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Proposal for exotic-hadron search by fragmentation functions

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It is proposed that fragmentation functions should be used to identify exotic hadrons. As an example, fragmentation functions of the scalar meson $f_0(980)$ are investigated. It is pointed out that the second moments and functional forms of the u- and s-quark fragmentation functions can distinguish the tetraquark structure from $q\bar{q}$. By the global analysis of $f_0(980)$ production data in electron-positron annihilation, its fragmentation functions and their uncertainties are determined. It is found that the current available data are not sufficient to determine its internal structure, while precise data in the future should be able to identify exotic quark configurations.

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In the hadron mass region below 1 GeV, there are scalar mesons, $f_0(600)$, $f_0(980)$, and $a_0(980)$, whose internal configurations are not obvious [1]. Their flavor compositions could be $f_0(600) = (u\bar{u} + d\bar{d})/\sqrt{2}$, $f_0(980) = s\bar{s}$, $a_0(980) = u\bar{d}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$, $\bar{u}d$ in a simple quark model by considering the mass relation, $m_u \sim m_d < m_s$. However, this assignment implies a mass sequence, $m(f_0(600)) \sim m(a_0(980)) < m(f_0(980))$, which contradicts with the observed one, $m(f_0(600)) < m(a_0(980)) \sim m(f_0(980))$. If $f_0(980)$ and $a_0(980)$ are exotic states such as tetraquark ones, the observed spectrum could be naturally understood. Since $f_0(980)$ and $a_0(980)$ are experimentally established resonances, they provide a good opportunity to study exotic mesons beyond a naive $q\bar{q}$-type quark model.

First, a brief outline of recent studies is given for the $f_0(980)$ structure. In a simple quark model, a light scalar meson f_0 with $J^{PC} = 0^{++}$ is identified as a ${}^3P_0$ quarkonium with the flavor structure $(u\bar{u} + d\bar{d})/\sqrt{2}$. However, if such an ordinary $q\bar{q}$ configuration is assigned for $f_0(980)$, the strong decay width is very large, $\Gamma(f_0 \to \pi\pi) =$ 500–1000 MeV, according to various theoretical calculations [2]. The small experimental width 40–100 MeV [3] cannot be consistently explained by simple quark models.

The strong-decay width suggests that $f_0(980)$ should not be an ordinary nonstrange $q\bar{q}$-type meson. The Fermilab-E791 collaboration measured the decay $D_s^+ \to \pi^-\pi^+\pi^+$ [4], which can proceed via intermediate states, for example, $D_s^+ \to f_0(980)\pi^+$ with $s\bar{s}$ quarks in $f_0(980)$. This experiment suggested a sizable strange-quark component in $f_0(980)$. The simplest configuration is a pure strange quarkonium $s\bar{s}$ for $f_0(980)$. In addition, since its mass is just below the $K\bar{K}$ threshold, it could be considered as a $K\bar{K}$ molecule [5]. If two color-singlet states of K and $\bar{K}$ are not well separated, it corresponds to a tetraquark state, $(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$, which was originally suggested in the MIT bag model [6]. Recent QCD-sum-rule studies support this idea of a tetraquark state [7]. Furthermore, there are lattice-QCD studies that $f_0(980)$ corresponds to the tetraquark state because the scalar tetraquark mass is about 1.1 GeV [8]. In addition, $f_0(980)$ used to be considered as a glueball candidate; however, recent lattice-QCD calculations rule out such a possibility because the mass of a 0^{++} glueball is estimated about 1700 MeV [9]. The situation of scalar mesons with masses in the 1 GeV region is summarized in Ref. [1]. All the possible $f_0(980)$ configurations are listed in Table I although the nonstrange-$q\bar{q}$ and glueball states seem to be unlikely according to the recent studies.

In the following, the notation f_0 indicates the $f_0(980)$ meson and $f_0(600)$ is not discussed. There were proposals to find the structure by a ϕ radiative decay into f_0 [10–12]. Since it is an electric dipole decay, the width should reflect information on its size, namely, its internal structure [10]. The experimental measurements of VEPP-2M [13] and DAΦNE [14] were reported for the decay $\phi \to f_0\gamma$. The data may suggest the tetraquark picture; however, there are still discussions on their interpretation [12]. Another possible experimental probe is the $\gamma\gamma \to \pi^+\pi^-$ process in the f_0 mass region. The two-photon decay width of $f_0(980)$ was recently reported as $0.205^{+0.095}_{-0.083}(\text{stat})^{+0.147}_{-0.117}(\text{syst})$ keV by the Belle collaboration [15]. Model calculations indicate 1.3–1.8 keV in the nonstrange $q\bar{q}$ picture; however, the measurement is consistent with the $s\bar{s}$ and $K\bar{K}$-molecule configurations. There are also ideas to use elliptic flow and nuclear modification ratios in heavy-ion reactions for finding exotic hadron structure [16].

There are compelling theoretical and experimental evidences that the scalar meson $f_0(980)$ is not an ordinary nonstrange $q\bar{q}$ meson. However, a precise configuration is not determined yet, and a clear experimental evidence is awaited. It is the purpose of this paper to show that the internal structure of exotic hadrons should be determined from their fragmentation functions by noting differences in favored and disfavored functions. We investigate $f_0(980)$ as an example in this work.

TABLE I. Possible $f_0(980)$ configurations and their features in fragmentation functions at small Q^2.

Type	Configuration	Second moments	Peak positions
Nonstrange $q\bar{q}$	$(u\bar{u} + d\bar{d})/\sqrt{2}$	$M_s < M_u < M_g$	$z_u^{\max} > z_s^{\max}$
Strange $q\bar{q}$	$s\bar{s}$	$M_u < M_s \lesssim M_g$	$z_u^{\max} < z_s^{\max}$
Tetraquark (or $K\bar{K}$)	$(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$	$M_u \sim M_s \lesssim M_g$	$z_u^{\max} \sim z_s^{\max}$
Glueball	gg	$M_u \sim M_s < M_g$	$z_u^{\max} \sim z_s^{\max}$

A fragmentation function is defined by a hadron-production cross section and the total hadronic cross section $F^h(z, Q^2) = \frac{1}{\sigma_{\rm tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$.: Here, the variable z is defined by the hadron energy E_h and the center-of-mass energy $\sqrt{s} = (\sqrt{Q^2})$ by $z \equiv E_h/(\sqrt{s}/2)$. The fragmentation occurs from primary partons, so that it is expressed by the sum of their contributions $F^h(z, Q^2) = \sum_i C_i(z, \alpha_s) \otimes D_i^h(z, Q^2)$,: where $\otimes$ indicates the convolution integral, $f(z) \otimes g(z) = \int_z^1 dy f(y) g(z/y)/y$, $D_i^h(z, Q^2)$ is the fragmentation function of the hadron h from a parton $i = (u, d, s, \cdots, g)$, $C_i(z, \alpha_s)$ is a coefficient function, and α_s is the running coupling constant. The favored fragmentation means a fragmentation from a quark or an antiquark which exists in a hadron as a constituent in a quark model, and the disfavored means a fragmentation from a sea quark. The favored and disfavored functions are assigned in the following discussions by considering the naive quark configurations in Table I.

We first consider a possible $s\bar{s}$ configuration for f_0. Then, the u- and d-quark fragmentation functions are disfavored ones and the s-quark function is a favored one. For example, the favored fragmentation from s is possible if a gluon is radiated from s, and then it splits into a $s\bar{s}$ pair to form the f_0 meson as shown in Fig. 1. The notations $O(g^2)$ and $O(g^3)$ indicate the second and third orders of the coupling constant g. In the disfavored process from u, there are processes in the order of $O(g^3)$ without an $O(g^2)$ term, so that its probability is expected to be smaller than the favored one from s. It leads to the relation for the second moments of fragmentation functions: $M_u < M_s$, where $M_i \equiv \int dz z D_i^{f_0}(z)$. The second moment M_i is the energy fraction for f_0 which is created from the parton i. In the same way, fragmentations occur from a gluon as shown in the figure. Since there are two processes in $O(g^2)$ with a soft gluon radiation, the second moment for the gluon is expected to be larger than the others. These considerations lead to the relation $M_u < M_s \lesssim M_g$ in Table I. Such a naive estimation should be a crude one, but it has been shown to work for the moments of the pion, kaon, and proton [17], so that it is also expected to be a reasonable guideline in other hadrons.

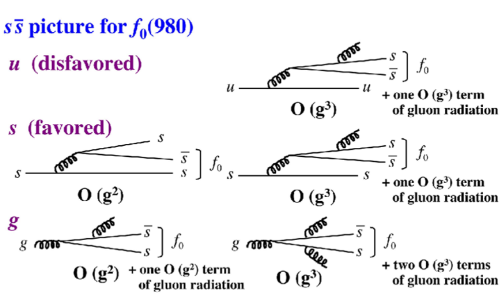

FIG. 1 (color online). Schematic diagrams for f_0 production in the $s\bar{s}$ configuration.

Next, functional forms are discussed in the $s\bar{s}$ picture. More energy is transferred to f_0 from the initial s in the $O(g^2)$ process than the one from the initial u due to an extra gluon emission in Fig. 1. It means that the fragmentation function $D_s^{f_0}(z)$ is distributed in the larger z region in comparison with $D_u^{f_0}(z)$ because the f_0 energy is directly proportional to z. Namely, they should have different functional forms and their peak positions are different at small Q^2 (~ 1 GeV^2). We express this situation as $z_u^{\max} < z_s^{\max}$ in Table I. The form of the gluon fragmentation function may not be simply compared with the quark processes.

In the same way, the second moments and functional forms are roughly estimated for the tetraquark picture. Since the fragmentations from u and s quarks are equally favored processes in this case, their moments and functions forms should be almost the same. The fragmentations into f_0 proceed by creating $u\bar{u}$ (or $d\bar{d}$) and $s\bar{s}$ pairs as shown in Fig. 2. There are more fragmentation processes from a gluon, so that the gluon moment is expected to be larger than the others. In this way, we obtain the relations, $M_u \sim M_s \lesssim M_g$ and $z_u^{\max} \sim z_s^{\max}$, in Table I. Since the flavor composition of f_0 is simply considered in the above discussions, this relation could be also applied to the $K\bar{K}$ case. However, the $K\bar{K}$ is a loose and extended bound state so that its production probability in the fragmentation is expected to be much smaller than that of the tetraquark state.

Although the nonstrange-$q\bar{q}$ and glueball configurations seem to be unlikely according to recent theoretical investigations, we also estimated possible relations in Table I. Since the estimation method is essentially the same, derivations are not explained here. If f_0 were a nonstrange-$q\bar{q}$ meson, the relations $M_s < M_u < M_g$ and $z_u^{\max} > z_s^{\max}$ are expected, whereas they are $M_u \sim M_s < M_g$ and $z_u^{\max} \sim z_s^{\max}$ if it were a glueball.

The fragmentation functions are determined by a global analysis of hadron-production data in e^+e^- annihilation [18]. There is recent progress on their analysis. Uncertainties of the fragmentation functions are determined in Ref. [17], and it was shown that the gluon and light-quark functions have large uncertainties for the pion,

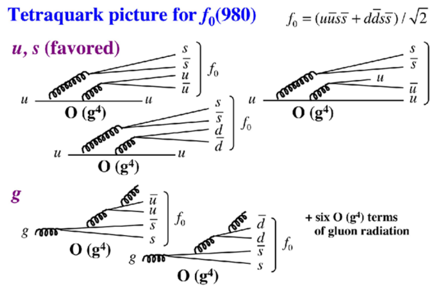

FIG. 2 (color online). Schematic diagrams for f_0 production in the tetraquark configuration.

kaon, and proton. Then, a global analysis with data in lepton scattering and proton-proton collisions was also reported [19]. This kind of global analysis is suitable for finding exotic hadrons by noting the typical features in the favored and disfavored functions.

All the possible configurations for f_0 indicate that up- and down-quark compositions are the same; however, they are generally different from the strange-quark and other ones. Therefore, a natural and model-independent parametrization is

$$\begin{aligned} D_u^{f_0}(z, Q_0^2) &= D_{\bar{u}}^{f_0}(z, Q_0^2) = D_d^{f_0}(z, Q_0^2) = D_{\bar{d}}^{f_0}(z, Q_0^2) \\ &= N_u^{f_0} z^{\alpha_u^{f_0}} (1-z)^{\beta_u^{f_0}}, \\ D_s^{f_0}(z, Q_0^2) &= D_{\bar{s}}^{f_0}(z, Q_0^2) = N_s^{f_0} z^{\alpha_s^{f_0}} (1-z)^{\beta_s^{f_0}}, \\ D_g^{f_0}(z, Q_0^2) &= N_g^{f_0} z^{\alpha_g^{f_0}} (1-z)^{\beta_g^{f_0}}, \\ D_c^{f_0}(z, m_c^2) &= D_{\bar{c}}^{f_0}(z, m_c^2) = N_c^{f_0} z^{\alpha_c^{f_0}} (1-z)^{\beta_c^{f_0}}, \\ D_b^{f_0}(z, m_b^2) &= D_{\bar{b}}^{f_0}(z, m_b^2) = N_b^{f_0} z^{\alpha_b^{f_0}} (1-z)^{\beta_b^{f_0}}, \end{aligned} \tag{1}$$

where N_i, α_i, and β_i are the parameters to be determined by a χ^2 analysis of the data for $e^+ + e^- \to f_0 + X$ [20]. The initial scale is taken $Q_0^2 = 1$ GeV^2, and the masses are $m_c = 1.43$ GeV and $m_b = 4.3$ GeV. The details of the analysis method in the next-to-leading order are explained in Ref. [17]. Uncertainties of the determined functions are estimated by the Hessian method [17], which has been used also in the studies of various parton distribution functions [21,22]:

$$[\delta D_i^{f_0}(z)]^2 = \Delta\chi^2 \sum_{j,k} \left(\frac{\partial D_i^{f_0}(z,\xi)}{\partial \xi_j}\right)_{\hat{\xi}} H_{jk}^{-1} \left(\frac{\partial D_i^{f_0}(z,\xi)}{\partial \xi_k}\right)_{\hat{\xi}}. \tag{2}$$

Here, $\delta D_i^{f_0}(z)$ is the uncertainty of the fragmentation function $D_i^{f_0}(z)$, $\Delta\chi^2$ value is taken so that the confidence level P becomes the one-σ-error range ($P = 0.6826$) by assuming the normal distribution in the multiparameter space, H_{ij} is the Hessian matrix, ξ_i is a parameter, and $\hat{\xi}$ indicates the optimum parameter set.

The number of f_0 data is very limited at this stage. In fact, available data are merely 20 three. This situation makes the analysis difficult in obtaining the minimum χ^2 point. There are irrelevant parameters which do not affect the total χ^2. We decided to fix three parameters at $\beta_g = 1$, $\alpha_u = 10$, and $\alpha_s = 10$ because of the lack of data. Then, the total number of parameters becomes 12. The minimum χ^2 is obtained $\chi^2/\text{d.o.f.} = 0.907$ in our analysis.

The determined functions are shown in Fig. 3. It is interesting to find that the up- and strange-quark functions are distributed relatively at large z, and both functions have similar shapes, whereas the gluon, charm-, and bottom-quark functions are distributed at smaller z. It may indicate that both functions, $D_u^{f_0}$ and $D_s^{f_0}$, are equally favored ones, which implies that the up-quark (and down-quark) is one of main components of f_0 as well as the strange-quark. Furthermore, they are peaked almost at the same points of z ($z_u^{\max} \sim z_s^{\max}$), which may be also considered as an evidence for the tetraquark structure according to Table I. However, if it is judged from their second moment ratio ($M_u/M_s = 0.43$), it looks like the $s\bar{s}$ configuration.

This conflict is mainly caused by the inaccurate determination of the fragmentation functions although it may be understood by admixture of the $s\bar{s}$ and tetraquark configurations. In Fig. 4, the uncertainties of $zD_u^{f_0}$, $zD_s^{f_0}$, and $zD_g^{f_0}$ are shown at $Q^2 = 1$ GeV^2 together with the functions themselves. We notice huge uncertainties which are an order of magnitude larger than the determined functions. If their moments are calculated, they have large errors $M_u = 0.0012 \pm 0.0107$, $M_s = 0.0027 \pm 0.0183$, and $M_g = 0.0090 \pm 0.0046$. From these results, the error of the moment ratio is estimated as $M_u/M_s = 0.43 \pm 6.73$, which makes it impossible to discuss the effect of the order of 50%. In this way, we find the structure of f_0 cannot be determined by the current e^+e^- data.

It is the purpose of this work to point out that structure of exotic hadrons should be determined by the fragmentation functions. Accurate measurements of hadron-production cross sections can be used for determining their internal quark and gluon configurations as explained in this paper by taking $f_0(980)$ as an example. We have shown that $s\bar{s}$ and tetraquark configurations, and also nonstrange-$q\bar{q}$ and glueball states, should be distinguished by the second moments and functional forms of the favored and disfavored

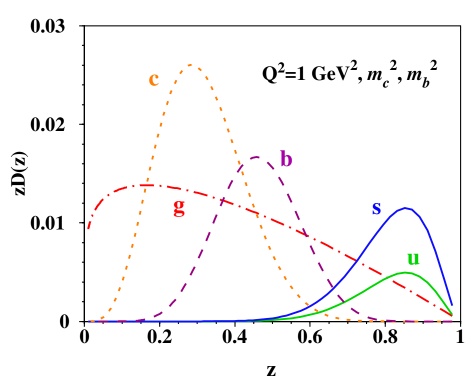

FIG. 3 (color online). Determined fragmentation functions of $f_0(980)$ by the global analysis. The functions $zD_u^{f_0}$, $zD_s^{f_0}$, and $zD_g^{f_0}$ are shown at $Q^2 = 1$ GeV^2, and the functions $zD_c^{f_0}$ and $zD_b^{f_0}$ are at $Q^2 = m_c^2$ and m_b^2, respectively.

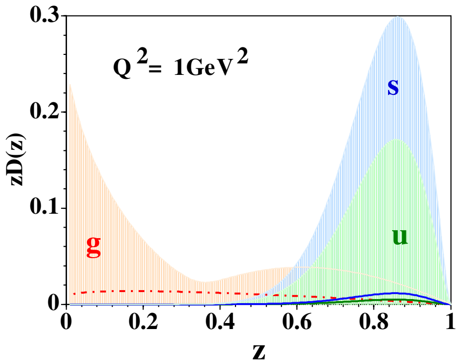

FIG. 4 (color online). Fragmentation functions, $zD_u^{f_0}$, $zD_s^{f_0}$, and $zD_g^{f_0}$, and their uncertainties are shown at $Q^2 = 1$ GeV^2. The uncertainties are shown by the shaded bands.

functions. Especially, the ratio of the u-quark moment to the s-quark one should be useful to judge the configuration.

In order to determine the internal structure, the flavor separation is important especially because the difference between the up- and strange-quark functions is the key to find the structure of f_0. First, charm- and bottom-quark tagged data should be provided for f_0 as they have been obtained for the pion, kaon, and proton. Then, the charm- and bottom-quark functions should be determined accurately. Second, semi-inclusive f_0-production data in lepton-proton scattering can be used for distinguishing between up- and strange-quark fragmentations because the initial quark distributions are different in the proton. These flavor separations will become possible by future experimental analyses. Our work is a starting point for exotic hadron search by suggesting the relations in the second moments and the functional forms and by indicating the current experimental situation as the uncertainty bands.

The fragmentation functions of f_0 and their uncertainties have been determined by the global analysis of f_0 production data. At this stage, the e^+e^- data are not precise enough; however, accurate experimental measurements could create a field of exotic hadrons which are beyond the naive $q\bar{q}$ and qqq type ones. Currently, analyses are in progress by the Belle collaboration [23] to provide accurate fragmentation functions. They are especially important because the functions are measured at small Q^2 ($\ll M_Z^2$), so that scaling violation can be investigated to find the gluon functions [17]. It is also important to have accurate measurements for ordinal mesons such as $\phi(1020)$ and $f_2(1270)$ in order to establish the f_0 configuration by comparing their favored and disfavored functions with the ones of f_0. We could investigate other exotic hadrons in the same way by their fragmentation functions.

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