Ultrahigh-energy cosmic-ray bounds on nonbirefringent modified Maxwell theory

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(Received 16 September 2007; published 11 January 2008)

A particularly simple Lorentz-violating modification of the Maxwell theory of photons maintains gauge invariance, *CPT*, and renormalization. This modified Maxwell theory, coupled to standard Dirac particles, involves 19 dimensionless "deformation parameters." Ten of these parameters lead to birefringence and are already tightly constrained by astrophysics. New bounds on the remaining nine nonbirefringent parameters can be obtained from the absence of vacuum Cherenkov radiation for ultrahigh-energy cosmic rays (UHECRs). Using selected UHECR events recorded at the Pierre Auger Observatory and assigning pseudorandom directions (i.e., assuming large-scale isotropy), Cherenkov bounds are found at the 10^{-18} level, which improve considerably upon current laboratory bounds. Future UHECR observations may reduce these Cherenkov bounds to the 10^{-23} level.

I. INTRODUCTION

It is possible that entirely new phenomena at the high-energy frontier of elementary particle physics (with characteristic energy $E_{\text{Planck}} \equiv \sqrt{\hbar c^5/G} \approx 1.2 \times 10^{19}$ GeV) lead to Lorentz-violating effects in the low-energy theory, in particular, the Maxwell theory of photons. The simplest *CPT*-invariant Lorentz-violating modification of the quadratic Maxwell action involves 19 dimensionless parameters [1–3].

Existing bounds on these 19 "deformation parameters" of modified Maxwell theory are as follows: ten birefringent parameters are bounded at the 10^{-32} level or better from astrophysics (see Refs. [4,5] and references therein) and nine nonbirefringent parameters are bounded at the 10^{-16} level or worse from laboratory experiments (see Refs. [6–8] and references therein).

Following earlier suggestions [9,10], it has been noted [11,12] that ultrahigh-energy cosmic rays (UHECRs) have the potential to place further limits on these nonbirefringent parameters by the inferred absence of "vacuum Cherenkov radiation." In fact, Ref. [12] already gave a bound at the 10^{-17} level on one nonbirefringent parameter from the existing UHECR data (cf. Refs. [13,14]). In this article, we obtain even better bounds for *all* nine nonbirefringent parameters from selected UHECR events recorded by the Pierre Auger Observatory [15] over the period January 2004 to February 2006 [16].

The present article is organized as follows. In Sec. II, we review the theory considered, quantum electrodynamics with a modified Maxwell term, and establish our notation. Also, we recall the Cherenkov threshold condition on

which our bounds will be based. In Sec. III, we discuss the selection of an appropriate set of UHECR events from two years running of the partially completed Auger experiment. In Sec. IV, we obtain from the Cherenkov threshold condition and the selected Auger events (with pseudorandom directions) the promised bounds on the nine deformation parameters of nonbirefringent modified Maxwell theory. In Sec. V, we summarize our findings and compare these new astrophysics bounds with previous laboratory bounds.

Throughout this article, we employ Cartesian coordinates $(x^\mu) = (x^0, \boldsymbol{x}) = (ct, x^1, x^2, x^3)$ and the Minkowski metric $(\eta_{\mu\nu}) = \text{diag}(+1, -1, -1, -1)$. Indices are lowered with the Minkowski metric $\eta_{\mu\nu}$ and raised with the inverse metric $\eta^{\mu\nu}$. Repeated upper and lower indices are summed over (Einstein summation convention). The direction of a 3-vector $\boldsymbol{x}$ is given by the unit 3-vector $\hat{\boldsymbol{x}} \equiv \boldsymbol{x}/|\boldsymbol{x}|$. Natural units with $c = \hbar = 1$ are used, unless stated otherwise.

II. THEORY

A. Nonbirefringent modified Maxwell theory

The action of modified Maxwell theory [1–3] is given by

$$\mathcal{S}_{\text{modM}} = \int_{\mathbb{R}^4} \mathrm{d}^4 x \left(-\frac{1}{4} (\eta^{\mu\rho} \eta^{\nu\sigma} + \kappa^{\mu\nu\rho\sigma}) F_{\mu\nu}(x) F_{\rho\sigma}(x) \right),$$
(1)

where $F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the standard Maxwell field strength of gauge fields $A_\mu(x)$ propagating over flat Minkowski spacetime with a metric $\eta_{\mu\nu}$ as defined in Sec. I. Here, $\kappa^{\mu\nu\rho\sigma}$ corresponds to a constant background tensor (with real and dimensionless components) having the same symmetries as the Riemann curvature tensor and a double trace condition $\kappa^{\mu\nu}{}_{\mu\nu} = 0$, so that

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there are $20 - 1 = 19$ independent components. All components of the κ-tensor in (1) are assumed to be very small, $|\kappa^{\mu\nu\rho\sigma}| \ll 1$, in order to ensure energy positivity [2]. The photonic action term (1) is gauge-invariant, *CPT*-even, and power-counting renormalizable (cf. Ref. [17]).

In order to restrict modified Maxwell theory to the nonbirefringent sector, the following *Ansatz* for $\kappa^{\mu\nu\rho\sigma}$ has been suggested [3,11]:

$$\kappa^{\mu\nu\rho\sigma} = \tfrac{1}{2}(\eta^{\mu\rho}\tilde{\kappa}^{\nu\sigma} - \eta^{\nu\rho}\tilde{\kappa}^{\mu\sigma} + \eta^{\nu\sigma}\tilde{\kappa}^{\mu\rho} - \eta^{\mu\sigma}\tilde{\kappa}^{\nu\rho}),$$

(2)

in terms of the nine components of a symmetric and traceless matrix $\tilde{\kappa}^{\mu\nu}$,

$$\tilde{\kappa}^{\mu\nu} = \tilde{\kappa}^{\nu\mu}, \qquad \tilde{\kappa}^{\mu}{}_{\mu} = 0.$$

(3)

The nine Lorentz-violating "deformation parameters" $\tilde{\kappa}^{\mu\nu}$, called "coupling constants" in Ref. [12], can be rewritten as follows:

$$(\tilde{\kappa}^{\mu\nu}) \equiv \overline{\kappa}^{00} \operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}) + (\delta\tilde{\kappa}^{\mu\nu}), \qquad \delta\tilde{\kappa}^{00} = 0, \quad (4)$$

with one independent parameter $\overline{\kappa}^{00}$ for the spatially isotropic part of $\tilde{\kappa}^{\mu\nu}$ and eight independent parameters in $\delta\tilde{\kappa}^{\mu\nu}$. For later use, we already define

$$\vec{\alpha} \equiv \begin{pmatrix} \alpha^0 \\ \alpha^1 \\ \alpha^2 \\ \alpha^3 \\ \alpha^4 \\ \alpha^5 \\ \alpha^6 \\ \alpha^7 \\ \alpha^8 \end{pmatrix} \equiv \begin{pmatrix} \tilde{\alpha}^{00} \\ \tilde{\alpha}^{01} \\ \tilde{\alpha}^{02} \\ \tilde{\alpha}^{03} \\ \tilde{\alpha}^{11} \\ \tilde{\alpha}^{12} \\ \tilde{\alpha}^{13} \\ \tilde{\alpha}^{22} \\ \tilde{\alpha}^{23} \end{pmatrix} \equiv \begin{pmatrix} (4/3)\overline{\kappa}^{00} \\ 2\delta\tilde{\kappa}^{01} \\ 2\delta\tilde{\kappa}^{02} \\ 2\delta\tilde{\kappa}^{03} \\ \delta\tilde{\kappa}^{11} \\ \delta\tilde{\kappa}^{12} \\ \delta\tilde{\kappa}^{13} \\ \delta\tilde{\kappa}^{22} \\ \delta\tilde{\kappa}^{23} \end{pmatrix}, \qquad (5)$$

where $\vec{\alpha}$ denotes the corresponding vector in the parameter space $\mathbb{R}^9$ with squared Euclidean norm

$$|\vec{\alpha}|^2 \equiv \sum_{l=0}^{8} (\alpha^l)^2. \qquad (6)$$

At this moment, it can also be mentioned that, for modified Maxwell theory, the phase velocity is given by $\mathbf{v}_{\text{phase}} \equiv \hat{\mathbf{k}}\omega/|\mathbf{k}| = c\hat{\mathbf{k}}(1 - \Xi(\hat{\mathbf{k}}))$, with a dimensionless function Ξ depending on the wave vector direction $\hat{\mathbf{k}}$ and the components of the κ-tensor [12]. This result implies that, for modified Maxwell theory, the phase velocity $\mathbf{v}_{\text{phase}}$ and the front velocity $\mathbf{v}_{\text{front}} \equiv \lim_{|\mathbf{k}|\to\infty} \mathbf{v}_{\text{phase}}(\mathbf{k})$ are equal for each direction $\hat{\mathbf{k}}$. Moreover, the phase and group velocities are equal to leading order in the Lorentz-violating parameters, $\mathbf{v}_{\text{group}} \equiv \partial\omega/\partial\mathbf{k} = \mathbf{v}_{\text{phase}} + O(\tilde{\kappa}^2)$. Recall that the front velocity is the relevant quantity for signal propagation and causality and that the group velocity at the dominant frequency component of a broad wave packet gives the velocity of energy transport; see, e.g., Refs. [18,19].

We now add spin-$\tfrac{1}{2}$ particles with electric charge e and mass M to the theory, taking the usual minimal coupling to the gauge field [17]. The action of these charged particles is assumed to be Lorentz invariant, so that, for the particular spacetime coordinates employed, the Lorentz violation of the combined theory resides solely in the κ term of (1). Specifically, the action with the standard Dirac term included is given by

$$\mathcal{S}_{\text{modM+standD}}$$
$$= \mathcal{S}_{\text{modM}} + \int_{\mathbb{R}^4} d^4x\, \bar{\psi}(x)(\gamma^\mu(i\partial_\mu - eA_\mu(x)) - M)\psi(x). \quad (7)$$

Note that the maximal attainable velocity of this charged particle equals c, which may or may not exceed the phase velocity of light discussed in the previous paragraph. See Refs. [20–22] for further discussion on microcausality in Lorentz-violating theories and, in particular, Eq. (50) of Ref. [21] for microcausality in the fermionic c_{00}-model, which is formally related to a special case of modified Maxwell theory (7) by an appropriate linear coordinate transformation [3,11].

As mentioned in the Introduction, new phenomena at the energy scale $E_{\text{Planck}} \approx 10^{19}$ GeV may lead to Lorentz violation in the low-energy theory, possibly described by modified Maxwell theory (1). The crucial point to realize is that, *a priori*, the Lorentz-violating parameters $\kappa^{\mu\nu\rho\sigma}$ need not be small (e.g., suppressed by inverse powers of E_{Planck}) but can be of order unity [23–25]. For this reason, it is of importance to obtain as strong bounds as possible on *all* deformation parameters $\kappa^{\mu\nu\rho\sigma}$.

B. Cherenkov threshold condition and UHECR bounds

Vacuum Cherenkov radiation for quantum electrodynamics (7) with the modified photonic action (1) and (2) has been studied in the classical approximation by Altschul [11] and at tree level by Kaufhold and Klinkhamer [12]. As explained in Ref. [12] (see also Ref. [10]), the radiated energy rate of a primary particle with point charge $Z_{\text{prim}}e$, mass $M_{\text{prim}} > 0$, momentum $\mathbf{q}_{\text{prim}}$, and ultrarelativistic energy $E_{\text{prim}} \sim c|\mathbf{q}_{\text{prim}}|$ is asymptotically given by

$$\frac{dW_{\text{modM}}(\mathbf{q}_{\text{prim}})}{dt} \sim Z_{\text{prim}}^2 \frac{e^2}{4\pi} \xi(\hat{\mathbf{q}}_{\text{prim}})E_{\text{prim}}^2/\hbar, \quad (8)$$

where the nonnegative dimensionless coefficient ξ results from appropriate contractions of the κ-tensor (2) with two rescaled q-vectors. The coefficient ξ may or may not depend on the primary particle direction $\hat{\mathbf{q}}_{\text{prim}}$.

The asymptotic behavior shown in (8) holds only for particle energies E_{prim} well above the Cherenkov threshold, which has the following order of magnitude:

$$E_{\text{thresh}} \sim M_{\text{prim}}c^2/\sqrt{\tilde{\kappa}}, \quad (9)$$

for an appropriate scale $\tilde{\kappa}$ obtained from the $\tilde{\kappa}^{\mu\nu}$ components written in terms of the parameters α^l (the scale $\tilde{\kappa}$ is

effectively set to zero if Cherenkov radiation is not allowed). For completeness, we mention that, at present, the most detailed study of vacuum Cherenkov radiation has been performed for another Lorentz-violating theory, Maxwell-Chern-Simons theory, and refer the reader to Refs. [12,26–29] for further discussion and references.

Continuing with nonbirefringent modified Maxwell theory, it is now possible to derive an upper bound on a particular combination of the deformation parameters α^l from the observation of a single UHECR event with a nuclear primary moving in the direction $\hat{\mathbf{q}}_{\mathrm{prim}}$ and having an ultrarelativistic energy $E_{\mathrm{prim}} \gg M_{\mathrm{prim}}c^2$. The argument is remarkably simple [9,10]: an UHECR proton or nucleus can arrive on Earth only if it does not lose energy by vacuum Cherenkov radiation and this requires the particle energy E_{prim} to be at or below threshold, $E_{\mathrm{prim}} \leq E_{\mathrm{thresh}}$.

The *caveat* of this simple argument is that the radiation rate (8) should not be suppressed by an extremely small numerical factor entering the coefficient ξ. But there is no reason to expect the presence of such an extremely small numerical factor; cf. Refs. [10,12]. In fact, a recent calculation [30] of the tree-level spinor-particle radiation rate for the special case of having only a single nonzero parameter $\alpha_0 > 0$ (i.e., case 2 of Appendix C of Ref. [12]) gives coefficient $\xi(\hat{\mathbf{q}}_{\mathrm{prim}}) = (7/24)\alpha_0$ on the right-hand side of (8). With ξ of order $\tilde{\kappa} > 0$, the particle would slow down from an initial energy $E_{\mathrm{prim}} \gg E_{\mathrm{thresh}}$ to the threshold energy E_{thresh} after having traveled a distance of the order of meters rather than parsecs (at least, for the values of $\tilde{\kappa}$ and E_{prim} considered in this article).

It may be worthwhile to repeat that expression (8) holds for an electric point charge, whereas we expect a form factor for a finite charge distribution with typical length scale $a \equiv \hbar c/\Lambda$; see also the discussion in Refs. [31,32]. But this modification of the theory would primarily affect the total radiation rate and not the energy threshold (9) on which our bounds are based, at least, for large enough cutoff Λ. Anyway, the theory considered in this article is precisely the one given in Sec. II A, without additional contact terms.

Using the explicit (direction-dependent) result [11] for the threshold energy (9), the condition $E_{\mathrm{prim}} \leq E_{\mathrm{thresh}}$ can then be written as the following upper bound on the deformation parameters (5) of modified Maxwell theory (7):

$$R(\alpha^0 + \alpha^j \hat{\mathbf{q}}_{\mathrm{prim}}^j + \tilde{\alpha}^{jk} \hat{\mathbf{q}}_{\mathrm{prim}}^j \hat{\mathbf{q}}_{\mathrm{prim}}^k) \leq (M_{\mathrm{prim}}c^2/E_{\mathrm{prim}})^2,$$
(10)

with each index j and k summed over 1 to 3 and ramp function $R(x) \equiv (x + |x|)/2$. The parameters $\tilde{\alpha}^{jk}$ appearing in the argument of the ramp function on the left-hand side of (10) are defined in (5), with $\tilde{\alpha}^{33} \equiv -\tilde{\alpha}^{11} - \tilde{\alpha}^{22}$ from the tracelessness condition (3).

Observe that, as expected on general grounds, bound (10) is invariant under a simultaneous rotation of $\hat{\mathbf{q}}_{\mathrm{prim}}$ and

appropriate redefinition of parameters $\alpha^1 \cdots \alpha^8$. This becomes especially clear if the argument of the ramp function on the left-hand side of (10) is written as $\alpha^0 + (\boldsymbol{\alpha} \cdot \hat{\mathbf{q}}_{\mathrm{prim}}) + (\boldsymbol{\beta} \cdot \hat{\mathbf{q}}_{\mathrm{prim}})(\boldsymbol{\gamma} \cdot \hat{\mathbf{q}}_{\mathrm{prim}})$, with two orthogonal 3-vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ replacing the 3-tensor $\tilde{\alpha}^{jk}$. In the same way, it is possible to absorb a parity-reflection of $\hat{\mathbf{q}}_{\mathrm{prim}}$ by a change of sign of the three parameters entering $\boldsymbol{\alpha}$, while leaving the other six parameters unchanged.

In the following, we will use (10) to obtain bounds on all nine deformation parameters α^l from a sufficiently large number N of UHECR events distributed over a large enough part of the sky (having primary energies E_n and directions $\hat{\mathbf{q}}_n$, for $n = 1, \ldots, N$) and an appropriate value of M_{prim}.

III. DATA SELECTION

The Pierre Auger Collaboration has published a table of 29 UHECR events with energies above 10 EeV $\equiv 10^{19}$ eV [16]. In addition, the atmospheric depth of the shower maximum, X_{max}, was determined for all of these events. Since the presently available Auger results indicate a mixed composition above 10 EeV [33] and our upper bound (10) scales with M_{prim}^2, we aim at selecting events initiated by *light* primaries such as protons or helium nuclei. For this reason, we apply the following selection criteria to the list of 29 events from Ref. [16]:

(1) $X_{\mathrm{max}}/(\mathrm{g\,cm^{-2}}) \geq 750 + 50(\log_{10}(E/\mathrm{EeV}) - 1)$;
(2) $\Delta_\gamma \geq +2$, where Δ_γ quantifies, in units of standard deviation, the difference between the observed X_{max} and the average X_{max}^γ expected from primary photons [see Eq. (1) of Ref. [16] for the definition of Δ_γ];
(3) primary energy between 10 and 30 EeV.

The first (and most selective) criterion aims at rejecting heavier nuclei. The chosen X_{max} parametrization roughly follows the average X_{max} values expected for helium nuclei. Thus, about half of the number of helium nuclei are rejected by this cut, while keeping the majority of protons. Oxygen and iron nuclei are rejected at a high level. In terms of the parameter Δ_{prim}, the selected events roughly have $\Delta_{\mathrm{oxygen}} \leq -1$ and $\Delta_{\mathrm{iron}} \leq -2$, respectively. Table I gives some calculated values for X_{max} as a function of primary energy and primary type (hydrogen, helium, oxygen, and iron nuclei and photon).

The second criterion is to ensure no contamination from primary photons, as neutral photons would not emit vacuum Cherenkov radiation at tree level. In fact, all 29 events from the original table of Ref. [16] already have $\Delta_\gamma \geq +2$.

The third criterion provides for a more homogeneous sample, but can certainly be relaxed in a future analysis. We have also accounted for the increased missing-energy correction in case of a nuclear primary, that is, the energies given in Ref. [16] (which refer to photon primaries) were increased by 7%, according to Ref. [39].

TABLE I. Simulated average shower-maximum atmospheric depth $X_{\max}$ in units of $\mathrm{g\,cm}^{-2}$, with root-mean square in brackets. Uncertainties are of the order of 3 $\mathrm{g\,cm}^{-2}$ for nuclear primaries and 5 $\mathrm{g\,cm}^{-2}$ for photons. The calculations were performed with CORSIKA [34] using two different hadronic interaction models, QGSJET 01 (abbreviated Q) [35] and SIBYLL 2.1 (abbreviated S) [36]. The asterisk (∗) indicates that, for primary photons above $10^{19.5}$ eV, the $X_{\max}$ values also depend on the direction of the event due to geomagnetic cascading (see, e.g., Ref. [37] and references therein). For instance, the $X_{\max}$ values at 10^{20} eV range between 940(85) and 1225(175) $\mathrm{g\,cm}^{-2}$, according to Ref. [38].

$\log_{10}(E/\mathrm{eV}) =$	17.5	18.0	18.5	19.0	19.5	20.0	20.5	21.0
Q—p	695(65)	725(65)	755(65)	775(65)	800(60)	820(60)	845(55)	865(55)
Q—He	665(45)	697(45)	725(45)	750(45)	775(40)	800(40)	825(38)	850(35)
Q—O			693(31)	717(30)	745(30)	780(30)	800(28)	
Q—Fe	600(25)	632(22)	664(22)	695(22)	725(20)	755(20)	782(20)	810(20)
S—p		740(65)		800(60)		860(55)	885(50)	
S—Fe		640(22)		700(22)		755(20)	785(20)	
Photon			915(60)	975(70)	1075(95)	(∗)	(∗)	(∗)

In total, 15 events remain which are listed in Table II. The corresponding event directions have not yet been published by the Pierre Auger Collaboration. Shown in Table II are pseudorandom directions chosen from uniform distributions of right ascension $\mathrm{RA}' \in [0°, 360°]$ and declination $\delta' \in [-70°, 25°]$, where the primes indicate the fictional nature of these directions. Note that the presently known UHECRs at energies $E \sim 10$ EeV are consistent with the hypothesis of large-scale isotropy; see, e.g., Refs. [13,14,40–44] for a selection of the available data.

All bounds of this paper are based on the values given in Table II, but, for the reason given above, we expect that the

TABLE II. Selected Auger events from Ref. [16]: event identification number, primary energy E [EeV], shower-maximum atmospheric depth $X_{\max}$ [$\mathrm{g\,cm}^{-2}$], and pseudorandom event directions with right ascension $\mathrm{RA}' \in [0°, 360°]$ and declination $\delta' \in [-70°, 25°]$. The primes on the ID numbers are to emphasize the nonreality of these event directions, which can later be replaced by the measured values RA and δ after their release by the Pierre Auger Collaboration (dropping the primes on the ID nos.).

ID no.	E	$X_{\max}$	(RA', δ')
668949′	18	765	(356, −29)
673409′	13	760	(344, −62)
828057′	14	805	(086, −34)
986990′	16	810	(152, −33)
1109855′	17	819	(280, −30)
1171225′	16	786	(309, −70)
1175036′	18	780	(228, +17)
1421093′	27	831	(079, +13)
1535139′	16	768	(006, −62)
1539432′	13	787	(153, −15)
1671524′	14	806	(028, −63)
1683620′	21	824	(024, −23)
1687849′	17	780	(031, −23)
2035613′	12	802	(079, −08)
2036381′	29	782	(158, −03)

same bounds are obtained from the actual event directions when they are made available by the Pierre Auger Collaboration. Just to be clear, the bounds of this article simply follow from having 15 UHECR events with more or less equal energies of order 10 EeV and more or less random directions over a significant part of the sky, the precise association of energy and direction being irrelevant.

The average energy of the sample given in Table II is $\langle E \rangle \approx 17$ EeV with an uncertainty of about 25% [45]. A larger uncertainty concerns the identity of the primary particle. We estimate an average mass for the selected events of $\langle M_{\mathrm{prim}} \rangle \approx f \times 5$ GeV/c^2, where the factor $f \in [0.2, 1.8]$ indicates the uncertainty ($f = 0.2$ refers to a pure proton beam, $f = 1$ to a mixed four-component primary composition which fits the Auger data [33], and $f = 1.8$ to the extreme case of no protons at all in the primary beam and a mixed three-component composition). In the next section, we will use an even larger value of f, namely $f = 3.2$, in order to obtain absolutely reliable bounds on the deformation parameters α^l.

IV. UHECR CHERENKOV BOUNDS

Following up on the remarks of the last paragraph of Sec. II B, it is now possible to determine numerically an exclusion domain in the $\vec{\alpha}$ parameter space $\mathbb{R}^9$ from the $N = 15$ events of Table II and the Cherenkov threshold condition (10). This exclusion procedure only works for those parameters $\vec{\alpha}$ for which the phase velocity of light is strictly less than the maximal attainable velocity c of the charged particle in theory (7), $v_{\mathrm{phase}} < c$. In fact, this phase-velocity condition corresponds to having a positive argument of the ramp function on the left-hand side of (10). Hence, the considered domain of parameter space is given by

$$D_{\mathrm{causal}}^{(\mathrm{open})} \equiv \{\vec{\alpha} \in \mathbb{R}^9: \quad \forall_{\hat{x} \in \mathbb{R}^3} \; (\alpha^0 + \alpha^j \hat{x}^j + \tilde{\alpha}^{jk} \hat{x}^j \hat{x}^k) > 0\},$$

(11)

with $\hat{x}$ an arbitrary unit vector in Euclidean 3-space, indices j and k summed over 1 to 3, and parameters $\tilde{\alpha}^{jk}$ defined by (5). As mentioned in Ref. [12] and Sec. II A, the condition $v_{\text{front}} = v_{\text{phase}} \leq c$ is, most likely, necessary for having a consistent and causal version of modified Maxwell theory, hence the suffix "causal" in (11). The other suffix "(open)" in (11) denotes the restriction of the causal domain by use of the open inequality "$(\alpha^0 + \cdots) > 0$" instead of the closed inequality "$(\alpha^0 + \cdots) \geq 0$," because no Cherenkov limits can be obtained for $v_{\text{phase}} = c$.

A hypersphere S_a^8 of finite radius a in the subspace (11) can be found numerically, so that for *each* $\vec{\alpha} \in S_a^8$ the inequality (10) is violated for *at least* one event from Table II. The excluded domain of parameter space then corresponds to the region on or outside this hypersphere. Observe, namely, that the left-hand side of (10), for a positive argument of the ramp function, increases by a factor λ under the scaling of $\vec{\alpha} \rightarrow \lambda\vec{\alpha}$ with $\lambda > 1$, so that inequality (10) is violated for $\lambda > 1$ if it is already for $\lambda = 1$.

In this way, the experimental data of Table II allow us to exclude the following region of parameter space at the 2σ level:

$$D_{\text{excluded}}^a = D_{\text{causal}}^{(\text{open})} \cap \{\vec{\alpha} \in \mathbb{R}^9 : |\vec{\alpha}| \geq a\}, \quad (12a)$$

$$a \approx 3 \times 10^{-18} \left(\frac{M_{\text{prim}}}{16 \text{ GeV}/c^2}\right)^2, \quad (12b)$$

where $|\vec{\alpha}|$ is the standard Euclidean norm (6) and where the reference value for the mass of the primary charged particle has conservatively been taken equal to that of oxygen (as mentioned in Sec. III, most primaries in the sample are expected to be protons and helium nuclei). The 2σ statistical error included in the number 3×10^{-18} on the right-hand side of (12b) is mainly due to the 25% energy uncertainty of the UHECRs.

The domain (12) gives only part of the excluded region, because at certain points on the hypersphere S_a^8 the excluded region pushes in. An interesting question, then, is how far the excluded region extends inwards. A partial answer is given by the following result. It is possible to establish numerically the existence of a hyperball B_b^9 with finite radius b, so that for *each* $\vec{\alpha} \in B_b^9$ the inequality (10) holds for *all* events from Table II. Specifically, the following domain of parameter space is found to be allowed:

$$D_{\text{allowed}}^b = D_{\text{causal}} \cap \{\vec{\alpha} \in \mathbb{R}^9 : |\vec{\alpha}| \leq b\}, \quad (13a)$$

$$b \approx (1.5 \pm 0.7) \times 10^{-19} \left(\frac{M_{\text{prim}}}{16 \text{ GeV}/c^2}\right)^2, \quad (13b)$$

with the approximate 1σ error in (13b) being mainly due to the energy uncertainty of the UHECRs and using the same reference value for the mass M_{prim} as in (12b). The complete allowed region may very well extend beyond the radius b, but not beyond the radius a as given by (12b).

Returning to the exclusion domain (12), the corresponding 2σ bound on $|\vec{\alpha}|$ is given by

$$\vec{\alpha} \in D_{\text{causal}}^{(\text{open})}:$$

$$|\vec{\alpha}|^2 \equiv \sum_{l=0}^{8} (\alpha^l)^2 < (3 \times 10^{-18})^2 \left(\frac{M_{\text{prim}}}{16 \text{ GeV}/c^2}\right)^4, \quad (14)$$

where, as argued above, a value of 16 GeV/c^2 for M_{prim} is entirely reasonable. With Lorentz invariance violated by the modified Maxwell theory, it is important to specify the reference frame in which bound (14) holds, namely, the solar-system frame in which the cosmic-ray energies are measured.

The new bound (14) is consistent with earlier expectations, as given by the one-sided bound (C12) of Ref. [12] in a somewhat implicit notation. Remark that our new bound on $\vec{\alpha}$ is also "one-sided" (vacuum Cherenkov radiation occurs only if the light velocity is less than that of the charged particle), but, now, the one-sidedness is in a 9-dimensional space. In order to be as clear as possible, the exclusion domain (12) has explicitly been given as an overlap with the domain (11) defined at the beginning of this section and bound (14) has been specified to hold for $\vec{\alpha}$ in the same domain.

The domains (12) and (13), together with the resulting upper bound (14), have been calculated from the measured (unprimed) energies and pseudorandom (primed) directions shown in Table II. But, as mentioned in Sec. III, we expect that the same results are obtained if the measured (unprimed) event directions are used.

In closing, it may be helpful to rephrase our main result (14) as follows: considering deformation parameters $\alpha^0, \ldots, \alpha^8$ with a corresponding phase velocity of light less than the maximal attainable velocity of charged particles, each of these nine parameters must separately have a modulus less than the value a given by (12b) at the 2σ level. A more detailed analysis of these bounds is left to the future when more UHECR events become available.

V. SUMMARY

In this article, we have established an UHECR Cherenkov bound (14) on the nine deformation parameters (5) of nonbirefringent modified Maxwell theory (1) and (2) coupled to a standard Dirac particle, with total action (7). Three remarks may be helpful to clarify the background and meaning of our result.

First, the derivation of bound (14) relies, strictly speaking, on the assumption of large-scale isotropy of the arrival directions of 10–30 EeV cosmic rays, which is, however, supported by all the available data (see Sec. III for further discussion). Second, this bound is essentially "one-sided" as it applies only to parameters belonging to domain (11), which may, however, correspond to a large part of the

''physical domain'' of the theory (see Secs. II A and IV for further discussion). Third, bound (14) holds for the theory as defined by the action (7) but also applies to a theory with additional Lorentz-violating parameters $c^{\mu\nu}$ in the Dirac sector [2], provided the parameters $\tilde{\kappa}^{\mu\nu}$ in bound (14) are replaced by the effective parameters $\tilde{\kappa}^{\mu\nu}_{\text{eff}} \equiv \tilde{\kappa}^{\mu\nu} - 2c^{\mu\nu}$ [3,11,32].

Having clarified the background of our Cherenkov bound (14), it is to be remarked that the corresponding bound at the 10^{-18} level for the spatially isotropic non-birefringent deformation parameter α^0 improves significantly upon the direct laboratory bound at the 10^{-7} level [6(c)] or the indirect (electron anomalous-magnetic moment) laboratory bound at the 10^{-8} level [7(c)]. But also the other Cherenkov bounds at the 10^{-18} level for the nonisotropic parameters α^l, for $l = 1, \ldots, 8$, improve considerably upon the direct laboratory bounds at the 10^{-12} to 10^{-16} levels [8]. While we note that a consistent comparison of bounds from laboratory and astrophysics data needs to be performed with great care (for example, Ref. [8(b)] allows for independent Lorentz-violating parameters in the photonic and fermionic sectors), the astrophysics bounds at the 10^{-18} level are certainly indicative.

With more and more UHECR events detected at the Pierre Auger Observatory over the coming years, these Cherenkov bounds at the 10^{-18} level can be expected to improve on at least two grounds. First, the availability of a larger number of events and further observables in addition to X_{max} will allow for more stringent cuts to obtain an essentially pure proton sample, which would give a reduction factor of 16^2 in (12b).[1] Second, higher energy events will become available for the analysis, perhaps up to values of order $E \sim 3 \times 10^{20}$ eV (the current highest known energy [46]), which would give a further reduction factor of approximately 10^3 in (12b).[2] Combined, the Cherenkov bounds on the nine deformation parameters α^l can perhaps be reduced by a factor of order 10^5 to the 10^{-23} level in the next decade.

Note added in proof.—The Pierre Auger Collaboration has recently published a list of 27 UHECR events with energies above 57 EeV and accurate event directions, which were recorded over the period 1 January 2004 to 31 August 2007 [49]. In order to provide better coverage of the northern celestial hemisphere, we add a 210 EeV event from the AGASA array [50] and the previously mentioned 320 EeV event from the Fly's Eye detector [46], both with reasonably accurate arrival directions. The observed energies and directions of these 29 events can now be used to sharpen bound (14) in terms of the square of the number a given by (12b).

In the absence of specific information on the chemical composition of all 29 events, we simply take $M_{\text{prim}} \approx 16 \text{ GeV}/c^2$. From these particular 29 UHECR events with an estimated energy uncertainty of the order of 25%, the improved value for a is found to be given by

$$A \equiv a_{\text{new}} \approx 2 \times 10^{-19} \left(\frac{M_{\text{prim}}}{16 \text{ GeV}/c^2}\right)^2, \qquad (15)$$

which gives the 2σ excluded region D^A_{excluded} as defined by (12a). With the 29 event directions known explicitly, there is no assumption entering the corresponding 2σ bound (14) with A^2 on the right-hand side, apart from the uncertainty in the primary mass and the *caveat* mentioned in Sec. II B.

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[1]If a sufficient number of 10 EeV cosmic-ray electrons could somehow be identified, there could be a further reduction factor in (12b) of approximately $(938/0.511)^2 \approx 3 \times 10^6$ compared to the proton case.

[2]The steeply falling flux spectrum and the expected suppression of protons from cosmologically distant sources with energies above 6×10^{19} eV [47,48] present an experimental challenge for pushing the energy to substantially higher values.

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