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$U(1)'$ solution to the μ-problem and the proton decay problem in supersymmetry without R-parity

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The minimal supersymmetric standard model (MSSM) is plagued by two major fine-tuning problems: the μ-problem and the proton decay problem. We present a simultaneous solution to both problems within the framework of a $U(1)'$-extended MSSM (UMSSM), without requiring R-parity conservation. We identify several classes of phenomenologically viable models and provide specific examples of $U(1)'$ charge assignments. Our models generically contain *either* lepton number violating *or* baryon number violating renormalizable interactions, whose coexistence is nevertheless automatically forbidden by the new $U(1)'$ gauge symmetry. The $U(1)'$ symmetry also prohibits the potentially dangerous and often ignored higher-dimensional proton decay operators such as $QQQL$ and $U^c U^c D^c E^c$ which are still allowed by R-parity. Thus, under minimal assumptions, we show that once the μ-problem is solved, the proton is sufficiently stable, even in the presence of a minimum set of exotics fields, as required for anomaly cancellation. Our models provide impetus for pursuing the collider phenomenology of R-parity violation within the UMSSM framework.

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I. INTRODUCTION

Supersymmetry (SUSY) at the terascale has been the leading candidate for physics beyond the standard model (SM). We do not know the concrete manifestation of supersymmetry at low energies, but the minimal supersymmetric standard model (MSSM) already incorporates most of the advantages of supersymmetry and has proved to be a useful playground for investigations of the possible SUSY signatures at high energy colliders such as the Tevatron and the Large Hadron Collider (LHC). In spite of its successes, however, the MSSM does not exhaust all possibilities and, given its shortcomings discussed below, it is certainly worth pursuing alternative, more general low-energy supersymmetric theories.

One of the most celebrated successes of low-energy supersymmetry is the resolution of the gauge hierarchy problem of the SM. SUSY protects the Higgs mass and the associated electroweak scale from the dangerous quadratically divergent radiative corrections. However, the MSSM itself suffers from its own fine-tuning problems. First, there is the so-called μ-problem [1], which is associated with the following superpotential coupling of the two MSSM Higgs doublets H_1 and H_2:

$$W_\mu = \mu H_2 H_1. \tag{1}$$

Since this coupling is allowed by both supersymmetry and gauge symmetry, there is no natural (i.e. in terms of a symmetry) explanation, at least within the MSSM, as to why the value of the μ parameter is so much smaller than the fundamental (Planck or string) scale. To fix this problem in a natural way, one has to introduce a symmetry which would prohibit the original μ term (1). However, in the end this symmetry needs to be broken, since a vanishing μ term would imply very light charginos, in violation of the LEP search limits [2]. Therefore, a viable model should dynamically generate an effective μ term. This is typically done by introducing a Higgs singlet S coupling to the MSSM Higgs doublets as

$$W_{\mu_{\rm eff}} = h S H_2 H_1. \tag{2}$$

The singlet S is charged under the new symmetry, so that the original μ term (1) is forbidden. The vacuum expectation value (VEV) of S would then break the symmetry and play the role of an effective μ parameter. Depending on the type of the new symmetry, the models can be classified into several categories [3]. For instance, when the symmetry is a $\boldsymbol{Z_3}$ discrete symmetry, one obtains the next-to-MSSM (NMSSM) [4], when the symmetry is an Abelian gauge symmetry $U(1)'$, we have the $U(1)'$-extended MSSM (UMSSM) [5], etc. [Other options include the minimal nonminimal SSM (MNSSM) [6] and the essential SSM (ESSM) [7].] In this study we shall work within the UMSSM framework, and we shall use the additional $U(1)'$ gauge interaction to forbid the original μ term (1) while allowing the effective μ term (2). We shall completely specify the particle content of the model and will demand that the new $U(1)'$ gauge symmetry is nonanomalous. An extra $U(1)$ symmetry is supported by many new physics paradigms including grand unified theories [8,9], extra dimensions [10], superstrings [11], little Higgs [12], dynamical symmetry breaking [13], and Stueckelberg mechanism [14].

The other fine-tuning problem of the MSSM is related to the existence of lepton number violating (LV) terms

$$W_{\rm LV} = \mu_i' H_2 L_i + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c \tag{3}$$

and baryon number violating (BV) terms

$$W_{\rm BV} = \lambda''_{ijk} U^c_i D^c_j D^c_k \tag{4}$$

in the superpotential. Here i, j, k are generation indices and summation over repeated indices is implied. The couplings (3) and (4) are again allowed by all gauge symmetries and supersymmetry and may even occur in the underlying grand unified theory [15]. The presence of both types of such terms would lead to unacceptably rapid proton decay unless certain combinations of couplings are tuned to be extremely small ($\lambda\lambda'' \lesssim 10^{-21}$, $\lambda'\lambda'' \lesssim 10^{-27}$ [16]). The standard practice for dealing with this fine-tuning problem is again to impose a new symmetry, the so-called R-parity [17], which is the only other new symmetry in the MSSM besides supersymmetry. R-parity forbids *both* types of problematic terms (3) and (4) and the proton appears to be safe.

At this point one might question whether it was really necessary to forbid *both* (3) and (4). Indeed, since proton decay requires both LV and BV interactions, forbidding either of them would be sufficient to stabilize the proton. In this sense, the imposition of R-parity is far from being the minimalist approach, since it eliminates a large chunk of potentially interesting phenomenology related to the physics of R-parity violation (RPV) [18]. In this study, we shall therefore utilize the $U(1)'$ gauge symmetry to forbid some, but not all R-parity violating interactions. More specifically, we shall look for models where the proton is stable in the presence of either LV interactions (3) or BV interactions (4). We shall find that, without ever demanding it, the LV and BV terms are in fact naturally separated in the sense that the $U(1)'$ symmetry may allow (3) or (4), but *not both* at the same time. This result, which we shall refer to as "LV-BV separation," is very general and relies only on the following three simple assumptions:

(1) The MSSM Yukawa couplings are allowed by the $U(1)'$ gauge symmetry.
(2) The μ-problem is solved as in the UMSSM, namely, the $U(1)'$ gauge symmetry forbids the original μ term (1) while allowing the effective μ term (2).
(3) There are no new exotic $SU(2)$ representations[1] beyond the field content of the MSSM.

The proof of the LV-BV separation is very simple and will be presented in Sec. II B.

At this point, giving up on R-parity may seem like a rather steep price to pay. After all, R-parity ensures that the lightest supersymmetric particle is stable and may provide a dark matter candidate. However, it is an under-publicized fact that R-parity by itself is not sufficient to stabilize the proton [19–22]. While R-parity does prevent the proton from decaying through the renormalizable operators (3) and (4), it still allows for potentially dangerous dimension five operators such as

$$W_5 = \frac{1}{\Lambda} C^L_{ijkl} Q_i Q_j Q_k L_l + \frac{1}{\Lambda} C^E_{ijkl} U^c_i U^c_j D^c_k E^c_l + \frac{1}{\Lambda} C^N_{ijkl} U^c_i D^c_j D^c_k N^c_l, \tag{5}$$

which violate both lepton number[2] and baryon number. Such operators are generically expected to appear at the cutoff scale Λ. The problem with R-parity is that if, as expected, Λ is on the order of the string scale or the Planck scale and the coefficients C are of order one, the proton would still decay too fast [19–22]. In this sense, R-parity does not provide a complete and satisfactory solution to the proton decay problem.[3] The presence of the additional $U(1)'$ symmetry, however, offers new possibilities for dealing with the dangerous higher-dimensional operators (5). In fact we shall see that under the same three simple assumptions listed above, not only are the renormalizable LV and BV interactions (3) and (4) naturally separated, but also the dangerous nonrenormalizable operators of the type (5) are automatically forbidden. In this sense, in comparison to R-parity, the $U(1)'$ gauge symmetry may provide a more attractive alternative solution to the proton decay problem.

Our work is complementary to a number of studies in the literature which have already considered an extra nonanomalous $U(1)$ gauge symmetry in lieu of R-parity to address the proton stability problem [19–22,24–30].[4] The more recent studies have adopted an even more economical approach, where the $U(1)'$ gauge symmetry is used to simultaneously solve both the μ-problem and the proton stability problem [27–30]. In those works the renormalizable R-parity violating interactions [as well as the nonrenormalizable interactions (5)] are forbidden by the $U(1)'$ symmetry.[5] The price to pay, however, was to allow for a relatively complicated spectrum, including e.g. $SU(2)_L$ exotics [27,28], several pairs of Higgs doublets (N_H) [29] or several singlet representations (N_S) [29,30]. Even though our motivation here was to allow for either LV or BV interactions, we have also analyzed cases where the $U(1)'$ symmetry forbids all RPV operators of lowest dimensions. Such examples are presented in Appendices A

[1] In general, our results also hold in the presence of a certain number of additional pairs of Higgs doublets—see Sec. II B.

[2] The lepton number of N^c is given by -1 in the presence of an $H_2 L N^c$ term in the superpotential, which will be one of our assumptions later on (Sec. II A). Strictly speaking, $W_{\rm LV}$ of Eq. (3) should also contain right-handed neutrino terms such as $N^c N^c$, $N^c N^c N^c$, and $S N^c N^c$ when a lepton number is assigned to N^c.

[3] See, for instance Ref. [23], to see how grand unified theories can help with this problem.

[4] For anomalous $U(1)$ approaches, see, for example, Ref. [31] and references therein.

[5] Previous studies [26] which considered R-parity violating interactions within the $U(1)'$ framework did not address the μ-problem.

and B. First, in Appendix A we consider the novel case of $N_H = 4$, while in Appendix B we treat the case of $N_H = 3$ which was previously discussed in Ref. [29]. We shall show that in both of those cases the nonlinear $U(1)'$ anomaly conditions factorize and *all* anomaly conditions essentially reduce to linear constraints. Furthermore, the case of $N_H = 3$, $N_S = 3$ exhibits an additional simplification: the quadratic and cubic $U(1)'$ anomaly conditions are not independent, and we find a three-parameter class of anomaly-free solutions which generalize the single model found in Ref. [29].

Previous studies found that the additional gauge symmetry usually also requires exotic fields for the cancellation of certain anomalies [27–30]. This tends to ruin the successful gauge coupling unification which is a hallmark of supersymmetry [32].[6] Here we do not require gauge coupling unification, and follow a bottom-up approach by introducing only the minimal set of exotic fields (three vectorlike pairs of colored triplets K_i and K_i^c, see Sec. III A) required for anomaly cancellation. For simplicity, we will also assume family-universal $U(1)'$ charges for all MSSM fields, including the right-handed neutrinos, but will let the exotics have family nonuniversal charges.

Our paper is organized as follows. In Sec. II we describe the general properties of our solutions. For this purpose, we shall only need to use the linear constraints on the $U(1)'$ charges following from the Yukawa-type couplings in the superpotential, plus the $U(1)' - [SU(2)_L]^2$ anomaly condition from Sec. III B. We begin by introducing our formalism and notation in Sec. II A and proceed to derive some of our main results in the remainder of Sec. II. In Sec. II B we explicitly show the LV-BV separation, namely, that the renormalizable LV terms and BV terms cannot coexist: if we allow for the LV terms (3) in the superpotential, then the BV terms (4) are automatically forbidden by the $U(1)'$ gauge symmetry, and vice versa. Then in Sec. II C we extend our discussion to the case of the nonrenormalizable RPV terms such as (5) and show that those are absent as well. In Sec. II D we derive a simple expression for the $U(1)'$ charge of the right-handed neutrino in terms of the $U(1)'$ charges of the other UMSSM fields, and discuss the origin of neutrino masses in our scenario. Finally, in Sec. II E we present the general solution to the linear constraints discussed in Sec. II A and then its specific form for the LV case or the BV case alone. In Sec. III we discuss the remaining constraints on the $U(1)'$ charges arising from the absence of gauge anomalies. We consider the anomaly conditions one at a time and discuss their implications for the model building to follow in the next three sections. In Sec. IV we present our simplest models ($N_H = 1$) with either LV or BV, but not both, types of interactions. We summarize and conclude in Sec. V. In Appendix A (Appendix B) we discuss models with $N_H = 4$ ($N_H = 3$) in which both types of RPV terms are forbidden by the $U(1)'$ symmetry. In Appendix C, we discuss a special case of $N_H = N_S = 1$ with an altered particle spectrum.

[6]Reference [33] considered an UMSSM with family nonuniversal charges which was free of exotics. However, in that case one cannot write down Yukawa couplings for all fermions at tree level, and in Ref. [33] nonholomorphic terms were introduced in order to radiatively generate the problematic Yukawa couplings.

II. GENERAL PROPERTIES OF THE $U(1)'$ MODELS

A. Setup and formalism

In the same spirit as the earlier works [27–30], we consider the $U(1)'$-extended MSSM where both the μ term and the R-parity violating terms in the superpotential are controlled by the $U(1)'$ gauge symmetry. In contrast to previous studies along these lines, we shall not forbid all renormalizable RPV terms from the very beginning. Instead, we shall in principle allow for the presence of either LV or BV terms in the superpotential. We will not be particularly concerned whether the RPV terms (3) and (4) arise at the renormalizable level or through a higher-dimensional operator. In fact, we shall find examples of both types below. We shall then demonstrate that, as a result of the $U(1)'$ symmetry, the proton is nevertheless still sufficiently stable, even at the nonrenormalizable level. Our result is quite general and relies only on our three simple assumptions listed in the introduction.

To set up our discussion, in Table I we list the particles of the UMSSM with their corresponding SM quantum numbers and $U(1)'$ charges. The first column lists the corresponding field, and the next two columns give its representation under $SU(3)_C$ and $SU(2)_L$. The last two columns show the hypercharge $y[F]$ and the $U(1)'$ charge $z[F]$ of a field F. In addition to the MSSM fields Q, U^c, D^c, L, E^c, H_1, and H_2, we also include three right-handed

TABLE I. Chiral fields in the model and their quantum numbers. $z[F]$ denotes the $U(1)'$ charge of a field F. In general, we consider N_H pairs of Higgs doublets H_1 and H_2 with identical quantum numbers, and N_S copies of SM Higgs singlets S.

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q	3	2	$\frac{1}{6}$	$z[Q]$
U^c	$\bar{3}$	1	$-\frac{2}{3}$	$z[U^c]$
D^c	$\bar{3}$	1	$\frac{1}{3}$	$z[D^c]$
L	1	2	$-\frac{1}{2}$	$z[L]$
E^c	1	1	1	$z[E^c]$
N^c	1	1	0	$z[N^c]$
H_2	1	2	$\frac{1}{2}$	$z[H_2]$
H_1	1	2	$-\frac{1}{2}$	$z[H_1]$
S	1	1	0	$z[S]$
K_i	3	1	$y[K_i]$	$z[K_i]$
K_i^c	$\bar{3}$	1	$-y[K_i]$	$z[K_i^c]$

neutrinos N^c. The Higgs singlet S is introduced in order to generate the effective μ term (2), and a successful solution to the μ-problem requires that

$$z[S] = -z[H_1] - z[H_2] \neq 0. \tag{6}$$

In what follows, we shall make repeated use of this equation which is nothing but the second of our three basic assumptions listed in the introduction. In general, we shall consider N_H pairs of Higgs doublets H_1 and H_2 with identical quantum numbers, and N_S SM Higgs singlets of type S. The Abelian gauge symmetry $U(1)'$ is assumed to be broken at the TeV scale where all Higgs fields (S, H_1, and H_2) get VEV's of that order. An effective μ term ($\mu_{\rm eff} = h\langle S\rangle$) is thus dynamically generated at the TeV scale, completing the solution to the μ-problem. This is very similar to the case of the NMSSM, but having the $U(1)'$ gauge symmetry of the UMSSM has the additional advantage of eliminating the domain wall problem associated with the discrete symmetry of the NMSSM [34].[7] As we mentioned earlier, a minimum set of vectorlike colored exotics K_i, K_i^c ($i = 1, 2, 3$) is also required for anomaly cancellation (see Sec. III A). At this point, the hypercharges of the exotics and the $U(1)'$ charges of all fields listed in Table I are yet to be determined.

In the remainder of this section we shall analyze the main properties of our solutions, based on a limited set of linear constraints for the $U(1)'$ charges. The remaining constraints will be analyzed in Sec. III. We shall first list the set of relevant equations, and proceed to analyze them in the subsequent subsections.

In addition to (2), we also require that the $U(1)'$ symmetry allows the usual Yukawa couplings in the superpotential

$$W_{\rm Yukawa} = y^D_{jk} H_1 Q_j D^c_k + y^U_{jk} H_2 Q_j U^c_k + y^E_{jk} H_1 L_j E^c_k + y^N_{jk}\left(\frac{S}{\Lambda}\right)^a H_2 L_j N^c_k. \tag{7}$$

Here capital letters denote the superfields of the MSSM whose quantum numbers are listed in Table I. Because of the observed smallness of the neutrino masses, we have in general allowed neutrino Yukawa couplings to arise from a nonrenormalizable operator suppressed by some high scale Λ [36]. However, in principle we do not exclude the possibility of $a = 0$. We discuss the possible appearance of a Majorana mass term for N^c in Sec. II D. The presence of the Yukawa terms (7) leads to the following constraints:

$$Y_D\colon\ z[H_1] + z[Q] + z[D^c] = 0, \tag{8}$$

$$Y_U\colon\ z[H_2] + z[Q] + z[U^c] = 0, \tag{9}$$

$$Y_E\colon\ z[H_1] + z[L] + z[E^c] = 0, \tag{10}$$

$$Y_N\colon\ z[H_2] + z[L] + z[N^c] + az[S] = 0. \tag{11}$$

We supplement these with Eq. (6) which we write as

$$Y_S\colon\ z[S] + z[H_1] + z[H_2] = 0 \tag{12}$$

and the $U(1)' - [SU(2)_L]^2$ anomaly condition from Sec. III B

$$A_2\colon\ 9z[Q] + 3z[L] + N_H(z[H_1] + z[H_2]) + A_2(\text{exotics}) = 0. \tag{13}$$

The set of six equations (8)–(13) is the starting point for our analysis in the remainder of this section. These six equations exactly correspond to our three basic assumptions listed in the introduction: the existence of the Yukawa terms (7) is guaranteed by Eqs. (8)–(11), the solution to the μ-problem is implied by Eq. (12), and the absence of $SU(2)_L$ exotics among our particle content in Table I simply means that there is no additional contribution to the $U(1)' - [SU(2)_L]^2$ anomaly and $A_2(\text{exotics}) = 0$ in Eq. (13).

B. LV-BV separation

Starting with Eqs. (8)–(13) and taking the linear combination $6Y_D + 3Y_U - 3Y_E + (N_H - 3)Y_S - A_2$ gives the following constraint among the $U(1)'$ charges

$$3(z[U^c] + 2z[D^c]) - 3(2z[L] + z[E^c]) + (N_H - 3)z[S] - A_2(\text{exotics}) = 0. \tag{14}$$

We find this equation particularly useful both in illustrating one of our main points, as well as in categorizing the existing $U(1)'$ models in the literature. Each term in Eq. (14) corresponds to a particular physical situation:

(1) The first term in Eq. (14) represents the baryon number violating interactions of Eq. (4). If this term is zero, BV interactions will be present in the model. In order to forbid (4), one must have $z[U^c] + 2z[D^c] \neq 0$, which would require at least one of the remaining three terms in Eq. (14) to be nonvanishing as well.

(2) The second term in Eq. (14) represents the lepton number violating interactions of Eq. (3). If this term is zero, LV interactions will be present in the model. In order to forbid (3), one must have $2z[L] + z[E^c] \neq 0$, which would require at least one of the remaining three terms in Eq. (14) to be nonvanishing as well.

(3) The third term in Eq. (14) simply counts the number N_H of Higgs doublet pairs in the model. This term would vanish only if $N_H = 3$, since the solution to the μ-problem requires $z[S] \neq 0$ [see Eq. (6)].

(4) The fourth term $A_2(\text{exotics})$ represents the contribution to the $U(1)' - [SU(2)_L]^2$ anomaly from states not listed in Table I. It is a model-builder's choice whether this term is present or not.

[7] In addition, quantum gravity effects may violate a global symmetry unless it is a remnant of a gauge symmetry [35].

Equation (14) allows us to categorize the existing $U(1)'$ models according to how many and which of these four terms are nonvanishing. For example, Ref. [29] forbids all renormalizable RPV terms, hence the first two terms in Eq. (14) are both nonzero. In fact, they cancel each other, since Ref. [29] assumes three pairs of Higgs doublets ($N_H = 3$) and no $SU(2)_L$ exotics, so that the last two terms in Eq. (14) are zero. On the other hand, the models of Refs. [27,28] illustrate the case where all four terms in Eq. (14) are nonvanishing: those models also forbid RPV interactions, but contain $SU(2)_L$ exotics and have $N_H \neq 3$. Finally, the models of Ref. [30] have $N_H = 1$ and no $SU(2)_L$ exotic representations, so they illustrate the intermediate case where three terms in Eq. (14) are nonvanishing.

According to our third basic assumption (see introduction), our approach will be to assume that there are no $SU(2)_L$ exotic representations so that $A_2(\text{exotics}) = 0$, in which case Eq. (14) becomes

$$3(z[U^c] + 2z[D^c]) - 3(2z[L] + z[E^c]) + (N_H - 3)z[S] = 0. \tag{15}$$

We shall be mostly interested in cases with $N_H \neq 3$, so that the third term in Eq. (15) is nonzero. For simplicity, we shall concentrate on $N_H = 1$ in Sec. IV (the case of $N_H = 4$ is treated in Appendix A). Under those circumstances, Eq. (15) reveals that, at least at the renormalizable level, the LV terms (3) and the BV terms (4) cannot coexist [i.e. the first two terms in Eq. (15) cannot vanish simultaneously], since we need at least one of them to cancel the nonvanishing third term proportional to $z[S]$. We refer to this mutual exclusion as the "LV-BV separation." The proton is then safe from decaying through renormalizable RPV interactions, even though R-parity is not present in the model. Furthermore, one does not need *both* of the first two terms in Eq. (15) in order to cancel the third one—only one of the first two terms will suffice. Therefore we are free to consider models where either the first or the second term in Eq. (15) is zero and the corresponding RPV interactions are allowed. For example, in the LV case, where $2z[L] + z[E^c] = 0$, Eq. (15) gives

$$z[U^c] + 2z[D^c] = \left(1 - \frac{N_H}{3}\right)z[S] \neq 0 \tag{16}$$

and the BV interactions (4) are not allowed. Similarly, in the BV case, where $z[U^c] + 2z[D^c] = 0$, Eq. (15) gives

$$2z[L] + z[E^c] = -\left(1 - \frac{N_H}{3}\right)z[S] \neq 0 \tag{17}$$

and the LV terms (3) are not allowed. It is straightforward to see that the LV-BV separation also holds if the corresponding LV and BV terms arise at the nonrenormalizable level—in that case, there are extra contributions to the right-hand side of Eqs. (16) and (17) which are integer multiples of $z[S]$, so that our argument still applies as long as $N_H = 1$.

C. Higher-dimensional operators and proton decay

As we already mentioned in the introduction, R-parity allows for potentially dangerous higher-dimensional operators like (5) which may still destabilize the proton. The new $U(1)'$ gauge symmetry can now be used to eliminate those as well [27–30]. It is interesting to note that simply by making use of Eqs. (8)–(13), and without specifying the further details of the model, we can readily compute the $U(1)'$ charge of any such operator and test whether it is allowed or not. For example, the linear combination $Y_D + 2Y_U + Y_E + (\frac{N_H}{3} - 2)Y_S - \frac{1}{3}A_2$ leads to

$$2z[U^c] + z[D^c] + z[E^c] + \left(\frac{N_H}{3} - 2\right)z[S] = 0, \tag{18}$$

which allows us to determine the $U(1)'$ charge of the $U^cU^cD^cE^c$ operator as

$$U^cU^cD^cE^c\colon\ 2z[U^c] + z[D^c] + z[E^c] = \left(2 - \frac{N_H}{3}\right)z[S]. \tag{19}$$

Since the solution to the μ-problem already implies $z[S] \neq 0$ [see Eq. (6)], this operator is forbidden, unless one allows for exactly six pairs of Higgs doublets in the model. Similarly, the operator $QQQL$ is also absent, since its charge can be obtained from the linear combination $\frac{1}{3}A_2 - \frac{N_H}{3}Y_S$:

$$QQQL\colon\ 3z[Q] + z[L] = \frac{N_H}{3}z[S]. \tag{20}$$

Because of Eq. (6), again it is clear that the $U(1)'$ symmetry does not allow this operator, since we already have at least one pair of Higgs doublets as in the MSSM. Finally, one can obtain the $U(1)'$ charge of the operator $U^cD^cD^cN^c$ from the combination $2Y_D + Y_U + Y_N + (\frac{N_H}{3} - 2)Y_S - \frac{1}{3}A_2$ as

$$U^cD^cD^cN^c\colon\ z[U^c] + 2z[D^c] + z[N^c] = \left(2 - a - \frac{N_H}{3}\right)z[S]. \tag{21}$$

Since a is an integer, we see that, in general, as long as the number N_H of Higgs doublet pairs is not divisible by three, this operator is also forbidden. Even when N_H is divisible by three, there will be only one special value of the integer a, namely, $a = 2 - \frac{N_H}{3}$, which would allow the existence of this operator. Since a must be positive, there are only two special cases that one should be worried about: ($N_H = 3, a = 1$) and ($N_H = 6, a = 0$). The case $N_H = 6$ is already disfavored by (19), while in the case $N_H = 3$ which we study in Appendix B, we shall consider only the case $a = 0$ as in Ref. [29].

To summarize, so far we have shown that in the simplest cases such as $N_H = 1, 2, 4, \cdots$ the conditions (8)–(13) are sufficient to rule out the dangerous dimension 5 operators (5) which simultaneously violate baryon and lepton number. This is already an important advantage of our models compared to the usual R-parity conserving scenario. However, since in our approach we are allowing some of the dimension 4 LV or BV interactions, we should also check for potentially dangerous *pairs* of dimension 4 and dimension 5 operators, which may in general arise from either F terms or D terms. For the case of the MSSM, the problematic combinations were identified in Ref. [37] as

$$\begin{aligned} &\{LQD^c, QQQH_1\}, \qquad \{U^cD^cD^c, QU^cE^cH_1\}, \\ &\{U^cD^cD^c, U^cD^{c\dagger}E^c\}, \qquad \{U^cD^cD^c, QU^cL^\dagger\}. \end{aligned} \tag{22}$$

Using Eqs. (8)–(13), it is easy to derive the following relations between the $U(1)'$ charges of the operators in each pair:

$$(z[L] + z[Q] + z[D^c]) + (3z[Q] + z[H_1]) = \frac{N_H}{3} z[S], \tag{23}$$

$$\begin{aligned} &(z[U^c] + 2z[D^c]) + (z[Q] + z[U^c] + z[E^c] + z[H_1]) \\ &= \left(2 - \frac{N_H}{3}\right) z[S], \end{aligned} \tag{24}$$

$$\begin{aligned} &(z[U^c] + 2z[D^c]) + (z[U^c] - z[D^c] + z[E^c]) \\ &= \left(2 - \frac{N_H}{3}\right) z[S], \end{aligned} \tag{25}$$

$$\begin{aligned} &(z[U^c] + 2z[D^c]) + (z[Q] + z[U^c] - z[L]) \\ &= \left(2 - \frac{N_H}{3}\right) z[S]. \end{aligned} \tag{26}$$

We see that all of the dangerous pairs of operators are forbidden by the $U(1)'$ symmetry, due to the condition (6). [The case $N_H = 6$ would in principle allow the last three pairs, but $N_H = 6$ was already disfavored by Eq. (19) and we shall not be considering it any further.]

So far we have shown that the proton is not destabilized by the potentially dangerous pairs of operators constructed out of MSSM fields only. Since our models have additional fields present (N^c, S, K_i, and K_i^c) beyond those of the MSSM, we still need to check that those extra fields do not give rise to dangerous pairs of operators analogous to (22). We systematically checked all relevant combinations of dimension 4 and/or dimension 5 operators involving N^c and S in addition to the usual MSSM fields, and verified that all combinations which violate lepton number and baryon number are forbidden by the $U(1)'$ symmetry when $z[S] \neq 0$, and $\frac{N_H}{3}$ is not an integer.[8]

It remains to discuss the effect of the colored exotics K, K^c on proton decay. Since they are heavy, they cannot appear among the proton decay products. However, they may still mediate proton decay. It is more difficult to see that the proton is safe from such processes because the $U(1)'$ charges and hypercharges of the colored exotics are not determined by Eqs. (8)–(13). One possible approach would be to choose the exotic hypercharges so that the lowest dimension operators coupling exotic quarks to the MSSM fields are absent [25,29]. Here we shall consider a more general setup, where the hypercharges of the colored exotics in principle may allow couplings to the MSSM fields (see Sec. III C). The proof of proton stability in that case will be presented in a separate publication [38] where we will discuss the discrete gauge symmetries [37,39] encoded in our models.

D. Majorana neutrino masses

Recent experiments show that the active neutrinos have masses. There are different possibilities regarding the origin of neutrino masses: e.g. Dirac neutrino masses may arise from the SM Higgs mechanism, and their smallness can be naturally explained through a seesaw mechanism with heavy right-handed Majorana neutrinos [40]. Other possibilities invoke extra dimensions [41] or higher-dimensional operators [42]. Since we allow for a neutrino Yukawa coupling [see Eq. (7)], our models can readily accommodate Dirac type neutrinos. In this subsection we investigate whether in addition to the neutrino Yukawa coupling, one could write down a Majorana term for the right-handed neutrinos, so that we can have some kind of a seesaw mechanism as well.

Taking the linear combination $Y_E + Y_N - Y_S$ allows us to express the $U(1)'$ charge of the right-handed neutrinos N^c as

$$\begin{aligned} z[N^c] &= -(2z[L] + z[E^c]) + (1 - a)z[S] \\ &= \begin{cases} (1 - a)z[S] & \text{(LV case);} \\ (2 - a - \frac{N_H}{3})z[S] & \text{(BV case).} \end{cases} \end{aligned} \tag{27}$$

We see that in the BV case, lepton number violating terms involving the N^c field (e.g. N^cN^c, $N^cN^cN^c$, and SN^cN^c) cannot be generated, unless $\frac{N_H}{3}$ is an integer. Therefore when $N_H \neq 3, 6, \cdots$, the LV-BV separation holds even

[8]This statement is strictly true in the LV case. In the BV case the only potentially troublesome pair of operators is $U^cD^cD^c$ and $N^cN^cN^cS$. The latter has $U(1)'$ charge $(7 - 3a - N_H)z[S]$ [see Eq. (30)] and is in principle allowed for the following three choices: $\{a, N_H\} = \{0, 7\}, \{1, 4\}, \{2, 1\}$. However, neither of these three options is a viable one: $N_H = 7$ is incompatible with the A_3 anomaly [see Eq. (42) below]; $a = 1$, $N_H = 4$ is inconsistent with the A_4 anomaly (see Appendix A); while $a = 2$ would imply too small neutrino masses.

in the presence of N^c fields with lepton number -1. While the BV case can then have only Dirac neutrino mass terms, the LV case may in general allow a Majorana neutrino mass term N^cN^c whenever $a = 1$. However, in the LV case the SN^cN^c term has a $U(1)'$ charge of $(3-2a)z[S]$ and is not allowed. The active neutrinos of the LV case may also get their masses without the RH neutrinos through f-$\tilde{f}$ loops involving the λ and λ' couplings, or through ν-$\tilde{H}_2^0$ mixing due to the $\mu'_{\rm eff}LH_2$ term in Eq. (3) [16].

E. General solution to the Yukawa constraints and the A_2 anomaly

In this subsection we present the general solution to the constraints (8)–(13) and then specify its particular form separately for the LV case and the BV case.

Since (8)–(13) are six constraints for nine variables, we find a three-parameter solution as

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \end{pmatrix} = \frac{\ell}{3}\begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \\ -3 \\ -3 \\ 0 \\ 0 \\ 0 \end{pmatrix} + h_1\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \frac{s}{9}\begin{pmatrix} N_H \\ 9-N_H \\ -N_H \\ 0 \\ 0 \\ 9(1-a) \\ -9 \\ 0 \\ 9 \end{pmatrix}, \quad (28)$$

where ℓ, h_1, and s are arbitrary coefficients. The notation for those is suggestive of their interpretation: $\ell = z[L]$, $h_1 = z[H_1]$, and $s = z[S]$.

In the LV case, we have an additional constraint, e.g. $2z[L] + z[E^c] = 0$, which implies the relation $h_1 = \ell$ and the solution (28) becomes

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \end{pmatrix} = 2\ell\begin{pmatrix} -\frac{1}{6} \\ \frac{2}{3} \\ -\frac{1}{3} \\ \frac{1}{2} \\ -1 \\ 0 \\ -\frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} + \frac{s}{9}\begin{pmatrix} N_H \\ 9-N_H \\ -N_H \\ 0 \\ 0 \\ 9(1-a) \\ -9 \\ 0 \\ 9 \end{pmatrix}. \quad (29)$$

Not surprisingly, we recognize in the first column vector on the right-hand side the hypercharge assignments of the UMSSM fields from Table I. Indeed, the constraints (8)–(12) arise from gauge-invariant operators, so clearly they will be satisfied by the hypercharges of the UMSSM fields. What is more important at this point is the additional remaining degree of freedom represented by the second term in the right-hand side of Eq. (29), which will allow us to find nontrivial solutions for the $U(1)'$ charges, different from the usual hypercharge.

In the BV case, the corresponding additional constraint $2z[U^c] + z[D^c] = 0$ implies $h_1 = \ell + (1 - \frac{N_H}{3})s$ and the solution (28) can be written as

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \end{pmatrix} = \left(2\ell - \frac{2}{3}N_H s\right)\begin{pmatrix} -\frac{1}{6} \\ \frac{2}{3} \\ -\frac{1}{3} \\ \frac{1}{2} \\ -1 \\ 0 \\ -\frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} + \frac{s}{3}\begin{pmatrix} 0 \\ 6 \\ -3 \\ N_H \\ -3-N_H \\ 6-N_H-3a \\ -6 \\ 3 \\ 3 \end{pmatrix}. \quad (30)$$

Just as in the LV case (29), the usual hypercharges appear as a particular solution to the constraints (8)–(13), but there is an additional class of solutions with nonzero $z[S]$, so that in general our solutions will be a linear combination of these two classes.

III. ANOMALIES

Table II summarizes the anomaly cancellation conditions for the $U(1)'$ charges of the fields in our model. In this section, we investigate these anomaly cancellation conditions one by one and discuss their implications for model building.

A. Anomaly A_1 ($U(1)' - [SU(3)_C]^2$)

We begin with the mixed $U(1)' - [SU(3)_C]^2$ anomaly which we denote with A_1. First we rederive the well-known result that the presence of the Yukawa couplings in the superpotential (8)–(12) requires exotic representations be-

TABLE II. Anomaly cancellation conditions for the $U(1)'$ charges of the particles in our model listed in Table I. The first column lists a shorthand identifier for each condition, which will be used throughout the text.

Identifier	Anomaly	Equation
A_1	$U(1)'$-$[SU(3)_C]^2$	$\mathrm{tr}[zt^at^b] = \frac{1}{4}\delta^{ab}\sum_q z = 0$ (color triplet fermions only)
A_2	$U(1)'$-$[SU(2)_L]^2$	$\mathrm{tr}[z\tau^a\tau^b] = \frac{1}{2}\delta^{ab}\sum_{f_L} z = 0$ (doublet fermions only)
A_3	$U(1)'$-$[U(1)_Y]^2$	$\mathrm{tr}[zy^2] = \sum_f zy^2 = 0$
A_4	$U(1)_Y$-$[U(1)']^2$	$\mathrm{tr}[yz^2] = \sum_f yz^2 = 0$
A_5	$[U(1)']^3$	$\mathrm{tr}[z^3] = \sum_f z^3 = 0$
A_6	$U(1)'$-$[\text{gravity}]^2$	$\mathrm{tr}[z] = \sum_f z = 0$

yond those of the MSSM. Denoting the contribution of such exotics to the $U(1)' - [SU(3)_C]^2$ anomaly by $A_1(\text{exotics})$, we can write A_1 as

$$A_1:\ 3(2z[Q] + z[U^c] + z[D^c]) + A_1(\text{exotics}) = 0. \quad (31)$$

The first term is the contribution of the three generations of quarks in the MSSM, while the second term is the potential colored exotics contribution. Now taking the linear combination $A_1 - 3Y_U - 3Y_D + 3Y_S$, we get

$$A_1(\text{exotics}) = -3z[S], \quad (32)$$

which, in light of Eq. (6), shows the need for colored exotic representations [27–30].

In this paper, we shall assume that the exotics are N_K vectorlike pairs of chiral fields K_i and K_i^c so that they do not alter the anomaly cancellation conditions among the SM gauge groups. More specifically, we assume that they are triplets and antitriplets of $SU(3)_C$ with equal and opposite $U(1)_Y$ hypercharges $\pm y[K_i]$ (see Table I). Perhaps most importantly, as already mentioned earlier in the introduction, we are assuming that the exotics which are needed to cancel the A_1 anomaly are $SU(2)_L$ singlets, so that $A_2(\text{exotics}) = 0$. With those assumptions, Eq. (31) becomes

$$A_1':\ 3(2z[Q] + z[U^c] + z[D^c]) + \sum_{i=1}^{N_K}(z[K_i] + z[K_i^c]) = 0. \quad (33)$$

In order to avoid conflict with experiment, the exotic quarks K_i and K_i^c must be sufficiently heavy [43]. If their masses arise from an ordinary mass term KK^c in the superpotential, then $U(1)'$ invariance implies $A_1(\text{exotics}) = \sum_{i=1}^{N_K}(z[K_i] + z[K_i^c]) = 0$ and the μ-problem cannot be solved because of the conflicting requirements of Eqs. (6) and (32). We therefore choose to generate masses for *all* colored exotics at the TeV scale, through superpotential couplings to the S field:

$$W_{\text{exotics}} = h''_{ij} S K_i K_j^c. \quad (34)$$

Assuming that the couplings in Eq. (34) are diagonal, we get the following constraint among the $U(1)'$ charges of the exotics

$$Y_{K_i}:\ z[S] + z[K_i] + z[K_i^c] = 0. \quad (35)$$

Since $z[S] \neq 0$, this equation reveals that K and K^c do *not* carry equal and opposite $U(1)'$ charges, even though their hypercharges are equal and opposite ($y[K_i] + y[K_i^c] = 0$).

Now taking the linear combination $A_1' - 3Y_U - 3Y_D + 3Y_S - \sum_{i=1}^{N_K} Y_{K_i}$ gives

$$(3 - N_K)z[S] = 0. \quad (36)$$

Combined with Eq. (6), this determines the number of exotic families as

$$N_K = 3. \quad (37)$$

Notice that the A_1 anomaly did not impose any constraints on the $U(1)'$ charges themselves, but simply fixed the number of allowed representations in the model. We shall see that the same phenomenon will take place when we consider some of the other anomaly conditions below. In the end, this will leave us with sufficient freedom to find sets of $U(1)'$ charges which will satisfy all of our model requirements.

B. Anomaly A_2 ($U(1)' - [SU(2)_L]^2$)

This anomaly condition was already introduced as Eq. (13) in Sec. II A. With our assumption that all exotics in the model are $SU(2)_L$ singlets, it becomes

$$A_2:\ 9z[Q] + 3z[L] + N_H(z[H_1] + z[H_2]) = 0. \quad (38)$$

C. Anomaly A_3 ($U(1)' - [U(1)_Y]^2$)

In general, the A_3 anomaly condition is given by

$$\begin{aligned} &9(2z[Q]y[Q]^2 + z[U^c]y[U^c]^2 + z[D^c]y[D^c]^2) \\ &\quad + 3(2z[L]y[L]^2 + z[E^c]y[E^c]^2) + 3\sum_{i=1}^{N_K}(z[K_i]y[K_i]^2 \\ &\quad + z[K_i^c]y[K_i^c]^2) + N_H(2z[H_1]y[H_1]^2 \\ &\quad + 2z[H_2]y[H_2]^2) = 0 \end{aligned} \quad (39)$$

where $y[F]$ is the $U(1)_Y$ hypercharge of a field F as given in

Table I, and we have omitted terms involving fields with vanishing hypercharge (N^c and S). Substituting the known hypercharges from Table I and using (35), we can rewrite it as

$$A_3:\ z[Q] + 8z[U^c] + 2z[D^c] + 3z[L] + 6z[E^c] + N_H(z[H_1] + z[H_2]) - 6z[S]\sum_{i=1}^{N_K} y[K_i]^2 = 0. \quad (40)$$

Now taking the linear combination $A_3 + A_2 - 8Y_U - 2Y_D - 6Y_E + (8 - 2N_H)Y_S$ leads to the following simple constraint

$$\left(4 - N_H - 3\sum_{i=1}^{N_K} y[K_i]^2\right) z[S] = 0. \quad (41)$$

Because of condition (6), this uniquely reduces to

$$\sum_{i=1}^{N_K} y[K_i]^2 = \frac{1}{3}(4 - N_H), \quad (42)$$

where the hypercharges are normalized as in Table I. We see that, just as was the case for A_1, the anomaly cancellation condition A_3 did not provide an additional constraint on the $U(1)'$ charges, but instead only limits the number of Higgs doublet pairs N_H and the choice for exotic hypercharges $y[K_i]$. Since the left-hand side of Eq. (42) must be positive-definite and N_H is an integer, there are only four possible choices for the number of Higgs doublet pairs: $N_H = 1$, 2, 3 or 4, that we need to consider. The case of $N_H = 3$ was already considered in Ref. [29] and we shall revisit it again in Appendix B. We shall also consider the case of $N_H = 4$ in Appendix A. Our main interest, however, will be in the minimal case of $N_H = 1$, which will be discussed below in Sec. IV.

Having fixed the number of Higgs doublet pairs N_H, Eq. (42) provides a guideline for choosing the hypercharges of the colored exotics. Since the A_1 anomaly already required $N_K = 3$ (see Sec. III A), it is clear that a family-universal choice with rational numbers is only possible for $N_H = 3$, with $y[K_i] = \pm\frac{1}{3}$, or for $N_H = 4$, with $y[K_i] = 0$. In the case of $N_H = 1$ or $N_H = 2$, one would have to choose exotic hypercharges in a family nonuniversal way. In general, there are many possible choices, but here we shall limit ourselves to those where the exotic hypercharges are the same (up to a sign) as the hypercharges of the corresponding $SU(2)_L$ singlet, color triplet representations in the MSSM (U^c and D^c):

$$N_H = 1 \Rightarrow y[K_i] = \{\pm\tfrac{1}{3}, \mp\tfrac{2}{3}, \mp\tfrac{2}{3}\}, \quad (43)$$

$$N_H = 2 \Rightarrow y[K_i] = \{\pm\tfrac{1}{3}, \pm\tfrac{1}{3}, \mp\tfrac{2}{3}\}, \quad (44)$$

$$N_H = 3 \Rightarrow y[K_i] = \{\pm\tfrac{1}{3}, \pm\tfrac{1}{3}, \pm\tfrac{1}{3}\}. \quad (45)$$

All three choices (43)–(45) satisfy the A_3 anomaly condition (42). The signs of the exotic hypercharges could be in general chosen arbitrarily. We have limited ourselves to two cases—with the upper signs in Eqs. (43)–(45) the exotics have the wrong quantum numbers to couple to the MSSM quarks and mediate proton decay. In that case, however, the lightest exotic would be stable and may pose problems for cosmology. This could be avoided, e.g. if the reheating temperature is very low, $T_{\rm RH} \lesssim 100$ GeV, which may still be compatible with baryogenesis [44]. On the other hand, choosing the lower signs in Eqs. (43)–(45) allows the exotics to couple to the MSSM quarks, thus avoiding problems with cosmology. Nevertheless, as we already discussed in Sec. II C, in that case the $U(1)'$ symmetry is sufficient to stabilize the proton. We shall therefore allow for both sets of signs for the exotic hypercharges in Eqs. (43)–(45).

D. Anomaly A_6 ($U(1)' - [\text{gravity}]^2$)

The gravitational anomaly $U(1)' - [\text{gravity}]^2$ is given as

$$A_6:\ 9(2z[Q] + z[U^c] + z[D^c]) + 3(2z[L] + z[E^c] + z[N^c]) + 2N_H(z[H_1] + z[H_2]) + N_S z[S] + 3\sum_{i=1}^{N_K}(z[K_i] + z[K_i^c]) = 0, \quad (46)$$

where N_S is the number of Higgs singlets S in the model. Taking the linear combination $A_6 - 9Y_U - 9Y_D - 3Y_E - 3Y_N + (12 - 2N_H)Y_S - 3\sum_{i=1}^{N_K} Y_{K_i}$, we get

$$(N_S - 2N_H - 3a + 3)z[S] = 0. \quad (47)$$

Because of Eq. (6), this implies

$$N_S = 2N_H + 3a - 3. \quad (48)$$

Once again, the anomaly condition did not constrain the $U(1)'$ charges, but just the number of representations. The simplest possibility appears to be $N_H = 1$, $a = 1$, $N_S = 2$, and this is the case we shall investigate in Sec. IV. Another example discussed in Appendix A is $N_H = 4$, $a = 1$, and $N_S = 8$. Finally, $N_H = 3$, $a = 0$, and $N_S = 3$ is the case considered in Ref. [29] and below in Appendix B. We see that Eq. (48) excludes the minimal (in the sense of total number $N_H + N_S$ of Higgs representations) possibility of $N_H = N_S = 1$ in our current setup.[9] However, this conclusion can be avoided with the addition of extra SM singlet exotic fields. Appendix C provides a specific example of such a model with $N_H = N_S = 1$.

E. The Anomalies A_4 ($U(1)_Y - [U(1)']^2$) and A_5 ($[U(1)']^3$)

The remaining anomaly conditions A_4 and A_5 are in general nonlinear equations for the $U(1)'$ charges:

[9]The gravitational anomaly A_6 was not taken into account in Ref. [30], which allowed building a model with $N_H = N_S = 1$.

$$A_4\colon\ 9(2y[Q]z[Q]^2 + y[U^c]z[U^c]^2 + y[D^c]z[D^c]^2) + 3(2y[L]z[L]^2 + y[E^c]z[E^c]^2) + 2N_H(y[H_1]z[H_1]^2 + y[H_2]z[H_2]^2) + 3\sum_{i=1}^{N_K}(y[K_i]z[K_i]^2 + y[K_i^c]z[K_i^c]^2) = 0, \tag{49}$$

$$A_5\colon\ 9(2z[Q]^3 + z[U^c]^3 + z[D^c]^3) + 3(2z[L]^3 + z[E^c]^3 + z[N^c]^3) + 2N_H(z[H_1]^3 + z[H_2]^3) + N_S z[S]^3 + 3\sum_{i=1}^{N_K}(z[K_i]^3 + z[K_i^c]^3) = 0. \tag{50}$$

Because of their nonlinearity, in the past A_4 and A_5 have typically been the stumbling blocks for finding anomaly-free solutions for the $U(1)'$ charges. Here we shall show, however, that under our previous assumptions (8)–(13), *both* of these equations factorize—each one is in fact proportional to $z[S]$ [which according to Eq. (6) is nonzero] so effectively we are able to reduce the power of Eq. (49) and (50) by one.[10] For example, the A_4 anomaly reduces to a *linear* constraint among the $U(1)'$ charges. The easiest way to see this is to substitute the general solution (28) into Eq. (49), which gives

$$\frac{1}{3}s\left\{(12N_H - 36)h_1 + (7N_H - 18)s - 12\ell - 9\sum_{i=1}^{N_K}y[K_i](s + 2z[K_i])\right\} = 0. \tag{51}$$

Since $s \neq 0$, the expression within the curly brackets must vanish, which allows us to solve e.g. for one of the exotic charges $z[K_i]$ in terms of the other two as well as s, h_1, and ℓ.

Similarly, substituting the general solution (28) into Eq. (50), and using Eq. (48), we get

$$s\Big\{-3[(3a + 4N_H - 12)h_1^2 - 6ah_1\ell + (3a - 4)\ell^2] + [3(3a^2 - 6a - 4N_H + 12)h_1 - (9a^2 - 18a + 2N_H)\ell]s - \frac{1}{3}[9a^3 - 27a^2 + 18a - N_H^2 + 9N_H]s^2 - 9\sum_{i=1}^{N_K}z[K_i](s + z[K_i])\Big\} = 0. \tag{52}$$

Once again, since $s \neq 0$, the expression within the curly brackets must vanish, which translates into only a *quadratic* constraint on the $U(1)'$ charges.

As we shall see later in Appendix B, a further drastic simplification of the above formulas (51) and (52) occurs for the case of $N_H = 3$, $a = 0$, and $y[K_i] = \pm\frac{1}{3}$, when the cubic anomaly completely factorizes, and effectively reduces to a linear constraint. Furthermore, this linear constraint turns out to be equivalent to the constraint implied by Eq. (51), so that in effect the cubic anomaly condition is automatically satisfied and in that case does not constrain the $U(1)'$ charges at all.

This completes our discussion of the anomaly cancellation conditions involving the new $U(1)'$. To recapitulate, in Sec. II we first considered the effect of the six constraints (8)–(13) on the $U(1)'$ charges of the nine nonexotic fields in our model (see Table I). This resulted in the general three-parameter solution given by Eq. (28). Then in Sec. III, we studied the remaining[11] five anomaly cancellation conditions A_1, A_3, A_4, A_5, and A_6, which involved three additional variables—the $U(1)'$ charges $z[K_i]$ of the exotic fields K_i. We found that only two out of these five new conditions actually restrict the values of the $U(1)'$ charges, so that there is still a lot of freedom remaining in the actual $U(1)'$ charge assignments. In the following we shall demonstrate this explicitly by presenting specific examples of anomaly-free charge assignments which satisfy all of the model-building constraints considered so far. In Sec. IV we shall find, as anticipated, that there exist solutions which allow for either LV or BV, but not both. Nevertheless, the proton will be stable in such models, as already discussed in Sec. II C, and the μ-problem will be solved by Eq. (6).

IV. MODELS WITH LEPTON OR BARYON NUMBER VIOLATION

In this section we shall concentrate on the simplest case of $N_H = 1$. In addition to the usual MSSM fields, the model also contains $N_S = 2$ Higgs singlets S_i and $N_K = 3$ vectorlike pairs (K_i, K_i^c) of exotic quarks introduced to cancel the A_1 anomaly (see Sec. III A). The R-parity conserving part of the superpotential is given by the combination of Eqs. (2), (7), and (34):

$$W_{\rm RPC} = y_{jk}^D H_1 Q_j D_k^c + y_{jk}^U H_2 Q_j U_k^c + y_{jk}^E H_1 L_j E_k^c + y_{ijk}^N \frac{S_i}{\Lambda} H_2 L_j N_k^c + h_i S_i H_2 H_1 + h''_{ijk} S_i K_j K_k^c. \tag{53}$$

[10] The factorization of the A_4 and A_5 anomalies has been previously noticed in Ref. [30] for the specific case of $N_H = 1$, $a = 0$ and a particular set of exotics.

[11] Recall that A_2 was already accounted for in Sec. II.

Recall that with $N_H = 1$ and $N_S = 2$, the A_6 anomaly condition (48) demands $a = 1$, so that the neutrino Yukawa couplings arise from a nonrenormalizable operator as shown. We assume diagonal couplings of the exotics to S (i.e. $z[K_i^c] = -z[K_i] - z[S]$) but off-diagonal terms may also exist if two or more exotic quarks have identical $U(1)'$ charges. As discussed in the introduction, the μ-problem is solved through an effective μ term $\mu_{\rm eff} = h_1\langle S_1\rangle + h_2\langle S_2\rangle$ by requiring $z[S] \neq 0$. This forbids not only the original μ term (1), but also mass terms for the exotics (KK^c) and Higgs singlet self-couplings S, S^2, and S^3.

The R-parity violating part of the renormalizable superpotential of the UMSSM is

$$W_{\rm RPV} = W_{\rm LV} + W_{\rm BV}, \tag{54}$$

where

$$W_{\rm LV} = \lambda_{ijk}L_iL_jE_k^c + \lambda'_{ijk}L_iQ_jD_k^c + h'_{ij}S_iH_2L_j, \tag{55}$$

$$W_{\rm BV} = \lambda''_{ijk}U_i^cD_j^cD_k^c. \tag{56}$$

It is easy to see that the $U(1)'$ symmetry either simultaneously allows all three terms $\{LLE^c, LQD^c, SH_2L\}$, in which case $z[L] = z[H_1]$, or simultaneously forbids all three. In the LV case, therefore, we shall expect to have all three terms appearing in Eq. (55) present.

A comment is in order regarding the possibility of a bare LV $\mu' H_2L_i$ term in the superpotential. Such a term is dangerous because it will reintroduce a hierarchy problem (μ'-problem) of the type we originally intended to avoid. Indeed, the general solution (28) in principle allows for this term. However, it is easy to see that in both the LV case and the BV case we are interested in, this term is absent and the μ'-problem is solved in exactly the same way as the μ-problem. For example, in the LV case the $U(1)'$ charge of H_2L_i from Eq. (29) is $z[H_2L_i] = z[S]$ which is not vanishing because of condition (6). In the BV case, from Eq. (30) we get $z[H_2L_i] = (N_H - 6)z[S]/3$. Since the case of $N_H = 6$ was already discarded (see Sec. II C), the H_2L_i is again forbidden by the $U(1)'$ symmetry. An effective μ' term will be nevertheless generated from the $S_iH_2L_j$ term in $W_{\rm LV}$, once the $U(1)'$ symmetry is broken by the VEV of S at the TeV scale.

The $U(1)'$ symmetry is broken when S gets a VEV $\langle S\rangle$ at the TeV scale. This generates the corresponding effective bilinear terms in the superpotential with coefficients

$$\mu_{\rm eff} \equiv h_i\langle S_i\rangle, \qquad \mu'_{i,\rm eff} \equiv h'_{ji}\langle S_j\rangle, \qquad m_{K,ij} \equiv h''_{kij}\langle S_k\rangle. \tag{57}$$

With the natural size of the couplings $\{h, h', h''\} \sim 1$, the effective μ and μ' parameters as well as the masses of the exotic quarks m_K are all of order a TeV. With the effective bilinear terms, the superpotential of the UMSSM becomes similar to that of the MSSM. First, the model predicts a new gauge boson, Z', near the $U(1)'$ symmetry breaking scale:

$$M_{Z'}^2 = g_{Z'}^2(z[H_1]^2v_1^2 + z[H_2]^2v_2^2 + z[S]^2v_{s1}^2 + z[S]^2v_{s2}^2). \tag{58}$$

Here, $g_{Z'}$ is the $U(1)'$ gauge coupling constant, $v_i = \sqrt{2}\langle H_i\rangle$ (with $v_1^2 + v_2^2 \simeq 246^2$ GeV^2), and $v_{si} = \sqrt{2}\langle S_i\rangle$. The direct constraint on the mass of the Z' comes from searches at the Tevatron in the dilepton channel ($Z' \to \ell^+\ell^-$). The typical bound is $M_{Z'} > 600 \sim 900$ GeV, depending on the $U(1)'$ charges of the quarks and leptons [45]. The VEV's of the Higgs doublets will also induce mixing between the Z and Z' gauge bosons. If the Z' is sufficiently heavy, this mixing is quite small, in accordance with the experimental constraints from LEP (per mil level) [46]. The supersymmetric partners of the Z' and S (Z'-ino and singlino) become extra components of the neutralinos. The S field gives one physical CP-even Higgs state, while the corresponding CP-odd Goldstone boson gets absorbed as the longitudinal component of the Z' gauge boson. For recent studies on phenomenology of the UMSSM, see Ref. [47].

We shall now present explicit examples where the $U(1)'$ symmetry allows for either $W_{\rm LV}$ or $W_{\rm BV}$, but not both at the same time. For simplicity, we assume the MSSM chiral fields (Q, U^c, D^c, L, E^c, N^c) to have family-universal $U(1)'$ charges,[12] but we allow family nonuniversal $U(1)'$ charges for the exotic quarks (K_i, K_i^c). The hypercharges of the exotic quarks may be family nonuniversal as well. In general, it is possible that there may be additional SM singlet fields which belong to the hidden sector, yet are charged under $U(1)'$ and thus contribute to the A_5 and A_6 anomalies. However, our primary intention was simply to demonstrate that an anomaly-free $U(1)'$ can be used and is sufficient to achieve all of our goals outlined in the introduction. Therefore, for concreteness and for simplicity, we shall assume only the field content listed in Table I.

In Table III, we show several examples of anomaly-free charge assignments (up to an arbitrary normalization factor) for $N_H = 1$, $N_S = 2$, $a = 1$, and $y[K_i] = \{\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\}$. We have classified our examples in two groups: the first five columns are LV models which allow for LV, but not BV terms in the superpotential, while the remaining six columns are BV models which allow for BV, but not LV terms in the superpotential. In LV models I–IV the LV terms appear already at the renormalizable level as in Eq. (55). In model V the terms of Eq. (55) appear at the nonrenormalizable level ($SLLE^c$, $SLQD^c$, and S^2H_2L) and in addition there are renormalizable LV terms involving exotics, e.g. $NK_1K_1^c$ and $EK_2K_1^c$. Similarly, BV models I–III already allow renormalizable BV couplings

[12]Family nonuniversal $U(1)'$ charges in the SM quark sector may induce dangerous flavor changing neutral currents [48]. (On the other hand, such a flavor changing Z' may provide an explanation of the discrepancies in rare B decays [49]).

TABLE III. Examples of anomaly-free $U(1)'$ charge assignments for $N_H = 1$, $N_S = 2$, $a = 1$, and $y[K_i] = \{\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\}$. These $U(1)'$ charges can be scaled by an arbitrary normalization factor, as well as rotated by hypercharge (see text for details).

	LV					BV					
	I	II	III	IV	V	I	II	III	IV	V	VI
$z[Q]$	1	3	3	3	4	1	3	15	0	0	0
$z[U^c]$	8	24	24	24	5	2	6	30	3	9	9
$z[D^c]$	−1	−3	−3	−3	−4	−1	−3	−15	0	0	0
$z[L]$	0	0	0	0	−9	−2	−6	−30	1	3	3
$z[E^c]$	0	0	0	0	9	2	6	30	−1	−3	−3
$z[N^c]$	0	0	0	0	9	2	6	30	−1	−3	−3
$z[H_2]$	−9	−27	−27	−27	−9	−3	−9	−45	−3	−9	−9
$z[H_1]$	0	0	0	0	0	0	0	0	0	0	0
$z[S]$	9	27	27	27	9	3	9	45	3	9	9
$z[K_1]$	−5	−13	−23	−25	−5	−1	−7	−17	−3	−7	−5
$z[K_2]$	−2	−4	−8	−7	−5	−1	−4	−20	0	−1	1
$z[K_3]$	1	2	1	−1	−5	−1	−4	−11	0	2	1
$z[K_1^c]$	−4	−14	−4	−2	−4	−2	−2	−28	0	−2	−4
$z[K_2^c]$	−7	−23	−19	−20	−4	−2	−5	−25	−3	−8	−10
$z[K_3^c]$	−10	−29	−28	−26	−4	−2	−5	−34	−3	−11	−10

as in Eq. (56), while BV models IV–VI allow only nonrenormalizable BV operators such as $QQD^{c\dagger}$, $QQQH_1$, and $H_1H_2U^cD^cD^c$.

A few comments are in order. First, each example in Table III in fact corresponds to a whole family of solutions. This is because hypercharge itself also satisfies all of our requirements, including the absence of mixed anomalies with $U(1)'$. Therefore, each one of our solutions can be "rotated" by hypercharge in an arbitrary normalization. More specifically, if $z_0[F_i]$ is any particular solution from Table III, then a family of anomaly-free $U(1)'$ charges is generated by the linear combination

$$z[F_i] = \alpha z_0[F_i] + \beta y[F_i], \tag{59}$$

where $y[F_i]$ are the hypercharge assignments of our fields F_i from Table I and α and β are arbitrary coefficients. Therefore, the numerical values for the $U(1)'$ charges in our models are subject to fixing the convention for Eq. (59). In Table III we only listed examples which are *not* equivalent in the sense of Eq. (59).

In spite of the freedom provided by Eq. (59), the numerical values of the $U(1)'$ charges are important for phenomenology, as they determine the couplings of the particles in our model to the Z'. For instance, our LV examples I–IV in Table III are completely leptophobic, as they have $z[L] = z[E^c] = z[N^c] = 0$. Under those circumstances, the standard collider bounds on the Z' mass are degraded, and a very light Z' can be allowed. However, this is not a general property of our LV models, since the hypercharge "rotation" (59) could generate nonzero $U(1)'$ charges for L, E^c, and H_1. On the other hand, $z[N^c] = 0$ *is* a general property of LV models I–IV in this particular case ($a = 1$), as already anticipated by Eq. (27). Similarly, the vanishing entries for the $U(1)'$ charges of Q, D^c, and H_1 in our BV models, can also be rotated away from zero using Eq. (59).

As we mentioned in Sec. III C, we also consider the case where the exotic hypercharges have the opposite sign: $y[K_i] = \{-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\}$. The actual solutions for the $U(1)'$ charges that we find in that case are given simply by those of Table III, with the replacement $z[K_i] \leftrightarrow z[K_i^c]$. In general, this choice of $y[K_i]$ appears dangerous, since hypercharge alone would then allow for LV and BV couplings involving exotic fields. However, we find that due to the general phenomenon of LV-BV separation discussed in Sec. II B, the $U(1)'$ symmetry is still sufficient to prevent the simultaneous appearance of LV and BV couplings in the superpotential, and in all but one case (namely, BV-IV with opposite exotic hypercharge) the proton turns out to be stable [38].

V. CONCLUSIONS

In this paper, we constructed a $U(1)'$-extended MSSM without R-parity, where the extra nonanomalous $U(1)$ gauge symmetry plays the dual role of solving the μ-problem and controlling the R-parity violating terms (3) and (4). The $U(1)'$ gauge symmetry provides a solid theoretical framework for discussing the phenomenology of R-parity violation. The most important implication of our models is the LV-BV separation: when the lepton number violating terms (3) are allowed by the $U(1)'$ symmetry, the baryon number violating terms (4) in the superpotential are automatically forbidden, and vice versa. Within our approach, the dangerous dimension 5 operators such as $QQQL$ or $U^cU^cD^cE^c$, which are allowed by R-parity and could still destabilize the proton, are also eliminated. This presents a very minimal solution to the proton decay problem which is alternative to R-parity. We showed that the LV-BV separation holds under very general circumstances. Perhaps the most stringent and least motivated was our assumption that there are no exotic $SU(2)_L$ representations. While one cannot judge the validity of this assumption without knowledge of the fundamental theory at high energies, it is certainly consistent with the principle of "Occam's razor."

While in our LV and BV examples the corresponding RPV couplings are allowed by the symmetries, the size of those couplings is still undetermined. The experimental upper bounds on the individual RPV couplings range from 10^{-3} for λ to 10^{-7} for λ''. We do not consider such small values particularly fine-tuned, especially when compared to the Yukawa couplings of the first generation fermions in the SM. In fact such small RPV couplings may naturally originate from higher-dimensional operators, without modifying the analysis and the conclusions of our paper [38].

An interesting feature of our setup is that all LV terms (λLLE^c, $\lambda' LQD^c$, $\mu'_{\rm eff} H_2 L$) must coexist, as long as one of them is allowed. This is phenomenologically interesting since, for instance, the observation of a sneutrino resonance in an s-channel at hadron colliders such as the Tevatron and the LHC requires both λ and λ' couplings. Besides the relation among the R-parity violating terms, our models also provide a connection between the phenomenology of R-parity violation and $U(1)'$ extensions of the MSSM. In this sense, a potential discovery of a Z' resonance at the Tevatron or LHC would motivate searches for R-parity violating SUSY signatures, and vice versa.

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APPENDIX A: MODELS WITH $N_H = 4$

In this appendix we briefly consider the case of $N_H = 4$. Again we shall choose $a = 1$, which fixes $N_S = 8$ in accordance with Eq. (48). The exotic hypercharges are uniquely determined from Eq. (42) to be $y[K_i] = 0$. For simplicity, in this appendix we shall assume that the exotic quarks also have the same $U(1)'$ charges as well: $z[K_1] = z[K_2] = z[K_3] \equiv k$. With those choices, the quadratic and cubic anomaly conditions (51) and (52) can be rewritten as

$$\frac{2}{3}s(6h_1 + 5s - 6\ell) = 0, \tag{A1}$$

$$-\frac{1}{12}s\{(6h_1 + 5s - 6\ell)(42h_1 + 7s + 6\ell) + 9(s + 6k)(5s + 6k)\} = 0. \tag{A2}$$

Notice that taking into account Eq. (A1) eliminates the first term in the curly brackets in Eq. (A2) and the A_5 anomaly condition completely factorizes:

$$-\frac{3}{4}s(s + 6k)(5s + 6k) = 0. \tag{A3}$$

This allows us to obtain explicitly a family of anomaly-free solutions for the $U(1)'$ charges of the fields in Table I. It turns out that all of these solutions forbid *both* the LV and BV terms, something which could not have been expected on the basis of Eq. (15) alone. Indeed, the A_4 anomaly constraint (A1) is inconsistent with the individual constraints for the LV case ($h_1 = \ell$) and the BV case ($h_1 = \ell + (1 - \frac{N_H}{3})s$) which were derived earlier in Sec. II E. In either case, compatibility with Eq. (A1) demands $s = 0$, which is not allowed by the condition (6).

The factorized constraints (A1) and (A2) can now be solved rather easily and the general solution (28) can be written as

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \\ z[K] \\ z[K^c] \end{pmatrix} = -2\ell \begin{pmatrix} \frac{1}{6} \\ -\frac{2}{3} \\ \frac{1}{3} \\ -\frac{1}{2} \\ 1 \\ 0 \\ \frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix} + \frac{s}{18} \begin{pmatrix} 8 \\ -5 \\ 7 \\ 0 \\ 15 \\ -15 \\ -3 \\ -15 \\ 18 \\ 18\rho \\ -18(1+\rho) \end{pmatrix}, \tag{A4}$$

where $\rho = -\frac{1}{6}$ ($\rho = -\frac{5}{6}$) in the case of $s + 6k = 0$ ($5s + 6k = 0$). As expected, we obtain a two-parameter family of solutions—one parameter (ℓ) corresponds to the usual hypercharge assignments while the second parameter (s) gives the nontrivial part of the $U(1)'$ solution.

APPENDIX B: MODELS WITH $N_H = 3$

In this appendix we shall consider $U(1)'$ models with $N_H = 3$, $N_S = 3$, and $a = 0$, as in Ref. [29]. As we already saw in Sec. II B, in that case one should either simultaneously allow or simultaneously forbid the LV and BV terms [see Eq. (15)]. Furthermore, $N_H = 3$ allows for family-universal hypercharges of the exotic quarks [see Eqs. (42) and (45)]. We shall consider two possible values for the exotic hypercharges: $y[K_i] = +\frac{1}{3}$ and $y[K_i] = -\frac{1}{3}$. For simplicity, in this appendix we shall again assume universal $U(1)'$ charges for the exotic quarks: $z[K_1] = z[K_2] = z[K_3] \equiv k$. The A_4 anomaly (51) can then be written as

$$A_4\colon \begin{cases} -2s(3k + 2\ell + s) = 0, & \text{for } y[K_i] = \frac{1}{3}; \\ 2s(3k - 2\ell + 2s) = 0, & \text{for } y[K_i] = -\frac{1}{3}; \end{cases} \tag{B1}$$

while the A_5 anomaly condition is independent of $y[K_i]$ and reads

$$A_5\colon \quad -3s(3k + 2\ell + s)(3k - 2\ell + 2s) = 0. \tag{B2}$$

We can see that A_5 completely factorizes into linear polynomials which already appear in the expression for A_4. Therefore, A_5 does not provide an additional restriction on the $U(1)'$ charges, i.e. A_5 will be automatically satisfied for *any* choice of $U(1)'$ charges which is consistent with A_4. Since A_4 is already a linear relation, this allows us to derive a three-parameter class of solutions which generalize the single model found in Ref. [29]. For $y[K_i] = +\frac{1}{3}$, from Eqs. (28) and (B1) we find the general solution

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \\ z[K] \\ z[K^c] \end{pmatrix} = \ell \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -3 \\ 2 \\ 0 \\ -2 \\ 0 \\ 2 \end{pmatrix} + h_1 \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \\ -3 \\ 3 \\ 0 \\ -3 \\ 1 \\ 2 \end{pmatrix}, \tag{B3}$$

in terms of the $U(1)'$ charges $\ell \equiv z[L]$, $h_1 \equiv z[H_1]$, and $k \equiv z[K_i]$. This solution is anomaly-free and satisfies all of the constraints discussed in Secs. II and III. As a special case, it also contains the solution found in Ref. [29], which we recover by imposing $8\ell = -7h_1 = -7k$. For example, $\ell = \frac{7}{12}$, $h_1 = k = -\frac{2}{3}$ gives

$$\begin{aligned} &z[Q, U^c, D^c, L, E^c, N^c, H_2, H_1, S, K, K^c] \\ &\quad = \{\tfrac{1}{12}, \tfrac{1}{12}, \tfrac{7}{12}, \tfrac{7}{12}, \tfrac{1}{12}, -\tfrac{5}{12}, -\tfrac{1}{6}, -\tfrac{2}{3}, \tfrac{5}{6}, -\tfrac{2}{3}, -\tfrac{1}{6}\}, \end{aligned} \tag{B4}$$

which is exactly the charge assignment in the model of Ref. [29]. In addition to our requirements listed in Secs. II and III, Ref. [29] demanded the presence of a Majorana mass term SN^cN^c in the superpotential. This would imply the constraint $8\ell - 2h_1 + 9k = 0$, which still leaves us with a two-parameter class of solutions

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \\ z[K] \\ z[K^c] \end{pmatrix} = \ell \begin{pmatrix} -1 \\ 3 \\ -3 \\ 1 \\ -5 \\ 1 \\ -2 \\ 4 \\ -2 \\ 0 \\ 2 \end{pmatrix} + \frac{k}{2} \begin{pmatrix} -2 \\ 5 \\ -7 \\ 0 \\ -9 \\ 3 \\ -3 \\ 9 \\ -6 \\ 2 \\ 4 \end{pmatrix} \tag{B5}$$

as a generalization of Eq. (B4).

For completeness, we shall also consider the other possible sign of the exotic hypercharges: $y[K_i] = -\frac{1}{3}$, since in that case A_5 is also automatically satisfied due to its factorization (B2), which makes it easy to obtain another class of solutions satisfying all Yukawa constraints and all anomaly cancellation conditions. Putting together Eqs. (28) and (B1), we find

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \\ z[K] \\ z[K^c] \end{pmatrix} = \ell \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ -1 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + h_1 \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \frac{k}{2} \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \\ -3 \\ 3 \\ 0 \\ -3 \\ 2 \\ 1 \end{pmatrix}. \tag{B6}$$

Unfortunately, this class of models does *not* solve the proton decay problem: as can be seen from Eq. (B6), the $U(1)'$ symmetry still allows R-parity violating couplings involving exotic fields, e.g. $U^cD^cK^c$ and LQK^c.

APPENDIX C: MODELS WITH $N_H = N_S = 1$

We have already seen that the A_6 anomaly condition (48) restricts the number of Higgs representations N_H and N_S. As we mentioned in Sec. III D, the minimal case of $N_H = 1$, $N_S = 1$ is not allowed within the model we have discussed so far. However, the constraint (48) varies with the particle spectrum, and here we provide an example with a slightly altered spectrum which can allow $N_H = 1$, $N_S = 1$. We simply add another SM singlet field X with superpotential

$$W_X = \frac{\xi}{2} SXX, \tag{C1}$$

so that the $U(1)'$ charge of X is given by $z[X] = -\frac{1}{2}z[S]$. The general solution (28) is then rewritten as

$$\begin{pmatrix} z[Q] \\ z[U^c] \\ z[D^c] \\ z[L] \\ z[E^c] \\ z[N^c] \\ z[H_2] \\ z[H_1] \\ z[S] \\ z[X] \end{pmatrix} = \frac{\ell}{3} \begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \\ -3 \\ -3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + h_1 \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \frac{s}{9} \begin{pmatrix} N_H \\ 9 - N_H \\ -N_H \\ 0 \\ 0 \\ 9(1-a) \\ -9 \\ 0 \\ 9 \\ -9/2 \end{pmatrix} \tag{C2}$$

with no additional free parameters.

The new X particles will modify the anomaly conditions A_6 ($U(1)' - [\text{gravity}]^2$) and A_5 ($[U(1)']^3$) which get additional contributions of $N_X z[X]$ and $N_X z[X]^3$, respectively.

Then Eq. (48) is modified as

$$N_S = 2N_H + 3a - 3 + \tfrac{1}{2}N_X, \tag{C3}$$

where N_X is the number of families of the X fields. $N_H = 1$, $N_S = 1$ is now allowed with $a = 0$, $N_X = 4$. As an existence proof, we provide an example of an anomaly-free LV model of this category with $y[K_i] = \{\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\}$:

$$\begin{aligned} &z[Q, U^c, D^c, L, E^c, N^c, H_2, H_1, S, K_1, K_2, K_3, K_1^c, K_2^c, K_3^c, X] \\ &= \{4, 8, 2, -6, 12, 18, -12, -6, 18, -6, -3, -15, \\ &\quad -12, -15, -3, -9\}. \end{aligned} \tag{C4}$$

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