Minimal flavor violation, seesaw mechanism, and *R* parity

Emanuel Nikolidakis* and Christopher Smith†

Institut für Theoretische Physik, Universität Bern, CH-3012 Bern, Switzerland

(Received 30 October 2007; published 30 January 2008)

The minimal flavor violation hypothesis (MFV) is extended to the *R*-parity violating minimal supersymmetric standard model, supplemented with a simple seesaw mechanism. The requirement of MFV is shown to suppress lepton- and baryon-number violating couplings sufficiently to pass all experimental bounds, in particular, those for proton decay, and is thus a viable alternative to *R* parity. The phenomenological consequences for flavor-changing neutral currents, lepton flavor violation, and colliders are briefly discussed. Typically, MFV predicts sizable baryon-number violation in some characteristic channels, like single stop resonant production.

DOI: 10.1103/PhysRevD.77.015021 PACS numbers: 12.60.Jv, 11.30.Fs, 11.30.Hv

I. INTRODUCTION

The presence of the scalar partners of the quarks and leptons in the minimal supersymmetric standard model (MSSM) directly allows for renormalizable interactions violating baryon (B) or lepton (L) numbers. Though only accidental in the standard model (SM), there is considerable experimental support for the near conservation of at least one of these quantum numbers. In particular, the current bound on the proton lifetime sets a very tough limit on certain combinations of $\Delta L = 1$ and $\Delta B = 1$ couplings. To recover a viable phenomenology, the MSSM incorporates R parity [1], which explicitly forbids $\Delta L = 1$ and $\Delta B = 1$ interactions. At the same time, this symmetry leads to a very distinctive phenomenology, and signatures at colliders, since supersymmetric particles can be produced only in pairs, and the lightest supersymmetric particle (LSP) is absolutely stable.

The MSSM is not thought to be the ultimate theory. In particular, to account for the observed small neutrino masses by the seesaw mechanism [2], its particle content has to be extended at some high-energy scale. In that context, R parity loses its main appeal since it no longer protects the proton from decaying rapidly. Indeed, together with the $\Delta L = 2$ Majorana mass operator, R-parity conserving $\Delta L = 1$, $\Delta B = 1$ operators can appear among the dimension-five effective interactions generated from the integration of the high-energy degrees of freedom [3]. To resolve this issue, many models have been proposed, which typically predict that either L or B is exactly conserved at low energy, but not necessarily R parity (for a review, see e.g. Ref. [4]).

Besides the issue of proton stability, the presence of flavored scalar particles seems at odds with the observed suppression of flavor-changing processes, especially neutral currents and CP-violating phenomena. In the SM, the Cabibbo-Kobayashi-Maskawa (CKM) matrix, with its hierarchical structure, is able to account for all experimental results with an impressive precision. Therefore, with squark masses at or below the TeV to avoid destabilizing the electroweak scale, the MSSM scalar sector must be precisely fine-tuned to preserve these delicate patterns. Note that no fine-tuning is required in the supersymmetric sector, where quarks and squarks are aligned, but it needs to be enforced by hand for the soft-breaking terms, the remnants of the unknown supersymmetry-breaking mechanism.

A particularly elegant procedure to maintain this alignment, and thus to keep squark-induced flavor breakings in check, is to enforce it through a symmetry principle, the so-called minimal flavor violation (MFV) hypothesis of Ref. [5]. Though the origin of this symmetry is unclear, it allows for precise and well-defined predictions. Further, if proven valid by comparison with experiment, it may give us some glimpses of the mechanism at the origin of the flavor structures, totally unexplained within the MSSM, and will constrain the supersymmetry-breaking mechanism.

The MFV hypothesis is thus a systematic symmetry principle well supported by data. Our goal in the present work is to show that it can also explain proton stability, without calling in any additional symmetry. To appreciate the problem at hand, let us recall the orders of magnitude at play. If baryon-number conservation is not enforced, and $\Delta L = 2$ effects arise through the seesaw mechanism, given the present limits on the proton lifetime of more than 10^{30} years [6], our goal is to reconcile the following magnitude estimates,

$$\begin{aligned} &\lambda_{\Delta B} \sim \mathcal{O}(1),\\ &\lambda_{\Delta L=2} \sim m_\nu/(100\text{ GeV}) \sim \mathcal{O}(10^{-12})\\ &\quad \overset{?}{\to} |\lambda_{\Delta B}\lambda_{\Delta L=1}| \lesssim \mathcal{O}(10^{-24}), \end{aligned} \tag{1}$$

for neutrino masses $m_\nu \lesssim 1$ eV and sparticle masses below 1 TeV. Assuming that the $\Delta L = 1$ and $\Delta L = 2$ scales are similar, the requirement of MFV alone will be seen to be able to bridge the remaining gap of more than 10 orders of magnitude, at least under certain circumstances.

*nikolida@itp.unibe.ch
†chsmith@itp.unibe.ch

Specifically, the suppression capabilities of MFV strongly depend on m_ν, the ratio of the vacuum expectation values of the two MSSM Higgses ($\tan\beta \equiv v_u/v_d$), and on the flavor directions in which baryon and lepton numbers are violated, a point which will be precisely defined later.

Finally, having replaced R parity by MFV has many consequences. For example, the LSP is able to decay, and single-sparticle events could be seen at colliders. In that context, MFV predicts specific hierarchies among R-parity violating couplings, and can thus tell us in which direction to look for supersymmetric effects. Also, the study of the conditions under which MFV is sufficient to stabilize the proton offers interesting indirect constraints on other sectors, for example, flavor-changing neutral currents (FCNC) or lepton flavor violation (LFV) effects, which also depend on $\tan\beta$ or m_ν.

The structure of the paper is as follows. In Sec. II, we establish the minimal spurion content, and apply the MFV principle to R-parity violating couplings. The properties of the MFV expansions under redefinitions of the Higgs and lepton fields are then analyzed, as well as the application to higher-dimensional operators. The phenomenological consequences are explored in Sec. III. First, the MFV predictions for the order of magnitude of all R-parity violating couplings are worked out. Then, the various bounds are checked, in particular, those from $\Delta B = 1$ nucleon decay, and the consequences for $\tan\beta$ and m_ν are discussed. Finally, the consequences for supersymmetric signals at colliders are briefly analyzed, and our results are summarized in the Conclusion.

II. THE MFV HYPOTHESIS FOR THE R-PARITY VIOLATING MSSM

In regard to their gauge interactions, the three generations of quarks and leptons are decoupled, and physics is invariant under redefinitions of these matter fields. This is the origin of the global $U(3)^5$ flavor symmetry [7].

In the MSSM, this symmetry is broken in many places. At the very least, it is broken in the superpotential so as to reproduce the known SM flavor sector, i.e. the quark and charged lepton masses and CKM mixings. *A priori*, the soft-breaking squark and slepton masses and trilinear terms represent new sources of flavor mixings [8].

If R parity is not enforced, there are additional $\Delta L = 1$ and $\Delta B = 1$ couplings, both in the superpotential and among the soft-breaking terms (see e.g. Ref. [4] for a review). All these new couplings break the $U(3)^5$ flavor symmetry, since at least the $U(1)$'s for the lepton and baryon numbers do not survive.

This is not yet the full story. In the MSSM, there is no right-handed neutrino, while the left-handed neutrinos are massless. Therefore, to fully account for the leptonic flavor sector, i.e. neutrino masses and mixings, additional sources of flavor breaking must be introduced. In the present work, neutrino masses are generated through a simple seesaw mechanism of type I, from integrating out right-handed neutrinos at some high-energy scale [2].

There are thus many sources of breaking of the $U(3)^5$ symmetry. To reduce them, the MFV principle is a very attractive hypothesis. Indeed, it starts from the requirement of minimality: only the simplest breakings of the $U(3)^5$ symmetry are allowed. By simplest we mean only the minimal sources of breaking able to generate a realistic flavor sector, i.e. the known fermion masses and mixings. Technically, these flavor-breaking sources are parametrized as spurions, i.e. nondynamical fields in definite $U(3)^5$ representations.

To enforce MFV, the first step is to identify these elementary spurions, and then to parametrize all the flavor-breaking sectors of the MSSM as invariants under the flavor group. At that stage, the freedom to choose the directions in which lepton and baryon numbers are broken will play a special role. Indeed, in practice, this freedom translates into the choice of which ε tensors among the numerical invariant tensors of the five $SU(3) \in U(3)^5$ are to be used to construct invariants. The purpose of the present section is to construct these MFV expansions, leaving numerical studies and phenomenological discussions for Sec. III.

A. The seesaw mechanism and MFV spurions

In the supersymmetric limit, and without R-parity violation, the MSSM superpotential [denoting quark and lepton superfields as $Q = (u_L, d_L)^T$, $U = u_R^\dagger$, $D = d_R^\dagger$, $L = (\nu_L, e_L)^T$, $E = e_R^\dagger$, generation indices by I, J, and the $SU(2)_L$ spinor products by parentheses],

$$W_{RPC} = U^I(\mathbf{Y}_u)^{IJ}(Q^J H_u) - D^I(\mathbf{Y}_d)^{IJ}(Q^J H_d) - E^I(\mathbf{Y}_\ell)^{IJ}(L^J H_d) + \mu(H_u H_d), \tag{2}$$

is the only source of breaking of the $U(3)^5$ flavor symmetry acting on the (s)quarks and (s)leptons:

$$G_f = \underbrace{SU(3)_Q \times SU(3)_U \times SU(3)_D}_{G_q} \times \underbrace{SU(3)_L \times SU(3)_E}_{G_\ell} \times G_1, \tag{3}$$

with the $U(1)$'s acting on individual fields moved into the group G_1. Upon rearranging them, it can be written as

$$G_1 = U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\rm PQ} \times U(1)_E. \tag{4}$$

The first three correspond to the conserved baryon number, lepton number,[1] and weak hypercharge. The $U(1)_{\rm PQ}$, acting on D and E, is conserved if the Higgs H_d is also transforming nontrivially, and then is equivalent to the Peccei-Quinn symmetry of the two Higgs doublet model [9].

[1]We use the same notation for the lepton number $U(1)_L$ and for the $U(1)_L$ acting on the left-handed lepton doublet. In the following, $U(1)_L$ always denotes the latter.

Finally, the remaining $U(1)$ acts only on E and is broken by the leptonic Yukawa.

The superpotential W_{RPC} is made formally invariant under G_f by promoting $\mathbf{Y}_{u,d,\ell}$ to spurion fields transforming as [5]

$$U\overset{G_f}{\rightarrow}Ug_U^\dagger, \qquad D\overset{G_f}{\rightarrow}Dg_D^\dagger, \qquad Q\overset{G_f}{\rightarrow}g_Q Q, \qquad E\overset{G_f}{\rightarrow}Eg_E^\dagger, \qquad L\overset{G_f}{\rightarrow}g_L L, \tag{5}$$

$$\mathbf{Y}_u\overset{G_f}{\rightarrow}g_U\mathbf{Y}_u g_Q^\dagger, \qquad \mathbf{Y}_d\overset{G_f}{\rightarrow}g_D\mathbf{Y}_d g_Q^\dagger, \qquad \mathbf{Y}_\ell\overset{G_f}{\rightarrow}g_E\mathbf{Y}_\ell g_L^\dagger, \tag{6}$$

or $\mathbf{Y}_u \sim (\bar{3},3,1)_{G_q}$, $\mathbf{Y}_d \sim (\bar{3},1,3)_{G_q}$, $\mathbf{Y}_\ell \sim (\bar{3},3)_{G_\ell}$. Therefore, in order to account for the quark masses, CKM mixings, and charged lepton masses, one needs at least three spurions with these transformation properties. The minimal case is when the basic sources of flavor breaking are only along the $(\bar{3},3,1)_{G_q}$, $(\bar{3},1,3)_{G_q}$, and $(\bar{3},3)_{G_\ell}$ directions.

Remaining are the light neutrino masses. For them, we supplement the MSSM with a seesaw mechanism [2], following Ref. [10]. We start by adding heavy right-handed (s)neutrinos:

$$W_{\nu_R} = W_{\rm RPC} + \tfrac{1}{2}N^I\mathbf{M}^{IJ}N^J + N^I(\mathbf{Y}_\nu)^{IJ}(L^J H_u), \tag{7}$$

corresponding to an enlarged flavor symmetry $G_f \times U(3)_N$ at the high-energy scale, with

$$N\overset{G_f\times U(3)_N}{\longrightarrow}Ng_N^\dagger, \qquad \mathbf{Y}_\nu\overset{G_f\times U(3)_N}{\longrightarrow}g_N\mathbf{Y}_\nu g_L^\dagger, \qquad \mathbf{M}\overset{G_f\times U(3)_N}{\longrightarrow}g_N\mathbf{M}g_N^T. \tag{8}$$

When the Majorana mass $\mathbf{M}$ is very large, right-handed neutrinos can be integrated out, leading to the well-known nonrenormalizable dimension-five term in the superpotential [11]

$$\begin{aligned} W_{\rm dim\text{-}5} &= \tfrac{1}{2}(\mathbf{Y}_\nu)^{IK}(L^K H_u)(\mathbf{M}^{-1})^{IJ}(\mathbf{Y}_\nu)^{JL}(L^L H_u) \\ &\overset{\rm SSB}{\rightarrow}\tfrac{1}{2}v_u^2\nu_L^I(\mathbf{Y}_\nu)^{IK}(\mathbf{M}^{-1})^{IJ}(\mathbf{Y}_\nu)^{JL}\nu_L^L. \end{aligned} \tag{9}$$

This term gives mass to the left-handed neutrinos after electroweak symmetry breaking, and we define the dimensionless neutrino mass spurion as

$$\mathbf{Y}_\nu \equiv v_u\mathbf{Y}_\nu^T\mathbf{M}^{-1}\mathbf{Y}_\nu \sim \mathcal{O}(m_\nu/v_u), \tag{10}$$

transforming as $\mathbf{Y}_\nu \sim (\bar{6},1)_{G_\ell}$. It is symmetric, $\mathbf{Y}_\nu = \mathbf{Y}_\nu^T$, since $\mathbf{M}^{-1}$ can be assumed diagonal without loss of generality. For simplicity, we further assume

$$\mathbf{M} = M_R\mathbf{1}. \tag{11}$$

This is the well-known seesaw mechanism: the heavy mass scale M_R bears the responsibility for the smallness of the neutrino masses, not the Yukawa $\mathbf{Y}_\nu$. For example, with $m_\nu \sim 1$ eV, $\mathbf{Y}_\nu \sim \mathcal{O}(1)$ when $M_R \sim 10^{13}$ GeV.

For consistency, we must include also the other spurion transforming as a singlet under $U(3)_N$, which is $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \sim (8,1)_{G_\ell}$. Compared to $\mathbf{Y}_\nu$, it is not suppressed by the heavy mass scale. It has to be included into our list of spurions because it is transforming differently than $\mathbf{Y}_\nu$. Further, it plays an important role in the generation of LFV effects if the supersymmetry-breaking scale is much higher than the M_R scale [12]. Therefore, the two neutrino spurions which have to be included are

$$\mathbf{Y}_\nu\overset{G_f}{\rightarrow}g_L^*\mathbf{Y}_\nu g_L^\dagger, \qquad \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\overset{G_f}{\rightarrow}g_L\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu g_L^\dagger. \tag{12}$$

What is important about the spurions in Eqs. (6) and (12) is their specific transformation properties under the flavor group, not their expressions in terms of the Yukawas. For instance, consider the $U(QH_u)$ term of W_{RPC}. As soon as the flavor symmetry is broken along the $(\bar{3},3,1)_{G_q}$ and $(\bar{3},1,3)_{G_q}$ directions, which we parametrize by the two spurions $\mathbf{Y}_u$ and $\mathbf{Y}_d$, MFV implies that we can write

$$\begin{aligned} W_{RPC} \ni\ & U^I(a_1\mathbf{Y}_u + a_2\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u + a_3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\ldots)^{IJ} \\ & \times (Q^J H_u), \end{aligned} \tag{13}$$

with MFV coefficients $a_i \sim \mathcal{O}(1)$. Since it is always possible to redefine the spurions as $a_1\mathbf{Y}_u + a_2\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u + a_3\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\ldots \rightarrow \mathbf{Y}_u$, which is neutral from the point of view of G_f, the Yukawa couplings can be assumed to take their usual forms, and there is no need to distinguish between spurion fields and Yukawa couplings. In addition, using the freedom to perform flavor rotations, they can be brought to their background values

$$\begin{aligned} &\mathbf{Y}_u = \mathbf{m}_u V/v_u, \qquad \mathbf{Y}_d = \mathbf{m}_d/v_d, \qquad \mathbf{Y}_\ell = \mathbf{m}_\ell/v_d, \\ &\mathbf{Y}_\nu = U^*\mathbf{m}_\nu U^\dagger/v_u, \qquad \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \overset{CP}{=} M_R\mathbf{Y}_\nu/v_u, \end{aligned} \tag{14}$$

where V is the CKM matrix, U the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, $\mathbf{m}_{u,d,\ell,\nu}$ are the diagonal fermion mass matrices, and $v_{u,d}$ the vacuum expectation values (VEV's) of the $H_{u,d}^0$ Higgs, with $v_u^2 + v_d^2 \approx (174\ \text{GeV})^2$ and $\tan\beta \equiv v_u/v_d$. The spurion $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ can be fixed only in the CP limit, by neglecting unknown phases (see Sec. III A). It is important to stress that we only take the CP limit in numerical estimates. For the purpose of enforcing the MFV hypothesis, it is essential to keep track of their different transformation properties under $SU(3)_L$, namely, $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \sim (8,1)_{G_\ell}$ and $\mathbf{Y}_\nu \sim (\bar{6},1)_{G_\ell}$.

B. The general MFV expansion

In the previous section, we established the minimal set of spurions needed to generate a phenomenologically viable quark and lepton flavor-breaking sector. Out of them, we now parametrize all the other flavor-breaking sectors of the MSSM, including both R-parity conserving (RPC) and R-parity violating (RPV) couplings (see e.g. Ref. [4] for a review). Specifically, the RPV superpotential terms are

$$W_{RPV} = \tfrac{1}{2}\boldsymbol{\lambda}^{IJK}(L^I L^J)E^K + \boldsymbol{\lambda}'^{IJK}(L^I Q^J)D^K + \mu'^I(H_u L^I) + \tfrac{1}{2}\boldsymbol{\lambda}''^{IJK}U^I D^J D^K, \tag{15}$$

while the RPC and RPV soft-breaking terms involving the scalar fields are

$$\mathcal{L}_{\text{soft}}^{RPC\text{-bilinear}} = -m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - (b(H_u H_d) + \text{H.c.}) - \tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q} - \tilde{U}\mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{D}\mathbf{m}_D^2 \tilde{D}^\dagger - \tilde{L}^\dagger \mathbf{m}_L^2 \cdot \tilde{L} - \tilde{E}\mathbf{m}_E^2 \tilde{E}^\dagger, \tag{16a}$$

$$\mathcal{L}_{\text{soft}}^{RPC\text{-trilinear}} = -\tilde{U}^I(\mathbf{A}_u)^{IJ}(\tilde{Q}^J H_u) + \tilde{D}^I(\mathbf{A}_d)^{IJ}(\tilde{Q}^J H_d) + \tilde{E}^I(\mathbf{A}_\ell)^{IJ}(\tilde{L}^J H_d) + \text{H.c.}, \tag{16b}$$

$$\mathcal{L}_{\text{soft}}^{\text{RPV-bilinear}} = -\mathbf{b}'^I(H_u \tilde{L}^I) - (\mathbf{m}_{Ld}^2)^I H_d^\dagger \cdot \tilde{L}^I + \text{H.c.}, \tag{16c}$$

$$\mathcal{L}_{\text{soft}}^{\text{RPV-trilinear}} = \tfrac{1}{2}\mathbf{A}^{IJK}(\tilde{L}^I\tilde{L}^J)\tilde{E}^K + \mathbf{A}'^{IJK}(\tilde{L}^I\tilde{Q}^J)\tilde{D}^K + \tfrac{1}{2}\mathbf{A}''^{IJK}\tilde{U}^I\tilde{D}^J\tilde{D}^K + \text{H.c.}, \tag{16d}$$

with the trilinear terms $\boldsymbol{\lambda}^{IJK}$ and $\mathbf{A}^{IJK}$ ($\boldsymbol{\lambda}''^{IJK}$ and $\mathbf{A}''^{IJK}$) antisymmetric under $I \leftrightarrow J$ ($J \leftrightarrow K$).

The MFV hypothesis is enforced by making all these couplings invariant under G_f, up to the $U(1)$'s which are *a priori* broken, using only the available spurions. As said before, there is no loss of generality in identifying these spurions such that Eq. (14) holds, since this corresponds to a mere $\mathcal{O}(1)$ redefinition for the MFV coefficients.

1. Algebraic reductions and application to RPC soft-breaking terms

To get overall singlets under $G_q \times G_\ell$, the two invariant tensors of the five $SU(3)$ can be used, namely, δ^{IJ} and ε^{IJK}. Given the large number of possible terms, we proceed in steps.

Let us first consider the two generic terms transforming as left-handed octets, $\mathbf{R}_\ell \sim (8,1)_{G_\ell}$ and $\mathbf{R}_q \sim (8,1,1)_{G_q}$. Making use of the Cayley-Hamilton relation

$$\mathbf{A}^3 - \text{Tr}(\mathbf{A})\mathbf{A}^2 + \tfrac{1}{2}\mathbf{A}(\text{Tr}(\mathbf{A})^2 - \text{Tr}(\mathbf{A}^2)) - \tfrac{1}{3}\text{Tr}(\mathbf{A}^3) + \tfrac{1}{2}\text{Tr}(\mathbf{A})\text{Tr}(\mathbf{A}^2) - \tfrac{1}{6}\text{Tr}(\mathbf{A})^3 = 0, \tag{17}$$

together with the third-generation dominance (valid to about 5%)

$$(\mathbf{Y}_{u,d,\ell}^\dagger \mathbf{Y}_{u,d,\ell})^2 \approx y_{t,b,\tau}^2 \mathbf{Y}_{u,d,\ell}^\dagger \mathbf{Y}_{u,d,\ell}, \tag{18}$$

with $y_t = m_t/v_u$ and $y_{b,\tau} = m_{b,\tau}/v_d$, reduces the number of relevant octet terms to

$$\mathbf{R}_q = \mathbf{1},\quad \mathbf{Y}_u^\dagger\mathbf{Y}_u,\quad \mathbf{Y}_d^\dagger\mathbf{Y}_d,\quad \mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u,\quad \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d, \tag{19}$$

$$\mathbf{R}_\ell = \mathbf{1},\quad \mathbf{Y}_\ell^\dagger\mathbf{Y}_\ell,\quad \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu,\quad \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_\ell^\dagger\mathbf{Y}_\ell,\quad \mathbf{Y}_\ell^\dagger\mathbf{Y}_\ell\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu,\quad (\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2,\quad \mathbf{Y}_\ell^\dagger\mathbf{Y}_\ell(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2,\quad (\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2\mathbf{Y}_\ell^\dagger\mathbf{Y}_\ell,\quad (\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2\mathbf{Y}_\ell^\dagger\mathbf{Y}_\ell\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu. \tag{20}$$

We also used identities involving two or more different matrices to reach this minimal basis. They can be found from Eq. (17) by expressing $\mathbf{A} = a_1\mathbf{A}_1 + a_2\mathbf{A}_2 + \ldots$ and extracting a given power of $a_1, a_2, \ldots$. Importantly, since the $y_{t,b,\tau}^2$ are at most of $\mathcal{O}(1)$, these identities as well as Eq. (18) do not generate large numerical coefficients.

The spurion $\mathbf{Y}_\nu$ was not used because it is very suppressed compared to the others. Also, there is no need to consider contractions with ε tensors. Indeed, all such terms necessarily involve an even number of ε tensors, which can be simplified to products or determinants of $\mathbf{R}_i$ monomials using

$$\varepsilon^{IJK}\varepsilon^{LMN} = \det\begin{pmatrix} \delta^{IL} & \delta^{IM} & \delta^{IN} \\ \delta^{JL} & \delta^{JM} & \delta^{JN} \\ \delta^{KL} & \delta^{KM} & \delta^{KN} \end{pmatrix},\quad \varepsilon^{LMN}\mathbf{A}^{LI}\mathbf{A}^{MJ}\mathbf{A}^{NK} = \det(\mathbf{A})\varepsilon^{IJK}. \tag{21}$$

The next step is to construct the skeleton decompositions of each coupling, and to dress them with all possible insertions of $\mathbf{R}_q$ and $\mathbf{R}_\ell$ monomials. For example, looking at $\mathbf{m}_U^2$, it transforms as $\mathbf{m}_U^2 \to g_U \mathbf{m}_U^2 g_U^\dagger$; hence its skeleton is $\mathbf{m}_U^2 = m_0^2(a_1\mathbf{1} + a_2\mathbf{Y}_u\mathbf{Y}_u^\dagger)$ with MFV coefficients $a_i \sim \mathcal{O}(1)$ and m_0^2 setting the supersymmetry-breaking scale, as in minimal supergravity model [8]. All the relevant MFV terms are then found by inserting $\mathbf{R}_q$ monomials between $\mathbf{Y}_u$ and $\mathbf{Y}_u^\dagger$. With the further requirement of hermicity for $\mathbf{m}_U^2$, we find

$$\mathbf{m}_U^2 = m_0^2(a_1\mathbf{1} + \mathbf{Y}_u(a_2\mathbf{1} + a_3\mathbf{Y}_u^\dagger\mathbf{Y}_u + a_4\mathbf{Y}_d^\dagger\mathbf{Y}_d + a_5(\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u + \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d))\mathbf{Y}_u^\dagger), \tag{22}$$

with MFV coefficients $a_i \sim \mathcal{O}(1)$. For compactness, we denote this expansion as $\mathbf{m}_U^2 = m_0^2(\mathbf{1} + \mathbf{Y}_u[\mathbf{R}_q]_{\text{Hc}}\mathbf{Y}_u^\dagger)$, where $[\ldots]_{\text{Hc}}$ stands for the Hermitian combination.

Proceeding similarly for the other RPC soft-breaking terms, we find, written in the compact form [arbitrary $\mathcal{O}(1)$ MFV coefficients are understood everywhere] [13],

$$\mathbf{m}_Q^2 = m_0^2[\mathbf{R}_q]_{\rm Hc}, \qquad \mathbf{m}_U^2 = m_0^2(\mathbf{1} + \mathbf{Y}_u[\mathbf{R}_q]_{\rm Hc}\mathbf{Y}_u^\dagger), \qquad \mathbf{m}_D^2 = m_0^2(\mathbf{1} + \mathbf{Y}_d[\mathbf{R}_q]_{\rm Hc}\mathbf{Y}_d^\dagger), \tag{23a}$$

$$(\mathbf{A}_u)^{IJ} = A_0((\mathbf{Y}_u\mathbf{R}_q)^{IJ} + \varepsilon^{LMN}\varepsilon^{ABC}(\mathbf{R}_u)^{IA}(\mathbf{R}_q\mathbf{Y}_u^\dagger)^{LB}(\mathbf{R}_q\mathbf{Y}_u^\dagger)^{MC}(\mathbf{R}_q)^{NJ}), \tag{23b}$$

$$(\mathbf{A}_d)^{IJ} = A_0((\mathbf{Y}_d\mathbf{R}_q)^{IJ} + \varepsilon^{LMN}\varepsilon^{ABC}(\mathbf{R}_d)^{IA}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{LB}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{MC}(\mathbf{R}_q)^{NJ}), \tag{23c}$$

$$\mathbf{m}_L^2 = m_0^2[\mathbf{R}_\ell]_{\rm Hc}, \qquad \mathbf{m}_E^2 = m_0^2(\mathbf{1} + \mathbf{Y}_\ell[\mathbf{R}_\ell]_{\rm Hc}\mathbf{Y}_\ell^\dagger), \tag{23d}$$

$$(\mathbf{A}_\ell)^{IJ} = A_0((\mathbf{Y}_\ell\mathbf{R}_\ell)^{IJ} + \varepsilon^{LMN}\varepsilon^{ABC}(\mathbf{R}_e)^{IA}(\mathbf{R}_\ell\mathbf{Y}_\ell^\dagger)^{LB}(\mathbf{R}_\ell\mathbf{Y}_\ell^\dagger)^{MC}(\mathbf{R}_\ell)^{NJ}), \tag{23e}$$

where the ε-terms have been reduced using Eq. (21) as long as only manifestly invariant terms under G_f are generated, and the $SU(3)_U$, $SU(3)_D$, and $SU(3)_E$ octets are defined as

$$\mathbf{R}_u = \mathbf{1} + \mathbf{Y}_u\mathbf{R}_q\mathbf{Y}_u^\dagger, \qquad \mathbf{R}_d = \mathbf{1} + \mathbf{Y}_d\mathbf{R}_q\mathbf{Y}_d^\dagger, \qquad \mathbf{R}_e = \mathbf{1} + \mathbf{Y}_\ell\mathbf{R}_\ell\mathbf{Y}_\ell^\dagger, \tag{24}$$

where, again, arbitrary $\mathcal{O}(1)$ coefficients are understood everywhere.

Apart from the additional ε structures for the trilinear terms $\mathbf{A}_{u,d}$, these expansions agree with those of Refs. [5,14], and their phenomenological consequences for FCNC were analyzed in Refs. [15]. In the leptonic sector, to our knowledge, they have not yet been written in this form. Note that $\mathbf{Y}_\nu$ does not enter these expansions, and all LFV effects arise from the nondiagonal $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ spurion, as in Ref. [12].

Concerning the ε-terms, their presence is unavoidable if one sticks to the MFV principle. Though it is clear that they cannot emerge from the renormalization group evolution (RGE) evolution of universal soft-breaking terms in the *RPC* MSSM, they are, in general, allowed once *RPV* couplings are introduced. Since their relevance for phenomenology has not yet been investigated, it is worth briefly describing their structure, leaving a detailed study for future work. The ε tensors being antisymmetric, their contributions have an inverted hierarchy compared to the Yukawa terms, and are always small, proportional to light-fermion masses. Anticipating on the results of numerical analyses of Sec. III, only $(\mathbf{A}_d)^{11}$ is significantly affected by the ε-terms, and this only at large $\tan\beta$ (see Appendix B).

2. Application to *RPV* couplings

The MFV expansions for the *RPV* terms of the superpotential are collected in Table I, where the intermediate spurion $\bar{\mathbf{Y}}_\nu$, transforming as $\bar{\mathbf{Y}}_\nu \to \det(g_L)\bar{\mathbf{Y}}_\nu g_L^\dagger$, is

$$\bar{\mathbf{Y}}_{\nu}^{I} \equiv \varepsilon^{QMJ}\mathbf{Y}_\nu^{\dagger PN}(\mathbf{R}_\ell)^{QN}(\mathbf{R}_\ell)^{MP}(\mathbf{R}_\ell)^{JI} = \varepsilon^{QMJ}(\mathbf{R}_\ell\mathbf{Y}_\nu^\dagger\mathbf{R}_\ell^T)^{MQ}(\mathbf{R}_\ell)^{JI}, \tag{25}$$

and the ϵ and ϵ' tensors,

$$\epsilon^{ABCDEF} \equiv \varepsilon^{ACE}\varepsilon^{BDF}, \varepsilon^{EFD}\varepsilon^{ABC}, \varepsilon^{EFA}\varepsilon^{BCD}, \tag{26}$$

$$\epsilon'^{ILMNPQR} \equiv \varepsilon^{LMN}\varepsilon^{PQR}, \varepsilon^{LMQ}\varepsilon^{NPR}, \varepsilon^{LMP}\varepsilon^{NQR}, \tag{27}$$

stand for the three inequivalent contractions in the $SU(3)_L$ and $SU(3)_Q$ space, respectively. For ϵ, there is a fourth possible contraction, $\varepsilon^{ABE}\varepsilon^{CDF}$, which gives back $\boldsymbol{\lambda}_3$. Structures involving more ε tensors can be reduced to those of Table I using Eq. (21).

We do not write down explicitly the MFV expansion for the *RPV* soft-breaking terms since they can be readily obtained from Table I: $\mathbf{b}'$ and $\mathbf{m}_{Ld}^2$ transform as $\boldsymbol{\mu}'$, while $\mathbf{A}$, $\mathbf{A}'$, and $\mathbf{A}''$ transform as $\boldsymbol{\lambda}$, $\boldsymbol{\lambda}'$, and $\boldsymbol{\lambda}''$, respectively. The normalization of the dimensionful couplings will be addressed in Sec. II C. For now, we just state that $\boldsymbol{\mu}'$, $\mathbf{b}'$, and $\mathbf{m}_{Ld}^2$ are normalized with respect to μ, b, and $m_{H_d}^2$, respectively, while trilinear soft-breaking terms are all normalized by the supersymmetry-breaking scale A_0 [see Eq. (23)]. The MFV coefficients are then dimensionless and assumed to be of $\mathcal{O}(1)$.

Reminiscent of the fact that *RPV* operators break either baryon or lepton number, each of them involves at least one ε tensor [the $\mathbf{R}_i$ are neutral under all $U(1)$'s]. However, in terms of the $U(1)$'s acting on the individual fields, we have the freedom to decide in which direction to break B and L. As shown in Table I, the $\Delta L = 1$ structures break $U(1)_L$ or both $U(1)_E$ and $U(1)_L$, while $\Delta B = 1$ structures break $U(1)_U$, $U(1)_D$, and/or $U(1)_Q$.

We are not forced to simultaneously break all these $U(1)$'s; only one per sector is needed. In particular, we can require the $U(1)$ for all the right-handed fields to remain exact. Alternatively, we can choose to maintain only $U(1)_D$ and $U(1)_E$, which are intimately connected with $U(1)_{\rm PQ}$. Indeed, if we do not assign a $U(1)_{\rm PQ}$ charge to H_d, it is then borne by the Yukawas $\mathbf{Y}_d$ and $\mathbf{Y}_\ell$. Terms which violate $U(1)_D$ or $U(1)_E$ are then precisely those which violate $U(1)_{\rm PQ}$.

In practice, enforcing one of the $U(1)$'s amounts to suppressing the structures which break it by powers of $\det(\mathbf{Y}_u)$, $\det(\mathbf{Y}_d)$, or $\det(\mathbf{Y}_\ell)$. These determinants always involve the light-fermion masses, and are very small even at large $\tan\beta$. For example, enforcing $U(1)_{U,D,E}$ suppresses all ε structures in the *RPC* soft-breaking terms of Eq. (23), while it leaves only $\boldsymbol{\mu}_1'$, $\boldsymbol{\lambda}_{1,2}$, $\boldsymbol{\lambda}_1'$, and $\boldsymbol{\lambda}_3''$ as dominant *RPV* structures. Finally, if we decide to enforce $U(1)_L$, all $\Delta L = 1$ structures get suppressed by at least one power of $\det(\mathbf{Y}_\ell)$. This global suppression of the $\Delta L = 1$ sector is possible because $\mathbf{Y}_\nu$ is transforming nontrivially only under $SU(3)_L$.

TABLE I. Superpotential *RPV* terms under the MFV hypothesis. For $\boldsymbol{\lambda}_i^{IJK}$, it is understood that contributions must be antisymmetrized under $I \leftrightarrow J$, while, similarly, $\boldsymbol{\lambda}_i^{\prime\prime IJK}$ must be antisymmetrized under $J \leftrightarrow K$. The explicit MFV expansion for each structure is obtained by summing over the possible insertions of the R_i terms of Eqs. (19), (20), and (24), and inserting arbitrary $\mathcal{O}(1)$ MFV coefficients in front of each term of this sum. Finally, the ε tensors each act in one of the five $SU(3)$, and hence break a specific $U(1)$, as indicated in the last column.

Structure	MFV terms	Broken $U(1)$
$\boldsymbol{\mu}_1^{\prime I}$	$\mu \bar{\mathbf{Y}}_\nu^I$	$\det(g_L)$
$\boldsymbol{\lambda}_1^{IJK}$	$\bar{\mathbf{Y}}_\nu^I(\mathbf{Y}_l\mathbf{R}_l)^{KJ}$	$\det(g_L)$
$\boldsymbol{\lambda}_2^{IJK}$	$\varepsilon^{LMN}(\mathbf{R}_\ell)^{LI}(\mathbf{Y}_\ell\mathbf{R}_\ell\mathbf{Y}_\nu^\dagger\mathbf{R}_\ell^T)^{KM}(\mathbf{R}_\ell)^{NJ}$	$\det(g_L)$
$\boldsymbol{\lambda}_3^{IJK}$	$\bar{\mathbf{Y}}_\nu^I\varepsilon^{LMN}\varepsilon^{ABC}(\mathbf{R}_e)^{KA}(\mathbf{R}_\ell\mathbf{Y}_\ell^\dagger)^{LB}(\mathbf{R}_\ell\mathbf{Y}_\ell^\dagger)^{MC}(\mathbf{R}_\ell)^{NJ}$	$\det(g_L^2 g_E^\dagger)$
$\boldsymbol{\lambda}_4^{IJK}$	$\varepsilon^{LMN}\epsilon^{ABCDEF}(\mathbf{R}_\ell\mathbf{Y}_\ell^\dagger)^{AL}(\mathbf{R}_\ell\mathbf{Y}_\ell^\dagger)^{BM}(\mathbf{R}_e)^{KN}(\mathbf{R}_\ell\mathbf{Y}_\nu^\dagger R_\ell^T)^{CD}(R_\ell)^{EI}(R_\ell)^{FJ}$	$\det(g_L^2 g_E^\dagger)$
$\boldsymbol{\lambda}_1^{\prime IJK}$	$\bar{\mathbf{Y}}_\nu^I(\mathbf{Y}_d\mathbf{R}_q)^{KJ}$	$\det(g_L)$
$\boldsymbol{\lambda}_2^{\prime IJK}$	$\bar{\mathbf{Y}}_\nu^I\varepsilon^{LMN}\varepsilon^{ABC}(\mathbf{R}_d)^{KA}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{LB}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{MC}(\mathbf{R}_q)^{NJ}$	$\det(g_L g_D^\dagger g_Q)$
$\boldsymbol{\lambda}_1^{\prime\prime IJK}$	$\varepsilon^{LMN}(\mathbf{Y}_u\mathbf{R}_q\mathbf{Y}_d^\dagger)^{IL}(\mathbf{R}_d)^{JM}(\mathbf{R}_d)^{KN}$	$\det(g_D^\dagger)$
$\boldsymbol{\lambda}_2^{\prime\prime IJK}$	$\varepsilon^{LMN}(\mathbf{R}_u)^{LI}(\mathbf{Y}_d\mathbf{R}_q\mathbf{Y}_u^\dagger)^{JM}(\mathbf{Y}_d\mathbf{R}_q\mathbf{Y}_u^\dagger)^{KN}$	$\det(g_U^\dagger)$
$\boldsymbol{\lambda}_3^{\prime\prime IJK}$	$\varepsilon^{LMN}(\mathbf{Y}_u\mathbf{R}_q)^{IL}(\mathbf{Y}_d\mathbf{R}_q)^{JM}(\mathbf{Y}_d\mathbf{R}_q)^{KN}$	$\det(g_Q^\dagger)$
$\boldsymbol{\lambda}_4^{\prime\prime IJK}$	$\varepsilon^{LMN}\varepsilon^{ABC}\varepsilon^{DEF}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{LD}(\mathbf{R}_q\mathbf{Y}_u^\dagger)^{MA}(\mathbf{R}_q\mathbf{Y}_u^\dagger)^{NB}(\mathbf{R}_u)^{IC}(\mathbf{R}_d)^{JE}(\mathbf{R}_d)^{KF}$	$\det(g_Q g_U^\dagger g_D^\dagger)$
$\boldsymbol{\lambda}_5^{\prime\prime IJK}$	$\varepsilon^{STU}\varepsilon^{ABC}\varepsilon^{DEF}\varepsilon^{ILMNPQR}(\mathbf{R}_q\mathbf{Y}_u^\dagger)^{LS}(\mathbf{R}_q\mathbf{Y}_u^\dagger)^{PT}(\mathbf{R}_u)^{IU}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{MA}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{QB}(\mathbf{R}_d)^{JC}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{ND}(\mathbf{R}_q\mathbf{Y}_d^\dagger)^{RE}(\mathbf{R}_d)^{KF}$	$\det(g_Q^2 g_U^\dagger g_D^{\dagger 2})$

Because of the antisymmetry of ε tensors, some of the terms vanish identically, so that the bases in Table I are not fully reduced algebraically. Nevertheless, the number of terms for each *RPV* couplings is much larger than their true degrees of freedom. However, in many cases, only a handful of operators are dominant and need to be kept, as we will see in the next section. For now, we just note that, if $\tan\beta$ is not large, one can neglect $\mathbf{Y}_d$ compared to $\mathbf{Y}_u$, while if M_R is smaller than about 10^{13} GeV, $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ is negligible and can be dropped everywhere. One then remains with about 10 complex parameters to describe $\boldsymbol{\mu}'$, $\boldsymbol{\lambda}$, $\boldsymbol{\lambda}'$, and $\boldsymbol{\lambda}''$ couplings (see Appendix A), depending on which of the $U(1)$'s remain exact.

C. Natural scales for the *RPV*-MFV coefficients

It is remarkable that all the *RPV* couplings can be generated out of the minimal set of spurions needed to account for the known fermion masses and mixings. In addition, it appears that there is a fundamental distinction between the baryon- and lepton-number violating terms. Indeed, $\Delta L = 1$ couplings are strictly forbidden as long as $m_\nu = 0$, since the ν_L Majorana mass, transforming as $(\bar{6}, 1)$, is definitely needed to get invariants under G_ℓ.[2] Then, the seesaw mechanism not only suppresses neutrino masses, it suppresses all $\Delta L \neq 0$ effects. On the other hand, $\Delta B = 1$ couplings can be readily parametrized in terms of the usual quark Yukawas.

Naturalness demands all MFV coefficients to be of $\mathcal{O}(1)$, but leaves open the overall normalization of dimensionful couplings like $\boldsymbol{\mu}'$ or *RPV* soft-breaking terms. In the present section, this issue will be analyzed from several perspectives, showing that the naive normalization of dimensionful *RPV* couplings in terms of their *RPC* counterparts is the most natural. In this respect, it is worth mentioning that RGE invariance cannot help much. Indeed, the MFV expansions are stable under the RGE, and, further, the $\Delta B = 1$ and $\Delta L = 1$ sectors are decoupled [17].

1. Basis independence, sneutrino VEV's, and neutrino masses

When lepton number is not conserved, the left-handed lepton doublet L^I and the Higgs doublet H_d have the same quantum numbers and can mix. *A priori*, the Lagrangian fields do not correspond to the physical Higgs and lepton states. In other words, defining the four-component vector $\phi^\alpha = (H_d, L^I)$, the physics is invariant if we carry out the field redefinition [18]

$$\phi^\alpha \to U_\beta^\alpha \phi^\beta, \tag{28}$$

with $U \in SU(4)$. Obviously, the gauge sector is invariant, but what we call the *RPC* and ($\Delta L = 1$) *RPV* sectors get mixed. Indeed, one can immediately see from Eqs. (2) and (15) that a change of basis modifies the relative size of $\mathbf{Y}_\ell$ and $\boldsymbol{\lambda}$, $\mathbf{Y}_d$ and $\boldsymbol{\lambda}'$, and μ and $\boldsymbol{\mu}'$ (the soft-breaking terms are similarly affected). Since we can, for example, choose a basis in which one bilinear term is rotated away, there are too many parameters and only some combinations of them are physical. We will now check that the expansions ob-

[2]This is not the only possible choice of spurions in the lepton sector. In Ref. [16], the Majorana mass arises from *RPV* couplings, promoted to spurions, while there is no need to extend the flavor group at high energy. However, that approach is not so predictive for LFV effects, and further, cannot explain proton stability. In the present work, the smallness of *RPV* effects originates from their MFV structures, and does not have to be imposed separately.

tained in the previous section satisfy the MFV principle despite these ambiguities.

If the *RPV* couplings take, in some basis, the MFV forms obtained in the previous section, the sneutrino VEV's $\langle \nu^I \rangle$ are, in general, nonvanishing. We must thus check that rotating these VEV's away only amounts to $\mathcal{O}(1)$ redefinitions of the MFV coefficients for all *RPV* couplings. Let us consider for now only the *RPC* and *RPV* bilinear terms, Eqs. (16a) and (16c), written in four-component notation

$$W \ni \bar{\mu}_\alpha (H_u \phi^\alpha), \quad \mathcal{L}_{\text{soft}} \ni -(\bar{b}_\alpha (H_u \phi^\alpha) + \text{H.c.}) - \bar{m}^2_{\alpha\beta} \phi^{\alpha\dagger} \phi^\beta, \tag{29}$$

where

$$\bar{\mu}^{\,\alpha} = (\mu, \boldsymbol{\mu}'), \qquad \bar{b}^\alpha = (b, \mathbf{b}'), \qquad \bar{m}^2_{\alpha\beta} = \begin{pmatrix} m^2_{H_d} & \mathbf{m}^2_{Ld} \\ (\mathbf{m}^2_{Ld})^\dagger & \mathbf{m}^2_L \end{pmatrix}. \tag{30}$$

In Ref. [19], it was shown that if $\bar{b}$ is proportional to $\bar{\mu}$, and $\bar{\mu}$ is an eigenvector of $\bar{m}^2$, then we can choose a basis in which the vacuum expectation values $\langle \phi^\alpha \rangle$ are aligned with $\bar{\mu}^\alpha$; in particular, we can choose $\bar{\mu}^\alpha = (\mu, 0, 0, 0)$, $\bar{b}^\alpha = (b, 0, 0, 0)$, and $\langle \phi^\alpha \rangle = (v_d, 0, 0, 0)$. Now, if the MFV expansions for $\boldsymbol{\mu}'$, $\mathbf{b}'$, and $\mathbf{m}^2_{Ld}$ are normalized with respect to μ, b, and $m^2_{H_d}$, respectively, we are in a situation of near-alignment and sneutrino VEV's are very small:

$$\bar{\mu}^\alpha = (\mu, \mu \bar{\mathbf{Y}}_\nu), \quad \bar{b}^\alpha = (b, b\bar{\mathbf{Y}}'_\nu), \quad \mathbf{m}^2_{Ld} = m^2_{H_d} \bar{\mathbf{Y}}''_\nu \rightarrow \bar{\mu} \times \bar{b} = \mathcal{O}(\bar{\mathbf{Y}}_\nu), \quad \bar{m}^2_{\alpha\beta} \bar{\mu}^\beta = m^2_{H_d} \bar{\mu}_\alpha + \mathcal{O}(\bar{\mathbf{Y}}_\nu), \tag{31}$$

and therefore $\langle \nu^I \rangle \sim \mathcal{O}(v_d \bar{\mathbf{Y}}^I_\nu)$. In other words, the misalignment is entirely due to the $\mathcal{O}(1)$ differences between the MFV coefficients of the $\bar{\mathbf{Y}}_\nu$ structures of $\bar{\mu}$, $\bar{b}$, and $\bar{m}^2$. To rotate away these VEV's, consider the change of basis

$$U = \begin{pmatrix} 1 & -\varepsilon^I \\ \varepsilon^{*I} & 1_{3\times3} \end{pmatrix}, \tag{32}$$

with $\varepsilon^I = \langle \nu^I \rangle / v_d = a \bar{\mathbf{Y}}^I_\nu$. The constant a is of $\mathcal{O}(1)$ and we set it to 1 for simplicity. The impact for all *RPC* terms is completely negligible, while the redefined *RPV* terms automatically satisfy their MFV expansions:

$$\delta \boldsymbol{\mu}'^I = \mu \bar{\mathbf{Y}}^I_\nu, \qquad \delta \boldsymbol{\lambda}^{IJK} = \bar{\mathbf{Y}}^I_\nu (\mathbf{Y}_\ell)^{KJ} - (I \leftrightarrow J), \qquad \delta \boldsymbol{\lambda}'^{IJK} = \bar{\mathbf{Y}}^I_\nu (\mathbf{Y}_d)^{KJ}, \tag{33}$$

$$\delta \mathbf{b}'^I = b \bar{\mathbf{Y}}^I_\nu, \qquad \delta \mathbf{m}^2_{Ld} = m^2_{H_d} \bar{\mathbf{Y}}_\nu, \qquad \delta \mathbf{A}^{IJK} = \bar{\mathbf{Y}}^I_\nu (\mathbf{A}_\ell)^{KJ} - (I \leftrightarrow J), \qquad \delta \mathbf{A}'^{IJK} = \bar{\mathbf{Y}}^I_\nu (\mathbf{A}_d)^{KJ}. \tag{34}$$

This rotation thus only induces $\mathcal{O}(1)$ shifts in the values of the MFV coefficients of *RPV* terms, provided the *RPV* soft-breaking trilinear terms are normalized by A_0. Note that, by the same reasoning, one can also see that the MFV expansion is stable if one of the bilinear terms is rotated away.

Given the freedom to rotate the *RPC* and *RPV* couplings, it could happen that the MFV structure is hidden. In other words, *RPV* couplings could be very large but fine-tuned with *RPC* terms, such that, moving to the $\langle \nu^I \rangle = 0$ basis, they would again assume their MFV forms. This latter form is more natural in the sense that the $\Delta L = 1$ couplings are then of the same order of magnitude as the physical, basis-independent parameters describing $\Delta L = 1$ effects[20]. Indeed, for example, the two basis-independent angles ξ and ζ tuning the lepton-Higgsino and slepton-Higgs mixings are both $\mathcal{O}(\bar{\mathbf{Y}}_\nu)$ [19,21]:

$$\cos\xi = \frac{1}{|\bar{\mu}| v_d} \sum_\alpha \mu_\alpha v^\alpha \rightarrow \sin\xi = \mathcal{O}(\bar{\mathbf{Y}}_\nu), \qquad \cos\zeta = \frac{1}{|\bar{b}| v_d} \sum_\alpha b_\alpha v^\alpha \rightarrow \sin\zeta = \mathcal{O}(\bar{\mathbf{Y}}_\nu). \tag{35}$$

Finally, since these angles are very small, the impact of *RPV* couplings on charged lepton or neutrino masses is negligible, and the background values for the spurions can be fixed as in Eq. (14). This is obvious for the charged leptons, while for the $\Delta L = 2$ neutrino masses, it is necessarily quadratic in $\Delta L = 1$ effects, i.e. $\mathcal{O}(\mathbf{Y}^2_\nu)$. For example, tree-level mixing induced by the *RPV* bilinear terms scales as $\tan^2\xi$ [19], while those generated at the loop level by the *RPV* trilinear terms scale as $\boldsymbol{\lambda}^2$ or $\boldsymbol{\lambda}'^2$.

2. High-energy scales and higher-dimensional operators

In the present work, the $(\bar{6}, 1)$ spurion is normalized as $\mathbf{Y}_\nu = U^* \mathbf{m}_\nu U^\dagger / v_u$, so that it lies on the same footing as the other fermion masses; see Eq. (14). Consequently, all $\Delta L = 1$ couplings are very suppressed if MFV holds, since at least one power of $\mathbf{Y}_\nu$ is needed to make them invariant under G_f. This is the most natural and model-independent assumption because, as said previously, the MSSM spurion content does not allow for $\Delta L = 1$ couplings in the $m_\nu = 0$ limit.

At the same time though, the seesaw mechanism is responsible for the smallness of m_ν, and calls for additional degrees of freedom at the scale $\Lambda_{\Delta L=2} \sim M_R$. Therefore, it is tempting to associate this scale also with $\Delta L = 1$ effects, i.e. to imagine that they arise from some nontrivial dynamics at the M_R scale. To concoct such a model is not trivial and lies beyond the purpose of the present article. However, it should be clear that if $\Lambda_{\Delta L=1} \sim \Lambda_{\Delta L=2}$, the same spurion $\mathbf{Y}_\nu$ can be used to parametrize both $\Delta L = 1$ and $\Delta L = 2$ couplings.

Alternatively, if $\Lambda_{\Delta L=2} \gg \Lambda_{\Delta L=1}$, large compensating factors would arise, since the spurion to be used is then $r \times \mathbf{Y}_\nu$ with $r = \Lambda_{\Delta L=2}/\Lambda_{\Delta L=1}$. Note that this corresponds to the rescaling

$$\bar{\mu}^{\alpha} = (\mu, r\mu\bar{\mathbf{Y}}_\nu), \qquad \bar{b}^{\alpha} = (b, rb\bar{\mathbf{Y}}_\nu),$$
$$\mathbf{m}^2_{Ld} = rm^2_{H_d}\bar{\mathbf{Y}}_\nu, \qquad \boldsymbol{\lambda}, \boldsymbol{\lambda}' \sim r\mathcal{O}(\mathbf{Y}_\nu), \qquad (36)$$
$$\mathbf{A}, \mathbf{A}' \sim rA_0\mathcal{O}(\mathbf{Y}_\nu),$$

which is compatible with the MFV expansion [the developments of the previous section remain essentially unchanged, now with $\langle \nu^I \rangle = \mathcal{O}(rv_d\bar{\mathbf{Y}}^I_\nu)$]. Of course, r should not be too large, otherwise $\Delta L = 1$ couplings would contribute to the neutrino mass, thereby invalidating Eq. (14).

In the present work, we stick to the minimal hypothesis and assume $\Lambda_{\Delta L=1} \sim \Lambda_{\Delta L=2}$. Ultimately, it is the comparison with experimental constraints which will tell us if this is viable, or give us clues as to the scale at which $\Delta L = 1$ effects arise, and hopefully about the dynamics going on there.

In the $\Delta B = 1$ sector, there is no seesaw mechanism at play and therefore no clue as to the mechanism behind their generation. In the present work, we treat $\Delta B = 1$ couplings on the same footing as *RPC* terms; i.e. we accept that baryon number is simply not conserved. However, one should keep in mind that all $\Delta B = 1$ MFV coefficients could very well be suppressed or enhanced by some ratio of scales, or suppressed by some gauge couplings, $a_i \sim \mathcal{O}(g^2/4\pi)$.

If we imagine that there is a nontrivial lepton-number violating dynamics going on at the high-energy scale, it is natural to expect that integrating out the heavy degrees of freedom leads to additional dimension-five operators[3]. Let us concentrate on the *RPC* dimension-five operators in the superpotential,

$$W_{\text{dim-5}} \ni \frac{\kappa_1^{IJKL}}{\Lambda_{\Delta L=1}}(Q^IQ^J)(Q^KL^L) + \frac{\kappa_2^{IJKL}}{\Lambda_{\Delta L=1}}(D^IU^JU^K)E^L + \frac{\kappa_5^{IJ}}{\Lambda_{\Delta L=2}}(L^IH_u)(L^JH_u). \qquad (37)$$

The operator $\boldsymbol{\kappa}_5$ corresponds to the one arising from the integration of the right-handed neutrinos, Eq. (9), with the scale $\Lambda_{\Delta L=2}$ then given by M_R.

The overall scale of $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ is simply $\Lambda_{\Delta L=1}$ even though they are also breaking baryon number, since we do not associate any particular scale to $\Delta B = 1$ effects. If we assume again that $\Lambda_{\Delta L=1} \sim \Lambda_{\Delta L=2}$, the operators $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ could induce proton decay at an unacceptable rate. However, we think that, if enforcing MFV is sufficient to separately suppress $\Delta L = 1$ and $\Delta B = 1$ interactions so as to pass experimental bounds on proton decay, the same should be true for $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$. Indeed, the flavor group G_f factorizes as $G_q \times G_\ell \times G_1$; hence it makes no difference whether MFV is used to parametrize a product of $\Delta L = 1$ and $\Delta B = 1$ operators, or a single $\Delta L = 1$, $\Delta B = 1$ operator. Explicitly, the MFV expansions are

$$\frac{\boldsymbol{\kappa}_1^{IJKL}}{\Lambda_{\Delta L=1}} = \frac{1}{v_u}\varepsilon^{MNP}(\mathbf{R}_q)^{MI}(\mathbf{R}_q)^{NJ}(\mathbf{R}_q)^{PK}\bar{\mathbf{Y}}^L_\nu, \qquad (38)$$

$$\frac{\boldsymbol{\kappa}_2^{IJKL}}{\Lambda_{\Delta L=1}} = \frac{1}{v_u}(\boldsymbol{\lambda}''^{IJK})_{u\leftrightarrow d}(\mathbf{Y}_\ell\bar{\mathbf{Y}}^\dagger_\nu)^L, \qquad (39)$$

where $(\boldsymbol{\lambda}''^{IJK})_{u\leftrightarrow d}$ is obtained from Table I by interchanging $\mathbf{Y}_d \leftrightarrow \mathbf{Y}_u$ and $\mathbf{R}_d \leftrightarrow \mathbf{R}_u$. Obviously, their structures are very similar to simple products of $\Delta L = 1$ and $\Delta B = 1$ couplings. To check that $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ pass the experimental bounds if $\boldsymbol{\lambda}$, $\boldsymbol{\lambda}'$, and $\boldsymbol{\lambda}''$ do would require a detailed analysis, which lies out of our main purpose. Indeed, contrary to the renormalizable *RPV* couplings, the $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ contributions to proton decay arise only at the loop level, since these interactions preserve R parity. They thus depend also on the parameters of the MSSM gauge sector (see Ref. [3] for a discussion). Further, the suppressions due to flavor mixing are not necessarily to be found in the $\boldsymbol{\kappa}_i$ themselves. For instance, the $\boldsymbol{\kappa}_1^{123L}$ coupling is not suppressed by CKM or light-quark mass factors, but necessarily involves a third-generation (s)quark; hence its contribution to proton decay will nevertheless involve some flavor mixings.

Because of these flavor mixings and loop-suppression factors, the contributions to proton decay from nonrenormalizable operators should be subleading compared to the $\boldsymbol{\lambda} \times \boldsymbol{\lambda}''$ and $\boldsymbol{\lambda}' \times \boldsymbol{\lambda}''$ tree-level contributions, and will not be considered anymore here. However, it is remarkable that the MFV principle can offer a common solution to both problems, irrespective of whether the couplings are *RPV* or *RPC*. As said, this is essentially because the flavor group factorizes as $G_q \times G_\ell$, while R parity acts additively for a given coupling.

III. PHENOMENOLOGICAL CONSEQUENCES FOR THE *R*-PARITY VIOLATING MSSM

When applied to *RPV* couplings, the MFV hypothesis necessarily makes use of ε tensors to contract the spurions to $G_q \times G_\ell$ singlets. Because of its antisymmetry, the couplings are then, in general, proportional to products of fermion masses of more than one generation. Given the strong hierarchy among these masses, MFV tends to strongly suppress all *RPV* couplings. It is the purpose of the present section to analyze under which circumstances this suppression is sufficient to pass experimental bounds on $\Delta B = 1$ and $\Delta L = 1$ processes, especially proton decay. Once established, the impact of these conditions on the possible *RPV* effects at colliders, and the connections with FCNC or LFV effects will be briefly commented.

It should be remarked also that naive expectations like $\lambda''^{IJK} \sim O(m_u^I m_d^J m_d^K/v_u v_d^2)$ are not valid in MFV, because the generation indices are twisted by the antisymmetric ε tensors. MFV thus implies a very peculiar form of helicity suppression with, for example, a coupling involving the up quark tuned by m_t/v_u. Also, MFV predicts that the $\Delta L =$

1 couplings are all proportional to products of neutrino and charged lepton masses.

A. Experimental information for the spurions

The flavor symmetry permits one to rotate the spurions to their background values, Eq. (14), which can be fixed in terms of experimental values. For the Yukawa $\mathbf{Y}_{u,d,\ell}$, we take the quark and charged lepton masses, as well as the CKM matrix elements from Ref. [6]. For the neutrino spurions $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ and $\mathbf{Y}_\nu$, we start from the neutrino mixing parameters of the best fit of Ref. [22],

$$\begin{aligned} \Delta m_{21}^2 &= \Delta m_\odot^2 = 7.9^{+0.27}_{-0.28} \times 10^{-5}\ \text{eV}^2, \\ |\Delta m_{31}^2| &= \Delta m_{\text{atm}}^2 = (2.6 \pm 0.2) \times 10^{-3}\ \text{eV}^2, \\ \theta_{12} &= \theta_\odot = (33.7 \pm 1.3)^\circ, \\ \theta_{23} &= \theta_{\text{atm}} = (43.3^{+4.3}_{-3.8})^\circ, \\ \theta_{13} &= (0^{+5.2}_{-0})^\circ. \end{aligned} \tag{40}$$

In a first approximation, since we are only interested in the order of magnitude of the RPV couplings, we can neglect the small θ_{13} as well as the CP-violating phase, and fix the atmospheric angle at $\theta_{\text{atm}} = 45^\circ$ (maximal mixing), such that the PMNS mixing matrix takes the simple form

$$U \simeq \begin{pmatrix} c_\odot & s_\odot & 0 \\ -s_\odot/\sqrt{2} & c_\odot/\sqrt{2} & 1/\sqrt{2} \\ s_\odot/\sqrt{2} & -c_\odot/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{41}$$

where $s_\odot = \sin\theta_\odot$, $c_\odot = \cos\theta_\odot$, and $\tan\theta_\odot \simeq 2/3$. Under these approximations,

$$\mathbf{Y}_\nu = \frac{1}{v_u}\left(m_\nu \mathbf{1} + \frac{\Delta m_{21}}{\sqrt{2}(1+t_\odot^2)} \begin{pmatrix} \sqrt{2}t_\odot^2 & t_\odot & -t_\odot \\ t_\odot & 1/\sqrt{2} & -1/\sqrt{2} \\ -t_\odot & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} + \frac{\Delta m_{31}}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \right), \tag{42}$$

where $m_{\nu 1} = m_\nu$, $m_{\nu 2} = m_\nu + \Delta m_{21}$, $m_{\nu 3} = m_\nu + \Delta m_{31}$. The neutrino mass scale m_ν is unknown, but should not exceed about 1 eV if the cosmological bound $\sum_i m_i \lesssim 1$ eV holds [23]. Depending on the spectrum (i.e., whether $m_{\nu 1}$ or $m_{\nu 3}$ is the lightest neutrino), the mass differences Δm_{21} and Δm_{31} are related to the mixing parameters as

$$\begin{aligned} &\Delta m_{21} = (\Delta m_\odot^2 + m_\nu^2)^{1/2} - m_\nu > 0, \\ &\begin{cases} \Delta m_{31} = (\Delta m_{\text{atm}}^2 + m_\nu^2)^{1/2} - m_\nu > 0 & \text{(Normal)}, \\ \Delta m_{31} = (m_\nu^2 - \Delta m_{\text{atm}}^2)^{1/2} - m_\nu < 0 & \text{(Inverted)}. \end{cases} \end{aligned} \tag{43}$$

Therefore, for fixed $\Delta m_\odot^2$ and Δm_{atm}^2, off-diagonal elements of $\mathbf{Y}_\nu$ quickly decrease with increasing m_ν, with the maximum being

$$m_{\nu_1} = 0\ \text{(Normal)}: \Delta m_{21} = (\Delta m_\odot^2)^{1/2}, \quad \Delta m_{31} = (\Delta m_{\text{atm}}^2)^{1/2}, \tag{44}$$

$$m_{\nu_3} = 0\ \text{(Inverted)}: \Delta m_{21} = \frac{\Delta m_\odot^2}{2(\Delta m_{\text{atm}}^2)^{1/2}}, \quad \Delta m_{31} = -(\Delta m_{\text{atm}}^2)^{1/2}. \tag{45}$$

For the normal spectrum, m_ν varies between 0 and about 1 eV, while in the inverted hierarchy, it varies between $(\Delta m_{\text{atm}}^2)^{1/2}$ and about 1 eV. Therefore, the off-diagonal elements of $\mathbf{Y}_\nu$ are typically smaller for the inverted spectrum, and we will not consider that case anymore.

Contrary to the other spurions, $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ cannot be fixed entirely in terms of experimentally known quantities, since neutrino masses and mixings only give us access to $\mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu$. Following Refs. [24,25], the overall ambiguity in $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ can be parametrized in terms of three phases:

$$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = \frac{M_R}{v_u^2} U \mathbf{m}_\nu^{1/2} \mathbf{H}^2 \mathbf{m}_\nu^{1/2} U^\dagger, \qquad \mathbf{H} = e^{i\mathbf{\Phi}}, \qquad \mathbf{\Phi} = \begin{pmatrix} 0 & \phi_1 & \phi_2 \\ -\phi_1 & 0 & \phi_3 \\ -\phi_2 & -\phi_3 & 0 \end{pmatrix}. \tag{46}$$

We work in the CP limit, $\phi_i = 0$, such that $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu =^{CP} M_R \mathbf{Y}_\nu / v_u$; i.e. both neutrino spurions are real, parallel, and symmetric. Corrections induced by the CP phases ϕ_i are assumed to be small,[3] while those due to δ_{13} are always suppressed by $\sin\theta_{13}$. In practice, taking $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ and $\mathbf{Y}_\nu$ aligned greatly reduces the number of structures, since, for example,

$$\varepsilon^{IJK} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)^{IJ} \overset{CP}{=} 0, \tag{47}$$

by symmetry. Also, the Cayley-Hamilton relation Eq. (17) can be used to discard products of two or more $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ with $\mathbf{Y}_\nu$. Further, the $m_\nu \mathbf{1}$ piece of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ can be dropped since the identity is already part of $\mathbf{R}_\ell$. Finally, for our perturbative expansion to make sense, we must ensure that $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \lesssim 1$, which translates as

$$\frac{\max[m_\nu, |\Delta m_{31}|]}{1\ \text{eV}} \frac{M_R}{10^{13}\ \text{GeV}} \lesssim 3. \tag{48}$$

For $m_\nu \simeq 0$, M_R can be at most $\sim 5 \times 10^{14}$, while for $M_R \lesssim 10^{13}$, the spurion $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is very suppressed as all its elements are tuned by Δm_{21} and Δm_{31}.

[3]Expanding $\mathbf{H} = \mathbf{1} + i\mathbf{\Phi}$, these phases can be introduced through a new spurion, $\Delta_{\mathbf{\Phi}} = M_R U \mathbf{m}_\nu^{1/2} \mathbf{\Phi} \mathbf{m}_\nu^{1/2} U^\dagger / v_u^2$, still suppressed by the neutrino masses, and presumably smaller than $M_R \mathbf{Y}_\nu / v_u$ if the phases are not large. For our leading order analysis, the perturbations induced by $\Delta_{\mathbf{\Phi}}$ are neglected.

B. The reduced MFV expansion and order-of-magnitude estimates

The first step to get the order of magnitude of the couplings is to reduce the MFV operator bases constructed in the previous section. Indeed, they all involve operators which are very suppressed once experimental values for the spurions are plugged in. Specifically, an operator can be neglected if it induces only small corrections to the entries of all possible linear combinations of the others, within a tolerance of about 5%–10%.

However, this reduction is not trivial because the operators to include, as well as the order of magnitude of the *RPV* couplings, crucially depend on $\tan\beta$, M_R, and m_ν. Generically, the number of operators increases with increasing $\tan\beta$, M_R, or decreasing m_ν, and having several nonaligned spurions makes the *RPV* couplings less hierarchical. Because of these dependences, giving a general and simultaneously minimal basis is not possible. For example, an operator can be dominant if $\tan\beta$ increases, but become subleading if m_ν decreases. Further, when $\tan\beta$, M_R are large and m_ν is small, the number of independent and dominant operators is, in general, comparable to the number of free parameters needed to fully specify the *RPV* couplings.

For these reasons, we prefer to analyze four extreme situations numerically:

$$\begin{aligned} &\text{Case I: } \tan\beta = 5, \quad M_R = 10^{12}\ \text{GeV}, \quad m_\nu = 0.5\ \text{eV},\\ &\text{Case II: } \tan\beta = 50, \quad M_R = 10^{12}\ \text{GeV}, \quad m_\nu = 0.5\ \text{eV},\\ &\text{Case III: } \tan\beta = 5, \quad M_R = 2\times 10^{14}\ \text{GeV}, \quad m_\nu = 0\ \text{eV},\\ &\text{Case IV: } \tan\beta = 50, \quad M_R = 2\times 10^{14}\ \text{GeV}, \quad m_\nu = 0\ \text{eV}. \end{aligned} \tag{49}$$

Cases II and IV maximize the impact of $\mathbf{Y}_d$ and $\mathbf{Y}_\ell$, while cases III and IV maximize that of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ and $\mathbf{Y}_\nu$. Note that, since we are only interested in order-of-magnitude estimates, we neglect the nonholomorphic corrections to the Yukawas induced at large $\tan\beta$ [26], and keep the background values fixed as in Eq. (14).

In Appendix A, we construct analytically the minimal basis for

$$\text{Case V: } \tan\beta \lesssim 20, \quad M_R \lesssim 2\times 10^{13}\ \text{GeV}, \quad m_\nu \gtrsim 0.05\ \text{eV}. \tag{50}$$

In that region of parameter space, $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is always subleading and the basis is rather simple.

In Tables II, III, and IV, the order of magnitude of the couplings is indicated, separately for each ε structure. These values correspond to the contribution of the numerically dominant operator for each individual coupling. In other words, after expanding the terms of each structure of Table I, plugging in the numerical values for the spurions, we find for each value of the I, J, K indices a numerical polynomial in the MFV coefficients. Assuming these coefficients are of $\mathcal{O}(1)$, and barring large interferences among operators, we take the largest term of this polynomial as the order of magnitude of the coupling. The notation in Table II for $\boldsymbol{\lambda}^{IJK}$, which is antisymmetric under $I \leftrightarrow J$, is to arrange its 9 degrees of freedom in a 3×3 matrix $\boldsymbol{\lambda}^{IJK} = \mathbf{X}^{JK}$, with $I = 2, 3, 1$ for $J = 1, 2, 3$, respectively. Similarly, $\boldsymbol{\lambda}''^{IJK}$ is antisymmetric under $J \leftrightarrow K$ and written in Table IV as $\boldsymbol{\lambda}''^{IJK} = \mathbf{X}^{IJ}$ with $K = 2, 3, 1$ for $J = 1, 2, 3$, respectively. Note that this coupling is insensitive to M_R and m_ν; only $\tan\beta$ is relevant.

It is immediately apparent from these tables that MFV is quite powerful to predict the overall scales in terms of $\tan\beta$, M_R, and m_ν, as well as the hierarchies within each structure. This is because the ε tensors twist the hierarchy of the $\mathbf{Y}_{u,d,\ell}$ spurions in a specific way. Of course, for terms

TABLE II. The order of magnitude of the intermediate spurion $\bar{\mathbf{Y}}_\nu$, Eq. (25), and of the $\boldsymbol{\lambda}^{IJK}(L^I L^J)E^K$ couplings, separately for each ε structure of Table I. Entries are to be understood as $x \equiv \mathcal{O}(10^{-x})$. The matrix entries correspond to the J, K indices, with I fixed as $I = 2, 3, 1$ for $J = 1, 2, 3$, respectively.

	$\bar{\mathbf{Y}}_\nu^I$	$\boldsymbol{\lambda}_1^{IJK}$	$\boldsymbol{\lambda}_2^{IJK}$	$\boldsymbol{\lambda}_3^{IJK}$	$\boldsymbol{\lambda}_4^{IJK}$
Scaling in $\tan\beta$	$\tan^2\beta$	$\tan^3\beta$	$\tan\beta$	$\tan^4\beta$	$\tan^2\beta$
Case I	$\begin{pmatrix}17\\19\\21\end{pmatrix}$	$\begin{pmatrix}23&19&23\\30&24&20\\26&24&18\end{pmatrix}$	$\begin{pmatrix}21&17&13\\16&18&17\\21&14&15\end{pmatrix}$	$\begin{pmatrix}22&23&28\\28&27&26\\25&27&24\end{pmatrix}$	$\begin{pmatrix}19&20&19\\15&22&23\\19&17&21\end{pmatrix}$
Case II	$\begin{pmatrix}15\\17\\19\end{pmatrix}$	$\begin{pmatrix}20&16&20\\27&21&17\\23&21&15\end{pmatrix}$	$\begin{pmatrix}20&16&12\\15&17&16\\20&13&14\end{pmatrix}$	$\begin{pmatrix}18&19&24\\24&23&22\\21&23&20\end{pmatrix}$	$\begin{pmatrix}17&18&17\\13&20&21\\17&15&19\end{pmatrix}$
Case III	$\begin{pmatrix}16\\17\\18\end{pmatrix}$	$\begin{pmatrix}21&18&18\\23&20&18\\22&19&17\end{pmatrix}$	$\begin{pmatrix}19&15&14\\19&16&15\\19&15&14\end{pmatrix}$	$\begin{pmatrix}20&21&24\\22&23&24\\21&22&23\end{pmatrix}$	$\begin{pmatrix}17&19&20\\17&20&21\\17&19&20\end{pmatrix}$
Case IV	$\begin{pmatrix}14\\15\\16\end{pmatrix}$	$\begin{pmatrix}18&15&15\\20&17&15\\19&16&14\end{pmatrix}$	$\begin{pmatrix}18&14&13\\18&15&14\\18&14&13\end{pmatrix}$	$\begin{pmatrix}16&18&20\\18&19&20\\17&18&19\end{pmatrix}$	$\begin{pmatrix}15&17&18\\15&18&19\\15&17&18\end{pmatrix}$

TABLE III. The order of magnitude of the $\boldsymbol{\lambda}'^{IJK}(L^I Q^J)D^K$ couplings, for each ε structure of Table I. Entries are to be understood as $x \equiv \mathcal{O}(10^{-x})$. The spurion $\bar{\mathbf{Y}}_\nu$ for each case is given in Table II.

	$\boldsymbol{\lambda}_1'^{IJK}$	$\boldsymbol{\lambda}_2'^{IJK}$
Scaling in $\tan\beta$	$\tan^3\beta$	$\tan^4\beta$
Case I/III	$\bar{\mathbf{Y}}_\nu^I \times \begin{pmatrix} 4 & 6 & 3 \\ 7 & 3 & 3 \\ 6 & 4 & 1 \end{pmatrix}^{JK}$	$\bar{\mathbf{Y}}_\nu^I \times \begin{pmatrix} 3 & 8 & 8 \\ 7 & 5 & 7 \\ 5 & 6 & 6 \end{pmatrix}^{JK}$
Case II/IV	$\bar{\mathbf{Y}}_\nu^I \times \begin{pmatrix} 3 & 5 & 2 \\ 6 & 2 & 1 \\ 5 & 3 & 0 \end{pmatrix}^{JK}$	$\bar{\mathbf{Y}}_\nu^I \times \begin{pmatrix} 1 & 6 & 6 \\ 5 & 3 & 5 \\ 3 & 4 & 4 \end{pmatrix}^{JK}$

TABLE IV. The order of magnitude of the $\boldsymbol{\lambda}''^{IJK}U^I D^J D^K$ couplings, for each ε structure of Table I. Entries are to be understood as $x \equiv \mathcal{O}(10^{-x})$. The matrix entries correspond to the I, J indices, with K fixed as $K = 2, 3, 1$ for $J = 1, 2, 3$, respectively.

	$\boldsymbol{\lambda}_1''^{IJK}$	$\boldsymbol{\lambda}_2''^{IJK}$	$\boldsymbol{\lambda}_3''^{IJK}$	$\boldsymbol{\lambda}_4''^{IJK}$
Scaling in $\tan\beta$	$\tan\beta$	$\tan^2\beta$	$\tan^2\beta$	$\tan\beta$
Case I/III	$\begin{pmatrix} 8 & 8 & 8 \\ 4 & 6 & 5 \\ 1 & 6 & 4 \end{pmatrix}$	$\begin{pmatrix} 11 & 6 & 7 \\ 12 & 9 & 9 \\ 13 & 12 & 13 \end{pmatrix}$	$\begin{pmatrix} 13 & 8 & 10 \\ 10 & 6 & 7 \\ 6 & 5 & 6 \end{pmatrix}$	$\begin{pmatrix} 5 & 5 & 5 \\ 7 & 9 & 7 \\ 7 & 12 & 10 \end{pmatrix}$
Case II/IV	$\begin{pmatrix} 7 & 7 & 7 \\ 3 & 5 & 4 \\ 0 & 5 & 3 \end{pmatrix}$	$\begin{pmatrix} 9 & 4 & 5 \\ 10 & 7 & 7 \\ 11 & 10 & 11 \end{pmatrix}$	$\begin{pmatrix} 11 & 6 & 8 \\ 8 & 4 & 5 \\ 4 & 3 & 4 \end{pmatrix}$	$\begin{pmatrix} 4 & 4 & 4 \\ 6 & 8 & 6 \\ 6 & 11 & 9 \end{pmatrix}$

involving many different structures like $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}''$, these hierarchies are somewhat blurred. However, if we enforce any one of the $U(1)$'s, some of the structures get suppressed by additional $\det(\mathbf{Y}_{u,d,\ell})$ factors, which are very small in all cases:

$$\tan\beta = 5:\ \det(\mathbf{Y}_u) \simeq 10^{-7}, \quad \det(\mathbf{Y}_d) \simeq 10^{-7}, \quad \det(\mathbf{Y}_\ell) \simeq 10^{-9}, \tag{51a}$$

$$\tan\beta = 50:\ \det(\mathbf{Y}_u) \simeq 10^{-7}, \quad \det(\mathbf{Y}_d) \simeq 10^{-4}, \quad \det(\mathbf{Y}_\ell) \simeq 10^{-6}. \tag{51b}$$

This can have important consequences for phenomenology, as we will see in the next subsection.

In Tables II, III, and IV, we also give the dominant $\tan\beta$ behavior, i.e. the behavior of the simplest term for each structure (i.e., the skeleton). Some individual elements may scale with higher powers of $\tan\beta$ than indicated if they are sensitive to the presence of $\mathbf{Y}_\ell$ or $\mathbf{Y}_d$. Anyway, MFV predicts that all RPV couplings scale at least linearly with $\tan\beta$. Indeed, in addition to the explicit powers of $\mathbf{Y}_{d,\ell}$ shown in Table I, Eq. (47) implies the presence of at least one power of $\mathbf{Y}_\ell^\dagger Y_\ell$ in $\bar{\mathbf{Y}}_\nu$. On the other hand, as $\tan\beta$ increases, the hierarchies within each structure are not much affected.

To maximize the impact of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ and $\mathbf{Y}_\nu$ we set $m_\nu = 0$. This may seem counterintuitive given Eq. (42). However, we already noticed that the $m_\nu \mathbf{1}$ piece is irrelevant for $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ because the identity is already part of $\mathbf{R}_\ell$. Concerning $\mathbf{Y}_\nu$, in most cases it occurs through $\bar{\mathbf{Y}}_\nu$, whose dominant term is

$$\bar{\mathbf{Y}}_\nu^I = \varepsilon^{QMI}(\mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell \mathbf{Y}_\nu^\dagger)^{MQ} + \ldots. \tag{52}$$

Therefore, the $m_\nu \mathbf{1}$ piece of $\mathbf{Y}_\nu$ again disappears since $\mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell$ is symmetric. Then, apart from $\boldsymbol{\lambda}_{2,4}^{213,321,132}$,[4] the $\Delta L = 1$ structures are all tuned essentially by Δm_{21} and Δm_{31}, and these mass differences are maximized when $m_\nu = 0$. In that case, the hierarchies are softened, because the neutrino spurions have large nondiagonal elements [see Eq. (42)]. Finally, we take M_R large enough to make $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ competitive. However, apart from softening the hierarchies, the presence of the $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ spurion has no impact on the order of magnitude of the dominant entries.

Let us look in more detail at each coupling. The bilinear terms are all proportional to $\bar{\mathbf{Y}}_\nu$, Eq. (25), which is smaller than naively expected from its proportionality to neutrino masses because of the presence of $\mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell$. Note that the hierarchy of $\mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell$ gets inverted by the ε tensor, resulting in the order of magnitudes shown in the first column of Table II.

For $\boldsymbol{\lambda}$, the first structure, proportional to $\bar{\mathbf{Y}}_\nu$, is, in general, smaller than the others. Then, if we impose $U(1)_E$, the third and fourth structures are suppressed by

[4]This can be seen by looking at the skeleton for these structures, and setting $\mathrm{Y}_\nu = m_\nu \mathbf{1}$. Then, $\lambda_2^{IJK} = \varepsilon^{IJN}(\mathbf{Y}_\ell)^{KN}$ and $\lambda_4^{IJK} = \varepsilon^{KLM}(\mathbf{Y}_\ell^\dagger)^{JL}(\mathbf{Y}_\ell^\dagger)^{IM}$, which are nonzero only for IJK, $JIK = 213, 321, 132$.

an additional $\det(\mathbf{Y}_\ell)$ factor and can be dropped. In that case, only $\boldsymbol{\lambda}_2$ needs to be kept, with its very definite hierarchy.

The $\boldsymbol{\lambda}'$ coupling is simply obtained as the direct product $\boldsymbol{\lambda}'^{IJK} = \bar{\mathbf{Y}}_\nu^I (\mathbf{A}_d)^{KJ}$. Therefore, besides the suppression brought in by $\bar{\mathbf{Y}}_\nu$, it is further suppressed by down-quark masses. In Table III, we only write explicitly $(\mathbf{A}_d)^{KJ}$ for $\tan\beta = 5$ and $\tan\beta = 50$. It is interesting to remark that the second structure, corresponding to the ε-terms for $\mathbf{A}_d$ in Eq. (23), has an inverted hierarchy compared to the first (see Appendix B for the numerical analysis of the *RPC* trilinear terms including the ε structures).

Finally, the $\Delta B = 1$ couplings, given in Table IV, are clearly much larger than those breaking lepton number. Nevertheless, *a priori*, one would have expected them to be all of $\mathcal{O}(1)$, so MFV suppresses them significantly. Only in the first structure, which involves the least number of Yukawas, there is an entry of $\mathcal{O}(1)$ when it involves the (s)top, $\boldsymbol{\lambda}''_{312}$. If we impose the $U(1)_D$ symmetry, $\Delta B = 1$ couplings are all much smaller, $\boldsymbol{\lambda}'' \lesssim 10^{-5}(10^{-3})$ for $\tan\beta = 5(50)$, respectively. The complicated structure $\boldsymbol{\lambda}''_5$ is smaller than $\boldsymbol{\lambda}''_4$ by at least 3 orders of magnitude, even at large $\tan\beta$, and can be safely neglected.

C. Bounds on $\Delta B = 1$ nucleon decays and consequences

The strongest experimental bounds come from the non-observation of $\Delta B = 1$ nucleon decay, setting constraints on certain combinations of the $\Delta L = 1$ and $\Delta B = 1$ couplings, and from $\Delta B = 2$ neutron-antineutron oscillations, directly constraining the $\boldsymbol{\lambda}''$ couplings. For their part, taken alone, the $\Delta L = 1$ couplings easily pass all experimental constraints since they are suppressed by the small neutrino masses. In the present section, we set all *RPV* soft-breaking terms to zero, and we assume that the $\Delta B = 1$ nucleon decays are only into quark and lepton final states.

Bounds from $\Delta B = 1$ nucleon decays.—Each bound arises from a particular mechanism and final state, and hence has its specific dependence on the intermediate sparticle masses. The strongest bounds, taken from Refs. [4,27], are listed in Table V. The numbers quoted for the MFV predictions are the maximum values attainable when the indices I, J (M) run over the three (first two) generations, respectively. Also, for processes involving external u_L quarks, the rotation to the mass-eigenstate basis is understood, i.e. $u'_L = V u_L$ according to Eq. (14). For example, the first bound in Table V stands for

$$|\boldsymbol{\lambda}'_{JKI}\boldsymbol{\lambda}''^{*}_{11I} V^{\dagger K1}|. \tag{53}$$

For leptons, no rotation is needed since neutrino flavors are not detected.

The first three bounds in Table V correspond to tree-level squark exchanges, and the factors

$$\tilde{q}_i^{\,2} \equiv (m_i/100\ \text{GeV})^2 \tag{54}$$

keep track of the mass suppressions. The following four make use of one LR mass insertion,

$$\delta_J^X \equiv (m_X^2)_{LR}^{JJ}/(m_X^2)_R^J. \tag{55}$$

At moderate $\tan\beta$, the $\mathbf{A}_d$ trilinear term is suppressed by

TABLE V. The MFV predictions for the combinations of couplings bounded from $\Delta B = 1$ nucleon decays ($I, J = 1, 2, 3$, $M = 1, 2$). The approximate bounds are discussed in the text. For the MFV predictions, entries are to be understood as $x \equiv \mathcal{O}(10^{-x})$. The scenarios A, B, C correspond to imposing $SU(3)^5$, $SU(3)^5 \times U(1)_D \times U(1)_E$, and $SU(3)^5 \times U(1)_U \times U(1)_D \times U(1)_E$, respectively. Finally, entries in bold are those missing the bounds by too many orders of magnitude to be compensated by making the squark as heavy as a few TeV.

Approximate bound	Case I			Case II			Case III			Case IV		
	A	B	C	A	B	C	A	B	C	A	B	C
$\vert\boldsymbol{\lambda}'_{J1I}\boldsymbol{\lambda}''^{*}_{11I}\vert \lesssim 10^{-27}\tilde{d}^2_{R,I}$	24	27	30	**20**	**22**	24	**23**	26	28	**19**	**21**	**23**
$\vert\boldsymbol{\lambda}'_{J2I}\boldsymbol{\lambda}''^{*}_{11I}\vert \lesssim 10^{-27}\tilde{d}^2_{R,I}$	24	26	29	**20**	**21**	24	**22**	25	28	**18**	**20**	**23**
$\vert\boldsymbol{\lambda}'_{M1I}\boldsymbol{\lambda}''^{*}_{12I}\vert \lesssim 10^{-27}\tilde{d}^2_{R,I}$	25	25	28	**20**	**20**	**23**	**23**	24	27	**18**	**19**	**22**
$\vert\boldsymbol{\lambda}'_{IJ1}\boldsymbol{\lambda}''^{*}_{11J}\vert \lesssim 10^{-27}\tilde{d}^2_{L,J}(\delta_J^D)^{-1}$	27	30	32	**22**	25	27	25	28	31	**20**	**23**	26
$\vert\boldsymbol{\lambda}'_{IJ2}\boldsymbol{\lambda}''^{*}_{11J}\vert \lesssim 10^{-27}\tilde{d}^2_{L,J}(\delta_J^D)^{-1}$	24	28	31	**20**	**23**	25	23	27	29	**19**	**21**	24
$\vert\boldsymbol{\lambda}'_{I31}\boldsymbol{\lambda}''^{*}_{123}\vert \lesssim 10^{-27}\tilde{b}^2_{L}(\delta_3^D)^{-1}$	27	28	31	**21**	**23**	26	26	27	29	**20**	**22**	24
$\vert\boldsymbol{\lambda}'_{MJ1}\boldsymbol{\lambda}''^{*}_{J12}\vert \lesssim 10^{-26}\tilde{u}^2_{L,J}(\delta_J^U)^{-1}$	23	29	29	**18**	23	23	**21**	27	27	**16**	**22**	**22**
$\vert\boldsymbol{\lambda}_{212}\boldsymbol{\lambda}''^{*}_{112}\vert < 10^{-20}(\tilde{m} \sim 1\ \text{TeV})$	21	27	30	**19**	23	26	20	26	28	**18**	22	25
$\vert\boldsymbol{\lambda}_{322}\boldsymbol{\lambda}''^{*}_{112}\vert < 10^{-20}(\tilde{m} \sim 1\ \text{TeV})$	23	29	31	21	25	28	21	27	29	**19**	23	26
$\vert\boldsymbol{\lambda}_{133}\boldsymbol{\lambda}''^{*}_{112}\vert < 10^{-21}(\tilde{m} \sim 1\ \text{TeV})$	20	26	28	**18**	22	25	**19**	25	27	**17**	21	24
$\vert\boldsymbol{\lambda}_{323}\boldsymbol{\lambda}''^{*}_{112}\vert < 10^{-21}(\tilde{m} \sim 1\ \text{TeV})$	22	27	30	**20**	24	27	**20**	25	28	**18**	22	25
$\vert\boldsymbol{\lambda}''_{112}\mu''^{I}/\mu\vert \lesssim 10^{-23}\tilde{u}^2_R$	22	27	30	**19**	23	26	20	26	29	**17**	21	24
$\vert\boldsymbol{\lambda}''_{312}\mu''^{I}/\mu\vert \lesssim 10^{-16}\tilde{d}^2_R$	18	23	23	**14**	18	18	16	22	22	**13**	17	17

the down-type Yukawa; hence the bounds in the fourth to sixth row are presumably less strict than indicated. On the other hand, the bound in the seventh row is more dangerous, both because of the potentially large stop left-right mixing, and because $|\boldsymbol{\lambda}'_{MJ1}\boldsymbol{\lambda}''^{*}_{J12}|$ involves the $\boldsymbol{\lambda}''_{312}$ coupling (see Table IV).

At the loop level, all combinations of $\boldsymbol{\lambda}'$ and $\boldsymbol{\lambda}''$ couplings become constrained, such that, conservatively [28],

$$|\boldsymbol{\lambda}'_{IJK}\boldsymbol{\lambda}''^{*}_{I'J'K'}| < \mathcal{O}(10^{-9}\text{–}10^{-11}). \tag{56}$$

Since $|\boldsymbol{\lambda}'| \lesssim \mathcal{O}(10^{-13})$, this bound is automatically satisfied.

There are also bounds involving $\boldsymbol{\lambda}$ or $\boldsymbol{\mu}'$ with $\boldsymbol{\lambda}''$, and the strongest are from Refs. [29,30], respectively. In Table V we quote only those which are not automatically satisfied under the MFV hypothesis.

Bounds from $n-\bar{n}$ oscillations.—The neutron-antineutron oscillations set constraints on the $\boldsymbol{\lambda}''_{11I}$ couplings at tree level [31],

$$|\boldsymbol{\lambda}''_{11I}| \lesssim (10^{-8} - 10^{-7})\frac{10^{8}s}{\tau_{\text{osc}}}\left(\frac{\tilde{m}}{100\text{ GeV}}\right)^{5/2}. \tag{57}$$

As commented in Ref. [4], these bounds are only indicative since the suppressions coming from LR mass insertions were ignored. This is especially true in MFV, which tends to strongly suppress such mass insertions. At loop level, $n-\bar{n}$ oscillations are constraining the largest element of $\boldsymbol{\lambda}''$ [32]:

$$|\boldsymbol{\lambda}''_{312}| \lesssim [10^{-3}, 10^{-2}]\left(\frac{200\text{ MeV}}{m_s}\right) \quad \text{for } m_{\tilde{q}} \sim [100\text{ GeV}, 200\text{ GeV}]. \tag{58}$$

However, this bound is rather weak for squark masses above 500 GeV, especially compared to the one in the seventh row of Table V.

Conservative upper bounds.—The order of magnitude for the combinations of couplings quoted in Table V should be understood as conservative upper bounds for several reasons. First, the MFV coefficients were assumed to be $\mathcal{O}(1)$. It would, however, still be natural to take them of the order of the Cabibbo angle, or suppressed by $1/4\pi$ loop factors, leading to additional suppressions by 1 or 2 orders of magnitude.

Second, throughout this work, the spurions are frozen at their background values at a very low scale, since the light-quark masses at about 2 GeV were used. If we had first run these masses to the electroweak scale or higher, the hierarchies within each coupling would be much stronger since light-fermion masses decrease rather quickly with the energy [33]. Further, the strong MFV suppressions occurring for the couplings in Table V precisely come from light-fermion mass factors.

The best strategy would probably have been to freeze the spurions at the scale at which the physics leading to the MFV symmetry is thought to be acting, presumably around the grand unified theory (GUT) scale. Then, the *RPV* couplings could have been run down to the electroweak scale. Such a study goes beyond the present order-of-magnitude analysis. Anyway, it is reasonable to expect that the running of the *RPV* couplings is smoother than those of the light-fermion masses; thus the bounds would again be easier to satisfy.

Finally, we never sum over the I, J, M indices, but rather take the largest value. We are thus disregarding the possibility of cancellations *à la* Glashow-Iliopoulos-Maiani [34]. Also, we work under the assumption that only one mechanism at a time is relevant; i.e. possible cancellations between the various processes for a given final state are neglected.

Behavior in terms of $\tan\beta$ *and* $U(1)_U$, $U(1)_D$.—All the couplings scale at least linearly with $\tan\beta$, so the combinations relevant for proton decay scale at least quadratically. Overall, the bounds become difficult to satisfy if $\tan\beta$ is too large, even with squark masses well above 1 TeV, and thus always require to impose either $U(1)_D$ or $U(1)_D \times U(1)_U$. Note also that if $\tan\beta$ is large and squarks are light, bounds from $\bar{n}-n$ oscillations are asking for $U(1)_D$, regardless of the possible suppression one could further impose on the $\Delta L = 1$ sector. In particular, the $\boldsymbol{\lambda}''_1$ structure, with its large 312 entry, cannot satisfy Eq. (58) at large $\tan\beta$ when $m_{\tilde{q}} \lesssim 200$ GeV. Away from these extreme situations, $n-\bar{n}$ oscillations are not very constraining.

If enforced, the $U(1)_U$ and $U(1)_D$ symmetries make the ε-terms of $\mathbf{A}_u$ and $\mathbf{A}_d$ negligibly small, since they are suppressed by $\det(\mathbf{Y}_u)$ and $\det(\mathbf{Y}_d)$. Therefore, probing for these structures allows one to test the directions in which B and L are violated. For example, if the presence of the ε-term in $\mathbf{A}_d$ is established at large $\tan\beta$, thus requiring $U(1)_D$ breaking, MFV would then have difficulties with $\bar{n}-n$ oscillations.

Behavior in terms of m_ν, M_R, *and LFV effects.*—In Fig. 1, we plot the order of magnitude predicted by MFV for $|\boldsymbol{\lambda}'_{M1I}\boldsymbol{\lambda}''^{*}_{12I}|$, as a function of $\tan\beta$ and m_ν. The behavior of the other bounds is similar. This plot permits us to interpolate between the various scenarios of Table V. Cases I, II, IV, and III correspond to the corners of the plot, starting from the upper left, in the clockwise direction. The three plots correspond to imposing $SU(3)^5$, $SU(3)^5 \times U(1)_D \times U(1)_E$, and $SU(3)^5 \times U(1)_U \times U(1)_D \times U(1)_E$, from left to right. The dependence on M_R is subleading, apart from restricting m_ν through the perturbative bound of Eq. (48). Therefore, we took $M_R = 10^{-12}$ in Fig. 1, which allows m_ν to reach about 1 eV.

The behavior of the bounds in terms of m_ν shown in Fig. 1 can be simply understood from the behavior of the off-diagonal elements of $\mathbf{Y}_\nu$; see Eqs. (42) and (43). Interestingly, the size of $\Delta L = 1$ couplings quickly decreases as m_ν increases from zero to about 0.1 eV. Therefore, in scenarios with large $\tan\beta$, it is better to have m_ν also quite large, somewhere in the range 0.1–1 eV.

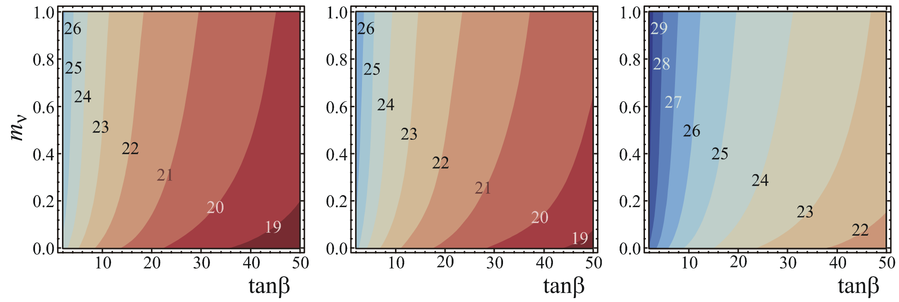

FIG. 1 (color online). The order of magnitude predicted by MFV for $|\boldsymbol{\lambda}'_{M1I}\boldsymbol{\lambda}''^{*}_{12I}|$, as a function of $\tan\beta$ and m_ν [numbers on the curves stand for $x \equiv \mathcal{O}(10^{-x})$]. The behavior of the other bounds is similar. Cases I, II, IV, and III correspond to the corners of the plot, starting from the upper left, in the clockwise direction. The three plots correspond to the scenarios A, B, and C of Table V, from left to right. The dependence on M_R is subleading, and we set $M_R = 10^{-12}$.

This then has some impact on LFV effects, and thus on rare leptonic processes like $\mu \to e\gamma$ or $\mu \to eee$. Indeed, generically, the bounds prefer large m_ν, which, given Eq. (48), can be attained only for small M_R. In that situation, the spurion $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ which tunes LFV effects [see Eq. (23)] is significantly suppressed.

Finally, imposing $U(1)_L$ would suppress all the $\Delta L = 1$ couplings by $\det(\mathbf{Y}_\ell)$ and make it trivial to satisfy all the bounds of Table V no matter the scenario. In that case, the only significant constraints come from $\bar{n} - n$ oscillations, and LFV effects can reach their maximum.

What to expect from $\Delta B = 1$ interactions at colliders and in FCNC's.—Except for nucleon decay, which, incidentally, may be just around the corner, lepton number can be considered as effectively conserved. On the other hand, purely $\Delta B = 1$ processes are not so suppressed and could offer competitive signals in the search for supersymmetry. There exists extensive literature on this subject, and we do not intend to review it here (see e.g. Ref. [4]). Instead, we discuss some selected effects of $\Delta B = 1$ *RPV* interactions, first for low-energy observables and then at hadron colliders.

At low energy, observables are necessarily *RPC*, hence quadratic in $\boldsymbol{\lambda}''$. In particular, the tree-level squark exchanges essentially correspond to diquark currents, with strength given by

$$\frac{|\boldsymbol{\lambda}''_{IJK}\boldsymbol{\lambda}''_{LMN}|}{m_{\tilde{q}}^2} \lesssim 10^{-8}\ \text{GeV}^{-2} \times \frac{(100\ \text{GeV})^2}{m_{\tilde{q}}^2}. \quad (59)$$

In the favorable situation of $\tan\beta$ not too large, so that $U(1)_D$ does not need to be imposed, the maximum is attained within MFV for the stop, $|\boldsymbol{\lambda}''_{312}\boldsymbol{\lambda}''_{331}| \sim 10^{-4}$–$10^{-5}$ and $|\boldsymbol{\lambda}''_{312}\boldsymbol{\lambda}''_{323}| \sim 10^{-5}$–$10^{-6}$. Many works have analyzed the possible impact of these currents, for example, in hadronic B decays [35], $b \to s\gamma$ [36], $D - \bar{D}$ and $B - \bar{B}$ mixing [37], and Δm_K, ε_K [38]. Not so surprisingly given that the SM contributions are tuned by $G_F \sim 10^{-5}\ \text{GeV}^{-2}$, the bounds obtained in these works are typically $|\boldsymbol{\lambda}''_{IJK}\boldsymbol{\lambda}''_{LMN}| \lesssim 10^{-2}$–$10^{-3}$ for $m_{\tilde{q}} \sim 100$ GeV. Therefore, if MFV correctly predicts the order of magnitude of *RPV* effects, to have any hope to see them in low-energy K, D, or B physics, the precision needed is rather challenging. Besides the experimental difficulties, tree-level *RPV* effects occur in hadronic channels only, whose accuracy is ultimately limited by QCD effects.

At hadron colliders, $\Delta B = 1$, *RPV* interactions would be easier to find because they can drastically change the phenomenology [39] (see also Refs. [4,40]).

First, the LSP can decay, mostly through hadronic channels [41], and since it is no longer stable, it can be colored and/or charged. If the neutralino is still the LSP, to identify the presence of *RPV* would require it to decay sufficiently quickly, within the detector. Looking back at Table IV, one can see that the largest $\boldsymbol{\lambda}''$ elements vary in the rather large range $10^{-5} \lesssim |\boldsymbol{\lambda}''|_{\max} \lesssim 10^{-1}$, depending on $\tan\beta$, and on the $U(1)_D$ and $U(1)_U$ symmetries. Then, depending also on the neutralino mass, the LSP may be effectively stable for the CERN LHC, or may decay very quickly. We refer to Refs. [41] for quantitative analyses and descriptions of the decay channels.

Second, single squark resonant production can occur, lowering the threshold for the discovery of supersymmetry. With the MFV prediction $\boldsymbol{\lambda}''_{312} \lesssim 10^{-1}$, one would expect mostly single stop production, which can have very distinctive signatures (see Ref. [42]). Also, the single gluino production through $pp \to \tilde{t} \to t\tilde{g}$ was advocated in Ref. [43] as a particularly clean channel in which to look for supersymmetric effects at the LHC. Similarly, the presence of the $\Delta B = 1$ couplings may be felt in top-quark production [44] or decay [45], which are also tuned by the dominant $\boldsymbol{\lambda}''_{312}$ coupling.

IV. CONCLUSION

In this paper, we have shown that imposing the MFV hypothesis can be sufficient to stabilize the proton. Further, MFV turns out to be more powerful than R parity, since it also suppresses the dangerous baryon- and lepton-number violating higher-dimensional operators. This symmetry principle, originally introduced to solve the MSSM flavor problem, can thus successfully replace R parity for building a viable model. In the GUT context, or when investigating supersymmetry-breaking mechanisms, ensuring the MFV criterium is satisfied below the TeV scale is a simple first step towards satisfying low-energy constraints. In this respect, depending on $\tan\beta$ and m_ν, one may need to allow the breaking of baryon and lepton numbers in only a few selected directions, by restricting the number of broken flavor $U(1)$'s.

Interestingly, the MFV suppression is not always sufficient to avoid a too rapid proton decay. First, this means that depending on the values of the parameters, proton decay and/or $n - \bar{n}$ oscillations can be very close to their current experimental bounds. Second, imposing these bounds gives indirect constraints on parameters relevant also for FCNC or LFV. Indeed, moderate $\tan\beta$ and large m_ν (or alternatively the inverted neutrino mass spectrum) are preferred, and are even compulsory if all the $U(1)$'s are broken. On the other hand, the seesaw scale M_R plays only a subleading role.

MFV predicts that lepton number can be considered as conserved in most cases, but not baryon number. The best signals of R-parity violation, or even of supersymmetry, are then expected at colliders. Indeed, the impact on low-energy observables is generically small compared to SM contributions. Further, at tree level, $\Delta B = 1$ effects contribute dominantly to hadronic processes, in which QCD uncertainties are quite challenging. On the other hand, at colliders, one could look for the decays of the lightest supersymmetric particle, not necessarily colorless and neutral. Also, MFV predicts significant couplings for resonant stop production, as well as for top production from down squarks. However, it should be noted that these couplings strongly depend on the $U(1)$'s enforced, with the favorable situation being $\lambda''_{312} \sim 10^{-1}$ when all of them are broken. Alternatively, if $U(1)_D$ is exact, $\Delta B = 1$ couplings are all less than about 10^{-5}–10^{-3}, depending on $\tan\beta$, and thus may not be readily accessible experimentally.

Though we performed our analysis in the MSSM, the viability of MFV for proton stability is, to some extent, model independent. Indeed, as long as the only $(\bar{6}, 1)$ spurion is related to the neutrino masses, $\mathbf{Y}_\nu \sim \mathcal{O}(m_\nu/v_u)$, lepton-number violating couplings remain very suppressed. In our work, the only model-dependent spurion is $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, a remnant of the specific type I seesaw with right-handed neutrinos. However, its impact is limited, since it only introduces a slight softening of the hierarchies within each RPV coupling, and this only when the seesaw scale is large, so that $\mathbf{Y}_\nu \sim \mathcal{O}(1)$.

Another important point is that the MSSM quark and lepton flavor groups are factorized. Indeed, no matter the precise form of the $\Delta B = 1$ and $\Delta L = 1$ operators inducing proton decay, MFV will separately suppress each sector, as for the dimension-five $QQQL$ and $UUDE$ effective interactions. Of course, this factorization no longer holds, in general, in GUT theories, where leptons and quarks can be in the same multiplet, but it may nevertheless reemerge at the electroweak scale. For example, in $SU(5)$, the RPV coupling $\bar{5}^I \bar{5}^J 10^K$ inducing both $\Delta B = 1$ and $\Delta L = 1$ couplings can be readily parametrized in terms of the $\mathbf{Y}_{\bar{5}} \sim \mathbf{Y}_d$ and $\mathbf{Y}_{10} \sim \mathbf{Y}_u$ Yukawa couplings (see e.g. Ref. [46] for the transformation rules), and is thus not suppressed by neutrino masses. However, the smaller flavor group $U(3)_{\bar{5}} \times U(3)_{10}$ restricts the possible directions in which lepton and baryon numbers are violated. If we require the flavor $U(1)_D \sim U(1)_{\bar{5}}$ and $U(1)_E \sim U(1)_{10}$ to remain exact, the coupling $\bar{5}^I \bar{5}^J 10^K$ as well as the bilinear $H_{\bar{5}} \bar{5}^I$ are forbidden, and R-parity violation may then arise only after the GUT symmetry is broken. At that stage, the nonrenormalizable dimension-five RPC operators may also appear, but, besides being suppressed by the GUT scale, they should still be suppressed by the MFV principle. To quantify this MFV suppression requires to specify the dynamics of the model, at least to some extent, and this goes beyond our bottom-up approach. Nevertheless, as said earlier, ensuring that the factorized $U(3)^5$ flavor group reemerges at low energy could offer an interesting strategy to keep both RPC and RPV flavor breakings in check.

Finally, in the present work, cosmological implications were not investigated. For instance, it would be interesting to analyze how the baryon asymmetry can survive the presence of $\Delta B = 1$ interactions, with the specific strengths predicted by MFV. Also, since the MSSM LSP is no longer stable, and given that there is always a $\Delta B = 1$ coupling larger than about 10^{-5}, it cannot be a viable dark matter candidate. Its nature then has to be resolved at a yet higher scale. In these contexts, ensuring that the MFV criterium is satisfied at low energy may offer interesting constraints on possible models.

ACKNOWLEDGMENTS

We would like to thank Gilberto Colangelo for his comments and support. This work is partially supported by the EU Contract No. MRTN-CT-2006-035482 (FLAVIAnet), and by the Schweizerischer Nationalfonds.

APPENDIX A: THE REDUCED BASIS FOR CASE V

The reduced basis in the region of parameter space corresponding to

$$\text{Case V}: \tan\beta \lesssim 20, \qquad M_R \lesssim 2 \times 10^{13}\ \text{GeV}, \qquad m_\nu \gtrsim 0.05\ \text{eV} \tag{A1}$$

is rather simple because, for these values of M_R and m_ν, the spurion $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is subleading and never occurs. Also, $\tan\beta$ is small enough to suppress all occurrences of $\mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell$, which then enters only when needed to get a nonvanishing contraction with ε tensors. On the other hand, $\tan\beta$ is large enough to feel the effects of $\mathbf{Y}_d^\dagger \mathbf{Y}_d$, and the basis we will construct for the $\Delta B = 1$ sector is in fact valid up to large $\tan\beta \approx 50$. As explained in Sec. II, the expansions for

RPV soft-breaking terms can be immediately obtained from those of the supersymmetric *RPV* terms, and will not be written down explicitly.

It should also be noted that the basis we construct can be useful in other situations as well. For example, if one insists on setting all $\Delta B = 1$ MFV coefficients to zero, but rescales $\Delta L = 1$ coefficients making them of $\mathcal{O}(M_R)$, the MFV expansions constructed here remain valid as long as $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \lesssim \mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell$, and would then offer a systematic framework for the phenomenological study of $\Delta L = 1$ effects.

Since $\boldsymbol{\mu}'^I = \mu \bar{\mathbf{Y}}_\nu^I$, we first work out the relevant operators in $\bar{\mathbf{Y}}_\nu$, and we are left with only

$$\mu'^I = \mu a_1 \varepsilon^{LMI} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell)^{LM}. \tag{A2}$$

Similarly, for the $\boldsymbol{\lambda}$ coupling, only two out of the several hundred operators are dominant,

$$\boldsymbol{\lambda}^{IJK} = a_2 \varepsilon^{IJL} (\mathbf{Y}_\ell \mathbf{Y}_\nu^\dagger)^{KL} + a_3 \varepsilon^{KLM} \varepsilon^{IJD} \varepsilon^{ABC} (\mathbf{Y}_\ell^\dagger)^{AL} \times (\mathbf{Y}_\nu^\dagger)^{BD} (\mathbf{Y}_\ell^\dagger)^{CM}.$$

For the $\boldsymbol{\lambda}'$ and $\boldsymbol{\lambda}''$ coupling, the reduced basis depends a lot on which $U(1)$ is imposed. Let us start with the $SU(3)^5$ case. After expanding $\mathbf{A}_d$, we find

$$\begin{aligned}
\boldsymbol{\lambda}'^{IJK} &= \varepsilon^{LMI} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\ell^\dagger \mathbf{Y}_\ell)^{LM} ((\mathbf{A}_d)_1 + (\mathbf{A}_d)_2)^{KJ}, \\
(\mathbf{A}_d)_1^{KJ} &= \mathbf{Y}_d (a_4 \mathbf{1} + a_5 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_1 \mathbf{Y}_d^\dagger \mathbf{Y}_d + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
&\quad + b_3 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d))^{KJ}, \\
(\mathbf{A}_d)_2^{KJ} &= \varepsilon^{LMN} \varepsilon^{KBC} (\mathbf{Y}_d^\dagger)^{LB} (\mathbf{Y}_d^\dagger)^{MC} (a_6 \mathbf{1} + a_7 \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
&\quad + b_4 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d)^{NJ},
\end{aligned} \tag{A3}$$

and

$$\begin{aligned}
\boldsymbol{\lambda}''^{IJK} &= \varepsilon^{LJK} (\mathbf{Y}_u (a_8 \mathbf{1} + a_9 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_5 \mathbf{Y}_d^\dagger \mathbf{Y}_d \\
&\quad + b_6 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u) \mathbf{Y}_d^\dagger)^{IL} + a_{10} \varepsilon^{IMN} (\mathbf{Y}_d \mathbf{Y}_u^\dagger)^{JM} \\
&\quad \times (\mathbf{Y}_d \mathbf{Y}_u^\dagger)^{KN} + \varepsilon^{LMN} (\mathbf{Y}_u (b_7 \mathbf{1} + b_8 \mathbf{Y}_u^\dagger \mathbf{Y}_u))^{IL} \\
&\quad \times (\mathbf{Y}_d)^{JM} (\mathbf{Y}_d)^{KN} + \varepsilon^{LMN} \varepsilon^{PJK} \varepsilon^{ABI} ((a_{11} \mathbf{1} \\
&\quad + b_9 \mathbf{Y}_d^\dagger \mathbf{Y}_d) \mathbf{Y}_d^\dagger)^{LP} (\mathbf{Y}_u^\dagger)^{MA} (\mathbf{Y}_u^\dagger)^{NB}.
\end{aligned} \tag{A4}$$

Altogether, we therefore have 20 operators, out of which the 9 b_i's can be dropped if $\tan\beta \lesssim 5$.

If we impose $SU(3)^5 \times U(1)_D \times U(1)_E$, the term a_3 and the whole $(\mathbf{A}_d)_2^{KJ}$ structure can be dropped, while

$$\begin{aligned}
\boldsymbol{\lambda}''^{IJK} &= \det(\mathbf{Y}_d) \varepsilon^{LJK} (\mathbf{Y}_u (a_6 \mathbf{1} + a_7 \mathbf{Y}_u^\dagger \mathbf{Y}_u) \mathbf{Y}_d^\dagger)^{IL} \\
&\quad + a_8 \varepsilon^{IMN} (\mathbf{Y}_d \mathbf{Y}_u^\dagger)^{JM} (\mathbf{Y}_d \mathbf{Y}_u^\dagger)^{KN} \\
&\quad + \varepsilon^{LMN} (\mathbf{Y}_u (a_9 \mathbf{1} + a_{10} \mathbf{Y}_u^\dagger \mathbf{Y}_u))^{IL} (\mathbf{Y}_d)^{JM} (\mathbf{Y}_d)^{KN} \\
&\quad + b_4 \det(\mathbf{Y}_d) \varepsilon^{LMN} \varepsilon^{PJK} \varepsilon^{ABI} (\mathbf{Y}_d^\dagger)^{LP} (\mathbf{Y}_u^\dagger)^{MA} (\mathbf{Y}_u^\dagger)^{NB}.
\end{aligned} \tag{A5}$$

In this case, we are left with 13 operators, out of which the 4 b_i's can be dropped if $\tan\beta \lesssim 5$.

Finally, if we impose $SU(3)^5 \times U(1)_U \times U(1)_D \times U(1)_E$, only four operators remain for $\boldsymbol{\lambda}''$:

$$\begin{aligned}
\boldsymbol{\lambda}''^{IJK} &= \det(\mathbf{Y}_d) \varepsilon^{LJK} (\mathbf{Y}_u (a_6 \mathbf{1} + a_7 \mathbf{Y}_u^\dagger \mathbf{Y}_u) \mathbf{Y}_d^\dagger)^{IL} \\
&\quad + \varepsilon^{LMN} (\mathbf{Y}_u (a_8 \mathbf{1} + a_9 \mathbf{Y}_u^\dagger \mathbf{Y}_u))^{IL} (\mathbf{Y}_d)^{JM} (\mathbf{Y}_d)^{KN},
\end{aligned} \tag{A6}$$

and we need 11 free parameters (eight when $\tan\beta \lesssim 5$) to describe all supersymmetric *RPV* couplings.

APPENDIX B: THE $\boldsymbol{\varepsilon}$ STRUCTURES FOR THE *RPC* TRILINEAR TERMS

For completeness, we give here the order of magnitude of the ε-terms of the *RPC* trilinear terms, Eq. (23). For all three, the basic effect is to create an inverted hierarchy compared to the usual Yukawa, though it can compete with it only at large $\tan\beta$.

For $\mathbf{A}_u$, whose sensitivity to $\tan\beta$ is only through $\mathbf{Y}_d^\dagger \mathbf{Y}_d$ and thus very small, we find

$$\mathbf{A}_u/A_0 = \begin{pmatrix} 10^{-5} & 10^{-6} & 10^{-7} \\ 10^{-3} & 10^{-2} & 10^{-4} \\ 10^{-2} & 10^{-2} & 1 \end{pmatrix} + \begin{pmatrix} 10^{-2} & 10^{-3} & 10^{-4} \\ 10^{-5} & 10^{-5} & 10^{-6} \\ 10^{-9} & 10^{-8} & 10^{-7} \end{pmatrix}_\varepsilon.$$

For $\mathbf{A}_d$, we already gave the result while analyzing $\boldsymbol{\lambda}'$ (which involves $\mathbf{A}_d^T$), see Table III, but we repeat the result here for clarity:

$$\mathbf{A}_d/A_0 \overset{\tan\beta=5}{=} \begin{pmatrix} 10^{-4} & 10^{-7} & 10^{-6} \\ 10^{-6} & 10^{-3} & 10^{-4} \\ 10^{-3} & 10^{-3} & 10^{-1} \end{pmatrix} + \begin{pmatrix} 10^{-3} & 10^{-7} & 10^{-5} \\ 10^{-8} & 10^{-5} & 10^{-6} \\ 10^{-8} & 10^{-8} & 10^{-6} \end{pmatrix}_\varepsilon,$$

$$\mathbf{A}_d/A_0 \overset{\tan\beta=50}{=} \begin{pmatrix} 10^{-3} & 10^{-6} & 10^{-5} \\ 10^{-5} & 10^{-2} & 10^{-3} \\ 10^{-2} & 10^{-1} & 1 \end{pmatrix} + \begin{pmatrix} 10^{-1} & 10^{-5} & 10^{-3} \\ 10^{-6} & 10^{-3} & 10^{-4} \\ 10^{-6} & 10^{-5} & 10^{-4} \end{pmatrix}_\varepsilon.$$

Finally, for $\mathbf{A}_\ell$, the situation is similar, though we have to distinguish the four cases of Eq. (49):

$$\mathbf{A}_{\ell}/A_0 \overset{\text{Case I}}{=} \begin{pmatrix} 10^{-5} & 10^{-11} & 10^{-11} \\ 10^{-9} & 10^{-3} & 10^{-7} \\ 10^{-8} & 10^{-6} & 10^{-2} \end{pmatrix} + \begin{pmatrix} 10^{-4} & 10^{-10} & 10^{-10} \\ 10^{-12} & 10^{-6} & 10^{-10} \\ 10^{-13} & 10^{-12} & 10^{-7} \end{pmatrix}_{\varepsilon},$$

$$\mathbf{A}_{\ell}/A_0 \overset{\text{Case II}}{=} \begin{pmatrix} 10^{-4} & 10^{-10} & 10^{-10} \\ 10^{-8} & 10^{-2} & 10^{-6} \\ 10^{-7} & 10^{-5} & 10^{-1} \end{pmatrix} + \begin{pmatrix} 10^{-2} & 10^{-8} & 10^{-8} \\ 10^{-10} & 10^{-4} & 10^{-8} \\ 10^{-11} & 10^{-10} & 10^{-5} \end{pmatrix}_{\varepsilon},$$

$$\mathbf{A}_{\ell}/A_0 \overset{\text{Case III}}{=} \begin{pmatrix} 10^{-5} & 10^{-7} & 10^{-7} \\ 10^{-4} & 10^{-3} & 10^{-4} \\ 10^{-3} & 10^{-2} & 10^{-2} \end{pmatrix} + \begin{pmatrix} 10^{-4} & 10^{-5} & 10^{-5} \\ 10^{-8} & 10^{-6} & 10^{-7} \\ 10^{-9} & 10^{-8} & 10^{-7} \end{pmatrix}_{\varepsilon},$$

$$\mathbf{A}_{\ell}/A_0 \overset{\text{Case IV}}{=} \begin{pmatrix} 10^{-4} & 10^{-6} & 10^{-6} \\ 10^{-3} & 10^{-2} & 10^{-3} \\ 10^{-2} & 10^{-1} & 10^{-1} \end{pmatrix} + \begin{pmatrix} 10^{-2} & 10^{-3} & 10^{-3} \\ 10^{-6} & 10^{-4} & 10^{-5} \\ 10^{-7} & 10^{-6} & 10^{-5} \end{pmatrix}_{\varepsilon}.$$

Therefore, the main impact of these ε-terms is to induce a larger LR mixing for the first-generation squark and slepton. This mixing is, however, quite small, even at large $\tan\beta$, and it remains to be seen if it can be singled out experimentally.

[1] G. R. Farrar and P. Fayet, Phys. Lett. **76B**, 575 (1978).
[2] P. Minkowski, Phys. Lett. **67B**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979; T. Yanagida, Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 1979; S. L. Glashow, Quarks and Leptons, Cargèse, 1979; R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).
[3] L. E. Ibanez and G. G. Ross, Nucl. Phys. **B368**, 3 (1992).
[4] R. Barbier *et al.*, Phys. Rep. **420**, 1 (2005).
[5] G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B645**, 155 (2002).
[6] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[7] R. S. Chivukula and H. Georgi, Phys. Lett. B **188**, 99 (1987).
[8] For an introductory review of the MSSM, see for example S. P. Martin, arXiv:hep-ph/9709356.
[9] R. D. Peccei and H. R. Quinn, Phys. Rev. D **16**, 1791 (1977).
[10] V. Cirigliano, B. Grinstein, G. Isidori, and M. B. Wise, Nucl. Phys. **B728**, 121 (2005).
[11] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
[12] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).
[13] G. Colangelo, E. Nikolidakis, and C. Smith (unpublished).
[14] L. J. Hall and L. Randall, Phys. Rev. Lett. **65**, 2939 (1990).
[15] G. Isidori, F. Mescia, P. Paradisi, C. Smith, and S. Trine, J. High Energy Phys. 08 (2006) 064; W. Altmannshofer, A. J. Buras, and D. Guadagnoli, J. High Energy Phys. 11 (2007) 065.
[16] S. Davidson and F. Palorini, Phys. Lett. B **642**, 72 (2006).
[17] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994).
[18] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).
[19] T. Banks, Y. Grossman, E. Nardi, and Y. Nir, Phys. Rev. D **52**, 5319 (1995).
[20] S. Davidson and J. R. Ellis, Phys. Lett. B **390**, 210 (1997).
[21] Y. Grossman and H. E. Haber, Phys. Rev. D **59**, 093008 (1999).
[22] M. C. Gonzalez-Garcia and M. Maltoni, arXiv:0704.1800.
[23] See e.g. J. Lesgourgues and S. Pastor, Phys. Rep. **429**, 307 (2006).
[24] S. Pascoli, S. T. Petcov, and C. E. Yaguna, Phys. Lett. B **564**, 241 (2003).
[25] V. Cirigliano, G. Isidori, and V. Porretti, Nucl. Phys. **B763**, 228 (2007).
[26] L. J. Hall, R. Rattazzi, and U. Sarid, Phys. Rev. D **50**, 7048 (1994).
[27] F. Vissani, Phys. Rev. D **52**, 4245 (1995).
[28] A. Y. Smirnov and F. Vissani, Phys. Lett. B **380**, 317 (1996).
[29] G. Bhattacharyya and P. B. Pal, Phys. Rev. D **59**, 097701 (1999).
[30] G. Bhattacharyya and P. B. Pal, Phys. Lett. B **439**, 81 (1998).
[31] F. Zwirner, Phys. Lett. **132B**, 103 (1983).
[32] D. Chang and W. Y. Keung, Phys. Lett. B **389**, 294 (1996).
[33] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
[34] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).
[35] C. E. Carlson, P. Roy, and M. Sher, Phys. Lett. B **357**, 99 (1995); D. Chakraverty and D. Choudhury, Phys. Rev. D **63**, 112002 (2001); D. K. Ghosh, X. G. He, B. H. J. McKellar, and J. Q. J. Shi, J. High Energy Phys. 07 (2002) 067; Y. D. Yang, R. M. Wang, and G. R. Lu, Phys. Rev. D **72**, 015009 (2005); R. Wang, G. R. Lu, E. K. Wang, and Y. D. Yang, Eur. Phys. J. C **47**, 815 (2006).
[36] D. Chakraverty and D. Choudhury, Phys. Rev. D **63**, 075009 (2001).

[37] S. L. Chen, X. G. He, A. Hovhannisyan, and H. C. Tsai, J. High Energy Phys. 09 (2007) 044.
[38] P. Slavich, Nucl. Phys. **B595**, 33 (2001).
[39] S. Dimopoulos and L. J. Hall, Phys. Lett. B **207**, 210 (1988).
[40] B. Allanach *et al.* (*R* parity Working Group Collaboration), arXiv:hep-ph/9906224.
[41] H. K. Dreiner and G. G. Ross, Nucl. Phys. **B365**, 597 (1991); H. Baer, C. H. Chen, and X. Tata, Phys. Rev. D **55**, 1466 (1997); B. C. Allanach *et al.*, J. High Energy Phys. 03 (2001) 048.
[42] S. Dimopoulos, R. Esmailzadeh, L. J. Hall, and G. D. Starkman, Phys. Rev. D **41**, 2099 (1990); E. L. Berger, B. W. Harris, and Z. Sullivan, Phys. Rev. Lett. **83**, 4472 (1999); Phys. Rev. D **63**, 115001 (2001).
[43] M. Chaichian, K. Huitu, and Z. H. Yu, Phys. Lett. B **490**, 87 (2000).
[44] A. Datta, J. M. Yang, B. L. Young, and X. Zhang, Phys. Rev. D **56**, 3107 (1997); R. J. Oakes, K. Whisnant, J. M. Yang, B. L. Young, and X. Zhang, Phys. Rev. D **57**, 534 (1998); P. Chiappetta, A. Deandrea, E. Nagy, S. Negroni, G. Polesello, and J. M. Virey, Phys. Rev. D **61**, 115008 (2000).
[45] J. M. Yang, B. L. Young, and X. Zhang, Phys. Rev. D **58**, 055001 (1998); K. J. Abraham, K. Whisnant, J. M. Yang, and B. L. Young, Phys. Rev. D **63**, 034011 (2001); G. Eilam, A. Gemintern, T. Han, J. M. Yang, and X. Zhang, Phys. Lett. B **510**, 227 (2001).
[46] B. Grinstein, V. Cirigliano, G. Isidori, and M. B. Wise, Nucl. Phys. **B763**, 35 (2007).