PHYSICAL REVIEW D **77,** 015001 (2008)

Unpolarized and longitudinally polarized lepton forward-backward asymmetries in $B \to K^* l^+ l^-$ decay in the fourth-generation standard model

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(Received 25 July 2007; revised manuscript received 6 November 2007; published 7 January 2008)

We investigate the influence of the fourth-generation quarks on the asymmetries for unpolarized and polarized leptons in $B \to K^* \ell^+ \ell^-$ decay. We obtain that for both (μ, τ) channels the magnitudes of the differential forward-backward (FB) asymmetries and the average forward-backward asymmetries are quite sensitive to the fourth-generation quarks' mass and mixing parameters. We also achieve that, among the double lepton polarization FB asymmetries, just longitudinally polarized leptons' FB asymmetry shows considerable discrepancy with respect to the third-generation standard model. Moreover, we find that longitudinally polarized leptons' FB asymmetry and its variation with new fourth-generation standard model parameters coincide with unpolarized FB when the lepton mass vanishes. The study of FB asymmetries can serve as a good tool to search for new-physics effects, precisely, to search for the fourth-generation quarks (t', b') via its indirect manifestations in the loop diagrams.

DOI: 10.1103/PhysRevD.77.015001 PACS numbers: 12.60.−i, 13.30.−a, 14.20.Mr

I. INTRODUCTION

New physics (NP) can be searched for in two ways: either by raising the available energy at colliders to produce new particles and reveal them directly, or by increasing the experimental precision on certain processes involving standard model (SM) particles as external states. The latter option, the indirect search for NP, should be pursued using processes that are forbidden, i.e., very rare or precisely calculable in the SM. In this respect, flavor changing neutral current (FCNC) processes are among the most powerful probes of NP, since in the SM they cannot arise at tree level, and even at the loop level, they are strongly suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism. Furthermore, in the quark sector they are all calculable in terms of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and, in particular, concerning $\bar{\rho}$ and $\bar{\eta}$ in the generalized Wolfenstein parametrization [1]. Unfortunately, in many cases a deep understanding of hadronic dynamics is required in order to be able to extract the relevant short-distance information from measured processes. Lattice QCD and QCD sum rules allow us to compute the necessary hadronic parameters in many processes. Indeed, the unitarity triangle analysis (UTA) with lattice QCD input is extremely successful in determining $\bar{\rho}$ and $\bar{\eta}$, and in constraining NP contributions [2–6].

It is well known that FCNC and *CP* violation are, indeed, the most sensitive probes of NP contributions to penguin operators. One of the possible extensions of the SM is the standard model with more than three generations. Nothing in the standard model itself fixes the number of quarks and leptons. While the up/down quarks are first generation quarks, the electron and e-neutrino are first-generation leptons. Since the first three generations are full, any new quarks and leptons would be members of a "fourth generation." In this sense, the SM may be treated as an effective theory of fundamental interactions rather than fundamental particles. The democratic mass matrix approach [7] is considered quite natural in the SM framework. It is interesting that flavors democracy favors the existence of the fourth SM family [8–10]. Any study related to the decay of fourth-generation quarks or indirect effects of those in FCNC requires the choice of the quark masses and mixings which are not free parameters, but rather they are constrained by the experimental value of ρ and S parameters [10]. The ρ parameter, in terms of the transverse part of the W- and Z-boson self-energies at zero momentum transfer, is given in [11],

$$\rho = \frac{1}{1 - \Delta\rho}; \qquad \Delta\rho = \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2}. \quad (1)$$

The common mass of the fourth quark ($m_{t'}$) lies between 320 GeV and 730 GeV, considering the experimental value of $\rho = 1.0002^{+0.0007}_{-0.0004}$ [12]. The last value is close to the upper limit on heavy quark masses, $m_q \leq 700$ GeV $\approx 4m_t$, which follows from partial-wave unitarity at high energies [13]. It should be noted that with the preferable value $a \approx g_w$ flavor democracy predicts $m_{t'} \approx 8m_w \approx 640$ GeV. The above-mentioned values for mass of $m_{t'}$ disfavors the fifth SM family since we expect that $m_t \leq m_{t'} \leq m_t''$ and also the experimental values of the ρ and S parameters [10] restrict the quark mass up to 700 GeV.

The study of production, decay channels, and CERN LHC signals of the fourth-generation quarks has been continuing. But, one of the efficient ways to establish the existence of the fourth-generation quarks is via their indirect manifestations in loop diagrams. Rare decays, induced by FCNC $b \to s(d)$ transitions, are at the forefront of our

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search to understand flavor and the origins of CP violation, offering one of the best probes for NP beyond the SM. Several hints for NP have emerged in the past few years. For instance, a large incompatibility is observed in direct CP asymmetries in $B \to K\pi$ decays [14],

$$\mathcal{A}_{K\pi} \equiv A_{CP}(B^0 \to K^+\pi^-) = -0.093 \pm 0.015,$$
$$\mathcal{A}_{K\pi^0} \equiv A_{CP}(B^+ \to K^+\pi^0) = +0.047 \pm 0.026, \tag{2}$$

or $\Delta\mathcal{A}_{K\pi} \equiv \mathcal{A}_{K\pi^0} - \mathcal{A}_{K\pi} = (14 \pm 3)\%$ [15]. As this percentage was not anticipated when first measured in 2004, it has stimulated discussion on the potential mechanisms which is not included in the third-generation standard model (SM3) calculations [16–18].

The $b \to s(d)\ell^+\ell^-$ decays have received considerable attention as a potential testing ground for the effective Hamiltonian describing FCNC in B and Λ_b decays. This Hamiltonian contains the one-loop effects of the electroweak interaction, which are sensitive to the quarks contributing to the loop [19–21]. In addition, there are important QCD corrections, which have recently been calculated in the next-to-next-to leading order [22]. Moreover, $b \to s(d)\ell^+\ell^-$ decays are also very sensitive to the new physics beyond SM3. New-physics effects manifest themselves in rare decays in two different ways: either through new combinations to the Wilson coefficients or through the new operator structure in the effective Hamiltonian, which is absent in the SM3. A significant issue in the new-physics search within flavor physics is the optimal separation of new-physics effects from uncertainties. It is well known that inclusive decay modes are dominated by partonic contributions; nonperturbative corrections are, in general, rather small [23]. Also, ratios of exclusive decay modes such as asymmetries for $B \to K(K^*, \rho, \gamma)\ell^+\ell^-$ decays [24–33] have already been studied for new-physics search. Here, large parts of the hadronic uncertainties are partially removed.

In this paper, we study the possibility of searching for new physics in the meson decays $B \to K^*\ell^+\ell^-$ using the SM with four generations of quarks (b', t'). The fourth quark (t'), like u, c, t quarks, contributes to the $b \to s(d)$ transition at loop level. Clearly, it would change the branching ratio and asymmetries. Note that fourth-generation effects on the branching ratio have been widely studied in baryonic and semileptonic B decays [34–47]. But, there are few works (lepton polarization asymmetries in $\Lambda_b \to \Lambda l^+ l^-$ [48,49]) related to the study of asymmetries either in heavy baryon to light baryon decay or in various B decay channels.

The main problem for the description of exclusive decays is to evaluate the form factors, i.e., matrix elements of the effective Hamiltonian between initial and final hadron states. It is well known that, in order to describe $B \to K^*\ell^+\ell^-$ decay, a number of form factors are needed (see for example [27]).

It should be mentioned here that the exclusive $B \to K^*\ell^+\ell^-$ decay rate, lepton polarization, and CP asymmetry are studied widely in the SM and beyond the SM, i.e., [27,39].

The sensitivity of the CP asymmetry to the existence of the fourth-generation quarks in the $B \to K^*\ell^+\ell^-$ decay is investigated in [39], and we obtain that the CP asymmetry is quite sensitive to the fourth-generation parameters ($m_{t'}$, $V_{t'b}V^*_{t's}$). In this connection, it is natural to ask whether unpolarized and polarized lepton pair forward-backward (FB) asymmetries are sensitive to the fourth-generation parameters, in the same decay. In the present study, we try to answer this question.

The paper is organized as follows: In Sec. II, using the effective Hamiltonian, the general expressions for the matrix element of $B \to K^*\ell^+\ell^-$ decay are derived. Section III is devoted to calculations of forward-backward asymmetries. In Sec. IV, we analyze the sensitivity of these functions to the fourth-generation parameters ($m_{t'}$, r_{sb}, ϕ_{sb}).

II. STRATEGY

With a sequential fourth generation, the Wilson coefficients C_7, C_9, and C_{10} receive contributions from the t' quark loop, which we will denote as $C^{\rm new}_{7,9,10}$. Because a sequential fourth generation couples in a similar way to the photon and W, the effective Hamiltonian relevant for $b \to s\ell^+\ell^-$ decay has the following form:

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V^*_{ts} \sum_{i=1}^{10} \mathcal{C}_i(\mu)\mathcal{O}_i(\mu), \tag{3}$$

where the full set of the operators $\mathcal{O}_i(\mu)$ and the corresponding expressions for the Wilson coefficients $\mathcal{C}_i(\mu)$ in the SM are given in [50–52]. As it has already been noted, the fourth-generation up-type quark t' is introduced in the same way as u, c, t quarks in the SM, and so new operators do not appear and clearly the full operator set is exactly the same as in the SM. The fourth generation changes the values of the Wilson coefficients $C_7(\mu)$, $C_9(\mu)$, and $C_{10}(\mu)$, via virtual exchange of the fourth-generation up-type quark t'. The above-mentioned Wilson coefficients will explicitly change as

$$\lambda_t C_i \to \lambda_t C_i^{\rm SM} + \lambda_{t'} C_i^{\rm new}, \tag{4}$$

where $\lambda_f = V^*_{fb}V_{fs}$. The unitarity of the 4×4 CKM matrix leads to

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \tag{5}$$

Since $\lambda_u = V^*_{ub}V_{us}$ is very small in strength compared to the others, then we can rewrite Eq. (5) as

$$\lambda_t = -\lambda_c - \lambda_{t'}$$

where $\lambda_c = V^*_{cb}V_{cs} \approx 0.04$, which is real by convention. It follows that

$$\lambda_t C_i^{\rm SM} + \lambda_{t'} C_i^{\rm new} = \lambda_c C_i^{\rm SM} + \lambda_{t'}(C_i^{\rm new} - C_i^{\rm SM}). \tag{6}$$

It is clear that, for $m_{t'} \to m_t$ or $\lambda_{t'} \to 0$, the $\lambda_{t'}(C_i^{\rm new} - C_i^{\rm SM})$ term vanishes, as required by the GIM mechanism. One can also write C_i's in the following form:

$$C_7^{\rm tot}(\mu) = C_7^{\rm SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_7^{\rm new}(\mu),$$
$$C_9^{\rm tot}(\mu) = C_9^{\rm SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_9^{\rm new}(\mu), \qquad (7)$$
$$C_{10}^{\rm tot}(\mu) = C_{10}^{\rm SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_{10}^{\rm new}(\mu),$$

where the last terms in these expressions describe the contributions of the t' quark to the Wilson coefficients. $\lambda_{t'}$ can be parametrized as

$$\lambda_{t'} = V_{t'b}^* V_{t's} = r_{sb} e^{i\phi_{sb}}. \qquad (8)$$

In deriving Eq. (7), we factored out the term $V_{tb}^* V_{ts}$ in the effective Hamiltonian given in Eq. (3). The explicit forms of the $C_i^{\rm new}$ can easily be obtained from the corresponding expression of the Wilson coefficients in the SM by substituting $m_t \to m_{t'}$ (see [50,51]). If the s quark mass is neglected, the above effective Hamiltonian leads to the following matrix element for the $b \to s\ell^+\ell^-$ decay:

$$\mathcal{H}_{\rm eff} = \frac{G\alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[C_9^{\rm tot} \bar{s}\gamma_\mu (1-\gamma_5) b \bar{\ell}\gamma_\mu \ell + C_{10}^{\rm tot} \bar{s}\gamma_\mu(1-\gamma_5) b \bar{\ell}\gamma_\mu\gamma_5 \ell - 2C_7^{\rm tot} \frac{m_b}{q^2} \bar{s}\sigma_{\mu\nu} q^\nu (1+\gamma_5) b \bar{\ell}\gamma_\mu \ell \Big], \qquad (9)$$

where $q^2 = (p_+ + p_-)^2$ and p_- and p_+ are the final leptons' four-momenta. The effective coefficient $C_9^{\rm tot}$ can be written in the following form:

$$C_9^{\rm tot} = C_9 + Y(s'), \qquad (10)$$

where $s' = q^2/m_b^2$ and the function $Y(s')$ denotes the perturbative part coming from one-loop matrix elements of four-quark operators and is given [50,52] as

$$Y_{\rm per}(s') = g(\hat{m}_c, s')(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \tfrac{1}{2} g(1, s')(4C_3 + 4C_4 + 3C_5 + C_6) - \tfrac{1}{2} g(0, s') \times (C_3 + 3C_4) + \tfrac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \qquad (11)$$

where $\hat{m}_c = \frac{m_c}{m_b}$. The explicit expressions for the g functions and the values of various Wilson coefficients (C_i) in the SM are given in [50,52].

In addition to the short-distance contribution, $Y_{\rm per}(s')$ also gets long-distance contributions, resulting from the real $c\bar{c}$ intermediate states, i.e., J/ψ, ψ', ... The J/ψ family is described by the Breit-Wigner distribution for the resonances by the following substitution [53–55]:

$$Y(s') = Y_{\rm per}(s') + \frac{3\pi}{\alpha^2} C^{(0)} \sum_{V_i = \psi_i} \kappa_i \frac{m_{V_i}\Gamma(V_i \to \ell^+\ell^-)}{m_{V_i}^2 - s' m_b^2 - i m_{V_i}\Gamma_{V_i}}, \qquad (12)$$

where $C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$. The phenomenological parameters κ_i can be fixed by the hadronic and semileptonic B decays, where the data for the right-hand side are given in [56]. For the lowest resonances J/ψ and ψ', $\kappa = 1.65$ and $\kappa = 2.36$ are used, respectively (see [57]).

After having an idea of the effective Hamiltonian and the relevant Wilson coefficients in the fourth-generation standard model (SM4), we now proceed to evaluate the transition matrix elements for the process $B(p_B) \to K^*(p_{K^*}) l^+(p_+) l^-(p_-)$. It follows from Eq. (9) that, in order to calculate the decay width and other physical observables of the exclusive $B \to K^*\ell^+\ell^-$ decay, the matrix elements $\langle K^* | \bar{s}\gamma_\mu(1-\gamma_5) b | B\rangle$ and $\langle K^* | \bar{s} i \sigma_{\mu\nu} q^\nu (1+\gamma_5) b | \rangle$ have to be calculated. In other words, the exclusive $B \to K^*\ell^+\ell^-$ decay, which is described in terms of the matrix elements of the quark operators given in Eq. (9) over meson states, can be parametrized in terms of form factors. For the vector meson K^* with the polarization vector ε_μ, the semileptonic form factors of the V-A current are defined as

$$\langle K^*(p,\varepsilon) | \bar{s}\gamma_\mu(1-\gamma_5) b | B(p_B)\rangle = -\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu} p^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} - i\varepsilon_\mu (m_B + m_{K^*}) A_1(q^2) + i(p_B + p_{K^*})_\mu (\varepsilon^* q) \times \frac{A_2(q^2)}{m_B + m_{K^*}} + i q_\mu \frac{2m_{K^*}}{q^2} (\varepsilon^* q)[A_3(q^2) - A_0(q^2)], \qquad (13)$$

where ε is the polarization vector of the K^* meson and $q = p_B - p_{K^*}$ is the momentum transfer. Using the equation of motion, the form factor $A_3(q^2)$ can be written in terms of the form factors $A_1(q^2)$ and $A_2(q^2)$ as follows:

$$A_3 = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1 - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2. \qquad (14)$$

In order to ensure the finiteness of (8) at $q^2 = 0$, we require $A_3(q^2 = 0) = A_0(q^2 = 0)$. The semileptonic form factors coming from the dipole operator $\sigma_{\mu\nu} q^\nu (1+\gamma_5) b$ are defined as

$$\langle K^*(p,\varepsilon) | \bar{s} i\sigma_{\mu\nu} q^\nu (1+\gamma_5) b | B(p_B)\rangle = 4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu} p^\rho q^\sigma T_1(q^2) + 2i[\varepsilon_\mu^*(m_B^2 - m_{K^*}^2) - (p_B + p_{K^*})_\mu (\varepsilon^* q)] T_2(q^2) + 2i(\varepsilon^* q)\Big[q_\mu - (p_B + p_{K^*})_\mu \frac{q^2}{m_B^2 - m_{K^*}^2}\Big] T_3(q^2). \qquad (15)$$

From Eqs. (7), (9), and (10) we observe that in calculating the physical observable at hadronic level, i.e., for the $B \to K^*\ell^+\ell^-$ decay, we face the problem of computing the form factors. This problem is related to the nonperturbative sector of QCD and it can be solved only in the framework of a nonperturbative approach. In the present work, we choose light cone QCD sum rule results for the form factors. Then, we will use the results of the works [58,59] in which the form factors are described by a three-parameter fit where the radiative corrections up to the leading twist contribution and SU(3)-breaking effects are taken into account. Considering

$$F(q^2) \in \{V(q^2), A_0(q^2), B_0(q^2), A_2(q^2), A_3(q^2), T_1(q^2), T_2(q^2), T_3(q^2)\},$$

the q^2 dependence of any of these form factors could be parametrized as [58,59]

$$F(s) = \frac{F(0)}{1 - a_F s + b_F s^2}, \tag{16}$$

TABLE I. The form factors for $B \to K^* \ell^+ \ell^-$ in a three-parameter fit [58].

	$F(0)$	a_F	b_F
$A_0^{B\to K^*}$	0.47	1.64	0.94
$A_1^{B\to K^*}$	0.35	0.54	−0.02
$A_2^{B\to K^*}$	0.30	1.02	0.08
$V^{B\to K^*}$	0.47	1.50	0.51
$T_1^{B\to K^*}$	0.19	1.53	1.77
$T_1^{B\to K^*}$	0.19	0.36	−0.49
$T_3^{B\to K^*}$	0.13	1.07	0.16

where the parameters $F(0)$, a_F, and b_F are listed in Table I for each form factor.

Using the form factors, the matrix element of the $B \to K^* \ell^+ \ell^-$ decay can be given as follows:

$$\begin{aligned}\mathcal{M}(B \to K^* \ell^+ \ell^-) = \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \{&\bar{\ell}\gamma^\mu(1-\gamma_5)\ell[-2B_0 \epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu} p_{K^*}^\lambda q^\sigma - iB_1 \varepsilon_\mu^* + iB_2(\varepsilon^* q)(p_B + p_{K^*})_\mu \\ &+ iB_3(\varepsilon^* q) q_\mu] + \bar{\ell}\gamma^\mu(1+\gamma_5)\ell[-2C_1 \epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu} p_{K^*}^\lambda q^\sigma - iD_1 \varepsilon_\mu^* \\ &+ iD_2(\varepsilon^* q)(p_B + p_{K^*})_\mu + iD_3(\varepsilon^* q) q_\mu]\},\end{aligned} \tag{17}$$

where

$$\begin{aligned}B_0 &= (C_9^{\rm tot} - C_{10}^{\rm tot})\frac{V}{m_B + m_{K^*}} + 4(m_b + m_s)C_7^{\rm tot}\frac{T_1}{q^2}, \\ B_1 &= (C_9^{\rm tot} - C_{10}^{\rm tot})(m_B + m_{K^*})A_1 + 4(m_b - m_s)C_7^{\rm tot}(m_B^2 - m_{K^*}^2)\frac{T_2}{q^2}, \\ B_2 &= \frac{C_9^{\rm tot} - C_{10}^{\rm tot}}{m_B + m_{K^*}}A_2 + 4(m_b - m_s)C_7^{\rm tot}\frac{1}{q^2}\left[T_2 + \frac{q^2}{m_B^2 - m_{K^*}^2}T_3\right], \\ B_3 &= 2(C_9^{\rm tot} - C_{10}^{\rm tot})m_{K^*}\frac{A_3 - A_0}{q^2} - 4(m_b - m_s)C_7^{\rm tot}\frac{T_3}{q^2}, \\ C_1 &= B_0(C_{10}^{\rm tot} \to -C_{10}^{\rm tot}), \qquad D_i = B_i(C_{10}^{\rm tot} \to -C_{10}^{\rm tot}), \qquad (i = 1, 2, 3).\end{aligned}$$

From this expression of the decay amplitude, for the differential decay width, we get the following result:

$$\frac{d\Gamma}{d\hat{s}}(B \to K^* \ell^+ \ell^-) = \frac{G^2 \alpha^2 m_B}{2^{14}\pi^5}|V_{tb}V_{ts}^*|^2 \lambda^{1/2}(1, \hat{r}, \hat{s}) v \Delta(\hat{s}), \tag{18}$$

with

$$\begin{aligned}\Delta = \frac{2}{3\hat{r}_{K^*}\hat{s}} m_B^2 {\rm Re}[&-12 m_B^2 \hat{m}_l{}^2 \lambda \hat{s}\{(B_3 - D_2 - D_3)B_1^* - (B_3 + B_2 - D_3)D_1^*\} + 12 m_B^4 \hat{m}_l{}^2 \lambda \hat{s}(1 - \hat{r}_{K^*})(B_2 - D_2)(B_3^* - D_3^*) \\ &+ 48 \hat{m}_l{}^2 \hat{r}_{K^*}\hat{s}(3B_1 D_1^* + 2m_B^4 \lambda B_0 C_1^*) - 16 m_B^4 \hat{r}_{K^*}\hat{s}\lambda(\hat{m}_l{}^2 - \hat{s})\{|B_0|^2 + |C_1|^2\} - 6 m_B^4 \hat{m}_l{}^2 \lambda \hat{s}\{2(2 + 2\hat{r}_{K^*} - \hat{s})B_2 D_2^* \\ &- \hat{s}|(B_3 - D_3)|^2\} - 4 m_B^2 \lambda\{\hat{m}_l{}^2(2 - 2\hat{r}_{K^*} + \hat{s}) + \hat{s}(1 - \hat{r}_{K^*} - \hat{s})\}(B_1 B_2^* + D_1 D_2^*) + \hat{s}\{6\hat{r}_{K^*}\hat{s}(3 + v^2) \\ &+ \lambda(3 - v^2)\}\{|B_1|^2 + |D_1|^2\} - 2m_B^4 \lambda\{\hat{m}_l{}^2[\lambda - 3(1 - \hat{r}_{K^*})^2] - \lambda \hat{s}\}\{|B_2|^2 + |D_2|^2\}],\end{aligned} \tag{19}$$

where $\hat{s} = q^2/m_B^2$, $\hat{r}_{K^*} = m_{K^*}^2/m_B^2$ and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, $\hat{m}_\ell = m_\ell/m_B$, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$.

The definition of the unpolarized and polarized FB asymmetries will be presented in the next section.

III. UNPOLARIZED AND POLARIZED FORWARD-BACKWARD ASYMMETRIES OF LEPTONS

In order to calculate the polarization asymmetries of both the leptons defined in the effective four-fermion interaction of Eq. (17), we must first define the orthogonal vectors S in the rest frame of ℓ^- and W in the rest frame of ℓ^+ (where these vectors are the polarization vectors of the leptons). Note that we will use the subscripts L, N, and T to correspond to the leptons which are polarized along the longitudinal, normal, and transverse directions, respectively [60–62].

$$S_L^\mu \equiv (0, \mathbf{e_L}) = \left(0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|}\right),$$
$$S_N^\mu \equiv (0, \mathbf{e_N}) = \left(0, \frac{\mathbf{p_{K^*}} \times \mathbf{p}_-}{|\mathbf{p_{K^*}} \times \mathbf{p}_-|}\right), \tag{20}$$
$$S_T^\mu \equiv (0, \mathbf{e_T}) = (0, \mathbf{e_N} \times \mathbf{e}_L),$$

$$W_L^\mu \equiv (0, \mathbf{w}_L) = \left(0, \frac{\mathbf{p}_+}{|\mathbf{p}_+|}\right),$$
$$W_N^\mu \equiv (0, \mathbf{w_N}) = \left(0, \frac{\mathbf{p_{K^*}} \times \mathbf{p}_+}{|\mathbf{p_{K^*}} \times \mathbf{p}_+|}\right), \tag{21}$$
$$W_T^\mu \equiv (0, \mathbf{w_T}) = (0, \mathbf{w_N} \times \mathbf{w}_L),$$

where $\mathbf{p}_+$, $\mathbf{p}_-$, and $\mathbf{p_{K^*}}$ are the three-momenta of the ℓ^+, ℓ^-, and K^* particles, respectively. On boosting the vectors defined by Eqs. (20) and (21) to the center of mass (CM) frame of the $\ell^-\ell^+$ system, only the longitudinal vector will be boosted, while the other two remain the same. The longitudinal vectors in the CM frame of the $\ell^-\ell^+$ system will become

$$S_L^\mu = \left(\frac{|\mathbf{p}_-|}{m_\ell}, \frac{E_\ell \mathbf{p}_-}{m_\ell |\mathbf{p}_-|}\right), \qquad W_L^\mu = \left(\frac{|\mathbf{p}_-|}{m_\ell}, -\frac{E_\ell \mathbf{p}_-}{m_\ell |\mathbf{p}_-|}\right). \tag{22}$$

The polarization asymmetries can now be calculated using the spin projector $\frac{1}{2}(1 + \gamma_5 \not{S})$ for ℓ^- and the spin projector $\frac{1}{2}(1 + \gamma_5 \not{W})$ for ℓ^+.

Regarding the above expressions, we now define the various forward-backward asymmetries of leptons. The definition of the unpolarized and normalized differential forward-backward asymmetry is (see for example [63])

$$\mathcal{A}_{\rm FB} = \frac{\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} - \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz}}{\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} + \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz}}, \tag{23}$$

where $z = \cos\theta$ is the angle between the B meson and ℓ^- in the center of mass frame of leptons.

Equipped with the above definition $\mathcal{A}_{\rm FB}$ can be obtained as

$$\mathcal{A}_{\rm FB} = 16 m_B^4 \hat{s} \frac{v\sqrt{\lambda}}{\Delta} \{\mathrm{Re}[B_0 D_1^*] - \mathrm{Re}[B_1^* C_1]\}. \tag{24}$$

When the spins of both leptons are taken into account, the $\mathcal{A}_{\rm FB}$ will be a function of the spins of the final leptons, and it is defined as

$$\begin{aligned}\mathcal{A}_{\rm FB}^{ij}(\hat{s}) &= \left(\frac{d\Gamma(\hat{s})}{d\hat{s}}\right)^{-1}\left\{\int_0^1 dz - \int_{-1}^0 dz\right\}\left\{\left[\frac{d^2\Gamma(\hat{s}, \vec{s}^- = \vec{i}, \vec{s}^+ = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^- = \vec{i}, \vec{s}^+ = -\vec{j})}{d\hat{s}dz}\right]\right. \\ &\quad \left. - \left[\frac{d^2\Gamma(\hat{s}, \vec{s}^- = -\vec{i}, \vec{s}^+ = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^- = -\vec{i}, \vec{s}^+ = -\vec{j})}{d\hat{s}dz}\right]\right\}, \\ &= \mathcal{A}_{FB}(\vec{s}^- = \vec{i}, \vec{s}^+ = \vec{j}) - \mathcal{A}_{\rm FB}(\vec{s}^- = \vec{i}, \vec{s}^+ = -\vec{j}) - \mathcal{A}_{\rm FB}(\vec{s}^- = -\vec{i}, \vec{s}^+ = \vec{j}) + \mathcal{A}_{\rm FB}(\vec{s}^- = -\vec{i}, \vec{s}^+ = -\vec{j}),\end{aligned} \tag{25}$$

where the subindices i and j can be either L, N, or T. Using these definitions for the double polarized FB asymmetries, we get the following results:

$$A_{\rm FB}^{LL} = 16 m_B^4 \hat{s} \frac{v\sqrt{\lambda}}{\Delta} \mathrm{Re}[(B_0 B_1^* - C_1 D_1^*)], \tag{26}$$

$$\begin{aligned}A_{\rm FB}^{LN} &= \frac{8}{3\hat{r}_{K^*}\Delta\hat{s}} m_B^2 \sqrt{\hat{s}} \lambda v\, \mathrm{Im}[-\hat{m}_l(B_1 D_1^* + m_B^4 \lambda B_2 D_2^*) \\ &\quad + 4 m_B^4 \hat{m}_l \hat{r}_{K^*} \sqrt{\hat{s}} B_0 C_1^* + m_B^2 \hat{m}_l (1 - \hat{r}_{K^*} - \hat{s}) \\ &\quad \times (B_1 D_2^* + B_2 D_1^*)],\end{aligned} \tag{27}$$

$$\begin{aligned}A_{\rm FB}^{NL} &= \frac{8}{3\hat{r}_{K^*}\Delta\hat{s}} m_B^2 \sqrt{\hat{s}} \lambda v\, \mathrm{Im}[-\hat{m}_l(B_1 D_1^* + m_B^4 \lambda B_2 D_2^*) \\ &\quad + 4 m_B^4 \hat{m}_l \hat{r}_{K^*} \sqrt{\hat{s}} B_0 C_1^* + m_B^2 \hat{m}_l (1 - \hat{r}_{K^*} - \hat{s}) \\ &\quad \times (B_1 D_2^* + B_2 D_1^*)],\end{aligned} \tag{28}$$

$$\begin{aligned}A_{\rm FB}^{LT} &= \frac{4}{3\hat{r}_{K^*}\Delta\hat{s}} m_B^2 \sqrt{\hat{s}} \lambda\, \mathrm{Re}[-\hat{m}_l\{|B_1 + D_1|^2 \\ &\quad + m_B^4 \lambda |B_2 + D_2|^2\} + 4 m_B^4 \hat{m}_l \hat{s} \hat{r}_{K^*}\{|B_0 + C_1|^2\} \\ &\quad + 2 m_B^2 \hat{m}_l (1 - \hat{r}_{K^*} - \hat{s})(B_1 + D_1)(B_2^* + D_2^*)],\end{aligned} \tag{29}$$

$$\begin{aligned}A_{\rm FB}^{TL} &= \frac{4}{3\hat{r}_{K^*}\Delta\hat{s}} m_B^2 \sqrt{\hat{s}} \lambda\, \mathrm{Re}[\hat{m}_l\{|B_1 + D_1|^2 \\ &\quad + m_B^4 \lambda |B_2 + D_2|^2\} - 4 m_B^4 \hat{m}_l \hat{s} \hat{r}_{K^*}\{|B_0 + C_1|^2\} \\ &\quad - 2 m_B^2 \hat{m}_l (1 - \hat{r}_{K^*} - \hat{s})(B_1 + D_1)(B_2^* + D_2^*)],\end{aligned} \tag{30}$$

$$A_{\rm FB}^{NT} = \frac{2}{\hat{r}_{K^*}\Delta\hat{s}} m_B^2\sqrt{\lambda} Im[-2m_B^4\hat{m}_l{}^2\hat{s}\lambda(B_2+D_2)(B_3^*-D_3^*) + 4m_B^4\hat{m}_l{}^2\lambda(1-\hat{r}_{K^*})B_2D_2^* + 2m_B^2\hat{m}_l{}^2\hat{s}(1+3\hat{r}_{K^*}-\hat{s})(B_1B_2^*-D_1D_2^*) + \hat{m}_l(1-\hat{r}_{K^*}-\hat{s})\{+2\hat{s}m_B^2\hat{m}_l(B_1+D_1)(B_3^*-D_3^*) + 4\hat{m}_lB_1D_1^*\} + 2m_B^2\hat{m}_l{}^2[\lambda+(1-\hat{r}_{K^*}-\hat{s})(1-\hat{r}_{K^*})](B_1^*D_2+B_2^*D_1)], \quad (31)$$

$$A_{\rm FB}^{TN} = \frac{2}{\hat{r}_{K^*}\Delta\hat{s}} m_B^2\sqrt{\lambda}\, {\rm Im}[-2m_B^4\hat{m}_l{}^2\hat{s}\lambda(B_2+D_2)(B_3^*-D_3^*) + 4m_B^4\hat{m}_l{}^2\lambda(1-\hat{r}_{K^*})B_2D_2^* + 2m_B^2\hat{m}_l{}^2\hat{s}(1+3\hat{r}_{K^*}-\hat{s})(B_1B_2^*-D_1D_2^*) + \hat{m}_l(1-\hat{r}_{K^*}-\hat{s})\{-2\hat{s}m_B^2\hat{m}_l(B_1+D_1)(B_3^*-D_3^*) + 4\hat{m}_lB_1D_1^*\} + 2m_B^2\hat{m}_l{}^2[\lambda+(1-\hat{r}_{K^*}-\hat{s})(1-\hat{r}_{K^*})](B_1^*D_2+B_2^*D_1)], \quad (32)$$

$$A_{\rm FB}^{NN} = 0, \quad (33)$$

$$A_{\rm FB}^{TT} = 0. \quad (34)$$

IV. NUMERICAL ANALYSIS

The dependence of the unpolarized and polarized FB symmetry on the fourth quark mass ($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^*V_{t's} = r_{sb}e^{i\phi_{sb}}$) are studied. The input parameters we use in our numerical calculations are as follows: $|V_{tb}V_{ts}^*| = 0.0385$, $m_{K^*} = 0.892$ GeV, $m_\tau = 1.77$ GeV, $m_\mu = 0.106$ GeV, $m_b = 4.8$ GeV, $m_B = 5.26$ GeV, and $\Gamma_B = 4.22\times10^{-13}$ GeV.

For the values of the Wilson coefficients, we use $C_7^{\rm SM} = -0.313$, $C_9^{\rm SM} = 4.344$, and $C_{10}^{\rm SM} = -4.669$. It should be noted that the above-presented value for $C_9^{\rm SM}$ corresponds only to short-distance contributions. In addition to the short-distance contributions, it receives long-distance contributions, resulting from the conversion of $\bar{c}c$ to the lepton pair. In this study, we neglect long-distance contributions. The reason for such a choice is dictated by the fact that in the SM the zero position of $\mathcal{A}_{\rm FB}$ for the $B\to K^*\ell^+\ell^-$ decay is practically independent of the form factors and is determined in terms of short-distance Wilson coefficients $C_9^{\rm SM}$ and $C_7^{\rm SM}$ (see [57,64]) and $s_0 = 3.9$ GeV^2. For the form factors, we have used the light cone QCD sum rules results [65,66]. As a result of the analysis carried out in this scheme, the q^2 dependence of the form factors can be represented in terms of three parameters as in (16), where the values of the parameters $F(0)$, a_F, and b_F for the $B\to K^*$ decay are listed in Table I.

In order to perform a quantitative analysis of the total branching ratio and the lepton polarizations, the values of the new parameters ($m_{t'}$, r_{sb}, ϕ_{sb}) are needed. Using the experimental values of $B\to X_s\gamma$ and $B\to X_s\ell^+\ell^-$, the bound on $r_{sb}\sim\{0.01-0.03\}$ has been obtained [39] for $\phi_{sb}\sim\{0-2\pi\}$ and $m_{t'}\sim\{300, 400\}$ (GeV). In our forthcoming study we will examine it in detail. Now, we have obtained that, in the case of the 1σ level deviation from the measured branching ratio, the maximum values of $m_{t'}$ are below the theoretical upper limits. The results are shown in Tables II, III, and IV [49]. In the foregoing numerical analysis, we vary $m_{t'}$ in the range $175\le m_{t'}\le 600$ GeV. The lower range is due to the fact that the fourth-generation up quark should be heavier than the third-generation ones ($m_t\le m_{t'}$) [10]. The upper range comes from the experimental bounds on the ρ and S parameters of the SM, which we mentioned above (see the Introduction).

(i) Except for $\mathcal{A}_{\rm FB}^{LL}$, the values of $\mathcal{A}_{\rm FB}^{ij}$ are quite small for the other components in the SM3 (see Table V), and with the SM4 the deviations are below 2%. Then, the measurement of the above-mentioned components in the experiments could practically be either problematic or impossible (see Table V). For this reason, we do not present the dependencies of $\mathcal{A}_{\rm FB}^{ij}$ on q^2 for different values of SM4 parameters for either the $B\to K^*\mu^+\mu^-$ or $B\to K^*\tau^+\tau^-$ decay.

(ii) There is almost no significant difference between the magnitude of the $\mathcal{A}_{\rm FB}^{LL}$ and ($\mathcal{A}_{\rm FB}$) and their averaged values either in SM3 or in SM4. In other words, $\mathcal{A}_{\rm FB}^{LL}$ coincide with unpolarized FB ($\mathcal{A}_{\rm FB}$) in the standard model and the consequential extension of the SM (SM4). A significant difference between $\mathcal{A}_{\rm FB}^{LL}$ and

TABLE II. The experimental limit on the maximum value of $m_{t'}$ for $\phi_{sb} = \pi/3$.

r_{sb}	0.005	0.01	0.02	0.03
$m_{t'}$ (GeV)	739	529	385	331

TABLE III. The experimental limit on the maximum value of $m_{t'}$ for $\phi_{sb} = \pi/2$.

r_{sb}	0.005	0.01	0.02	0.03
$m_{t'}$ (GeV)	511	373	289	253

TABLE IV. The experimental limit on the maximum value of $m_{t'}$ for $\phi_{sb} = 2\pi/3$.

r_{sb}	0.005	0.01	0.02	0.03
$m_{t'}$ (GeV)	361	283	235	217

TABLE V. The averaged values of unpolarized and polarized lepton FB asymmetries.

$A_{\rm FB}^{ij}(B \to K^* \ell^+ \ell^-)$	μ	τ
$\langle \mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB}) \rangle$	0.201	0.214
$\langle \mathcal{A}_{\rm FB}^{LN} \rangle = \langle \mathcal{A}_{\rm FB}^{NL} \rangle$	0.001	0.014
$\langle \mathcal{A}_{\rm FB}^{LT} \rangle \approx -\langle \mathcal{A}_{\rm FB}^{TL} \rangle$	0.002	0.032
$\langle \mathcal{A}_{\rm FB}^{NT} \rangle \approx \langle \mathcal{A}_{\rm FB}^{TN} \rangle$	0.001	0.016

($\mathcal{A}_{\rm FB}$) happens when the new types of interactions are taken into account in the effective Hamiltonian; i.e., the tensor type and scalar type interactions [27].

(iii) In Figs. 1–18, we present the dependence of the $\mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB})$ on q^2 for $B \to K^* \mu^+ \mu^-$ where $m_{t'}$: 300, 400, 500, 600 GeV, r_{sb}: 0.01, 0.02, 0.03, and ϕ_{sb}: 60°, 90°, 120°, respectively. From these figures, we see that the above-mentioned values of the SM4 parameters slightly shift the zero position of $\mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB})$ corresponding to the SM3 result. In other words, our analysis shows that the zero position of $\mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB})$ for the $B \to K^* \mu^+ \mu^-$ decay is practically independent of the existence of such SM4 parameters. The magnitude of the $\mathcal{A}_{\rm FB}^{LL}$ is the suppression function of SM4 parameters. The greater the values of $m_{t'}$ and r_{sb}, the smaller the magnitude of the $\mathcal{A}_{\rm FB}^{LL}$. On the other hand, the smaller the values of ϕ_{sb}, the smaller the magnitude of the $\mathcal{A}_{\rm FB}^{LL}$. The same situation holds for the $B \to K^* \tau^+ \tau^-$ decay (see Figs. 4–6, 10–12, and 16–18). But, in the case of the $B \to K^* \tau^+ \tau^-$ decay, the zero position for the FB asymmetries $\mathcal{A}_{\rm FB}^{LL}$ is absent both in SM3 and SM4 (see Figs. 4–6, 10–12, and 16–18).

Before performing a numerical analysis, a few words about FB asymmetries are in order. From explicit expres-

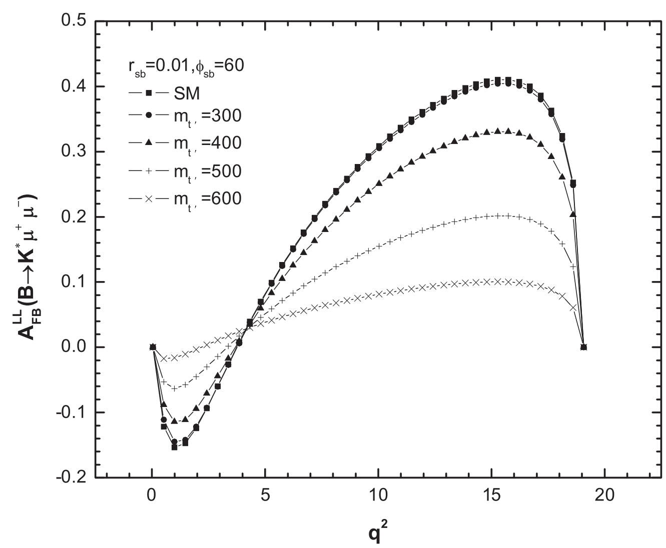

FIG. 1. The dependence of $\mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB})$ on q^2 where $m_{t'}$: 300, 400, 500, 600 GeV, $\phi_{sb} = 60°$, and $r_{sb} = 0.01$.

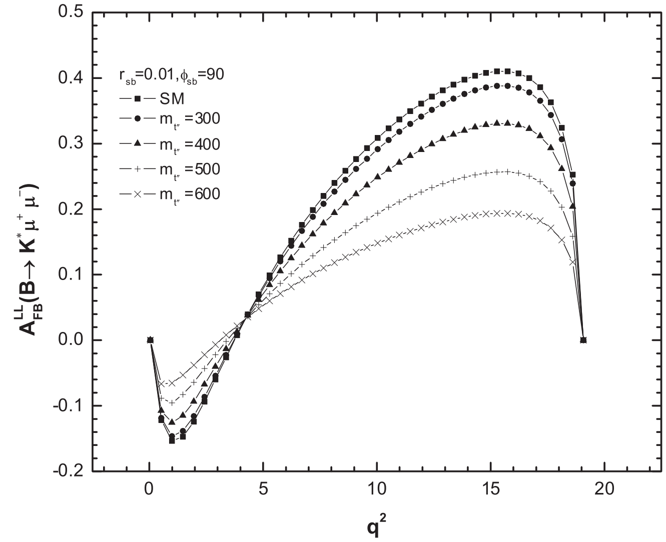

FIG. 2. The same as in Fig. 1, but for $r_{sb} = 0.02$.

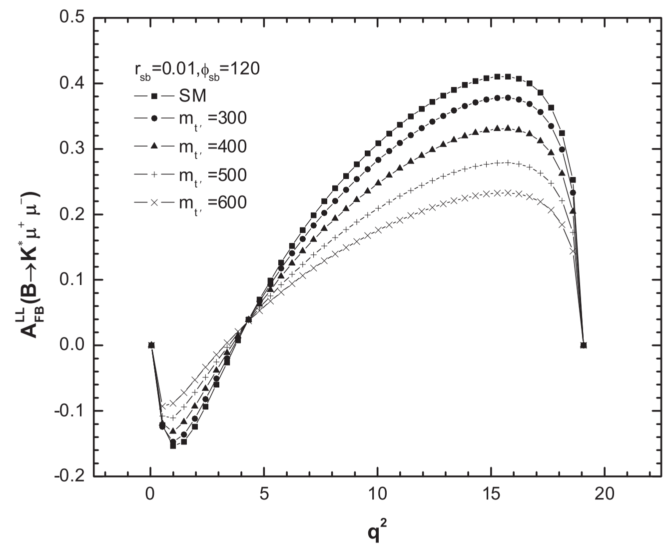

FIG. 3. The same as in Fig. 1, but for $r_{sb} = 0.03$.

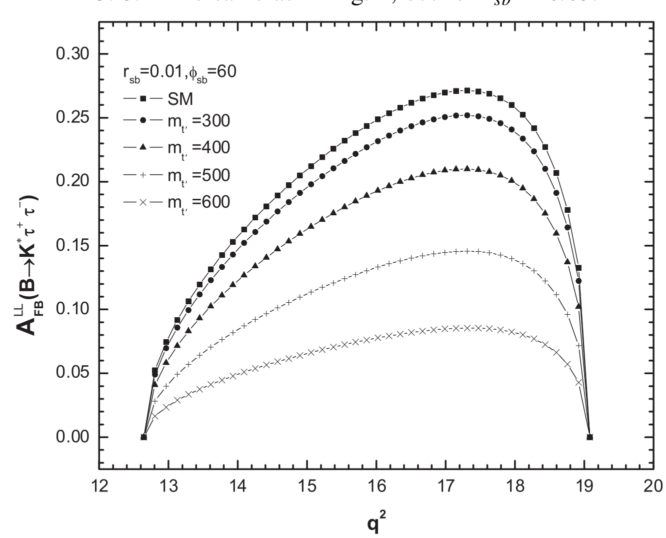

FIG. 4. The same as in Fig. 1, but for the τ lepton.

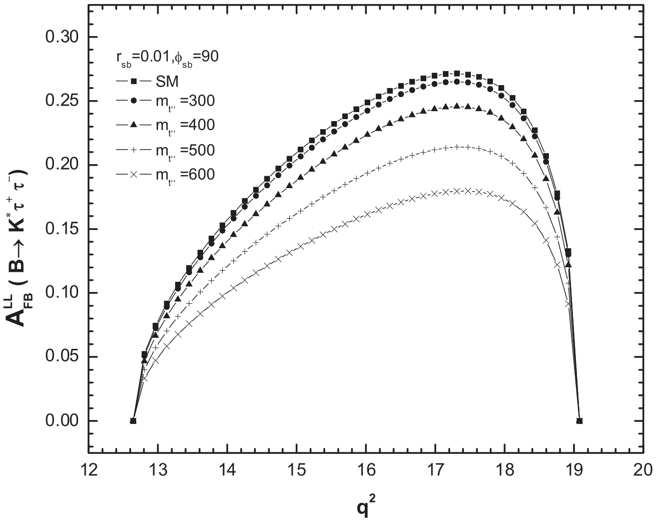

FIG. 5. The same as in Fig. 2, but for the τ lepton.

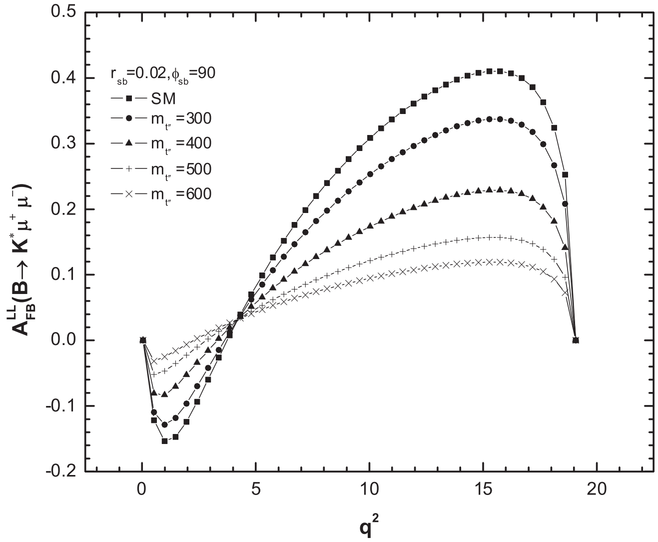

FIG. 8. The same as in Fig. 7, but for $r_{sb} = 0.02$.

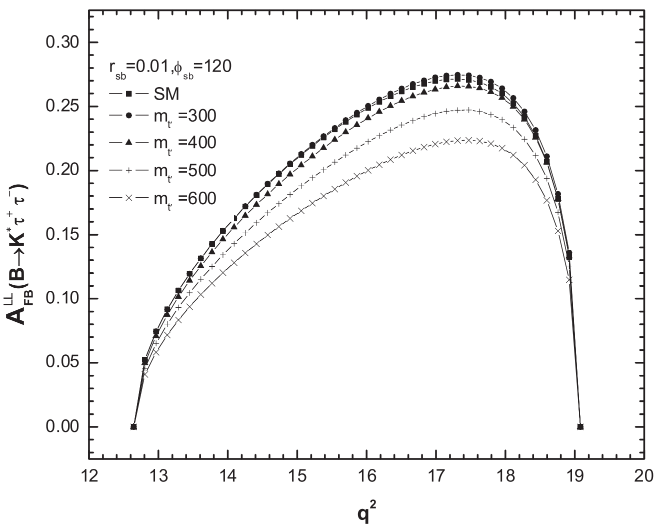

FIG. 6. The same as in Fig. 3, but for the τ lepton.

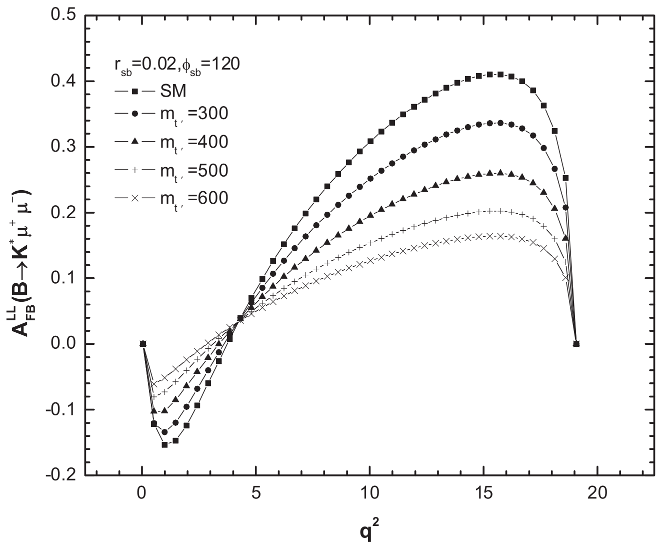

FIG. 9. The same as in Fig. 7, but for $r_{sb} = 0.03$.

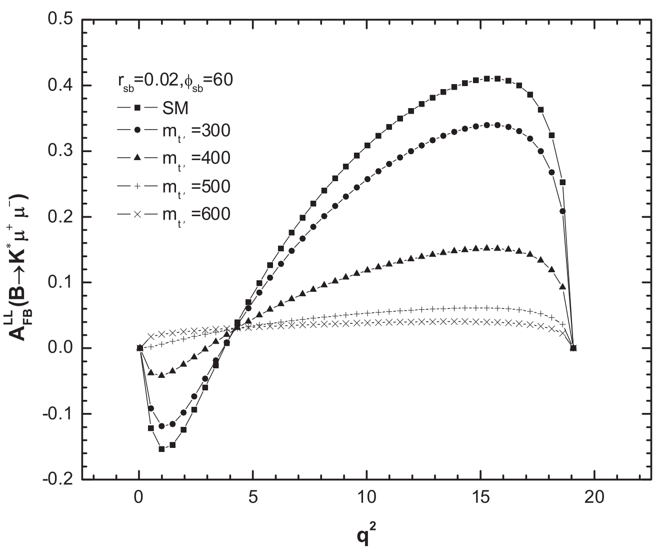

FIG. 7. The same as in Fig. 1, but for $\phi_{sb} = 90°$.

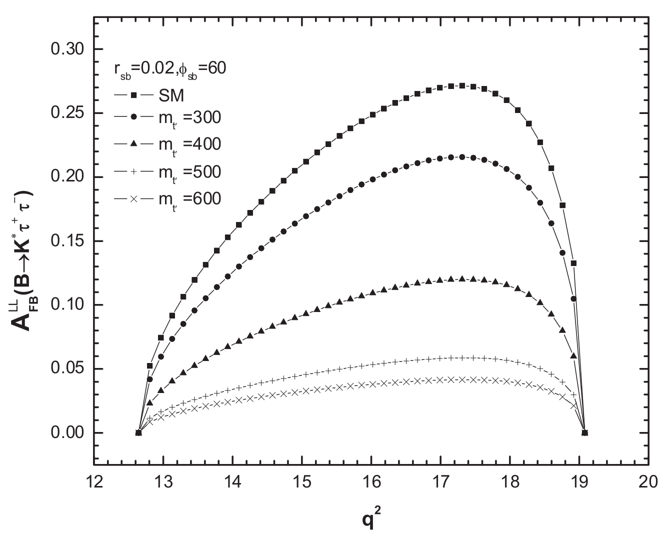

FIG. 10. The same as in Fig. 7, but for the τ lepton.

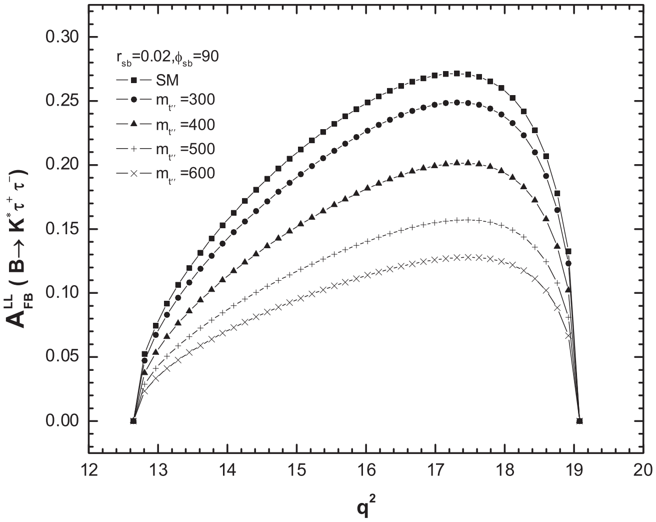

FIG. 11. The same as in Fig. 8, but for the τ lepton.

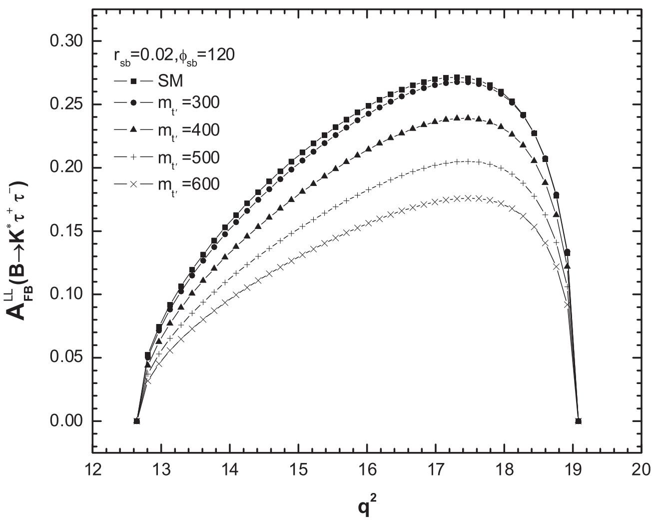

FIG. 12. The same as in Fig. 9, but for the τ lepton.

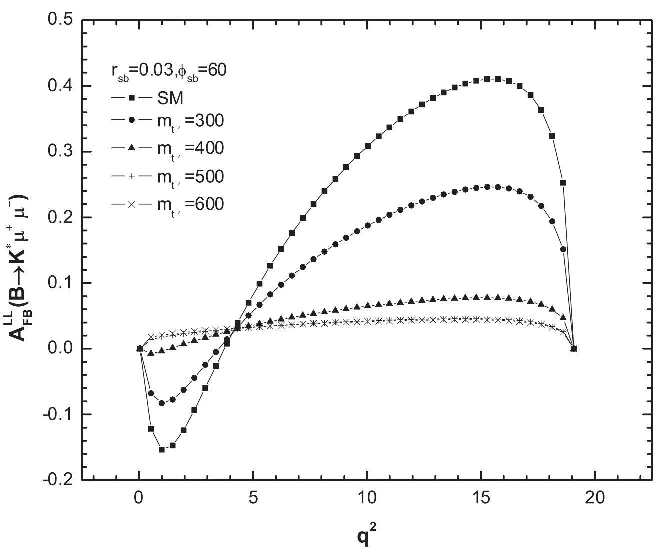

FIG. 13. The same as in Fig. 1, but for $\phi_{sb} = 120°$.

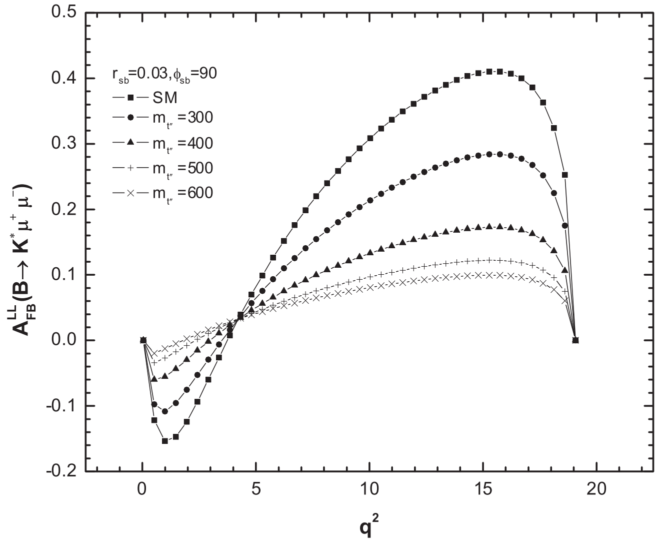

FIG. 14. The same as in Fig. 13, but for $r_{sb} = 0.02$.

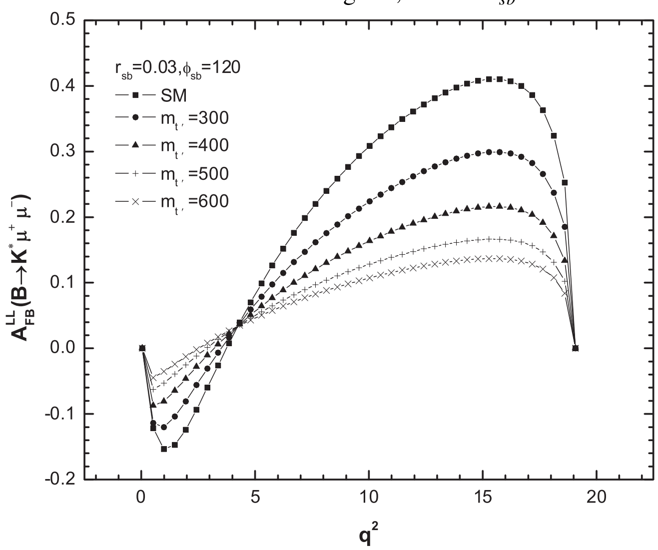

FIG. 15. The same as in Fig. 13, but for $r_{sb} = 0.03$.

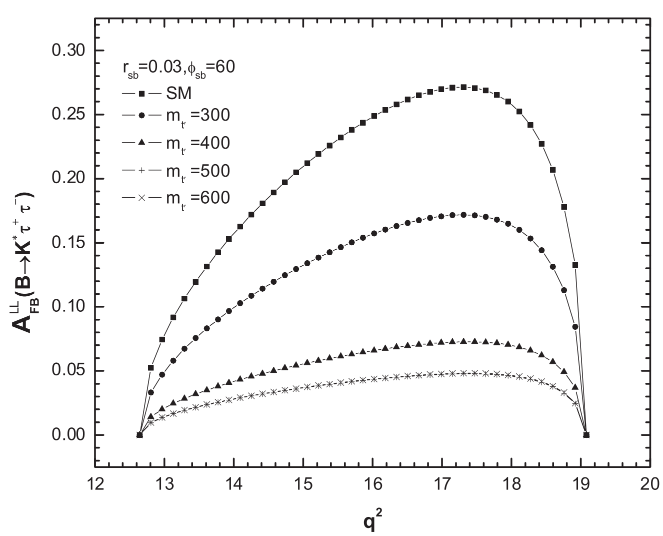

FIG. 16. The same as in Fig. 13, but for the τ lepton.

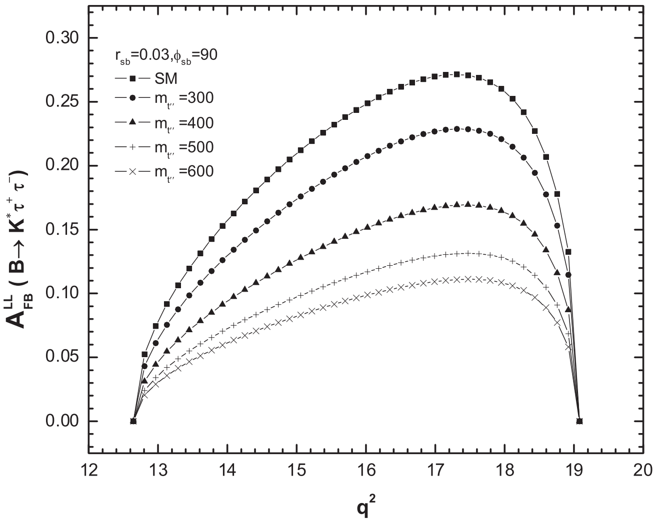

FIG. 17. The same as in Fig. 14, but for the τ lepton.

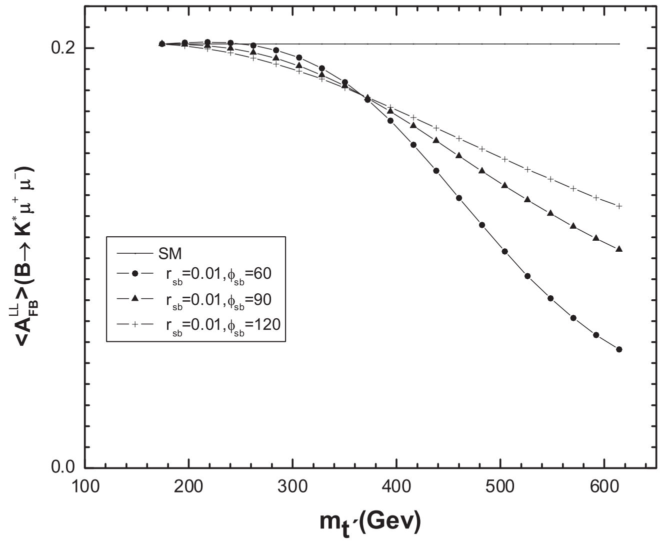

FIG. 19. The dependence of $\langle \mathcal{A}^{LL}_{\rm FB}(\mathcal{A}_{\rm FB})\rangle$ on $m_{t'}$, where ϕ_{sb}: 60°, 90°, 120°, and $r_{sb} = 0.01$.

sions of the FB asymmetries, one can easily see that they depend on both $\hat{s}$ and the new parameters ($m_{t'}$, r_{sb}). We should eliminate the dependence of the lepton polarization on one of the variables. We eliminate the variable $\hat{s}$ by performing integration over $\hat{s}$ in the allowed kinematical region. The averaged polarized and unpolarized FB asymmetries are defined as

$$\mathcal{B}_r = \int_{4m_\ell^2/m_B^2}^{(1-\sqrt{\hat{r}_{K^*}})^2} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s},$$
$$\langle A^{ij}_{\rm FB}\rangle = \frac{\int_{4m_\ell^2/m_B^2}^{(1-\sqrt{\hat{r}_{K^*}})^2} A^{ij}_{\rm FB} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\mathcal{B}_r}. \tag{35}$$

Equipped with the above expressions, we have calculated all averaged double-spin FB asymmetries (see Table V) in the decay $B \to K^* \ell^+ \ell^-$ for both μ and τ channels in the SM. We see that only one of these ($\langle \mathcal{A}^{LL}_{\rm FB}(\mathcal{A}_{\rm FB})\rangle$) is predicted to be measurable, i.e. have values larger than 10% (see Table V). (Indeed some asymmetries are expected to vanish or be small in the SM.) If any of these small asymmetries with large values have been found, this would be an effect of new physics beyond the SM.

$\langle \mathcal{A}^{LL}_{\rm FB}(\mathcal{A}_{\rm FB})\rangle$ strongly depends on the fourth quark mass ($m_{t'}$) and sensitivities to the product of quark mixing matrix elements (r_{sb}) for both μ and τ channels (see Figs. 19–24). Furthermore, for both channels, $\langle \mathcal{A}^{LL}_{\rm FB}(\mathcal{A}_{\rm FB})\rangle$ is a decreasing function of both $m_{t'}$ and r_{sb} (see Figs. 19–24).

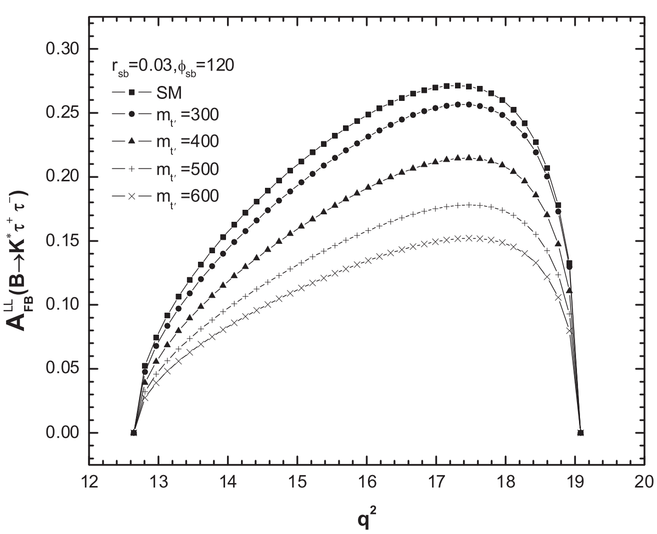

FIG. 18. The same as in Fig. 15, but for the τ lepton.

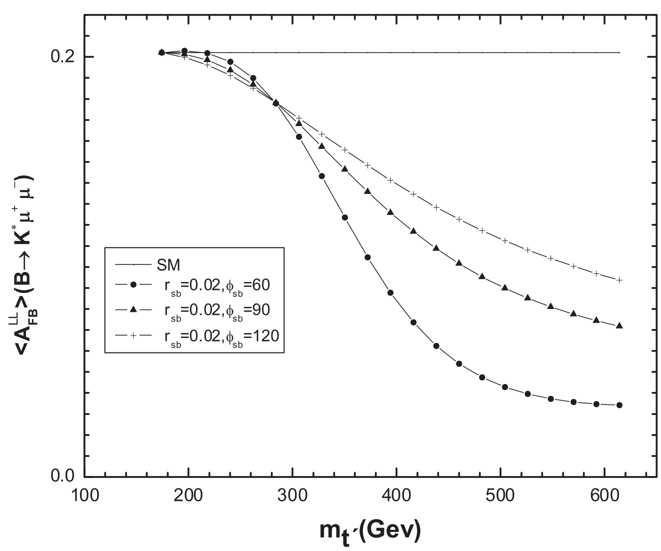

FIG. 20. The same as in Fig. 19, but for $r_{sb} = 0.02$.

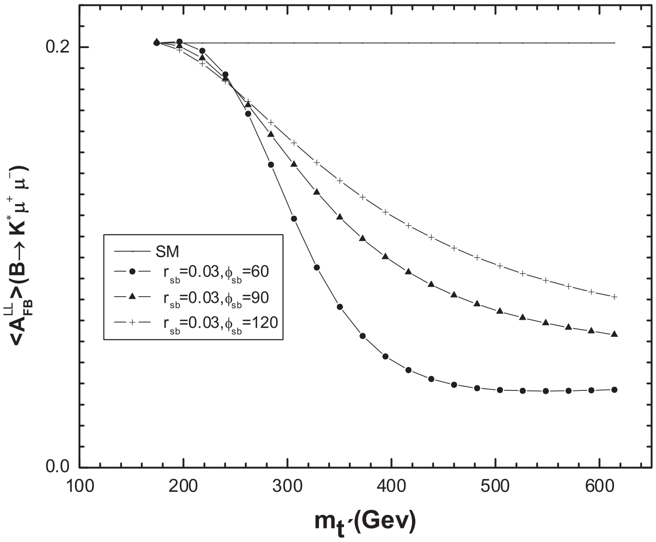

FIG. 21. The same as in Fig. 19, but for $r_{sb} = 0.03$.

The measurement of the magnitude and sign of the $\langle \mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB})\rangle$ can be used as a good tool in studying the fourth-generation quarks and NP beyond the SM.

From these analyses, we can conclude that the measurement of the magnitude and the zero position of $\mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB})$ and the measurement of the magnitude of $\langle \mathcal{A}_{\rm FB}^{LL}(\mathcal{A}_{\rm FB})\rangle$ asymmetries are an indication of the existence of new physics beyond the SM.

Let us discuss the problem of the measurement of the FB asymmetries in experiments. Experimentally, to measure an asymmetry $\langle A_{\rm FB}^{ij}\rangle$ of the decay with the branching ratio $\mathcal{B}$ at $n\sigma$ level, the required number of events (i.e., the number of $B\bar{B}$ pairs) is given by the expression

$$N = \frac{n^2}{\mathcal{B} s_1 s_2 \langle A_{\rm FB}^{ij}\rangle^2},$$

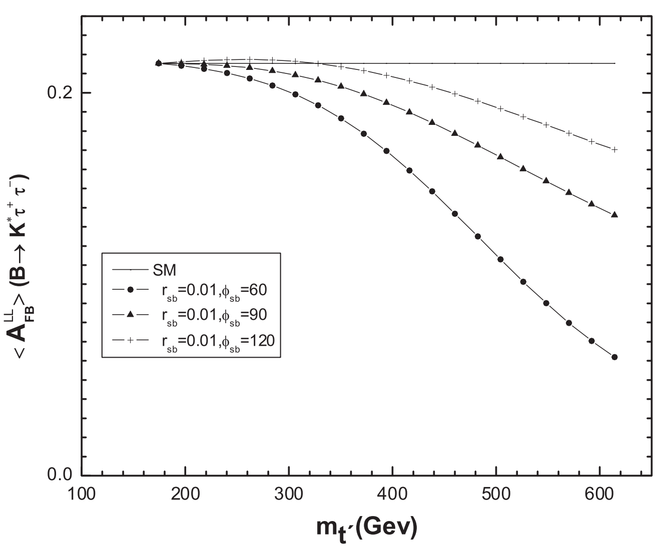

FIG. 22. The same as in Fig. 19, but for the τ lepton.

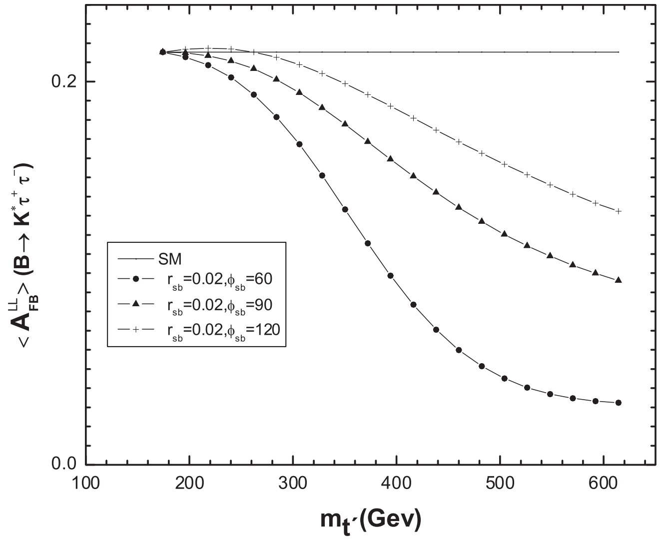

FIG. 23. The same as in Fig. 20, but for the τ lepton.

where s_1 and s_2 are the efficiencies of the leptons. Typical values of the efficiencies of the τ leptons range from 50% to 90% for their various decay modes (see for example [67] and references therein), and the error in τ-polarized FB asymmetries is estimated to be about (10–15)% [68]. As a result, the error in the measurement of the τ-lepton asymmetries is about (20–30)%, and the error in obtaining the number of events is about 50%. From the expression for N we see that, in order to observe the FB asymmetries in $B \to K^*\mu^+\mu^-$ and $B \to K^*\tau^+\tau^-$ decays at 3σ level, the minimum number of required events are as follows:

(i) for the $B \to K^*\mu^+\mu^-$ decay

$$N \sim \begin{cases} 10^8 & (\text{for } \langle A_{\rm FB}^{LL}\rangle), \\ 10^{12} & (\text{for } \langle A_{\rm FB}^{LT}\rangle, \langle A_{\rm FB}^{TL}\rangle), \end{cases}$$

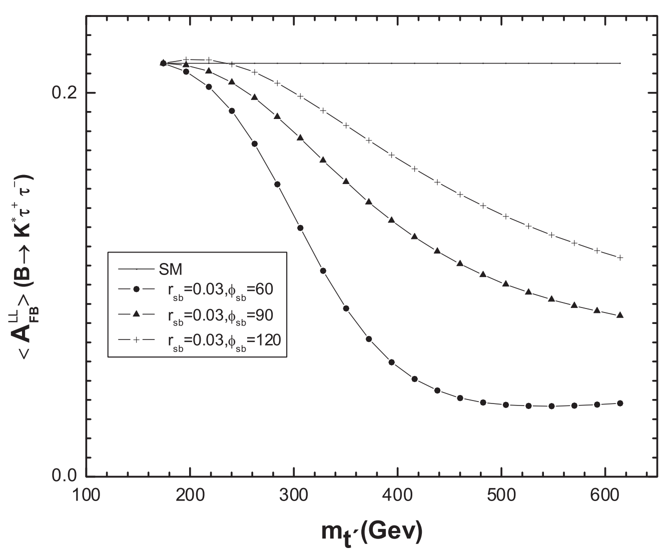

FIG. 24. The same as in Fig. 21, but for the τ lepton.

(ii) and for the $B \to K^* \tau^+ \tau^-$ decay

$$N \sim \begin{cases} 10^8 & (\text{for } \langle A_{\rm FB}^{LL} \rangle), \\ 10^{11} & (\text{for } \langle A_{\rm FB}^{LN} \rangle, \langle A_{\rm FB}^{NL} \rangle, \langle A_{\rm FB}^{NT} \rangle, \langle A_{\rm FB}^{TN} \rangle), \\ 10^{10} & (\text{for } \langle A_{\rm FB}^{TL} \rangle, \langle A_{\rm FB}^{TL} \rangle). \end{cases}$$

The number of $b\bar{b}$ pairs that are produced at B factories and the CERN LHC are about $\sim 5 \times 10^8$ and 10^{11}, respectively. As a result of a comparison of these numbers and N, we conclude that only $\langle A_{\rm FB}^{LL} \rangle$ in the $B \to K^* \ell^+ \ell^-$ decay and probably $\langle A_{\rm FB}^{LT} \rangle$ and $\langle A_{\rm FB}^{TL} \rangle$ in the $B \to K^* \tau^+ \tau^-$ decay can be detectable at the LHC.

In conclusion, we presented the analysis of unpolarized and polarized, lepton pair, forward-backward asymmetries and their averaged values in the exclusive $B \to K^* \ell^- \ell^+$ decay, by using the SM with four generations of quarks. The sensitivity of the unpolarized and polarized forward-backward asymmetries and their averaged values on the new parameters that come out of fourth-generation quarks were studied. First, we found out that unpolarized and longitudinally polarized FB coincide with each other. Second, unpolarized and longitudinally polarized FB asymmetries and their averaged values depicted a strong dependence on the fourth quark ($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^* V_{t's} = r_{sb} e^{i\phi_{sb}}$). We obtained that the study of the FB asymmetries for both μ and τ cases and the zero position of the polarized FB for the μ case can serve as a good tool to look for physics beyond the SM. More precisely, the results can be used to study the fourth generation of quarks indirectly.

ACKNOWLEDGMENTS

The authors would like to thank T. M. Aliev for his useful discussions.

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