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More visible effects of the hidden sector

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There is a growing appreciation that hidden sector dynamics may affect the supersymmetry breaking parameters in the visible sector (supersymmetric standard model), especially when the dynamics is strong and superconformal. We point out that there are effects that have not been previously discussed in the literature. For example, the gaugino masses are suppressed relative to the gravitino mass. We discuss their implications in the context of various mediation mechanisms. The issues discussed include anomaly mediation with singlets, the μ ($B\mu$) problem in gauge and gaugino mediation, and distinct mass spectra for the superparticles that have not been previously considered.

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I. INTRODUCTION

Supersymmetry has been widely recognized as an excellent solution to the hierarchy problem, as long as the superparticle masses are below the TeV scale. However, such low scale supersymmetry is in conflict with the data from flavor physics, unless the spectrum is (close to) that of minimal flavor violation, i.e., violation of the $U(3)^5$ flavor symmetry comes only from the standard model Yukawa couplings. There are several promising ways to mediate supersymmetry breaking effects preserving this property, for example, gauge mediation [1,2], anomaly mediation [3,4], and gaugino mediation [5].

One interesting possibility for the origin of a hierarchically small scale for supersymmetry breaking is dynamical supersymmetry breaking [6]. Supersymmetry breaking is triggered at low energies by nontrivial infrared gauge dynamics of the hidden sector, which is then transmitted to the supersymmetric standard model (SSM) sector through a mediation mechanism preserving flavor. Traditionally, the spectrum of the superparticles has been calculated using the SSM renormalization group equations below the scale of mediation. There is, however, a growing appreciation that the dynamics of the hidden sector may affect the supersymmetry breaking parameters in the SSM sector through renormalization group evolution between the mediation scale and the scale where the hidden sector fields decouple.

One of the most drastic examples of hidden sector dynamics is conformal sequestering [7], which occurs when the hidden sector exhibits strong superconformal dynamics. (For a discussion on the effects of the hidden sector outside of the conformal regime, see [8].) This achieves the suppression of certain (dangerous) local operators connecting the hidden sector and SSM sector fields, and helps one to mediate supersymmetry breaking in a flavor universal manner. The construction is motivated by the AdS/CFT correspondence [9]. If the SSM sector is located on the ''ultraviolet brane'' of a truncated AdS space [10], while the hidden sector is on the ''infrared brane,'' the physical separation between the two sectors due to the AdS bulk can be interpreted in terms of conformal dynamics in four dimensions. This helps us see that purely four-dimensional theories can achieve apparent sequestering due to the strong conformal dynamics of the hidden sector. This class of dynamics has been further discussed in [11–13].

In this paper, we point out that hidden sector conformal dynamics has additional effects on the SSM sector parameters that have not been discussed in the literature. Namely, operators that are linear in a singlet field in the hidden sector are sequestered by the wavefunction renormalization factor relative to the gravitino mass. Note that the authors of Ref. [11] stated that these operators are not sequestered, which we do not agree with. There are at least three immediate consequences of this observation. (1) Anomaly mediation does not require the absence of singlets in the hidden sector, as the gaugino masses are sequestered and the anomaly mediated piece can dominate. (2) Conformal hidden sector dynamics can make gravity mediated contributions more harmful in gauge and gaugino mediated models, depending on the dynamics. (3) The μ ($B\mu$) problem in gauge and gaugino mediation can in principle be solved by strong conformal dynamics, although it requires certain assumptions on the hidden sector dynamics. In addition, if the sequestering effects are sufficiently strong, we find very specific mass spectra for the superparticles that have not been discussed in the literature and can be tested at future experiments.

The organization of the paper is as follows. In Sec. II, we provide a general discussion on the effects of a strong hidden sector on local operators connecting the hidden and SSM sector fields. Section III summarizes the consequences of these effects on the SSM parameters. In Sec. IV we discuss possible scenarios in which the dominant mediation mechanism is gauge mediation. It is shown that strong conformal dynamics can provide a solution to the μ ($B\mu$) problem and/or lead to distinct spectra for the superparticles. Gaugino and anomaly mediation are considered

in Secs. V and VI. Finally, discussion and conclusions are given in Sec. VII.

II. GENERAL DISCUSSIONS

In this section, we present general discussions on the renormalization of operators that couple the hidden and SSM sector fields due to strong conformal dynamics in the hidden sector.

In many models of supersymmetry breaking, there are both gauge nonsinglet and singlet fields in the hidden sector. We generically call them q and S, respectively, without referring to particular models. They may be ''elementary'' or ''composite,'' but this distinction is not very clear in superconformal theories as they may allow for several inequivalent descriptions (duality). To keep the discussion uniform, we always take the normalization such that these fields have mass dimension $+1$.[1] Note, however, that we are interested in models where the q and S fields participate in strong conformal dynamics, and hence their scaling properties are not dictated by their classical dimensions but rather their conformal dimensions. We will generically refer to the chiral superfields of the SSM sector as ϕ.

The direct couplings between the hidden and SSM sector fields can come in various local operators. They are all higher dimension operators and suppressed by some energy scale M. In gauge mediation models, it is related (but not necessarily equal to) the messenger scale. In anomaly and gaugino mediated models, it is (generically) close to the Planck scale.

One class of direct interaction operators is quadratic in the hidden sector fields. For example, operators that contribute to the scalar squared masses are

$$\mathcal{O}_\phi: \int d^4\theta c_\phi^q \frac{q^\dagger q}{M^2}\phi^\dagger\phi, \qquad \int d^4\theta c_\phi^S \frac{S^\dagger S}{M^2}\phi^\dagger\phi. \quad (1)$$

Other operators of interest are

$$\mathcal{O}_{B\mu}: \int d^4\theta c_{B\mu}^q \frac{q^\dagger q}{M^2} H_u H_d + \text{H.c.}, \quad \int d^4\theta c_{B\mu}^S \frac{S^\dagger S}{M^2} H_u H_d + \text{H.c.}, \quad (2)$$

that contribute to the $B\mu$ parameter (the holomorphic supersymmetry breaking mass squared) in the Higgs sector. Here and below, the coefficients c's are dimensionless.

Using the singlet fields, we can also consider operators linear in the hidden sector fields. The gaugino mass operator is

$$\mathcal{O}_\lambda: \int d^2\theta c_\lambda^S \frac{S}{M}\mathcal{W}^{a\alpha}\mathcal{W}_\alpha^a + \text{H.c.}, \quad (3)$$

where $\mathcal{W}_\alpha^a$ ($a = 1, 2, 3$) are the field-strength superfields for the standard model gauge group. The operators

$$\mathcal{O}_A: \int d^4\theta c_A^S \frac{S}{M}\phi^\dagger\phi + \text{H.c.} \quad (4)$$

contribute to the A and B parameters (the parameters associated with holomorphic supersymmetry breaking scalar trilinear and bilinear interactions), as well as the scalar masses $|A|^2$. Finally, the operator

$$\mathcal{O}_\mu: \int d^4\theta c_\mu^S \frac{S^\dagger}{M} H_u H_d + \text{H.c.}, \quad (5)$$

contributes to the μ parameter (the supersymmetric Higgs mass).[2]

Note that we have used the formalism of global supersymmetry in the above expressions. This is sufficient for the purpose of discussing operators that arise from integrating out a set of messenger fields, e.g., gauge mediation. Later, we will discuss gravity and anomaly mediated contributions, which require a formulation with local supersymmetry. The terms integrated over a half of the superspace above will then include the conformal compensator field Φ as $\int d^2\theta\Phi^3$, while the terms over the full superspace as $\int d^4\theta\Phi^\dagger\Phi$ [14]. The latter should be regarded not as a part of the Kähler potential K, but rather the superspace density $f = -3M_{\rm Pl}^2 e^{-K/3M_{\rm Pl}^2}$ before the Weyl scaling that removes the field dependence in the Planck scale. Here, $M_{\rm Pl}$ is the reduced Planck scale. After the Weyl scaling, each chiral superfield needs to be further rescaled by $1/\Phi$ to obtain the usual kinetic terms, leaving a nontrivial Φ dependence in the various mass parameters. In vacua with supersymmetry breaking and no cosmological constant, $\Phi = 1 + \theta^2 m_{3/2}$, where $m_{3/2}$ is the gravitino mass. As we continue our discussion, it should be understood that there is an implicit compensator dependence in all of the mass parameters, and that any sequestering effects are occurring in f, and not in K.

In many cases, some of the operators Eqs. (1)–(5) are unwanted. The operators $\mathcal{O}_\phi$ in Eq. (1) and $\mathcal{O}_A$ in Eq. (4) are potential sources of flavor changing neutral currents. All of them are potential sources of CP violation. Both of these are constrained tightly by the data. The purpose of conformal sequestering, then, is to help suppress any unwanted operators.

The main point is that, as long as the relevant fixed point is infrared attractive, conformal field theories can help achieve this suppression. To see this, we can regard the

[1] For example, the meson field $\bar{Q}Q$ in supersymmetric QCD naturally has mass dimension $+2$, while we normalize it as $S = \bar{Q}Q/\Lambda_*$, with Λ_* being the strong scale.

[2] In principle, one may also consider direct superpotential couplings between the hidden and SSM sector fields, such as $\int d^2\theta S H_u H_d$ or $\int d^2\theta S Q_i U_j H_u/M_*$. We assume their absence throughout the paper.

SSM sector fields as background fields, and rescale the hidden sector fields to absorb the operators $\mathcal{O}_{\phi,B\mu}$ in Eqs. (1) and (2) into coupling constants of the theory. As long as the fixed point is stable against deformations of the dimensionless coupling constants, the coupling constants flow to their infrared fixed point values by power laws, losing ''memory'' of the initial conditions. Therefore, the unwanted operators can be suppressed by powers of energy scales. If we can suppress all unwanted operators by power laws, while at the same time keeping those we need, the conformal sequestering is a success.

Most of the discussions on conformal sequestering so far have focused on the operators quadratic in the hidden sector fields. However, it is important to consider operators linear in the hidden sector fields as well. To the best of the current authors' knowledge, the only paper that has addressed this class of operators is Ref. [11]. They stated that this class of operators is not suppressed relative to the gravitino mass. This observation would have allowed for an easy solution to the $B\mu$ problem in gauge mediation, since the unwanted operator $\mathcal{O}_{B\mu}$ of Eq. (2) would then be power suppressed at low energies while keeping the necessary operators $\mathcal{O}_{\lambda,\mu}$ of Eqs. (3) and (5) (see Sec. IV for more detail). Unfortunately, we disagree with this statement. We instead find that the conformal sequestering is more complete than what they suggested; operators linear in the hidden sector fields are also suppressed relative to the gravitino mass.

To make the discussion more concrete, let us introduce a couple of energy scales. We already defined M as the scale appearing in the higher dimension operators that couple the hidden and SSM sector fields. This may be close to the Planck scale for anomaly or gaugino mediated supersymmetry breaking, or it may be a combination of energy scales in general, such as in gauge mediated supersymmetry breaking. We also define the energy scale Λ_* as the scale where the hidden sector enters into the conformal regime.

Since S is singlet under the hidden sector gauge group, the superconformal algebra requires that it must have an R charge greater than $2/3$ to preserve unitarity [15]. The anomalous dimension is given in terms of the R charge by $3R/2 - 1$, and hence the wavefunction renormalization factor

$$\mathcal{L} = \int d^4\theta Z_S(\mu_R) S^\dagger S \tag{6}$$

always satisfies

$$Z_S(\mu_R) = \left(\frac{\Lambda_*}{\mu_R}\right)^{3R(S)-2} > 1, \tag{7}$$

for $\mu_R < \Lambda_*$, where μ_R is the renormalization scale, $R(S)$ the R charge of S, and we have taken $Z_S(\Lambda_*) = 1$. There are no 1PI diagrams that renormalize operators linear in S, and hence $\mathcal{O}_\lambda$ in Eq. (3), $\mathcal{O}_A$ in Eq. (4), and $\mathcal{O}_\mu$ in Eq. (5) receive only the wavefunction renormalization $Z_S^{-1/2}(\mu_R)$. Note that this effect is always a *suppression* of the operators. Therefore, their respective forms at the energy scale $\mu_R \ll \Lambda_*$ are

$$\int d^2\theta Z_S^{-1/2}(\mu_R) c_\lambda^S \frac{S}{M} \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha + \text{H.c.} \tag{8}$$

for the gaugino masses,

$$\int d^4\theta Z_S^{-1/2}(\mu_R) c_A^S \frac{S}{M} \phi^\dagger \phi + \text{H.c.} \tag{9}$$

for the A, B parameters, and the $|A|^2$ part of the scalar squared masses, and

$$\int d^4\theta Z_S^{-1/2}(\mu_R) c_\mu^S \frac{S^\dagger}{M} H_u H_d + \text{H.c.} \tag{10}$$

for the μ parameter.

The S field acquires an F-component vacuum expectation value (VEV) if there is a linear term in the superpotential, i.e., if there is an operator

$$\int d^2\theta f^2 S + \text{H.c.}, \tag{11}$$

where f has mass dimension one. In the basis where the S field is canonically normalized, this linear term is also suppressed in the infrared as

$$\int d^2\theta Z_S^{-1/2}(\mu_R) f^2 S + \text{H.c.} \tag{12}$$

The F-component VEV for the canonically normalized S is

$$F_S = -Z_S^{-1/2}(\mu_R) f^{*2}, \tag{13}$$

and the vacuum energy $V_0 = |Z_S^{-1/2}(\mu_R) f^2|^2$, and hence the gravitino mass, is

$$m_{3/2} \approx Z_S^{-1/2}(\mu_R) \frac{|f|^2}{M_{\text{Pl}}}. \tag{14}$$

The apparent suppression $Z_S^{-1/2}(\mu_R)$ of Eq. (14), however, does not have much physical meaning, since it suppresses all the μ and supersymmetry breaking parameters equally. For example, the gaugino masses are given by

$$|M_a| \approx Z_S^{-1/2}(\mu_R) \frac{|c_\lambda^S F_S|}{M} = Z_S^{-1}(\mu_R) \frac{|c_\lambda^S f^2|}{M}. \tag{15}$$

In this last expression, one factor of $Z_S^{-1/2}(\mu_R)$ comes from that of Eq. (14), but the other $Z_S^{-1/2}(\mu_R)$ from the suppression of the coefficient of Eq. (8). It is this latter $Z_S^{-1/2}(\mu_R)$ that provides the suppression of the gaugino masses relative to the gravitino mass: $M_a/m_{3/2} \sim Z_S^{-1/2}(\mu_R)$. Similar analyses also apply to the μ and A parameters. Therefore, the gaugino masses, μ, and A parameters receive a stronger suppression than the gravitino mass, affecting phenomenology and model building as we will discuss later.

In contrast to the operators linear in S, the operators $\mathcal{O}_{\phi,B\mu}$ in Eqs. (1) and (2) receive corrections from 1PI diagrams in addition to the wavefunction renormalization factors. Note that the R charges of gauge nonsinglet fields can be less than $2/3$ because they do not appear as asymptotic states, and hence the standard representation theory does not apply. To simplify the discussion, let us ignore operator mixing at this moment, and pretend that these operators renormalize by themselves. We then find

$$\begin{aligned} &\int d^4\theta \left(\frac{\mu_R}{\Lambda_*}\right)^{\alpha_q} Z_q^{-1}(\mu_R) c_\phi^q \frac{q^\dagger q}{M^2} \phi^\dagger \phi, \\ &\int d^4\theta \left(\frac{\mu_R}{\Lambda_*}\right)^{\alpha_S} Z_S^{-1}(\mu_R) c_\phi^S \frac{S^\dagger S}{M^2} \phi^\dagger \phi, \end{aligned} \tag{16}$$

for the scalar squared masses, and

$$\begin{aligned} &\int d^4\theta \left(\frac{\mu_R}{\Lambda_*}\right)^{\alpha_q} Z_q^{-1}(\mu_R) c_{B\mu}^q \frac{q^\dagger q}{M^2} H_u H_d + \text{H.c.}, \\ &\int d^4\theta \left(\frac{\mu_R}{\Lambda_*}\right)^{\alpha_S} Z_S^{-1}(\mu_R) c_{B\mu}^S \frac{S^\dagger S}{M^2} H_u H_d + \text{H.c.}, \end{aligned} \tag{17}$$

for the $B\mu$ term, where $Z_q(\mu_R)$ is defined analogously to $Z_S(\mu_R)$; see Eq. (7).[3] The exponents α_q and α_S are common to the operators in Eqs. (16) and (17), since the dependence of these operators on the hidden sector fields is the same. Note that here we defined $\alpha_{q,S}$ to parameterize the 1PI corrections; for example, if one of the operators in Eqs. (16) and (17) corresponds to a conserved current in the hidden sector, the $(\mu_R/\Lambda_*)^{\alpha_{q,S}}$ factor exactly cancels the wavefunction renormalization factor $Z_{q,S}^{-1}(\mu_R)$.

An interesting and often crucial question is the relative speed of suppression (sequestering) between the operators quadratic and linear in S. Suppose that there is no mixing between operators quadratic in S and those quadratic in q, and that only S has a supersymmetry breaking VEV. Then, if there were no extra exponent α_S, all the μ and soft parameters would receive similar suppressions as $M_a \sim \mu \sim A \propto Z_S^{-1/2} F_S$ and $m_I^2 \sim B\mu \propto Z_S^{-1} F_S^2$, while $m_{3/2} \propto F_S$. Here, m_I^2 represent the supersymmetry breaking scalar squared masses. Realistically, however, the situation is not that simple. The operators of the form $\mathcal{O}_\phi$ in Eq. (1) [and $\mathcal{O}_{B\mu}$ in Eq. (2)] in general mix with each other, and $\alpha_{q,S}$ are nonzero. In this case, the suppression of the operators quadratic in S is controlled by the smallest eigenvalue of the $2\gamma_i \delta_{ij} + \alpha_{ij}$ matrix, which we define as $2\gamma_S + \hat{\alpha}_S$. Here, i, j runs over q and S, and $\gamma_q \equiv 3R(q)/2 - 1$ and $\gamma_S \equiv 3R(S)/2 - 1$ are the anomalous dimensions of the q and S fields. (For a detailed discussion on operator mixing, see Appendix A.)

[3]If the operators $\mathcal{O}_{\phi,B\mu}$ in Eqs. (1) and (2) are generated at a scale $m_f < \Lambda_*$, the factors $(\mu_R/\Lambda_*)^{\alpha_{q,S}}$ in Eqs. (16) and (17) should be replaced by $(\mu_R/m_f)^{\alpha_{q,S}}$.

One additional subtlety is that the operators quadratic in S also mix in a calculable way with the operators linear in S. In particular, for the non-Higgs fields it is really the combination $c_\phi^S - |c_A^S|^2$ that ends up being suppressed by the exponent $2\gamma_S + \hat{\alpha}_S$ (after potentially mixing with other quadratic operators), and this is the same combination of operators that contributes to the scalar squared masses. Similarly, for the Higgs fields it is the combination $c_\phi^S - |c_A^S|^2 - |c_\mu^S|^2$ that ends up being suppressed by the same exponent, and this is the operator that contributes to $m_{H_{u,d}}^2 + \mu^2$. Finally, the combination of operators that contributes to the $B\mu$ parameter, $c_{B\mu}^S - c_\mu^S (c_{A,H_u}^S + c_{A,H_d}^S)$, is renormalized in the same way.

One can then obtain the qualitatively different outcomes:

$$\begin{aligned} &\text{Case 1: } M_a^2 \sim \mu^2 \sim A^2 \gg m_{Q_i,U_i,D_i,L_i,E_i}^2 \sim B\mu \\ &\qquad\qquad \sim m_{H_{u,d}}^2 + \mu^2 \quad (\hat{\alpha}_S > 0), \\ &\text{Case 2: } M_a^2 \sim \mu^2 \sim A^2 \ll m_{Q_i,U_i,D_i,L_i,E_i}^2 \sim B\mu \\ &\qquad\qquad \sim m_{H_{u,d}}^2 \quad (\hat{\alpha}_S < 0), \end{aligned} \tag{18}$$

depending on the sign of the exponent $\hat{\alpha}_S$. (In the absence of the operator mixing, $\hat{\alpha}_S = \alpha_S$.) In addition, since all the soft parameters are suppressed *relative to* the gravitino mass (except for those that correspond to conserved currents), it is also possible that they are all subdominant relative to the gravitino mass, in which case anomaly mediation may be dominant (Case 3). Unfortunately, for a given strongly coupled conformal theory, it is not possible to work out the signs or magnitudes of the exponents $\alpha_{q,S}$ with the currently available technology. We will therefore discuss all three cases on equal footing in the rest of the paper.

Note that we are only considering operators linear or quadratic in the hidden sector fields because they are the lowest dimension operators that contribute to the soft supersymmetry breaking parameters in the SSM sector. However, due to the incalculable strong dynamics, we cannot exclude the possibility that even higher dimension operators, i.e., cubic, quartic, or beyond in the hidden sector fields, receive anomalously large enhancements relative to the lower dimension operators and become as important. See Sec. VII for more on this point.

III. CONSEQUENCES ON THE SSM PARAMETERS

What effect does the strong hidden sector renormalization, discussed in the previous section, have on the μ and supersymmetry breaking parameters in the SSM sector? As we have seen, terms linear in the S field are power suppressed in a way that is controlled exactly by the R charge of S. On the other hand, particular combinations of terms quadratic in the S field and terms linear in the S field are suppressed (assuming the fixed point is infrared attractive)

by an incalculable amount, determined by the rate at which the theory flows back towards the conformal fixed point. Rather generically, if the effects are strong, we expect that one of these classes of operators will completely dominate over the other. An important point here is that the *relative* strengths of the operators linear in S remain fixed, since they are all suppressed by the same amount. Similarly, the relative strengths of the operators quadratic in S (in combination with linear operators) also do not change.[4]

The operators linear in S, $\mathcal{O}_{\lambda,A,\mu}$ in Eqs. (3)–(5), contribute to the gaugino masses M_a, μ parameter, scalar squared masses m_I^2, and A and B parameters. Note, however, that because the scalar masses, A parameters, and B parameter are all generated by the single operator $\mathcal{O}_A$ in Eq. (4), there are simple relations among them. On the other hand, the operators quadratic in S, $\mathcal{O}_{\phi,B\mu}$ in Eqs. (1) and (2), also independently contribute to the scalar masses and $B\mu$ term, and the dynamics may actually drive these to cancel the contributions from the linear operators. Thus, if the hidden sector effects are strong, we are generically led to one of the following situations.

A. Case 1: Linear operator dominance

In this case, any initial conditions in the quadratic operators are suppressed, and they are dynamically driven to cancel out the contributions to the soft parameters from the linear operators. As long as these linear operators are all generated at approximately the same size, we obtain the following spectrum at the scale where the hidden sector exits from the conformal fixed point:

$$\begin{aligned} &m^2_{Q_i,U_i,D_i,L_i,E_i} = 0, \qquad m^2_{H_{u,d}} = -\mu^2, \\ &a_{IJK} = y_{IJK}(A_I + A_J + A_K), \qquad B = 0, \end{aligned} \tag{19}$$

where I, J, K runs over the SSM matter and Higgs fields, Q_i, U_i, D_i, L_i, E_i, H_u, H_d ($i = 1, 2, 3$), and A_I represent the coefficients of the operators $\int d^4\theta S\phi_I^\dagger\phi_I$ times F_S, which are of the same order as the gaugino masses and the μ parameter

$$M_a \approx \mu \approx A_I. \tag{20}$$

The soft parameters m_I^2 and a_{IJK} are defined by $\mathcal{L}_{\text{soft}} = -m_I^2\phi_I^\dagger\phi_I - (a_{IJK}\phi_I\phi_J\phi_K + \text{H.c.})$, and y_{IJK} are the Yukawa couplings: $W = y_{IJK}\phi_I\phi_J\phi_K$. Here, we have neglected, for simplicity, possible mixings between different generations in A_I, which may be present in general.

[4]In the case that there is operator mixing and/or multiple singlets with F-component VEVs, the relative strengths among linear operators and/or quadratic operators can in principle change depending on how they project onto the "eigenvectors" of the renormalization group evolution. These effects, however, are typically of $O(1)$ if present, and do not affect the arguments below.

In order to avoid excessive flavor changing processes, the parameters A_I must take a special form in flavor space. One simple possibility is that the A_I operators are generated only for the Higgs and third generation matter fields. In this case, we obtain

$$m_I^2 = 0, \tag{21}$$

$$a_{IJH_u} = y_{IJH_u}A_{H_u}, \qquad a_{IJH_d} = y_{IJH_d}A_{H_d}, \tag{22}$$

for the first two generation matter fields, and

$$m^2_{Q_3,U_3,D_3,L_3,E_3} = 0, \qquad m^2_{H_{u,d}} = -\mu^2, \tag{23}$$

$$\begin{aligned} a_t &= y_t(A_{Q_3} + A_{U_3} + A_{H_u}), \\ a_b &= y_b(A_{Q_3} + A_{D_3} + A_{H_d}), \\ a_\tau &= y_\tau(A_{L_3} + A_{E_3} + A_{H_d}), \end{aligned} \tag{24}$$

$$B = 0, \tag{25}$$

for the third generation matter and Higgs fields. Here, y_t, y_b, y_τ are the top, bottom, and tau Yukawa couplings, a_t, a_b, a_τ the corresponding scalar trilinear interactions, and

$$M_a \approx \mu \approx A_I, \tag{26}$$

where $I = Q_3$, U_3, D_3, L_3, E_3, H_u, H_d. A special case of this spectrum is obtained if only the Higgs fields have the A_I operators: $A_{Q_3} = A_{U_3} = A_{D_3} = A_{L_3} = A_{E_3} = 0$.

The spectra given above represent the running parameters evaluated at the scale where the hidden sector exits from the conformal regime, which is generically much larger than the weak scale. The low-energy superparticle masses are then obtained by evolving these parameters down to the weak scale using the renormalization group equations. Since the hidden sector already leaves the strong conformal regime, these evolutions are dominated by loops of the SSM states, i.e., the running is well approximated by the standard SSM renormalization group equations.

B. Case 2: Quadratic operator dominance

In this case, the quadratic operators (or at least one of them in the case that there are operator mixings) are suppressed more slowly than the linear operators. This can easily be the case, for example, if the quadratic operators contain a global symmetry current(s) of the hidden sector, which does not receive any suppression factor. This leads to the split spectrum

$$m_I^2, B\mu \gg M_a^2, \mu^2, a_{IJK}^2, \tag{27}$$

at the scale where the hidden sector exits the conformal regime. This splitting is preserved by renormalization group evolution at lower energies, so if the splitting is very large, the spectrum requires a severe fine-tuning in electroweak symmetry breaking.

The spectrum, however, does not require fine-tuning if the splitting is not very large. The exact spectrum is determined by the mediation mechanism, and may show a distinct pattern which is not common to the scenarios in which the hidden sector dynamics are not taken into account. If we take gauge mediation, for example, we obtain a somewhat interesting spectrum in which the number of messenger fields appears to be *fractional*, as we will see in Sec. IV.

C. Case 3: Anomaly mediation dominance

In both of the previous situations, it is worth emphasizing that *all* the parameters are being suppressed relative to the gravitino mass $m_{3/2} \approx F_S/M_{\rm Pl}$, except for operators corresponding to conserved currents of the hidden sector, which we assume to be absent here. If the suppressions of both types of operators are strong enough, then, we will be led to the situation where the anomaly mediated contribution dominates. It is also possible, depending on the amount of suppressions, that the dominant contributions to the SSM sector parameters come both from anomaly mediation and some of the local operators involving the S field. These points will be discussed further in Sec. VI.

IV. GAUGE MEDIATION

In this section, we take gauge mediation as the dominant mediation mechanism generating the local operators in Eqs. (1)–(5), and consider the possible implications of the hidden sector dynamics discussed in the previous sections. The situation is different depending on which of Case 1 or Case 2 is realized as a result of the hidden sector dynamics. We first discuss the implications of Case 1 in Sec. IV A, and then discuss those of Case 2 in Sec. IV B. Finally, we discuss the competition with gravity and anomaly mediation in Sec. IV C.

A. Solution to the μ ($B\mu$) problem with conformal dynamics

A major difficulty of the gauge mediation scenario is the so-called μ problem—it is difficult to obtain phenomenologically acceptable values for the μ and $B\mu$ parameters. In fact, a careful look at the problem shows that it is really a $B\mu$ problem, rather than a μ problem (see, e.g., [16]). In gauge mediation, the gaugino masses, M_a, and the scalar squared masses, m_I^2, arise at one and two loops, respectively, so that these masses have the comparable size $M_a \approx m_I \approx (g^2/16\pi^2)(F_{\rm mess}/M_{\rm mess})$, where $F_{\rm mess}/M_{\rm mess} \approx$ (10–100) TeV is the scale characterizing the strength of the mediation. Now, it is not so difficult to come up with a model in which the μ term is generated at one loop, $\mu \approx (1/16\pi^2)(F_{\rm mess}/M_{\rm mess})$, so that it is comparable to the gaugino and scalar masses. However, such a model also tends to generate the $B\mu$ term at one loop, $B\mu \approx (1/16\pi^2)(F_{\rm mess}/M_{\rm mess})^2$, leading to the parameter B being one-loop *enhanced* relative to the other supersymmetry breaking masses, $B \equiv B\mu/\mu \approx F_{\rm mess}/M_{\rm mess}$. Since the size of B should be smaller than or of the order of the weak scale to obtain successful phenomenology, this is not acceptable.

We point out here that this problem can be solved if the hidden sector has strong conformal dynamics exhibiting the property described as Case 1 in Sec. III. Suppose that gauge mediation arises due to vectorlike messenger superfields f, $\bar{f}$ having a mass m_f and a coupling to the hidden sector superfield S in the superpotential [17]:

$$W = -m_f \bar{f} f + \lambda S \bar{f} f. \tag{28}$$

Here, S is a superfield responsible for supersymmetry breaking, $\langle S \rangle = \theta^2 F_S$, and the coupling λ encodes the information on the classical dimension of the (composite) operator S:

$$\lambda = O\left(\left(\frac{\Lambda_*}{M_*}\right)^{d_S - 1}\right). \tag{29}$$

Here, M_* is the cutoff scale of the theory, which can be taken to be around the Planck scale $M_* \approx M_{\rm Pl}$, and d_S the classical mass dimension of S. The parameter m_f can be taken real and positive without loss of generality.

At the scale m_f, the messenger fields are integrated out. This generates the operators

$$\mathcal{L} = \frac{1}{2} D_f \int d^2\theta \sum_a \frac{\lambda(m_f)}{16\pi^2 m_f} S \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha + \text{H.c.}, \tag{30}$$

where $a = 1, 2, 3$ represents the standard model gauge groups, $\mathcal{W}^a_\alpha$ the corresponding field-strength superfields, and D_f the Dynkin index of the messengers (1 for $\mathbf{5} + \mathbf{5}^*$, 3 for $\mathbf{10} + \mathbf{10}^*$ etc.), and

$$\mathcal{L} = -D_f \int d^4\theta \sum_I \sum_a \frac{2 g_a^4 C_I^a |\lambda(m_f)|^2}{(16\pi^2)^2 m_f^2} S^\dagger S \phi_I^\dagger \phi_I, \tag{31}$$

where g_a are the standard model gauge couplings evaluated at m_f, and C_I^a the quadratic Casimir coefficients. Here, $\lambda(m_f)$ is the physical coupling λ evaluated at $\mu_R \approx m_f$:

$$\lambda(m_f) = \left(\frac{m_f}{\Lambda_*}\right)^{\gamma_S} \lambda. \tag{32}$$

After S acquires a supersymmetry breaking VEV, the operators of Eqs. (30) and (31) become the gaugino masses and scalar squared masses, respectively.

In order to solve the μ problem, the μ parameter must be generated with a size comparable to the gaugino masses. It is, in fact, not very difficult to generate the μ term also at one loop as

$$\mathcal{L} \approx \int d^4\theta \frac{\lambda(m_f)^*}{16\pi^2 m_f} S^\dagger H_u H_d + \text{H.c.}, \tag{33}$$

(for an example of such models, see Appendix B). This leads to a μ parameter of the same order as the gaugino and scalar masses generated by Eqs. (30) and (31):

$$M_a \approx m_I \approx \mu \approx \left| \frac{\lambda(m_f) F_S}{16\pi^2 m_f} \right|. \tag{34}$$

The problem is that in any simple models producing the operator Eq. (33) at one loop, the same one-loop diagram also generates another operator

$$\mathcal{L} \approx \int d^4\theta \frac{|\lambda(m_f)|^2}{16\pi^2 m_f^2} S^\dagger S H_u H_d + \text{H.c.}, \tag{35}$$

which leads to a large B parameter

$$B \equiv \frac{B\mu}{\mu} \approx \left| \frac{\lambda(m_f) F_S}{m_f} \right| \gg M_a, m_I, \mu. \tag{36}$$

This is nothing but the $B\mu$ problem in gauge mediation discussed earlier. Models that generate the operator Eq. (33) at one loop also typically generate the operators

$$\mathcal{L} \approx \int d^4\theta \left(\frac{\lambda(m_f)}{16\pi^2 m_f} S H_u^\dagger H_u + \frac{\lambda(m_f)}{16\pi^2 m_f} S H_d^\dagger H_d + \text{H.c.} \right), \tag{37}$$

which contribute to A_{H_u}, A_{H_d}, B, $m_{H_u}^2$, and $m_{H_d}^2$. These operators, however, are harmless, since the generated soft masses are of the same order as the gaugino masses, and the A terms induced preserve flavor, i.e., a_{IJK} are proportional to the Yukawa matrices, y_{IJK}, in flavor space.

The operators of Eqs. (30), (31), (33), (35), and (37) are the ones generated at m_f and relevant for the μ and supersymmetry breaking parameters in the SSM sector. They lead to an unacceptably large B parameter. Note, however, that these correspond to the μ and supersymmetry breaking masses evaluated at the scale m_f. If the hidden sector interactions are strong below the scale m_f down to some scale m_X where conformality is broken, as we are assuming here, then the operators of Eqs. (30), (31), (33), (35), and (37) receive strong renormalization effects in the energy interval between m_f and m_X. The pattern of the soft masses at m_X ($\ll m_f$) depends on which of Case 1 and Case 2 is realized, and here we assume that Case 1 is realized. In this case, the operators Eqs. (31) and (35) (in particular combinations with the linear operators) are damped compared with the operators Eqs. (30), (33), and (37), with the relative strengths of the operators Eqs. (30), (33), and (37) preserved.

The μ and supersymmetry breaking parameters at m_X then satisfy the pattern of Eqs. (21)–(26) with $A_{Q_3,U_3,D_3,L_3,E_3} = 0$:

$$m^2_{Q_i,U_i,D_i,L_i,E_i} = 0, \tag{38}$$

$$(a_u)_{ij} = (y_u)_{ij} A_{H_u}, \qquad (a_d)_{ij} = (y_d)_{ij} A_{H_d}, \qquad (a_e)_{ij} = (y_e)_{ij} A_{H_d}, \tag{39}$$

$$m^2_{H_u} = -\mu^2, \qquad m^2_{H_d} = -\mu^2, \qquad B = 0, \tag{40}$$

$$M_a \approx \mu \approx A_{H_u} \approx A_{H_d}, \tag{41}$$

where $(y_u)_{ij}$, $(y_d)_{ij}$, and $(y_e)_{ij}$ are the up-type quark, down-type quark, and charged lepton Yukawa matrices, and $(a_u)_{ij}$, $(a_d)_{ij}$, and $(a_e)_{ij}$ the corresponding scalar trilinear interactions. The low-energy superparticle masses are obtained by evolving these parameters from m_X down to the weak scale. This evolution is well approximated by the standard SSM renormalization group equations. Since both μ and B at the weak scale are the same order as the other soft masses, the μ ($B\mu$) problem is solved.

The pattern in Eqs. (38)–(41) resembles that of gaugino mediation with a low compactification scale, or standard gauge mediation with a very large number of messenger fields. These theories, however, lead to a Landau pole for the standard model gauge couplings below the unification scale, and thus are not compatible with perturbative gauge coupling unification. Our theory is fully compatible with perturbative gauge coupling unification. Moreover, the present scenario leads to particular relations for $m_{H_u}^2$, $m_{H_d}^2$, μ, and B, which can be tested at future collider experiments.

B. Spectrum with a fractional number of messenger fields

We now consider the case that the hidden sector exhibits the dynamics of Case 2, rather than Case 1. This happens, for example, if one or more of the $S^\dagger S$ operators corresponds to a conserved global current(s) of the hidden sector dynamics. In this case, renormalization of the operators Eqs. (30), (31), (33), (35), and (37) below m_f is different from that discussed in the previous subsection. Specifically, at the scale m_X where the hidden sector leaves the conformal fixed point, the operators Eqs. (30), (33), and (37) are suppressed compared with Eqs. (31) and (35). This leads to a split spectrum

$$m_I^2 \approx B\mu \gg M_a^2 \approx \mu^2 \approx A_{H_u}^2 \approx A_{H_d}^2, \tag{42}$$

at $\mu_R \approx m_X$. The amount of the splitting depends on the explicit model as well as the distance of the conformal running, m_f/m_X.

If the splitting is very large, it leads to an extremely severe fine-tuning for electroweak symmetry breaking. This will then be interesting (only) in the sense of Ref. [18]. One interesting point about obtaining the split spectrum in this way is that the gaugino and Higgsino masses are naturally expected to be the same order, $M_a \approx \mu$.

On the other hand, if the splitting is not so large, the spectrum does not require an extreme fine-tuning, so the scenario may be interesting in the context of weak scale supersymmetry. It shows an interesting feature—the gaugino masses are suppressed compared to the scalar masses, and yet relative values of the gaugino masses, as well as those of the scalar masses, exactly stay as in the standard gauge mediation models. This implies that we effectively obtain a fractional number of messengers

$$N_{\rm mess} < 1, \tag{43}$$

in the standard gauge mediation formula for the gaugino and scalar masses

$$M_a = N_{\rm mess} \frac{g_a^2}{16\pi^2} \frac{F_{\rm mess}}{M_{\rm mess}}, \tag{44}$$

$$m_I^2 = 2N_{\rm mess} \sum_a C_I^a \left(\frac{g_a^2}{16\pi^2}\right)^2 \left|\frac{F_{\rm mess}}{M_{\rm mess}}\right|^2, \tag{45}$$

where $F_{\rm mess}/M_{\rm mess} \approx \lambda(m_f) F_S/m_f$ in our context. This feature of the spectrum can be tested at future collider experiments.

C. Competition with gravity and anomaly mediation

In this subsection we discuss the competition of the gauge mediated contribution with both gravity and anomaly mediation. In order for the predictions in the previous subsections to persist, the former must dominate over both of the latter.

Let us begin by estimating the size of the contributions to the supersymmetry breaking parameters from gravity mediation. There are two classes of contributions for this: those coming from local operators directly connecting the hidden and SSM sector fields and those arising from supergravity terms (the contributions arising from the F-term VEV of the compensator field).[5] The largest contribution for the first class typically comes from local operators of the form

$$\int d^4\theta \frac{1}{M_*^2} S^\dagger S \phi^\dagger \phi, \tag{46}$$

with an $O(1)$ coefficient, where $M_* \sim M_{Pl}$ is the cutoff scale.[6] If S is an elementary singlet, we also have a contribution from

$$\int d^4\theta \frac{1}{M_*} S \phi^\dagger \phi. \tag{47}$$

Which of Eqs. (46) and (47) gives the dominant contribution is then determined by the renormalization group scaling of these operators.

When we run Eq. (46) down to the scale m_X where conformality is broken, we obtain

$$\int d^4\theta \frac{1}{M_*^2} \left(\frac{m_X}{\Lambda_*}\right)^{\alpha_S} Z_S^{-1}(m_X) S^\dagger S \phi^\dagger \phi, \tag{48}$$

where α_S is the *same* exponent that appears in the evolution of the gauge mediated quadratic operators. (If there are multiple exponents due to operator mixing, α_S is the exponent leading to the least amount of damping, $\hat{\alpha}_S$). This gives a contribution to the supersymmetry breaking mass squared of ϕ of size

$$m_{\rm grav}^2 \approx \frac{Z_S^{-1}(m_X)}{M_*^2} \left(\frac{m_X}{\Lambda_*}\right)^{\alpha_S} |F_S|^2. \tag{49}$$

In the case that S is an elementary singlet, Eq. (47) leads to a contribution $m_{\rm grav}^2|_{\rm sing}$, which is given by Eq. (49) with α_S set to zero.

The second class of contributions for gravity mediation arises from supergravity terms, i.e., the F-term VEV of the compensator field Φ. This gives a contribution to the B parameter of the order of the gravitino mass

$$B_{\rm grav} \approx m_{3/2} \approx \frac{|F_S|}{M_{\rm Pl}}. \tag{50}$$

The other soft masses do not arise from this source (the classical contribution from F_Φ). However, we still need $B_{\rm grav} \lesssim m_I$ at low energies for successful electroweak symmetry breaking.

Now we can compare Eqs. (49) and (50) to the contribution from gauge mediation. In the case that $\alpha_S > 0$ (Case 1), we should compare these to the mass scale generated from the operators linear in S. Comparing with Eq. (49) gives

$$\frac{m_{\rm grav}^2}{m_{\rm gauge}^2} \approx \frac{(16\pi^2)^2}{\lambda^2} \frac{m_f^2}{M_*^2} \left(\frac{m_X}{\Lambda_*}\right)^{\alpha_S}, \tag{51}$$

which can easily be small if α_S is $O(1)$ and λ is not too small (i.e., d_S is not too large). Note that the contribution $m_{\rm grav}^2$ is *suppressed*, making it more harmless due to the conformal dynamics. In the case that S is an elementary singlet ($d_S = 1$), we must also consider $m_{\rm grav}^2|_{\rm sing}/m_{\rm gauge}^2$. This, however, can also easily be small, since λ is then expected to be of order unity.

The comparison with Eq. (50), on the other hand, gives

$$\frac{B_{\rm grav}^2}{m_{\rm gauge}^2} \approx \frac{(16\pi^2)^2}{\lambda^2} \frac{m_f^2}{M_{\rm Pl}^2} \left(\frac{\Lambda_*}{m_X}\right)^{2\gamma_S}. \tag{52}$$

This can be smaller than or of $O(1)$, i.e., the B parameter is

[5]The term "gravity mediation" is a misnomer for the first class, as it is not due to gravity. It simply refers to contributions from local operators at a scale M_* of the order of the Planck scale. We, however, stick to this common terminology.

[6]If the field S is an n-body composite operator, the cutoff scale operator is suppressed by $(\Lambda_*/M_*)^{2n-2}$. However, there are lower dimension operators $\int d^4\theta Q^\dagger Q \phi^\dagger \phi/M_*^2$, where Q represents the elementary fields contained in the composite field S. We expect they are matched at the strong scale as $Q^\dagger Q \approx S^\dagger S$. Therefore, we still obtain the operator of the size given in Eq. (46).

not too large, if λ is not too small and $(\Lambda_*/m_X)^{2\gamma_S}$ not too large. As long as Eq. (52) is smaller than or of $O(1)$, the contribution from anomaly mediation $m^2_{\rm anom}$ is always subdominant to $m^2_{\rm gauge}$, since $m_{\rm anom} \approx B_{\rm grav}/16\pi^2$.

In the case that $\alpha_S < 0$ (Case 2), we should compare the gravity and anomaly mediated pieces to that coming from the operators quadratic in S. For the contribution of Eq. (49), we find

$$\frac{m^2_{\rm grav}}{m^2_{\rm gauge}} \approx \frac{(16\pi^2)^2}{\lambda^2}\frac{m_f^2}{M_*^2}\left(\frac{\Lambda_*}{m_f}\right)^{|\alpha_S|}. \tag{53}$$

In this case the sequestering effect coming from α_S actually *enhances* the gravity mediated contribution relative to the gauge mediated contribution, and so the gravity mediated contribution dominates in a much larger portion of parameter space. If $\lambda = O(\Lambda_*/M_*)$ (i.e., $d_S = 2$), for example, and we require $m^2_{\rm grav}/m^2_{\rm gauge} \lesssim 10^{-3}$, then $|\alpha_S| = 1$ implies that $m_f \lesssim 10^{-7}\Lambda_*$. The contribution from $m^2_{\rm grav}|_{\rm sing}$ is always subdominant.

For the contribution of Eq. (50), we obtain

$$\frac{B^2_{\rm grav}}{m^2_{\rm gauge}} \approx \frac{(16\pi^2)^2}{\lambda^2}\frac{m_f^2}{M_{\rm Pl}^2}\left(\frac{\Lambda_*}{m_X}\right)^{2\gamma_S}\left(\frac{m_X}{m_f}\right)^{|\alpha_S|}. \tag{54}$$

This also allows $B^2_{\rm grav}/m^2_{\rm gauge} \lesssim O(1)$. Note that, in contrast to $m^2_{\rm grav}$, $B^2_{\rm grav}$ does not have to be much smaller than the gauge mediated contribution, $m^2_{\rm gauge}$, since it does not contribute to flavor violation. Again, as long as $B^2_{\rm grav}/m^2_{\rm gauge} \lesssim O(1)$, the contribution from anomaly mediation $m^2_{\rm anom}$ is subdominant because $m_{\rm anom} \approx B_{\rm grav}/16\pi^2$.

V. GAUGINO MEDIATION

An important ingredient for the solution to the μ $(B\mu)$ problem discussed in Sec. IVA is to have control over the operators of the form Eq. (4), which lead to A terms (as well as B and m_I^2 terms). Since these operators are not suppressed relative to the gaugino masses, their existence with random $O(1)$ coefficients would lead to large flavor violation at low energies. Gauge mediation allows us to have these operators under control—in minimal gauge mediation (without the dynamics generating μ)—these operators are not generated at the leading order in loop or $F_{\rm mess}/M^2_{\rm mess}$ expansions. We then only have to require that the dynamics generating μ does not induce these operators in such a way that they excessively violate flavor.

The argument above implies that, as long as the operators $\mathcal{O}_A$ in Eq. (4) are under control, the mechanism of Sec. IVA can apply (not necessarily in the context of gauge mediation). Interestingly, many theories in which the $\mathcal{O}_A$ operators are under control have a $B\mu$ problem similar to that in gauge mediation. Consider, for example, the gaugino mediation scenario [5], in which the gauge and Higgs fields propagate in the bulk of an extra dimension. The extra dimension is compactified on an S^1/Z_2 with length L, and the matter fields and hidden sector are localized on different branes. This allows us to control the $\mathcal{O}_A$ operators. Since the supersymmetry breaking field S and matter fields are localized on different branes, there can be no direct interaction between them, including the operators of the form Eq. (4) (taking ϕ to be the matter fields).

The μ and supersymmetry breaking parameters are generated only by the operators of the form Eqs. (1)–(5), localized on the hidden sector brane, with $\phi = H_u, H_d$.[7] Scaling the coefficients of these operators by naive dimensional analysis in higher dimensions [19], the generated μ and $B\mu$ parameters are

$$\mu \approx \frac{16\pi^2}{CM_*L}M_a, \qquad B\mu \approx \frac{(16\pi^2)^2}{C^2M_*L}M_a^2, \tag{55}$$

where M_* is the cutoff scale of the theory, C the group theory factor related to the size of the gauge group, and M_a the gaugino masses. Now, by choosing $M_*L \approx 16\pi^2/C$, we can easily have $\mu \approx M_a$. This is what we would expect if the 5D gauge couplings also follow naive dimensional analysis, since the 4D gauge couplings g_4 are then given by $g_4^2 \approx 16\pi^2/CM_*L \approx O(1)$. However, this gives

$$B = \frac{B\mu}{\mu} \approx \frac{16\pi^2}{C}M_a, \tag{56}$$

which is too large. The origin of this is that since the μ and $B\mu$ operators both contain H_uH_d, they are suppressed by the same volume factor M_*L. This, however, implies that the suppression is canceled out in $B = B\mu/\mu$, so that B is enhanced relative to the other soft masses. This is analogous to the situation in gauge mediation where both μ and $B\mu$ are suppressed by the same one-loop factor. Note that M_*L must be larger than unity in order for the effective theory to make sense, so this will always enhance B relative to μ.

A possible solution to this problem can now be given in the same way as before. Let us consider that the hidden sector becomes strongly interacting at the scale Λ_*, which we take to be close to M_*. Now, if we assume that the strong conformal dynamics realizes Case 1 in Sec. III, then B is suppressed relative to μ at the scale m_X, where the hidden sector leaves the conformal regime. This solves the $B\mu$ problem. The spectrum at m_X is given by Eqs. (38)–(41). The low-energy superparticle masses are then obtained by evolving these parameters down to the weak scale by the SSM renormalization group equations.

In order to solve the $B\mu$ problem in this way, the contribution of Eqs. (38)–(41) must be larger than or at least of the same order as the B parameter arising from gravity mediation $B_{\rm grav} \approx m_{3/2}$. This gives the condition

[7]We assume that the superpotential operators $W \sim H_uH_d$ and SH_uH_d are absent as before.

$$\frac{B_{\rm grav}^2}{m_{\rm gaugino}^2} \approx \frac{16\pi^2}{C}\frac{M_*^2}{M_{\rm Pl}^2}\left(\frac{\Lambda_*}{m_X}\right)^{2\gamma_S} \lesssim O(1), \qquad (57)$$

where we have taken $M_a \approx \sqrt{C}F_S/4\pi M_*$ at $\mu_R \approx \Lambda_*$, following naive dimensional analysis. (We have taken the group theory factor C appearing in loops to be common for all the fields.) This implies that the conformal running distance Λ_*/m_X cannot be large. One application of this mechanism arises when the gauge groups of the standard model are unified into a grand unified group in the higher dimensional bulk, in which case successful gauge coupling unification can be preserved even if the compactification scale L^{-1} is (slightly) below the conventional unification scale [20]. In this case we can take, for example, $C \simeq 5$ and $M_* \approx 10^{17}$ GeV, which allows $\Lambda_*/m_X \approx O(10\text{–}100)$ for $\gamma_S \sim 0.5$, a sufficient energy interval to suppress the B parameter (assuming that the relevant exponent $\hat{\alpha}_S$ is of order unity).

VI. ANOMALY MEDIATION

Anomaly mediation of supersymmetry breaking [3,4] is a subtle quantum effect in which the soft supersymmetry breaking parameters are induced due to the superconformal anomaly. The mediation is due to the presence of the F-component VEV of the Weyl compensator, which is required to cancel the cosmological constant once supersymmetry is broken. The remarkable feature of anomaly mediation is its ultraviolet insensitivity. Namely, no matter how complicated and flavor violating the theory is at high energies, once all supersymmetric thresholds are integrated out, the supersymmetry breaking effects at a given energy scale are determined only by physics at that energy scale, as was shown explicitly in [4,21]. As a result, the flavor changing effects are virtually absent in the soft parameters.

For the anomaly mediated supersymmetry breaking effects to dominate, direct operators that couple the hidden and SSM sector fields in Eqs. (1)–(5) must be suppressed relative to the gravitino mass. (In this context, we assume that the scale M in these operators is close to $M_{\rm Pl}$.) The original proposal in [3] was to physically separate the two sectors in an extra dimension, while that in [4] was to require the absence of elementary singlet fields so that the operators Eqs. (3)–(5) would be suppressed by simple dimensional reasons. The motivation for conformal sequestering was for the purpose of suppressing the direct coupling operators using a four-dimensional conformal field theory [7].

Our new observation that the operators in Eqs. (3)–(5) are suppressed by the wavefunction renormalization makes anomaly mediation possible in an even wider class of hidden sector models than was previously considered. For example, the models of Ref. [22] have gauge singlet fields that acquire F-component VEVs, and can be made superconformal once a sufficient number of extra flavors is added. If the operators linear in the gauge singlet fields are not sequestered, as originally claimed in Ref. [11], they would be dominant over the anomaly mediated contribution. Especially the A parameters from operators in Eq. (4) do not respect flavor in general, and the resulting model would be generically excluded by the flavor physics data. However, these operators actually *are* suppressed, and hence the anomaly mediated contribution dominates despite the presence of singlet fields in the hidden sector.

Depending on the amount of suppression, it is possible that either of the operators Eqs. (1) and (2) or Eqs. (3)–(5) give comparable contributions to the anomaly mediated contribution. Suppose, for example, that the operators of Eq. (1) are generated by gauge mediation and that the hidden sector shows the behavior of Case 2. In this case, if the contribution from these operators are comparable to the anomaly mediated one, then the well-known problem of tachyonic sleptons in anomaly mediation can be solved. In addition, in the minimal supersymmetric standard model (MSSM), gravity mediation gives a too large B parameter: $B_{\rm grav} \approx m_{3/2} \approx 100$ TeV. This may be solved if the μ term of the MSSM is generated by a VEV of a singlet field, or if gauge mediation generates the operator of Eq. (2) at one loop, leading to a large B parameter ($\approx 16\pi^2 m_I \approx$ 100 TeV) that cancels $B_{\rm grav}$ at a percent level.

VII. DISCUSSION AND CONCLUSIONS

In this paper, we have discussed the impact of strong hidden sector dynamics on the soft supersymmetry breaking parameters on general grounds. While the importance of the renormalization effects on the operators quadratic in the hidden sector fields had been known, we have shown they are also important on the operators linear in the hidden sector fields, despite what has been stated in the literature. This observation has implications both on theories of supersymmetry breaking and its mediation, as well as on phenomenology which may be probed in the near future at collider experiments.

In particular, conformal dynamics can sequester *both* scalar and gaugino masses. However, the relative speed of sequestering is not calculable in general, and it is not clear which one is more important at the end of the conformal dynamics in a given model. In the context of gauge mediation models, our result can be summarized as follows. If the scalar masses are suppressed faster than the gaugino masses, we obtain a spectrum that resembles gaugino mediation at a low compactification scale. Unlike genuine gaugino mediation, however, there is no issue with Landau poles before reaching the unification scale. In addition, A terms exist, as well as Higgs soft masses that cancel the μ^2 mass contribution. The gravity mediated contribution that is potentially flavor violating is less harmful than in the case without the conformal dynamics. This case also offer a solution to the outstanding μ ($B\mu$) problem in the supersymmetric standard model. On the other hand, if the gaugino masses are suppressed faster than the scalar masses, the spectrum looks as if the ''num-

ber of messengers'' is less than unity. In this case, the gravity mediated contribution is more harmful than in the case without the conformal dynamics.

In the context of gaugino mediation, the volume suppression factor tends to give $B \approx 16\pi^2 M_a$, which is unacceptable. The same mechanism as in the case of gauge mediation can lead to a solution to the problem. Finally, anomaly mediation may be dominant with conformal sequestering even if the hidden sector has a singlet field with gaugino mass and A term operators, because they are sequestered as well.

We point out, however, that our analysis is limited by the lack of understanding of Kähler potential renormalizations in strongly coupled theories. Not only can we not work out whether the scalar masses or gaugino masses are sequestered more, but we could also worry about operators at even higher dimensions. In this paper, we considered only the lowest dimension operators that can contribute to the soft supersymmetry breaking parameters. However, higher dimension operators, such as those at cubic or quartic orders in the hidden sector fields, may be as important if the strong renormalization effects overcome the naive suppression in power counting when fields acquire VEVs. Without detailed knowledge of the dynamics, we cannot exclude this possibility. In addition, we assumed that the wavefunction renormalization factors are given solely by those in the superconformal limit determined by the R charges. However, realistic theories are necessarily perturbed by relevant operators to break supersymmetry, and it is possible that their impact on the wavefunction factors is anomalously enhanced by strong dynamics. Note that these two issues are related, because one can always redefine the fields such that they do not acquire VEVs, but this will induce new relevant operators into the theory. These effects are not possible near the Banks-Zaks fixed point [23], and hence are impossible to study using perturbation theory.

Once the LHC discovers supersymmetry, and the ILC determines the spectrum of superparticles precisely, it would be exciting to see if it shows any impact of strong hidden sector dynamics. For this program, it will be important to better understand the consequence of strong dynamics on the renormalization of various operators, including higher dimension ones. We hope that our work provides a step towards achieving this goal.

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Note added.—While completing this paper, we received a paper by Roy and Schmaltz [25]. It proposes to solve the $B\mu$ problem in gauge mediation using conformal dynamics of the hidden sector, which overlaps with our discussion in Sec. IV A. They claim to obtain a spectrum identical to minimal gaugino mediation [26], while we find that finite A terms and Higgs soft masses that cancel the μ^2 mass contribution are generic.

APPENDIX A: OPERATOR MIXING

In this appendix we will give some explicit examples of how different Kähler potential operators can mix with one another. In general, there will be certain linear combinations of operators that evolve by power laws with definite exponents α. Some of these linear combinations may contain global symmetry currents of the conformal field theory, and will not be renormalized at all. In the notation of Sec. II, this means that the exponents α precisely cancel the known wavefunction renormalizations that are determined by the R charges of the fields.

Supersymmetric $SU(N_c)$ QCD with $\frac{3}{2}N_c < N_f < 3N_c$ gives an unusually simple example, where N_f is the number of vectorlike flavors. In this theory, there are four linear combinations of quadratic operators one can write down

$$\begin{array}{ll} Q^\dagger Q + \bar{Q}^\dagger \bar{Q}, & Q^\dagger Q - \bar{Q}^\dagger \bar{Q}, \\ Q^\dagger T^a Q, & \bar{Q}^\dagger T^a \bar{Q}. \end{array} \tag{A1}$$

The latter three correspond to the conserved $U(1)_B$, $SU(N_f)_Q$, and $SU(N_f)_{\bar{Q}}$ currents, and hence if these combinations appear in Eq. (1), the operators are not sequestered. On the other hand, the first one corresponds to the $U(1)_A$ current that is anomalous under the strong $SU(N_c)$ dynamics, and therefore runs with an exponent α_A. Unfortunately, we have no means to calculate α_A. In particular, we do not know whether it is positive or negative.

The corresponding situation in the magnetic dual theory [24] is somewhat more complicated. In addition to the dual quarks q, $\bar{q}$, there are mesons M with two indices, and hence there are many more combinations of operators that one can write down. Mixing between operators containing the dual quarks and mesons can happen because this theory has the superpotential coupling $\mathrm{Tr}(M\bar{q}q)$.

We can classify the quadratic operators according to their representation under the $SU(N_f)_Q \times SU(N_f)_{\bar{Q}}$ symmetry of the theory. For example, the operator proportional to

$$\mathrm{Tr}\,(T^a M T^b M^\dagger), \tag{A2}$$

transforms as (adjoint, adjoint) under the flavor group. There are no other quadratic operators of the same symmetry properties, and hence it does not mix with any others. It is not a conserved current and hence is renormalized. Because it does not mix, it renormalizes on its own

with a single exponent. One cannot prove on general grounds that it is suppressed at low energies, but one can do explicit calculations close to the Banks-Zaks fixed point $N_f \approx \frac{3}{2}N_c$.[8] Note that weakly gauging the vectorlike $SU(N_f)$ flavor symmetry [7] would still allow this operator.

The operators

$$\begin{matrix} N_c \mathrm{Tr}(T^a M M^\dagger) + N_f (q^\dagger T^{a*} q), \\ \mathrm{Tr}(T^a M M^\dagger) - (q^\dagger T^{a*} q), \end{matrix} \tag{A3}$$

transform as (adjoint, singlet) under the flavor group. The latter contains a conserved current, and hence is not renormalized (not sequestered). The former, however, does not correspond to a symmetry because of the superpotential coupling, and is hence renormalized. Again, we do not have a general proof, but explicit calculations suggest that it is sequestered, with $\alpha > 0$ at the one-loop level. Note that the former linear combination is the ''eigenvector'' of the mixing only at the one-loop level, while the precise linear combination is unknown at all orders. The situation with the operators in which q's are replaced by $\bar{q}$'s is identical.

Finally, there are three (singlet, singlet) operators

$$\mathrm{Tr}\,(M M^\dagger), \qquad q^\dagger q + \bar{q}^\dagger \bar{q}, \qquad q^\dagger q - \bar{q}^\dagger \bar{q}. \tag{A4}$$

The last one corresponds to the conserved $U(1)_B$ current and hence is not renormalized. The first two operators mix with unknown relative coefficients. Neither of them are conserved currents and hence should be renormalized. At the one-loop level, the eigenvectors of this mixing are

$$\begin{matrix} 2\,\mathrm{Tr}(M M^\dagger) - (q^\dagger q + \bar{q}^\dagger \bar{q}), \\ N_c\,\mathrm{Tr}(M M^\dagger) + N_f (q^\dagger q + \bar{q}^\dagger \bar{q}). \end{matrix} \tag{A5}$$

The latter is sequestered already at the one-loop level with $\alpha > 0$. The former is accidentally conserved at the one-loop level, while it should receive renormalization at higher orders. Therefore, $\alpha < 0$ at the lowest order for this operator. It is not clear at all what the signs of the α exponents are in the strongly coupled situation.

The situation becomes even more complicated in theories with additional matter content, such as the model with an additional adjoint used in Ref. [13].

In general, operators of the same symmetry properties mix and the degree of sequestering (if any) is determined by the eigenvalues of the mixing matrix. Once the theory is strongly coupled, we do not have the techniques to work them out. Even when the sequestering is plausible in theories believed to be infrared attractive, the signs of the exponents α beyond the wavefunction renormalization are incalculable.

[8]Up to three loops, there are no 1PI diagrams that renormalize this operator, and hence the renormalization is given solely in terms of the wavefunction renormalization. There is a 1PI four-loop diagram, which should generate a nonzero exponent α, yet we do not know its sign.

APPENDIX B: GENERATING THE μ TERM IN GAUGE MEDIATION

In this appendix we present one simple way to generate a μ parameter of the same order as the gaugino masses in gauge mediation. We take the messenger superfields f and $\bar{f}$ to transform under the $\mathbf{10} + \mathbf{10}^*$ representation of the $SU(5)_{\mathrm{SM}}$ symmetry containing the standard model gauge group as a subgroup, and introduce the superpotential interactions

$$W = y f f H_u + \bar{y}\, \bar{f}\, \bar{f}\, H_d. \tag{B1}$$

Here, we have imposed a Z_2 parity under which f and $\bar{f}$ are odd while the other fields are even. This has the advantage that mixings between the messenger and matter superfields are forbidden, so that the problem of flavor is not reintroduced.[9] The absence of a tree level μ term is assumed.

The interactions of Eq. (B1) generate operators responsible for the μ and $B\mu$ parameters at one loop. Integrating out f, $\bar{f}$ with the interactions Eq. (B1) generates

$$\mathcal{L} = 3 \int d^4\theta \frac{y \bar{y} \lambda(m_f)^*}{16\pi^2 m_f} S^\dagger H_u H_d + \mathrm{H.c.}, \tag{B2}$$

and

$$\mathcal{L} = 3 \int d^4\theta \frac{y \bar{y} |\lambda(m_f)|^2}{16\pi^2 m_f^2} S^\dagger S H_u H_d + \mathrm{H.c.}, \tag{B3}$$

at the scale m_f, where $\lambda(m_f)$ is defined in Eq. (32). Integrating out f, $\bar{f}$ also generates

$$\mathcal{L} = 3 \int d^4\theta \frac{\lambda(m_f)}{16\pi^2 m_f} S(|y|^2 H_u^\dagger H_u + |\bar{y}|^2 H_d^\dagger H_d) + \mathrm{H.c.} \tag{B4}$$

These operators contribute to A_{H_u}, A_{H_d}, B, $m_{H_u}^2$, and $m_{H_d}^2$. Assuming $y \sim \bar{y} \sim O(1)$, these provide the operators discussed in Sec. IV: Eqs. (33), (35), and (37).

[9]The Z_2 parity makes the lightest messenger particle stable, which may overclose the universe. We can, however, simply assume that the reheating temperature is low enough so that these particles are not produced thermally. Alternatively, we can (slightly) modify the model. For example, we can eliminate Z_2 and introduce messenger matter mixings, whose sizes, however, are controlled by a $U(1)$ flavor symmetry. This modifies the third generation and Higgs mass spectra (c.f. Sec. III). Another possibility is to use a messenger field that is adjoint under $SU(5)_{\mathrm{SM}}$ and even under matter parity. This allows us to avoid the introduction of the flavor problem as well as the cosmological problem, without an additional discrete symmetry.

[1] M. Dine and W. Fischler, Phys. Lett. **110B**, 227 (1982); Nucl. Phys. **B204**, 346 (1982); L. Alvarez-Gaumé, M. Claudson, and M. B. Wise, Nucl. Phys. **B207**, 96 (1982); S. Dimopoulos and S. Raby, Nucl. Phys. **B219**, 479 (1983).

[2] M. Dine, A. E. Nelson, and Y. Shirman, Phys. Rev. D **51**, 1362 (1995); M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D **53**, 2658 (1996).

[3] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999).

[4] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, J. High Energy Phys. 12 (1998) 027.

[5] D. E. Kaplan, G. D. Kribs, and M. Schmaltz, Phys. Rev. D **62**, 035010 (2000); Z. Chacko, M. A. Luty, A. E. Nelson, and E. Ponton, J. High Energy Phys. 01 (2000) 003.

[6] E. Witten, Nucl. Phys. **B188**, 513 (1981).

[7] M. A. Luty and R. Sundrum, Phys. Rev. D **65**, 066004 (2002); **67**, 045007 (2003).

[8] A. G. Cohen, T. S. Roy, and M. Schmaltz, J. High Energy Phys. 02 (2007) 027.

[9] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).

[10] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[11] M. Dine, P. J. Fox, E. Gorbatov, Y. Shadmi, Y. Shirman, and S. D. Thomas, Phys. Rev. D **70**, 045023 (2004).

[12] R. Sundrum, Phys. Rev. D **71**, 085003 (2005); M. Ibe, K. I. Izawa, Y. Nakayama, Y. Shinbara, and T. Yanagida, Phys. Rev. D **73**, 015004 (2006); **73**, 035012 (2006); S. Kachru, L. McAllister, and R. Sundrum, arXiv:hep-th/0703105.

[13] M. Schmaltz and R. Sundrum, J. High Energy Phys. 11 (2006) 011.

[14] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, and P. van Nieuwenhuizen, Nucl. Phys. **B147**, 105 (1979); E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Nucl. Phys. **B212**, 413 (1983).

[15] G. Mack, Commun. Math. Phys. **55**, 1 (1977); M. Flato and C. Fronsdal, Lett. Math. Phys. **8**, 159 (1984); V. K. Dobrev and V. B. Petkova, Phys. Lett. B **162**, 127 (1985).

[16] G. R. Dvali, G. F. Giudice, and A. Pomarol, Nucl. Phys. **B478**, 31 (1996).

[17] H. Murayama and Y. Nomura, Phys. Rev. D **75**, 095011 (2007); Phys. Rev. Lett. **98**, 151803 (2007).

[18] N. Arkani-Hamed and S. Dimopoulos, J. High Energy Phys. 06 (2005) 073; G. F. Giudice and A. Romanino, Nucl. Phys. **B699**, 65 (2004); **B706**, 65(E) (2005).

[19] Z. Chacko, M. A. Luty, and E. Ponton, J. High Energy Phys. 07 (2000) 036; Y. Nomura, Phys. Rev. D **65**, 085036 (2002).

[20] L. J. Hall and Y. Nomura, Phys. Rev. D **64**, 055003 (2001); **65**, 125012 (2002).

[21] E. Boyda, H. Murayama, and A. Pierce, Phys. Rev. D **65**, 085028 (2002).

[22] K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **95**, 829 (1996); K. A. Intriligator and S. D. Thomas, Nucl. Phys. **B473**, 121 (1996).

[23] T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).

[24] N. Seiberg, Nucl. Phys. **B435**, 129 (1995).

[25] T. S. Roy and M. Schmaltz, arXiv:0708.3593.

[26] M. Schmaltz and W. Skiba, Phys. Rev. D **62**, 095005 (2000).