{ij}^n\dot{Q}_{ij}^n - \omega_n^2 Q_{ij}^n Q_{ij}^n \right]. \quad (5)$$

Here M and μ are the total and reduced masses, and $\mathcal{E}_{ij} = -m_2\partial_i\partial_j(1/r)$ is the tidal field. This action is valid to leading order in the orbital potential but to all orders in the internal potentials of the NSs, except that it neglects GW dissipation, because Q_{ij} and $\mathcal{E}_{ij}$ are defined in the star's local asymptotic rest frame [20].

Using the action (5), adding the leading order, Burke-Thorne GW dissipation terms, and defining the total quadrupole $Q_{ij}^T = Q_{ij} + \mu x_i x_j - \mu r^2\delta_{ij}/3$ with $Q_{ij} = \Sigma_n Q_{ij}^n$, gives the equations of motion

$$\ddot{x}^i + \frac{M}{r^2}n^i = \frac{m_2}{2\mu}Q_{jk}\partial_i\partial_j\partial_k\frac{1}{r} - \frac{2}{5}x_j\frac{d^5 Q_{ij}^T}{dt^5}, \quad (6a)$$

$$\ddot{Q}_{ij}^n + \omega_n^2 Q_{ij}^n = m_2\lambda_{1,n}\omega_n^2\partial_i\partial_j\frac{1}{r} - \frac{2}{5}\lambda_{1,n}\omega_n^2\frac{d^5 Q_{ij}^T}{dt^5}. \quad (6b)$$

By repeatedly differentiating Q_{ij}^T and eliminating second order time derivative terms using the conservative parts of Eqs. (6), we can express $d^5 Q_{ij}^T/dt^5$ in terms of x^i, $\dot{x}^i$, Q_{ij}^n and $\dot{Q}_{ij}^n$ and obtain a second order set of equations; this casts Eqs. (6) into a numerically integrable form.

When GW damping is neglected, there exist equilibrium solutions with $r = \text{const}$, $\Phi = \Phi_0 + \omega t$ for which Q_{ij}^T is static in the rotating frame. Working to leading order in $\lambda_{1,n}$, we have $Q_{11}^T = \mathcal{Q}' + \mathcal{Q}\cos(2\Phi)$, $Q_{22}^T = \mathcal{Q}' - \mathcal{Q}\cos(2\Phi)$, $Q_{12}^T = \mathcal{Q}\sin(2\Phi)$, $Q_{33}^T = -2\mathcal{Q}'$, where

$$\mathcal{Q} = \frac{1}{2}\mu r^2 + \sum_n \frac{3m_2\lambda_{1,n}}{2(1 - 4x_n^2)r^3},$$
$$\mathcal{Q}' = \frac{1}{6}\mu r^2 + \sum_n \frac{m_2\lambda_{1,n}}{2r^3} \quad (7)$$

and $x_n = \omega/\omega_n$. Substituting these solutions back into the action (5), and into the quadrupole formula $\dot{E} = -\frac{1}{5} \times \langle \dddot{Q}_{ij}^T \dddot{Q}_{ij}^T \rangle$ for the GW damping, provides an effective description of the orbital dynamics for quasicircular inspirals in the adiabatic limit. We obtain for the orbital radius, energy and energy time derivative

$$r(\omega) = M^{1/3}\omega^{-2/3}\left[1 + \frac{3}{4}\sum_n \chi_n g_1(x_n) \right], \quad (8a)$$

$$E(\omega) = -\frac{\mu}{2}(M\omega)^{2/3}\left[1 - \frac{9}{4}\sum_n \chi_n g_2(x_n) \right], \quad (8b)$$

$$\dot{E}(\omega) = -\frac{32}{5}M^{4/3}\mu^2\omega^{10/3}\left[1 + 6\sum_n \chi_n g_3(x_n) \right], \quad (8c)$$

where $\chi_n = m_2\lambda_{1,n}\omega^{10/3}m_1^{-1}M^{-5/3}$, $g_1(x) = 1 + 3/(1 - 4x^2)$, $g_2(x) = 1 + (3 - 4x^2)(1 - 4x^2)^{-2}$, and $g_3(x) = (M/m_2 + 2 - 2x^2)/(1 - 4x^2)$. Using the formula $d^2\Psi/d\omega^2 = 2(dE/d\omega)/\dot{E}$ for the phase $\Psi(f)$ of the Fourier transform of the GW signal at GW frequency $f = \omega/\pi$ [21] now gives for the tidal phase correction

$$\delta\Psi(f) = -\frac{15m_2^2}{16\mu^2 M^5}\sum_n \lambda_{1,n}\int_{v_i}^v dv' v'(v^3 - v'^3)g_4(x_n'),$$
$$g_4(x) = \frac{2M}{m_2(1 - 4x^2)} + \frac{22 - 117x^2 + 348x^4 - 352x^6}{(1 - 4x^2)^3}. \quad (9)$$

Here $v = (\pi M f)^{1/3}$, v_i is an arbitrary constant related to the initial time and phase of the waveform, and $x_n' = (v')^3/(M\omega_n)$. In the limit $\omega \ll \omega_n$ assumed in most previous analyses [8,9,11,12], we get

$$\delta\Psi = -\frac{9}{16}\frac{v^5}{\mu M^4}\left[\left(11\frac{m_2}{m_1} + \frac{M}{m_1} \right)\lambda_1 + 1 \leftrightarrow 2 \right], \quad (10)$$

which depends on internal structure only through λ_1 and λ_2. Here we have added the contribution from star 2. The phase (10) is formally of post-5-Newtonian (P5N) order, but it is larger than the point-particle P5N terms (which are currently unknown) by $\sim (R/M)^5 \sim 10^5$.

V. ACCURACY OF MODEL

We will analyze the information contained in the portion of the signal before $f = 400$ Hz. This frequency was chosen to be at least 20% smaller than the frequency of the innermost stable circular orbit [22] for a conservatively large polytropic NS model with $n = 1.0$, $M = 1.4M_\odot$, and $R = 19$ km. We now argue that in this frequency band, the simple model (10) of the phase correction is sufficiently accurate for our purposes.

We consider six types of corrections to (10). For each correction, we estimate its numerical value at the frequency $f = 400$ Hz for a binary of two identical $m = 1.4M_\odot$, $R = 15$, $n = 1.0$ stars: (i) Corrections due to modes with $l \geq 3$ which are excited by higher order tidal tensors $\mathcal{E}_{ijk}, \ldots$. The $l = 3$ correction to $E(\omega)$, computed using the above methods in the low frequency limit, is smaller than the $l = 2$ contribution by a factor of $65k_3 R^2/(45k_2 r^2)$, where k_2, k_3 are apsidal constants. For Newtonian polytropes we have $k_2 = 0.26$, $k_3 = 0.106$ [8], and the ratio is $0.58(R/r)^2 = 0.04(R/15 \text{ km})^2$. (ii) To as-

sess the accuracy of the $\omega \ll \omega_n$ limit underlying (10) we simplify the model (5) by taking

$$\omega_n = \omega_0 \quad \text{for all } n, \tag{11}$$

so that $Q_{ij}^n/\lambda_{1,n}$ is independent of n. This simplification does not affect (10) and *increases* the size of the finite frequency corrections in (9) since $\omega_n \gtrsim \omega_0$[1]. This will yield an upper bound on the size of the corrections. (Also the $n \geq 1$ modes contribute typically less than 1%–2% of the Love number [9].) Fig. 2 shows the phase correction $\delta\Psi$ computed numerically from Eqs. (6), and the approximations (9) and (10) in the limit (11). We see that the adiabatic approximation (9) is extremely accurate, to better than 1%, and so the dominant error is the difference between (9) and (10). The fractional correction to (10) is $\sim 0.7x^2 \sim 0.2(f/f_0)^2$, where $f_0 = \omega_0/(2\pi)$, neglecting unobservable terms of the form $\alpha + \beta f$. This ratio is $\lesssim 0.03$ for $f \leq 400$ Hz and for $f_0 \gtrsim 1000$ Hz as is the case for f-mode frequencies for most NS models [13]. (iii) We have linearized in λ_1; the corresponding fractional corrections scale as $(R/r)^5 \sim 10^{-3}(R/15 \text{ km})^5$ at 400 Hz. (iv) The leading nonlinear hydrodynamic corrections can be computed by adding a term $-\alpha Q_{ij}^0 Q_{jk}^0 Q_{ki}^0$ to the Lagrangian (5), where α is a constant. This corrects the phase shift (10) by a factor $1 - 285\alpha\lambda_{1,0}^2\omega^2/968 \sim 0.9995$, where we have used the models of Ref. [23] to estimate α. (v) Fractional corrections to the tidal signal due to spin scale as $\sim f_{spin}^2/f_{max}^2$, where f_{spin} is the spin frequency and f_{max} the maximum allowed spin frequency. These can be neglected as $f_{max} \gtrsim 1000$ Hz for most models and f_{spin} is expected to be much smaller than this. (vi) Post-1-Newtonian corrections to the tidal signal (10) will be of order $\sim M/r \sim 0.05$. However these corrections will depend only on λ_1 when $\omega \ll \omega_n$, and can easily be computed and included in the data analysis method we suggest here.

Thus, systematic errors in the measured value of λ due to errors in the model should be $\lesssim 10\%$, which is small compared to the current uncertainty in λ (see Fig. 1).
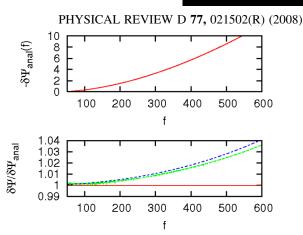

FIG. 2 (color online). [Top] Analytic approximation (10) to the tidal perturbation to the gravitational wave phase for two identical $1.4M_\odot$ neutron stars of radius $R = 15$ km, modeled as $n = 1.0$ polytropes, as a function of gravitational wave frequency f. [Bottom] A comparison of different approximations to the tidal phase perturbation: the numerical solution (lower dashed, green curve) to the system (6), and the adiabatic analytic approximation (9) (upper dashed, blue), both in the limit (11) and divided by the leading order approximation (10).

VI. MEASURING THE LOVE NUMBER

The binary's parameters are extracted from the noisy GW signal by integrating the waveform $h(t)$ against theoretical inspiral templates $h(t, \theta^i)$, where θ^i are the parameters of the binary. The best-fit parameters $\hat{\theta}^i$ are those that maximize the overlap integral. The probability distribution for the signal parameters for strong signals and Gaussian detector noise is $p(\theta^i) = \mathcal{N} \exp(-1/2\Gamma_{ij}\Delta\theta^i\Delta\theta^j)$ [24], where $\Delta\theta^i = \theta^i - \hat{\theta}^i$, $\Gamma_{ij} = (\partial h/\partial\theta^i, \partial h/\partial\theta^j)$ is the Fisher information matrix, and the inner product is defined by Eq. (2.4) of Ref. [24]. The rms statistical measurement error in θ^i is then $\sqrt{(\Gamma^{-1})^{ii}}$.

Using the stationary phase approximation and neglecting corrections to the amplitude, the Fourier transform of the waveform for spinning point masses is given by $\tilde{h}(f) = \mathcal{A}f^{-7/6}\exp(i\Psi)$. Here the phase Ψ is

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3M}{128\mu}(\pi M f)^{-5/3}\left[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)v^2 - 4(4\pi - \beta)v^3 + 10\left(\frac{3058673}{1016064} + \frac{5429}{1008}\frac{\mu}{M}\right.\right.$$
$$+ \frac{617}{144}\frac{\mu^2}{M^2} - \sigma\right)v^4 + \left(\frac{38645\pi}{252} - \frac{65}{3}\frac{\mu}{M}\right)\ln v + \left(\frac{11583231236531}{4694215680} - \frac{640\pi^2}{3} - \frac{6848\gamma}{21}\right)v^6$$
$$+ \frac{\mu}{M}\left(\frac{15335597827}{3048192} + \frac{2255\pi^2}{12} + \frac{47324}{63} - \frac{7948}{9}\right)v^6 + \left(\frac{76055}{1728}\frac{\mu^2}{M^2} - \frac{127825}{1296}\frac{\mu^3}{M^3} - \frac{6848}{21}\ln(4v)\right)v^6$$
$$+ \pi\left(\frac{77096675}{254016} + \frac{378515}{1512}\frac{\mu}{M} - \frac{74045}{756}\frac{\mu^2}{M^2}\right)v^7\right], \tag{12}$$

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[1] Buoyancy forces and associated g-modes for which $\omega_n \leq \omega_0$ have a negligible influence on the waveform's phase [14].

where $\nu = (\pi M f)^{1/3}$, β and σ are spin parameters, and γ is Euler's constant [25]. The tidal term (10) adds linearly to this, yielding a phase model with 7 parameters $(t_c, \phi_c, M, \mu, \beta, \sigma, \tilde{\lambda})$, where $\tilde{\lambda} = [(11m_2 + M)\lambda_1/m_1 + (11m_1 + M)\lambda_2/m_2]/26$ is a weighted average of λ_1 and λ_2. We incorporate the maximum spin constraint for the NSs by assuming a Gaussian prior for β and σ as in Ref. [24].

Figure 1 [bottom panel] shows the 90% confidence upper limit $\tilde{\lambda} \leq 20.1 \times 10^{36} \, \text{g cm}^2 \, \text{s}^2$ we obtain for LIGO II (horizontal line) for two nonspinning $1.4 M_\odot$ NSs at a distance of 50 Mpc (signal-to-noise of 95 in the frequency range 20–400 Hz) with cutoff frequency $f_c = 400$ Hz, as well as the corresponding values of λ for relativistic polytropes with $n = 0.5$ (dashed curve) and $n = 1.0$ (solid line). The corresponding constraint on radius assuming identical $1.4 M_\odot$ stars would be $R \leq 13.6$ km (15.3 km) for $n = 0.5$ ($n = 1.0$) polytropes. Current NS models span the range 10 km $\lesssim R \lesssim$ 15 km.

Our phasing model (12) is the most accurate available model, containing terms up to post-3.5-Newtonian (P3.5N) order. We have experimented with using lower order phase models (P2N, P2.5N, P3N), and we find that the resulting upper bound on $\tilde{\lambda}$ varies by factors of order ~ 2. Thus there is some associated systematic uncertainty in our result. To be conservative, we have adopted the most pessimistic (largest) upper bound on $\tilde{\lambda}$, which is that obtained from the P3.5N waveform.

In conclusion, even if the internal structure signal is too small to be seen, the analysis method suggested here could start to give interesting constraints on NS internal structure for nearby events.

ACKNOWLEDGMENTS

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