Canonical charmonium interpretation for $Y(4360)$ and $Y(4660)$

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In this work, we consider the canonical charmonium assignments for $Y(4360)$ and $Y(4660)$. $Y(4660)$ is a good candidate of 5^3S_1 $c\bar{c}$ state, the possibility of $Y(4360)$ as a 3^3D_1 $c\bar{c}$ state is studied, and the charmonium hybrid interpretation of $Y(4360)$ cannot be excluded completely. We evaluate the e^+e^- leptonic widths, $E1$ transitions, $M1$ transitions and the open flavor strong decays of $Y(4360)$ and $Y(4660)$. Experimental tests for the charmonium assignments are suggested.

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I. INTRODUCTION

In the last five years we have witnessed a revival of interest in charm spectroscopy, the B factories (*BABAR* and Belle) and other machines have reported a large number of new states with hidden charm: $h_c(1P)$ [1], $\eta_c(2S)$ [2], $X(3872)$ [3], $X(3940)$ [4], $Y(3940)$ [5], $Z(3930)$ [6], and $Y(4260)$ [7]. Some of them can be understood as $c\bar{c}$ states, while a conventional assignment for some are elusive (for a recent review, see, e.g., [8]). These discoveries are enriching and also challenging our knowledge for the hadron spectroscopy and the underlining theory for strong interactions.

Recently the Belle Collaboration has observed two charmoniumlike states $Y(4360)$ and $Y(4660)$ in $e^+e^- \to \pi^+\pi^-\psi(2S)$ via initial state radiation [9]. The mass of $Y(4360)$ is $4361 \pm 9 \pm 9\ \mathrm{MeV}/c^2$ with a width of $74 \pm 15 \pm 10\ \mathrm{MeV}/c^2$ and the statistical significance is of more than 8σ. The mass of $Y(4660)$ is $4664 \pm 11 \pm 5\ \mathrm{MeV}/c^2$ with a width of $48 \pm 15 \pm 3\ \mathrm{MeV}/c^2$ and statistical significance 5.8σ. Both these two structures are known to be produced in initial state radiation from e^+e^- annihilation and hence to have $J^{PC} = 1^{--}$. They were seen in the decays with products of $\pi^+\pi^-\psi(2S)$. It has been determined by the Belle Collaboration that

$$\begin{aligned}\Gamma(Y(4360) \to e^+e^-)\mathcal{B}(Y(4360) \to \pi^+\pi^-\psi(2S)) &= 10.4 \pm 1.7 \pm 1.5(11.8 \pm 1.8 \pm 1.4)\ \mathrm{eV}/c^2\\ \Gamma(Y(4660) \to e^+e^-)\mathcal{B}(Y(4660) \to \pi^+\pi^-\psi(2S)) &= 3.0 \pm 0.9 \pm 0.3(7.6 \pm 1.8 \pm 0.8)\ \mathrm{eV}/c^2,\end{aligned} \tag{1}$$

where the numbers in the bracket are the solution II fit performed by Belle. In order to understand the nature of $Y(4360)$ and $Y(4660)$, it is worth to note that the *BABAR* Collaboration has observed a broad structure $Y(4325)$ in the process $e^+e^- \to \gamma_{\mathrm{ISR}}\pi^+\pi^-\psi(2S)$ at $4324 \pm 24\ \mathrm{MeV}/c^2$ with a width $172 \pm 33\ \mathrm{MeV}/c^2$ [10]. The mass of $Y(4360)$ is close to that of $Y(4325)$, the main difference between $Y(4325)$ and $Y(4360)$ is their widths, and it seems very difficult to observe these two structures simultaneously because of the large width of $Y(4325)$. If both $Y(4325)$ and $Y(4360)$ are not experimental artifacts, they could be the same structure and the width difference is due to the experimental error, or they are two different resonances. If one assumed that they were two different structures, it would be very difficult to assign both of them as conventional charmoniums simultaneously. So, at least one of them should be exotic state. It possibly may be produced by $D_1\bar{D}$ (or $D_{s1}\bar{D}_s$) rescattering effect or some other mechanism, therefore it would indicate the necessity of refinements in the naive "quenched" $q\bar{q}$ quark models or the inclusion of additional dynamical effects. In this paper, we assume $Y(4325)$ and $Y(4360)$ are exactly the same resonance for simplicity. Possible noncharmonium assignment of $Y(4360)$ and the relation between $Y(4325)$ and $Y(4360)$ will be considered in detail in future work. Finally we would like to mention that the Belle Collaboration claims the broad $Y(4325)$ is comprised of two narrower peaks $Y(4360)$ and $Y(4660)$ [11].

In order to understand the structure of $Y(4360)$ and $Y(4660)$, i.e., whether they are conventional charmonium states or other exotic structures, it is very necessary to first consider the canonical charmonium assignments and the characteristic signals. With $J^{PC} = 1^{--}$, a conventional $c\bar{c}$ state is either a S-wave state or a D-wave state. There are already reasonably well established $c\bar{c}$ candidates for $1S$, $2S$, $1D$, $3S$, $2D$, and $4S$ [12], therefore new 1^{--} charmonium states can only belong to $3D$, $4D$, $5S$, or $6S$, and a natural assignment for $Y(4360)$ will be a 3^3D_1 $c\bar{c}$ state, and $Y(4660)$ as a 5^3S_1 charmonium. However, this assignment has the problem that the mass of the $Y(4360)$ is somewhat lower than the nonrelativistic potential model prediction for the 3^3D_1 $c\bar{c}$ state, which will be shown in the next section.

In this work, we study the properties of $Y(4360)$ and $Y(4660)$ under the hypothesis of $Y(4360)$ as a 3^3D_1 $c\bar{c}$ state and $Y(4660)$ as a 5^3S_1 $c\bar{c}$ state. We first briefly review the

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nonrelativistic potential model and give its prediction for the masses of 3^3D_1, 4^3D_1, 5^3S_1, and 6^3S_1 $c\bar{c}$ states. The e^+e^- leptonic widths, $E1$ transitions, $M1$ transitions and open-charm strong decays of both $Y(4360)$ and $Y(4660)$ are studied in Sec. III and IV respectively. From these results, we suggest adequate measurements which can verify the canonical charmonium assignments and distinguish the $c\bar{c}$ structure from other non-$c\bar{c}$ possibilities. Finally we present our summary and some discussions. Possible charmonium hybrid assignment of $Y(4360)$ and its crucial decay modes are suggested.

II. REVIEW ON NONRELATIVISTIC POTENTIAL MODEL AND THE CHARMONIUM ASSIGNMENTS FOR $Y(4360)$ AND $Y(4660)$

The quark potential models have successfully described the charmonium spectrum, which generally assumes shorted-ranged color coulomb interaction and long-ranged linear scalar confining interaction plus spin-dependent part coming from one gluon exchange and the confining interaction. The potential model is closely related to QCD, which can be derived from the QCD effective field theory [13,14]. Here we shall use the simple nonrelativistic potential model proposed by Barnes, Godfrey, and Swanson [15], the zeroth-order Hamiltonian is,

$$H_0 = \frac{\mathbf{p}^2}{m_c} - \frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_c^2}\tilde{\delta}_\sigma(r)\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}, \quad (2)$$

where $\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$, which is a Gaussian-smeared hyperfine interaction. The solution of the Schrödinger equation with the above H_0 gives our zeroth-order charmonium wave functions. The splitting within the multiplets is then determined by taking the matrix element of the spin-dependent Hamiltonian H_{sd} between these zeroth-order wave functions. The spin-dependent Hamiltonian is taken from the one-gluon-exchange Breit-Fermi Hamiltonian (which gives spin-orbit and tensor terms) and an inverted spin-orbit term, which follows from the assumption of a Lorentz scalar confining interaction. The $H_{\rm sd}$ is as follows,

$$H_{sd} = \frac{1}{m_c^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{4\alpha_s}{r^3}\mathbf{T}\right]. \quad (3)$$

This simple potential model consists of four parameters: the strong coupling constant α_s which is taken to be a constant for simplicity, the string tension b, the charm quark mass m_c, and the hyperfine interaction smearing parameter σ. Fitting the masses of the 11 reasonably well established experimental charmonium states, the values of these four parameters are already fixed as follows: $\alpha_s = 0.5461$, $b = 0.1425\ \text{GeV}^2$, $m_c = 1.4794$ GeV, and $\sigma = 1.0946$ GeV [15]. Solving the Schrödinger equation with the zeroth-order Hamiltonian H_0 numerically by the MATHEMATICA program [16] and treating the spin-dependent terms H_{sd} as mass shifts by the leading order perturbation, we obtain the masses and wave funtions of the canonical $c\bar{c}$ states. The masses of 3^3D_1, 4^3D_1, 5^3S_1, and 6^3S_1 are predicted as,

$$M(3^3D_1) = 4455\ \text{MeV}, \qquad M(4^3D_1) = 4740\ \text{MeV},$$
$$M(5^3S_1) = 4704\ \text{MeV}, \qquad M(6^3S_1) = 4977\ \text{MeV}. \quad (4)$$

Comparing with the masses of $Y(4360)$ and $Y(4660)$, it is natural to assign $Y(4360)$ as a 3^3D_1 and $Y(4660)$ as a 5^3S_1 canonical charmonium states. Although the mass of $Y(4360)$ is somewhat smaller than the theoretical prediction, however, we notice that the mass predictions of various potential model for the high charmonium may differ by 10–100 MeV [8], therefore $Y(4360)$ as a 3^3D_1 $c\bar{c}$ state is not irrational. In this work we assume that the discrepancy in the spectrum is due to the theoretical uncertainties or other effects such as the coupled channel effects. It is interesting to refit the parameters α_s, b, m_c, and σ including both $Y(4360)$ and $Y(4660)$ or only $Y(4660)$.

III. ELECTRONMAGNETIC TRANSITIONS OF $Y(4360)$ AND $Y(4660)$

A. The e^+e^- leptonic decay of $Y(4360)$ and $Y(4660)$

The decay of the quarkonium state into a lepton pair proceeds via a single virtual photon, as long as the mass of the initial quarkonium is sufficiently small that the contribution of a virtual Z can be ignored. The leptonic partial decay widths probe the compactness of the quarkonium system, and they provide useful information about the wave functions of the 1^{--} quarkonium states. The leptonic width of n^3S_1 charmonium is given by [17,18],

$$\Gamma(n^3\mathrm{S}_1 \to e^+e^-) = \frac{4\alpha^2 e_c^2|\psi_n(0)|^2}{M_n^2}\left(1 - \frac{16\alpha_s}{3\pi} + \ldots\right), \quad (5)$$

where $e_c = 2/3$ is the charm quark electric charge, M_n is the mass of the n^3S_1 state, and the second term is the QCD correction. $\psi_n(0)$ is the n^3S_1 wave function at the origin, and the radial wave function is normalized according to $\int_0^\infty dr r^2|\psi_n(r)|^2 = 1$. At the leading order, the width of D-wave $c\bar{c}$ states to e^+e^- is proportional to $|\psi_n''(0)|^2$,

$$\Gamma(n^3\mathrm{D}_1 \to e^+e^-) = \frac{25\alpha^2 e_c^2}{2m_c^4 M_n^2}|\psi_n''(0)|^2, \quad (6)$$

which is generally smaller than the corresponding widths of the n^3S_1 states. Using the nonrelativistic quark model wave functions calculated in the previous section, we evaluate these leptonic decay widths of $Y(4360)$ and $Y(4660)$ at both the experimental values and the theoretical predictions of nonrelativistic potential model. The width

TABLE I. The e^+e^- partial widths of $Y(4360)$ and $Y(4660)$.

Initial state	Mass (GeV)	$\Gamma_{e^+e^-}$ (keV)
$Y(4360)(3^3D_1)$	4.361	0.87
	4.455	0.83
$Y(4660)(5^3S_1)$	4.664	1.34
	4.704	1.32

predictions are given in Table I, where we choose $\alpha_s \approx 0.23$ [12].

Using Eq. (1), we can estimate that

$$\mathcal{B}(Y(4360) \to \pi^+\pi^-\psi(2S)) \sim 1.20 \times 10^{-2} \quad (\text{or } 1.36 \times 10^{-2})$$
$$\mathcal{B}(Y(4660) \to \pi^+\pi^-\psi(2)) \sim 2.24 \times 10^{-3} \quad (\text{or } 5.67 \times 10^{-3}). \tag{7}$$

The numbers in the bracket are the results corresponding to the solution II fit by the Belle Collaboration. Generally we expect the branch fraction for $c\bar{c}(3^3D_1) \to \pi^+\pi^-\psi(2S)$ should be of the order 10^{-3}, e.g., $\mathcal{B}(\psi(4160) \to \pi^+\pi^-\psi(2S)) < 4 \times 10^{-3}$ [12], where $\psi(4160)$ is a good candidate of 2^3D_1 $c\bar{c}$ state. Therefore $\mathcal{B}(Y(4360) \to \pi^+\pi^-\psi(2S))$ seems a little larger, which may be because of the QCD radiative corrections to $\Gamma(Y(4360) \to e^+e^-)$, and nonvalence components may also contribute, which deserves investigating further. $\mathcal{B}(Y(4660) \to \pi^+\pi^-\psi(2S)) \sim 10^{-3}$, which indicates that $Y(4660)$ may be a good candidate of 5^3S_1 $c\bar{c}$ state.

B. Radiative transitions of $Y(4360)$ and $Y(4660)$

Radiative decay of higher-mass charmonium states is an important way to produce lower charmonium states, and it plays a significant role in charmonium physics. By means of the radiative transitions one can probe the internal charge structure of hadrons, hence it is useful for determining the quantum numbers and hadronic structures of heavy quark mesons. The radiative transition amplitude is determined by the matrix element of the electromagnetic current between the initial quarkonium state i and the final state f, i.e., $\langle f|j^\mu_{em}|i\rangle$. Expanding in powers of photon momentum generates the electric and magnetic multipole moments, the leading order transition amplitudes are electric dipole ($E1$) transition or magnetic dipole ($M1$) transition. They are quite straightforward to be evaluated in the potential model.

1. $E1$ transitions

The partial width for $E1$ transitions between states $n^{2S+1}L_J$ and $n'^{2S'+1}L'_{J'}$ $c\bar{c}$ state in the nonrelativistic quark model is given by [19–22],

$$\Gamma_{E1}(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4\alpha e_c^2 E_\gamma^3}{3}(2J'+1) \times \mathcal{S}_{fi}\delta_{S,S'}|\langle n'^{2S'+1}L'_{J'}|r| \times n^{2S+1}L_J\rangle|^2 \frac{E_f}{M_i} \tag{8}$$

where E_γ is the photon energy, E_f is the energy of final state $n'^{2S'+1}L'_{J'}$, and M_i is the mass of the initial state $n^{2S+1}L_J$. We have included the relativistic phase factor $\frac{E_f}{M_i}$, and the statistical factor $\mathcal{S}_{fi}$ is

$$\mathcal{S}_{fi} = \max(L, L') \cdot \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix}^2. \tag{9}$$

The matrix element $\langle n'^{2S'+1}L'_{J'}|r|n^{2S+1}L_J\rangle$ can be straightforwardly evaluated using the nonrelativistic Schrödinger wave functions of the model described in the previous section, and the resulting $E1$ transition widths of $Y(4360)$ and $Y(4660)$ together with the photon energies are given in Tables II, III, IV, and V, where the $E1$ transition widths predictions for initial state assuming both the ex-

TABLE II. $E1$ radiative transitions of $Y(4360)$ (3^3D_1), and the $Y(4360)$ mass is taken from experiment.

Final meson	E_γ (MeV)	Γ (keV)
$\chi_2(3^3P_2)$	44	0.12
$\chi_1(3^3P_1)$	89	15.9
$\chi_0(3^3P_0)$	156	112
$\chi_2(2^3P_2)$	371	0.39
$\chi_1(2^3P_1)$	414	8.00
$\chi_0(2^3P_0)$	479	16.2
$\chi_2(2^3F_2)$	10	0.062
$\chi_2(1^3P_2)$	731	0.17
$\chi_1(1^3P_1)$	767	2.94
$\chi_0(1^3P_0)$	843	5.11
$\chi_2(1^3F_2)$	319	0.053

TABLE III. $E1$ radiative transitions of $Y(4360)(3^3D_1)$, and the $Y(4360)$ mass is the prediction of the nonrelativistic potential model, which is 4.455 GeV.

Final meson	E_γ (MeV)	Γ (keV)
$\chi_2(3^3P_2)$	135	3.65
$\chi_1(3^3P_1)$	180	128
$\chi_0(3^3P_0)$	246	425
$\chi_2(2^3P_2)$	456	0.71
$\chi_1(2^3P_1)$	498	13.7
$\chi_0(2^3P_0)$	562	25.6
$\chi_2(2^3F_2)$	103	58.3
$\chi_2(1^3P_2)$	808	0.23
$\chi_1(1^3P_1)$	844	3.85
$\chi_0(1^3P_0)$	918	6.49
$\chi_2(1^3F_2)$	405	0.11

TABLE IV. $E1$ radiative transitions of $Y(4660)(5^3S_1)$, and the $Y(4360)$ mass is taken from experiment.

Final meson	E_γ (MeV)	Γ (keV)
$\chi_2(4^3P_2)$	42	7.92
$\chi_1(4^3P_1)$	87	42.8
$\chi_0(4^3P_0)$	152	75.5
$\chi_2(3^3P_2)$	334	0.34
$\chi_1(3^3P_1)$	377	0.29
$\chi_0(3^3P_0)$	439	0.15
$\chi_2(2^3P_2)$	640	0.68
$\chi_1(2^3P_1)$	681	0.48
$\chi_0(2^3P_0)$	741	0.20
$\chi_2(1^3P_2)$	976	0.37
$\chi_1(1^3P_1)$	1010	0.24
$\chi_0(1^3P_0)$	1082	0.097

periment observed masses and the nonrelativistic potential model predictions are given. The masses of the involved final state charmoniums are taken from the Particle Data Group [12] if the state is included in the meson summary table. If it is not, then the masses predicted in the nonrelativistic potential model described in the previous section are used. The exceptions are $h_c(^1P_1)$ and $\eta_c(3^1S_0)$, we assume $M(\eta_c(3^1S_0)) = 4.011$ GeV (the mass of the known $\psi(4040)$ minus the theoretical $3S$ splitting) and $M(h_c(^1P_1)) = 3.525$ GeV, which is the spin-averaged mass of the ^3P_J χ_J states.

From Tables II and III, we can see that $Y(4360)$ should have very small $E1$ radiative widths to the triplet member of the $1P$ multiplet and 1^3F_2 state, if $Y(4360)$ is a pure 3^3D_1 state. The radiative widths to the unknown $3P$ and $2P$ triplet states $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\chi_0(2^3P_0)$, and $\chi_1(2^3P_1)$ are theoretically large, so the radiative decays of $Y(4360)$ can be used to produce these states. Since the structures of both $X(3940)$ and $Y(3940)$ are still unclear, they possibly belong to the 2^3P_J multiplet [4,5]. Consequently the $E1$ transitions of $Y(4360)$ into $\chi_0(2^3P_0)$, $\chi_1(2^3P_1)$ are especially of interests, which maybe helpful to clarifying the issue of $X(3940)$ and $Y(3940)$.

TABLE V. $E1$ radiative transitions of $Y(4660)(5^3S_1)$, and the $Y(4660)$ mass is the prediction of the nonrelativistic potential model, which is 4.704 GeV.

Final meson	E_γ (MeV)	Γ (keV)
$\chi_2(4^3P_2)$	81	57.7
$\chi_1(4^3P_1)$	126	129
$\chi_0(4^3P_0)$	191	147
$\chi_2(3^3P_2)$	371	0.46
$\chi_1(3^3P_1)$	413	0.38
$\chi_0(3^3P_0)$	475	0.19
$\chi_2(2^3P_2)$	675	0.78
$\chi_1(2^3P_1)$	714	0.55
$\chi_0(2^3P_0)$	774	0.23
$\chi_2(1^3P_2)$	1008	0.40
$\chi_1(1^3P_1)$	1042	0.26
$\chi_0(1^3P_0)$	1112	0.10

Next we consider the $E1$ transition of $Y(4660)$ as a 5^3S_1 $c\bar{c}$ state. As is shown evidently in Tables IV and V, the strong suppression of $Y(4660)$ $E1$ decays to n^3P_J ($n = 1$, 2, 3) states are predicted. The radiative width to 4^3P_J multiplet is large, which can provide access to the spin-triplet members of $4P$ multiplet.

2. *M*1 transitions

$M1$ transitions flip the quark spin, and $M1$ transitions are generally suppressed relative to the $E1$ transitions, and it has been observed in the charmonium system. $M1$ transitions between different radial multiplets are only nonzero due to the small relativistic corrections to a vanishing lowest order $M1$ transition matrix element, therefore there may be serious inaccuracy in some channels. Analogous to the $E1$ transitions in the previous subsection, the $M1$ transitions width is given by [19–22]

$$\Gamma_{M1}(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4\alpha e_c^2 E_\gamma^3}{3m_c^2}\frac{2J'+1}{2L+1} \times \delta_{L,L'}\delta_{S,S'\pm 1}\Big| \times \langle n'^{2S'+1}L'_{J'}|j_0\Big(\frac{E_\gamma r}{2}\Big) \times |n^{2S+1}L_J\rangle|^2 \frac{E_f}{M_i}, \tag{10}$$

where the meaning of the notations is the same as that in the $E1$ transition case. The above formula has included the recoil factor $j_0(E_\gamma r/2)$ with $j_0(x) = \sin x/x$. Using the wave functions of the nonrelativistic potential model in Sec. II, the $M1$ transitions width both with and without the recoil factor are calculated straightforwardly, theoretical predictions with the corresponding photon energies are

TABLE VI. $M1$ radiative transitions of $Y(4360)(3^3D_1)$, and $Y(4360)$ mass is the experimental value 4.361 GeV.

Final meson	E_γ (MeV)	Γ (keV)	$\Gamma_{\rm rec}$ (keV)
$\eta_{c2}(2^1D_2)$	199	0.000 31	0.082
$\eta_{c2}(1^1D_2)$	525	0.000 67	0.19

TABLE VII. $M1$ radiative transitions of $Y(4360)(3^3D_1)$, and $Y(4360)$ mass is the theoretical prediction of the nonrelativistic potential model, which is 4.455 GeV.

Final meson	E_γ (MeV)	Γ (keV)	$\Gamma_{\rm rec}$ (keV)
$\eta_{c2}(2^1D_2)$	287	0.000 93	0.24
$\eta_{c2}(1^1D_2)$	607	0.0010	0.29

TABLE VIII. $M1$ radiative transitions of $Y(4660)(5^3S_1)$, and $Y(4660)$ mass is the experimental value 4.664 GeV.

Final meson	E_γ (MeV)	Γ (keV)	$\Gamma_{\rm rec}$ (keV)
$\eta_c(4^1S_0)$	272	0.15	0.95
$\eta_c(3^1S_0)$	607	0.47	3.45
$\eta_c(2^1S_0)$	913	0.82	4.26
$\eta_c(1^1S_0)$	1381	2.65	9.36

TABLE IX. $M1$ radiative transitions of $Y(4660)5^3S_1$, and $Y(4660)$ mass is the theoretical prediction of the nonrelativistic potential model, which is 4.704 GeV.

Final meson	E_γ (MeV)	Γ (keV)	$\Gamma_{\rm rec}$ (keV)
$\eta_c(5^1S_0)$	19	0.013	0.013
$\eta_c(4^1S_0)$	309	0.23	1.39
$\eta_c(3^1S_0)$	642	0.55	4.05
$\eta_c(2^1S_0)$	945	0.90	4.69
$\eta_c(1^1S_0)$	1409	2.80	9.89

shown in Tables VI, VII, VIII, and IX. Obviously the $M1$ transitions of $Y(4360)$ and $Y(4660)$ strongly depend on the recoil factors, and it may be too small to be observed.

IV. STRONG DECAYS OF $Y(4360)$ AND $Y(4660)$

Strong decays of mesons are driven by nonperturbative gluodynamics, which are a sensitive probe of hadron structure. However, it is very difficult to be calculated from the first principle. For charmonium above the $D\bar{D}$ threshold, the dominant decay modes usually are the open-charm strong decays, in which the initial c and $\bar{c}$ separate into different final states. Okubo-Zweig-Iizuka forbidden decays are expected to be small, e.g., experimental indications are that $\mathcal{B}(\psi(3770) \to J/\psi\pi\pi) \sim 2.15 \times 10^{-3}$–$3.31 \times 10^{-3}$, hence we shall focus on the open-charm strong decays of $Y(4360)$ and $Y(4660)$ in this section.

Although open flavor decays are poorly understood from the QCD dynamics so far, a number of phenomenological models have been proposed to deal with this issue, the most popular are the $^3P_0$ model (quark pair creation model) [23–26], the flux-tube model [27,28] and the Cornell model [19,20]. In the flux-tube model, a meson consists of a quark and antiquark connected by a tube of chromoelectric flux, which is treated as a vibrating string. For conventional mesons the string is in its vibrational ground state. The flux-tube breaking decay model [28] is similar to the $^3P_0$ model, but extends it by including the dynamics of the flux tubes. This is done by considering the overlap of the flux tube of the initial meson with those of the two outgoing mesons. The $^3P_0$ model is a limiting case of the flux-tube breaking model (the $^3P_0$ model emerges in the case of infinitely thick flux tube), which greatly simplifies the calculations and gives similar results. The Cornell model [19,20] assumes that strong decays take place through pair production from the linear confining potential, which transforms as the time component of a Lorentz vector j^0, rather than the Lorentz scalar in the $^3P_0$ model. The Cornell model has the advantage of unifying the description of the spectrum and decays and completely specifies the strength of the decay. Recently it has been used to discuss the possible charmonium assignments of $X(3872)$ [29].

The Orsay group has evaluated the open-charm strong decays of three $c\bar{c}$ states $\psi(3770)$, $\psi(4040)$, and $\psi(4415)$ in the $^3P_0$ model [30], later this work was extended by taking into account flux-tube breaking [31]. Recently Barnes *et al.* used the $^3P_0$ model to study the strong decays of both various candidates of $X(3872)$ [32] and higher charmoinum up to the mass of the $4S$ multiplet [33]. In the following we shall consider the open flavor strong decays of $Y(4360)$ and $Y(4660)$ as 3^3D_1 and 5^3S_1 canonical charmonium in the simple harmonic oscillator wave function approximation in the framework of the flux-tube model, this approximation enables analytical studies of amplitudes, and it is known to be an excellent approximation for charmed mesons and light flavor mesons. Here we assume the harmonic oscillator parameter β of final states mesons are identical, which is different from β_A of the initial charmoinum. We will calculate the decay width following the procedure outlined in Refs. [28,34]. Previous attempts on exploring the charmonium strong decay in the $^3P_0$ model, flux-tube model, and Cornell model suggest that the typical error of the partial width predictions is 30% and can reach factors of 2 or even 3.

In the rest frame of A, the decay amplitude for an initial meson A into two final mesons B and C is,

$$\begin{aligned}\mathcal{M}(A \to B + C) = \int d^3\mathbf{r}_A \int d^3\mathbf{y}\psi_A(\mathbf{r}_A) \\ \times \exp\left(i\frac{M}{m+M}\mathbf{p}_B \cdot \mathbf{r}_A\right)\gamma(\mathbf{r}_A, \mathbf{y}) \\ \times \left(i\nabla_{\mathbf{r}_B} + i\nabla_{\mathbf{r}_C} + \frac{2m\mathbf{p}_B}{m+M}\right) \\ \times \psi_B^*(\mathbf{r}_B)\psi_C^*(\mathbf{r}_C) + (B \leftrightarrow C),\end{aligned} \tag{11}$$

where both the flavor and spin overlap have been omitted in the above amplitude, and $\gamma(\mathbf{r}_A, \mathbf{y})$ is the flux-tube overlap function, which measures the spatial dependence of the pair creation amplitude. $\mathbf{y}$ is the pair creation position, $\mathbf{r}_A$, $\mathbf{r}_B$, and $\mathbf{r}_C$ are, respectively, the quark-antiquark axes of A, B, and C mesons, they are related by $\mathbf{r}_B = \mathbf{r}_A/2 + \mathbf{y}$, $\mathbf{r}_C = \mathbf{r}_A/2 - \mathbf{y}$. The initial quark (antiquark) in A is of mass M with m the mass of the created quark pair. For charmonium decay concerned here, $M = m_c$, $m = m_q$ ($q = u, d, s$). When the flux tube is in its ground states (conventional mesons), the flux-tube overlap function is [28]

$$\gamma(\mathbf{r}_A, \mathbf{y}) = A_{00}^0\sqrt{\frac{fb}{\pi}}\exp\left(-\frac{fb}{2}\mathbf{y}_\perp^2\right). \tag{12}$$

As usual, we take the string tension $b = 0.18\ \mathrm{GeV}^2$, and the constituent quark mass $m_u = m_d = 0.33$ GeV, $m_s = 0.55$ GeV, and $m_c = 1.5$ GeV, and the estimated value $f = 1.1$ and $A^0_{00} = 1.0$ are used in our calculation. The final D meson masses used to determined phase space and final state momentum are taken from the Particle Data Group [12] and from recent Belle results [35], and if not available, the estimated mass motivated by the spectroscopy predictions are used [36]. These masses are $M(D) = 1.8694$ GeV, $M(D^*) = 2.0078$ GeV, $M(D_0^*) = 2.308$ GeV (Belle), $M(D_1) = 2.444$ GeV, $M(D_1') = 2.422$ GeV, $M(D_2) = 2.459$ GeV, $M(D^{*'}) = 2.64$ GeV, $M(D_s) = 1.9683$ GeV, $M(D_{s0}^*) = 2.317$ GeV, $M(D_{s1}) = 2.459$ GeV, $M(D_{s2}) = 2.572$ GeV, $M(D_s^{*'}) = 2.73$ GeV.

Heavy-light mesons are not charge conjugation eigenstates and so mixing can occur among states with the same J^P. The $J^P = 1^+$ axial vector $c\bar{n}$ and $c\bar{s}$ mesons D_1 and D_1' are the coherent superpositions of quark model $^3P_1$ and $^1P_1$ states,

$$|D_1\rangle = \cos\theta|^1P_1\rangle + \sin\theta|^3P_1\rangle$$
$$|D_1'\rangle = -\sin\theta|^1P_1\rangle + \cos\theta|^3P_1\rangle. \tag{13}$$

Little is known about the $^3P_1 - {}^1P_1$ mixing angle θ at present, however, in the heavy quark limit, the mixing angle is predicted to be $-54.7°$ or $35.3°$ if the expectation of heavy quark spin-orbit interaction is positive or negative [37,38]. Since the former implies that the 2^+ state mass is larger than the 0^+ state mass, and this agrees with the experiment, we assume $\theta = -54.7°$ in the following. We note that generally finite quark mass will modify this mixing angle, and we can extract how large the mixing angle is by studying the dependence of the strong decay amplitudes on the mixing angle θ.

When we calculate the decay widths from the amplitudes, there are ambiguities around the choice of phase space. The first choice is the fully relativistic phase space, which leads to a factor of $\frac{E_B E_C}{M_A}$ in the final expression for the width in the center of mass frame, where E_B and E_C are, respectively, the energies of mesons B and C, and M_A is the mass of meson A. The second choice is fully nonrelativistic phase space, then the energy factor is replaced by $M_B M_C/M_A$, which is smaller than the relativistic phase space. A third possibility employed by Kokoski and Isgur, is the "mock meson" method, they suggest that the energy factor should be $\tilde{M}_B\tilde{M}_C/\tilde{M}_A$, where $\tilde{M}_i$ $(i = A, B, C)$ is the "mock meson" mass, which are the calculated masses of the meson i in the spin-independent quark-antiquark potential [28]. In practice, the numerical result is little different from the relativistic phase space except for the pseudoscalar Goldstone bosons involved in the final states. Therefore we shall give our partial width predictions for the relativistic phase space and nonrelativistic phase space in the following.

Theoretical estimates for the harmonic oscillator parameters β and β_A scatter in a relative large region 0.3–0.7 GeV. Many recent quark model studies of mesons [26,39,40] and baryon [41] decays in $^3P_0$ model use the value 0.4 GeV. Moreover, the harmonic oscillator parameters of D, D^*, and $D(^3P_J)$ etc. are predicted to be 0.45–0.66 GeV, and mostly center around 0.5 GeV [28]. Therefore we take $\beta_A = 0.4$ GeV, $\beta = 0.5$ GeV in our calculation as an illustration, the outgoing mesons center of mass (CM) momentum p_B, the partial widths and strong decay amplitudes for the kinematically allowed open-charm decay modes of $Y(4360)$ and $Y(4660)$ are shown in Tables X, XI, XII, XIII, XIV, XV, XVI, and XVII. We shall discuss some interesting and characteristic aspects about the strong decays of $Y(4360)$ and $Y(4660)$.

A. Discussions about $Y(4360)$ strong decay

From Tables X, XI, and XII, we can see that if $Y(4360)$ is a pure 3^3D_1 $c\bar{c}$ state with mass 4.361 GeV, there are ten open-charm strong decay modes: DD, D^*D, D^*D^*, D_2^*D, $D_0^*D^*$, D_1D, $D'D$, D_sD_s, $D_s^*D_s$, $D_s^*D_s^*$, and the total width is predicted to be 67.69 MeV (RPS) or 53.24 MeV (NRPS), comparing with the Belle experimental measurement $74 \pm 15 \pm 10$ MeV. Provided that $Y(4360)$ mass is the prediction of nonrelativistic potential model (4.455 GeV), then the additional decay modes D_1D^*, $D_1'D^*$, $D_{s0}D_s^*$, and $D_{s1}D_s$ become available. There is a relative large difference between the relative phase space normalization and

TABLE X. Open-charm strong decay of $Y(4360)(3^3D_1)$, a factor of $+i$ has been suppressed in all old partial waves. $Y(4360)$ mass is the experimental value 4.361 GeV.

Mode	p_B (GeV)	Γ (MeV) RPS	NRPS	Amps($\mathrm{GeV}^{-1/2}$)
DD	1.12	39.03	28.69	$M_{P0} = 0.3877$
D^*D	1.00	18.02	14.24	$M_{P1} = -0.1977$
D^*D	0.85	6.83	5.79	$M_{P0} = 0.0705$
				$M_{P1} = 0$
				$M_{P2} = -0.0315$
				$M_{F2} = 0.1695$
D_2^*D	0.26	0.26	0.25	$M_{D2} = -0.0463$
$D_0^*D^*$	0.31	0.67	0.66	$M_{S1} = 0$
				$M_{D1} = -0.0684$
D_1D	0.32	0.62	0.60	$M_{S1} = 0$
				$M_{D1} = -0.0653$
$D_1'D$	0.38	1.56	1.50	$M_{S1} = 0.0776$
				$M_{D1} = -0.0534$
D_sD_s	0.94	1.23	1.01	$M_{P0} = 0.1066$
$D_s^*D_s$	0.77	0.00	0.00	$M_{P1} = 4.3 \times 10^{-4}$
$D_s^*D_s^*$	0.54	0.11	0.10	$M_{P0} = -0.0135$
				$M_{P1} = 0$
				$M_{P2} = 0.0060$
				$M_{F2} = -0.0384$
Total		68.33	52.84	

TABLE XI. Open-charm strong decay of $Y(4360)(3^3D_1)$, $Y(4360)$ mass is 4.455 GeV the prediction of the nonrelativistic potential model.

Mode	p_B (GeV)	Γ (MeV) RPS	Γ (MeV) NRPS	Amps(GeV$^{-1/2}$)
DD	1.21	51.33	36.15	$M_{P0} = 0.4234$
D^*D	1.10	35.42	26.81	$M_{P1} = -0.2615$
D^*D^*	0.96	18.22	14.80	$M_{P0} = 0.1411$
				$M_{P1} = 0$
				$M_{P2} = -0.0631$
				$M_{F2} = 0.2367$
D_2^*D	0.52	0.02	0.02	$M_{D2} = -0.0089$
$D_0^*D^*$	0.55	0.00	0.00	$M_{S1} = 0$
				$M_{D1} = 0.0034$
D_1D^*	0.08	0.01	0.01	$M_{S1} = 0$
				$M_{D1} = 0.0117$
				$M_{D2} = 0.0003$
D_1D	0.55	0.00	0.00	$M_{S1} = 0$
				$M_{D1} = 0.0004$
$D_1'D^*$	0.24	0.50	0.49	$M_{S1} = -0.0323$
				$M_{D1} = -0.0304$
				$M_{D2} = -0.0506$
$D_1'D$	0.59	0.34	0.31	$M_{S1} = -0.0046$
				$M_{D1} = 0.0345$
D_sD_s	1.04	4.70	3.67	$M_{P0} = 0.1953$
$D_s^*D_s$	0.89	0.55	0.46	$M_{P1} = -0.0509$
$D_s^*D_s^*$	0.71	0.02	0.02	$M_{P0} = -0.0117$
				$M_{P1} = 0$
				$M_{P2} = 0.0052$
				$M_{F2} = 0.0091$

the nonrelativistic phase space normalization in the DD mode, since the outgoing CM momentum p_B is comparable to the D meson mass in this case.

The leading decay mode of $Y(4360)$ is predicted to be DD with a branching ratio $\approx 57\%$, the second-largest decay mode is $D^*D(\approx 26\%)$, and the D^*D^* mode also has sizable branching ratio. The relative partial wave amplitudes in the D^*D^* final state are predicted to have a very interesting pattern, $M_{P0}/M_{P2} = -\sqrt{5}$, and M_{F2} is predicted to be dominant, whereas it is zero for an S-wave charmonium decay. Measuring the relative branching ratio

TABLE XII. Open-charm strong decay of $Y(4360)(3^3D_1)$, $Y(4360)$ mass is 4.455 GeV the prediction of the nonrelativistic potential model (continued).

Mode	p_B (GeV)	Γ (MeV) RPS	Γ (MeV) NRPS	Amps(GeV$^{-1/2}$)
$D_{s0}^*D_s^*$	0.24	0.12	0.12	$M_{S1} = 0$
				$M_{D1} = -0.0462$
$D_{s1}D_s$	0.25	0.11	0.10	$M_{S1} = 0$
				$M_{D1} = -0.0428$
Total		111.35	82.96	

experimentally can determine whether $Y(4360)$ is D-wave charmonium or S-wave charmonium. In addition, we find the following relation,

$$M_{S1}(3^3D_1 \to {}^3P_1 + {}^1S_0) = \frac{1}{\sqrt{2}} M_{S1}(3^3D_1 \to {}^1P_1 + {}^1S_0)$$
$$M_{S1}(3^3D_1 \to {}^3P_1 + {}^3S_1) = \frac{1}{\sqrt{2}} M_{S1}(3^3D_1 \to {}^1P_1 + {}^3S_1). \quad (14)$$

Therefore for the heavy mixing angle $\theta = -54.7°$, we have the following relations,

$$M_{S1}(Y(4360) \to D_1 + D) = M_{S1}(Y(4360) \to D_1 + D^*) = 0. \quad (15)$$

Thus, if $Y(4360)$ is a pure 3^3D_1 $c\bar{c}$ state, the decays of $Y(4360)$ to D_1D or D_1D^* (are in D-wave rather than in S-wave, where D_1 is the broader of the 1^+ $c\bar{q}$ $(q = u, d)$ axial vector mesons. To test the robustness of our conclusions, we study the stability of our results with respect to the variation of β. The β dependence of the partial decay width and the total decay width are, respectively, shown in and Figs. 1 and 2. In Fig. 1 we showed the variation of DD, D^*D, D^*D^*, D_2^*D, $D_0^*D^*$, D_1D, and $D_1'D$ partial decay widths with the harmonic oscillation parameter β, and we see that the partial decay widths into $S + P$ final states are small.

B. Discussions about $Y(4660)$ strong decay

Since the mass of $Y(4660)$ is large, many open-charm strong decay modes are allowable, which are listed obviously in Tables XIII, XIV, XV, XVI, and XVII. For $Y(4660)$ mass being the experimental value 4.664 GeV, the total width is predicted to be 45.04 MeV (RPS) or 32.78 MeV (NRPS) by our parameters, which is in agreement with the

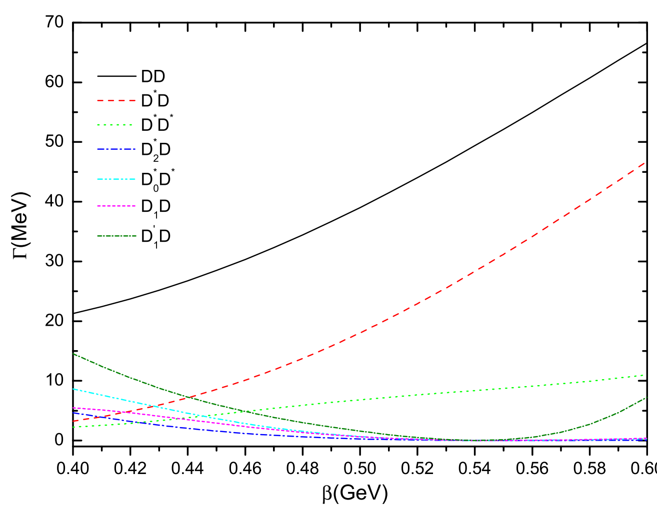

FIG. 1 (color online). The variation of DD, D^*D, D^*D^*, D_2^*D, $D_0^*D^*$, D_1D, and $D_1'D$ partial decay widths with β for $Y(4360)$ as a 3^3D_1 charmonium state.

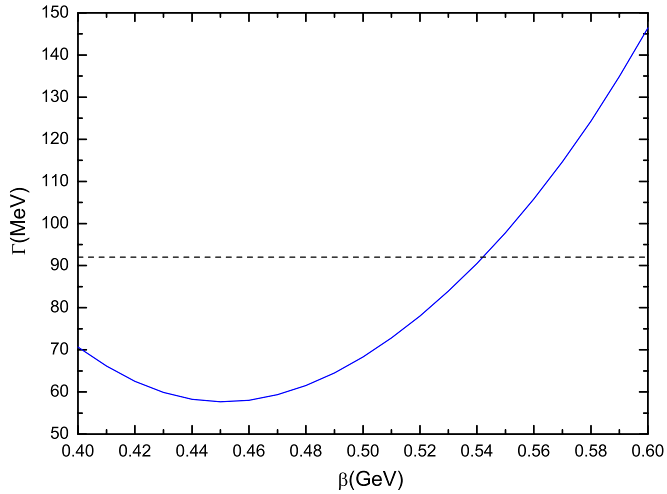

FIG. 2 (color online). $Y(4360)$ total width dependence on β as a 3^3D_1 $c\bar{c}$ state in the flux-tube model, the horizontal line denotes the current experimental upper bound [9].

Belle's measurement $48 \pm 15 \pm 3$ MeV. $Y(4660)$ dominantly decays into D^*D, D^*D^* with branching ratios about

TABLE XIII. Open-charm strong decay of $Y(4660)(5^3S_1)$. M_{LJ} is the partial wave amplitude, where $L = S, P, D, \ldots$ is the relative angular momentum and J is their total spin. Note that a factor of $+i$ has been suppressed in all old partial waves. $Y(4660)$ mass is the experimental value 4.664 GeV.

Mode	p_B (GeV)	Γ (MeV) RPS	NRPS	Amps(GeV$^{-1/2}$)
DD	1.39	5.29	3.40	$M_{P0} = 0.1238$
D^*D	1.30	17.47	12.06	$M_{P1} = 0.1651$
D^*D^*	1.19	15.32	11.36	$M_{P0} = 0.0499$
				$M_{P1} = 0$
				$M_{P2} = -0.2230$
				$M_{F2} = 0$
$D_2^*D^*$	0.67	0.23	0.21	$M_{S1} = 0$
				$M_{D1} = 0.0042$
				$M_{D2} = 0.0055$
				$M_{D3} = -0.0258$
				$M_{G3} = 0$
D_2^*D	0.86	0.75	0.64	$M_{D2} = -0.0422$
$D_0^*D^*$	0.88	1.00	0.86	$M_{S1} = -0.0480$
				$M_{D1} = 0$
D_1D^*	0.69	0.22	0.20	$M_{S1} = -0.0256$
				$M_{D1} = 0$
				$M_{D2} = 0$
D_1D	0.88	0.94	0.80	$M_{S1} = -0.0467$
				$M_{D1} = 0$
$D_1'D^*$	0.73	0.01	0.01	$M_{S1} = 0$
				$M_{D1} = -0.0032$
				$M_{D2} = 0.0056$
$D_1'D$	0.91	1.78	1.50	$M_{S1} = 0$
				$M_{D1} = 0.0634$
$D^{*\prime}D^*$	0.19	0.54	0.54	$M_{P0} = 0.0166$

TABLE XIV. Open-charm strong decay of $Y(4660)(5^3S_1)$. $Y(4660)$ mass is the experimental value 4.664 GeV (continued).

Mode	p_B (GeV)	Γ (MeV) RPS	NRPS	Amps(GeV$^{-1/2}$)
$D^{*\prime}D^*$				$M_{P1} = 0$
				$M_{P2} = -0.0743$
				$M_{F2} = 0$
$D^{*\prime}D$	0.59	0.28	0.26	$M_{P1} = -0.0313$
$D'D^*$	0.42	0.06	0.05	$M_{P1} = 0.0165$
$D'D$	0.69	0.04	0.04	$M_{P0} = 0.0107$
D_sD_s	1.25	0.50	0.35	$M_{P0} = 0.0567$
$D_s^*D_s$	1.13	0.39	0.30	$M_{P1} = 0.0373$
$D_s^*D_s^*$	0.99	0.00	0.00	$M_{P0} = 0.0014$
				$M_{P1} = 0$
				$M_{P2} = -0.0061$
				$M_{F2} = 0$
$D_{s2}^*D_s$	0.53	0.00	0.00	$M_{D2} = 0.0031$
$D_{s0}^*D_s^*$	0.73	0.06	0.05	$M_{S1} = 0.0180$
				$M_{D1} = 0$
$D_{s1}D_s^*$	0.46	0.02	0.02	$M_{S1} = 0.0121$
				$M_{D1} = 0$
				$M_{D2} = 0$
$D_{s1}D_s$	0.73	0.06	0.05	$M_{S1} = 0.0180$
				$M_{D1} = 0$
$D_{s1}'D_s^*$	0.20	0.01	0.01	$M_{S1} = 0$
				$M_{D1} = 0.0052$
				$M_{D2} = -0.0090$
$D_{s1}'D_s$	0.60	0.02	0.02	$M_{S1} = 0$
				$M_{D1} = -0.0106$
$D_s'D_s$	0.24	0.05	0.05	$M_{P0} = 0.0280$
Total		45.04	32.78	

38% and 34%, respectively, and DD is also an important mode. Some partial width of $S + P$ final states can be comparable to the DD partial width for certain parameters. We would like to mention that $Y(4660)$ as a 5^3S_1 charmonium has four nodes in the radial wave function, consequently some modes of smaller branching ratios could be sensitive to the node's positions.

It is interesting to note that the flux-tube model predicts the following relations between amplitudes,

$$\begin{aligned}
M_{S1}(5^3S_1 \to {}^3P_1 + {}^3S_1) &= -\sqrt{2} M_{S1}(5^3S_1 \to {}^1P_1 + {}^3S_1)\\
M_{D1}(5^3S_1 \to {}^3P_1 + {}^3S_1) &= \frac{1}{\sqrt{2}} M_{D1}(5^3S_1 \to {}^1P_1 + {}^3S_1)\\
M_{D2}(5^3S_1 \to {}^3P_1 + {}^3S_1) &= \frac{1}{\sqrt{2}} M_{D2}(5^3S_1 \to {}^1P_1 + {}^3S_1)\\
M_{S1}(5^3S_1 \to {}^3P_1 + {}^1S_0) &= -\sqrt{2} M_{S1}(5^3S_1 \to {}^1P_1 + {}^1S_0)\\
M_{D1}(5^3S_1 \to {}^3P_1 + {}^1S_0) &= \frac{1}{\sqrt{2}} M_{D1}(5^3S_1 \to {}^1P_1 + {}^1S_0),
\end{aligned} \tag{16}$$

then the following interesting relation appears,

TABLE XV. Open-charm strong decay of $Y(4660)(5^3S_1)$. M_{LJ} is the partial wave amplitude, where $L = S, P, D, \ldots$ is the relative angular momentum and J is their total spin. Note that a factor of $+i$ has been suppressed in all old partial waves. $Y(4660)$ mass is the prediction of the nonrelativistic potential model, which is 4.704 GeV.

Mode	p_B (GeV)	Γ (MeV) RPS	NRPS	Amps(GeV$^{-1/2}$)
DD	1.43	5.13	3.24	$M_{P0} = 0.1200$
D^*D	1.33	19.82	13.45	$M_{P1} = 0.1728$
D^*D^*	1.23	20.93	15.25	$M_{P0} = 0.0571$
				$M_{P1} = 0$
				$M_{P2} = -0.2554$
				$M_{F2} = 0$
$D_2^*D^*$	0.73	0.01	0.01	$M_{S1} = 0$
				$M_{D1} = 0.0009$
				$M_{D2} = 0.0012$
				$M_{D3} = -0.0057$
				$M_{G3} = 0$
D_2^*D	0.91	2.00	1.68	$M_{D2} = -0.0666$
$D_0^*D^*$	0.93	3.37	2.83	$M_{S1} = -0.0852$
				$M_{D1} = 0$
D_1D^*	0.76	0.06	0.05	$M_{S1} = -0.0122$
				$M_{D1} = 0$
				$M_{D2} = 0$
D_1D	0.93	3.19	2.67	$M_{S1} = -0.0834$
				$M_{D1} = 0$
$D_1'D^*$	0.79	0.12	0.11	$M_{S1} = 0$
				$M_{D1} = 0.0089$
				$M_{D2} = -0.0155$
$D_1'D$	0.96	3.54	2.93	$M_{S1} = 0$
				$M_{D1} = 0.0865$

$$M_{D1}(Y(4660) \to D_1 + D^*) = -\frac{1}{\sqrt{3}} M_{D2}(Y(4660) \to D_1 + D^*)$$
$$M_{D1}(Y(4660) \to D_1' + D^*) = -\frac{1}{\sqrt{3}} M_{D2}(Y(4660) \to D_1' + D^*). \quad (17)$$

The above two relations are independent of the $^3P_1 - {}^1P_1$ mixing angle θ. For the heavy quark mixing angle $\theta = -54.7°$, we have the following relations,

$$M_{D1}(Y(4660) \to D_1 + D) = M_{D1}(Y(4660) \to D_1 + D^*) = M_{D2}(Y(4660) \to D_1 + D^*) = 0$$
$$M_{S1}(Y(4660) \to D_1' + D) = M_{S1}(Y(4660) \to D_1' + D^*) = 0. \quad (18)$$

The above relations imply that $Y(4660)$ decays into both D_1D and D_1D^* in S-wave, while into $D_1'D$ and $D_1'D^*$ in D-wave, if it is purely a 5^3S_1 $c\bar{s}$ state. These predictions can also be used to test whether the $^3P_1 - {}^1P_1$ mixing is consistent with the heavy quark prediction.

As remarked in the previous discussion of $Y(4360)$ decay, the D^*D^* mode is especially interesting. There are four partial wave amplitudes in this final state M_{P0}, M_{P1}, M_{P2}, M_{F2}, both M_{P1} and M_{F2} are zero, and the ratio of two nonzero P-wave amplitudes is $M_{P2}/M_{P0} = -2\sqrt{5}$.

TABLE XVI. Open-charm strong decay of $Y(4660)(5^3S_1)$, $Y(4660)$ mass is 4.704 GeV the prediction of the nonrelativistic potential model (continued).

Mode	p_B (GeV)	Γ (MeV) RPS	NRPS	Amps(GeV$^{-1/2}$)
$D^{*'}D^*$	0.36	0.01	0.01	$M_{P0} = 0.0013$
				$M_{P1} = 0$
				$M_{P2} = -0.0058$
				$M_{F2} = 0$
$D^{*'}D$	0.66	0.00	0.00	$M_{P1} = -0.0029$
$D'D^*$	0.52	0.35	0.33	$M_{P1} = 0.0372$
$D'D$	0.75	0.69	0.62	$M_{P0} = 0.0433$
D_sD_s	1.29	0.70	0.49	$M_{P0} = 0.0659$
$D_s^*D_s$	1.17	0.76	0.57	$M_{P1} = 0.0509$
$D_s^*D_s^*$	1.03	0.06	0.05	$M_{P0} = 0.0048$
				$M_{P1} = 0$
				$M_{P2} = -0.0217$
				$M_{F2} = 0$
$D_{s2}^*D_s^*$	0.22	0.01	0.01	$M_{S1} = 0$
				$M_{D1} = -0.0026$
				$M_{D2} = -0.0034$
				$M_{D3} = 0.0160$
				$M_{G3} = 0$
$D_{s2}^*D_s$	0.61	0.02	0.02	$M_{D2} = 0.0113$
$D_{s0}^*D_s^*$	0.79	0.05	0.04	$M_{S1} = 0.0159$
				$M_{D1} = 0$
$D_{s1}D_s^*$	0.55	0.00	0.00	$M_{S1} = -4.3 \times 10^{-5}$
				$M_{D1} = 0$
				$M_{D2} = 0$
$D_{s1}D_s$	0.79	0.05	0.04	$M_{S1} = 0.0160$
				$M_{D1} = 0$

TABLE XVII. Open-charm strong decay of $Y(4660)(5^3S_1)$, $Y(4660)$ mass is 4.704 GeV the prediction of the nonrelativistic potential model (continued).

Mode	p_B (GeV)	Γ (MeV) RPS	NRPS	Amps(GeV$^{-1/2}$)
$D_{s1}'D_s^*$	0.36	0.02	0.02	$M_{S1} = 0$
				$M_{D1} = 0.0073$
				$M_{D2} = -0.0126$
$D_{s1}'D_s$	0.67	0.04	0.03	$M_{S1} = 0$
				$M_{D1} = -0.0149$
$D_s^{*'}D_s$	0.11	0.03	0.03	$M_{P1} = 0.0314$
$D_s'D_s$	0.39	0.00	0.00	$M_{P0} = 0.0051$
Total		60.96	44.48	

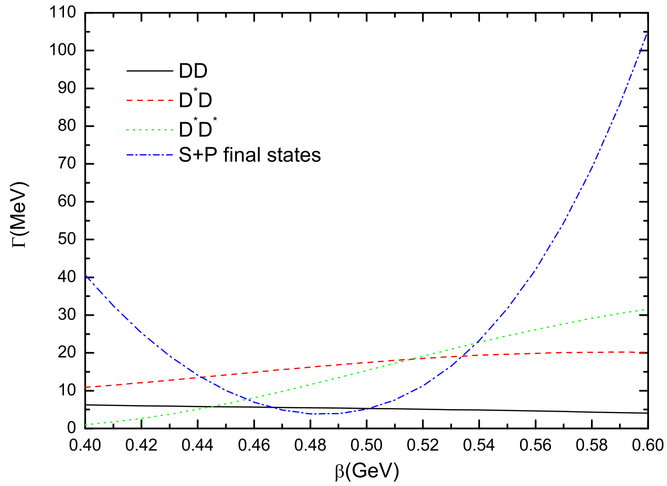

FIG. 3 (color online). The variation of DD, D^*D, D^*D^* and total $S+P$ final states partial decay widths with β for $Y(4660)$ as a 5^3S_1 charmonium state.

However, this ratio is $-1/\sqrt{5}$ in the case of D-wave charmonium decay, as is emphasized in the $Y(4360)$ decay. Experimentally measuring these ratios are essential for understand the nature of $Y(4660)$, and it is also an important test of the flux-tube decay model.

Moreover. $Y(4660)$ can decay into $D_2^*D^*$, which is allowed by phase space. Five partial wave amplitudes are allowed for this process M_{S1}, M_{D1}, M_{D2}, M_{D3}, M_{G3}, both M_{S1} and M_{G3} amplitudes are predicted to be zero, whereas it is nonzero for a D-wave $c\bar{c}$ state decay. The ratios of the three D-wave amplitudes areM_{D1}:M_{D2}:M_{D3} $=$ 1:$\sqrt{\frac{5}{3}}$: $-4\sqrt{\frac{7}{3}}$. These predictions can be used to test whether $Y(4660)$ is an S-wave charmonium, D-wave charmonium, or some other non-$c\bar{c}$ structure. In order to illustrate the parameter dependence of our predictions, we show the β dependence of the DD, D^*D, D^*D^* and total $S+P$ final states partial decay widths and the total width in Figs. 3 and 4 for $Y(4660)$ as a $5^3S_1c\bar{c}$ state. There are thirteen channels whose final states are S-wave and P-wave D mesons, and each partial decay width into $S+P$ final state is at most close to the DD partial width for a large part of the parameter regions.

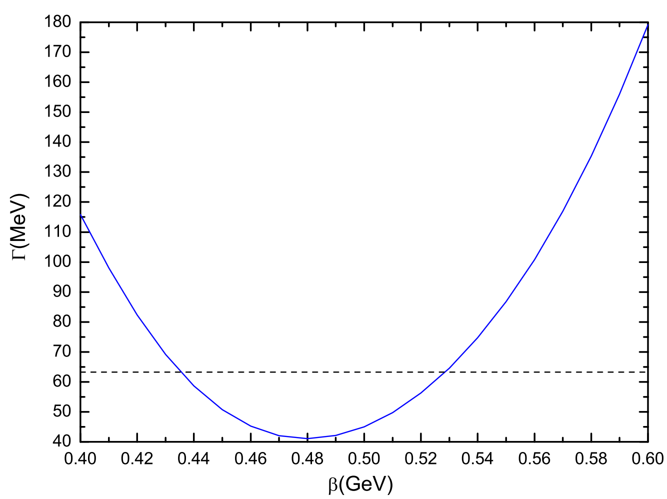

FIG. 4 (color online). $Y(4660)$ total width dependence on β as a 5^3S_1 $c\bar{c}$ state in the flux-tube model, the horizontal line denotes the current experimental upper bound [9].

In short summary, if $Y(4360)$ and $Y(4660)$ are 3^3D_1 and 5^3S_1 $c\bar{c}$ states, respectively, the DD, D^*D, and D^*D^* are expected to be the dominant decay modes, even if the variation of parameter is included. Some $S+P$ final state may have a comparable branching ratio for certain parameters. Careful measurements of these modes are crucial in testing these charmonium assignments.

V. DISCUSSION AND CONCLUSION

In this paper, we consider the canonical charmonium assignments of $Y(4360)$ and $Y(4660)$. Since these two structures are produced in initial state radiation from e^+e^- annihilation and hence to have $J^{PC}=1^{--}$, so they are S-wave or D-wave states if they are canonical charmoniums. From the mass spectrum prediction of the nonrelativistic potential model, we suggest $Y(4360)$ is a 3^3D_1 $c\bar{c}$ state, and $Y(4660)$ is a 5^3S_1 $c\bar{c}$ state, if they are both a conventional charmonium state, although $Y(4360)$ mass is somewhat smaller than the nonrelativistic potential model prediction. We have investigated the e^+e^- leptonic decay, $E1$ transitions, $M1$ transitions, and open-charm strong decay of both $Y(4360)$ and $Y(4660)$ in detail.

Although the mass of $Y(4360)$ is consistent with $Y(4325)$ observed by the *BABAR* collaboration, it is much narrower. Thus more data are required to clarify whether they are the same structure. From the e^+e^- partial width of $Y(4360)$, we estimate $\mathcal{B}(Y(4360)\to\pi^+\pi^-\psi(2S))\sim 1.20\times10^{-2}$ (or 1.36×10^{-2}), which is a little larger than the corresponding branching ratio for a conventional D-wave $c\bar{c}$ state decay. It possibly may be due to large QCD radiative corrections or some other non-$c\bar{c}$ components. It also indicates we should examine other possible interpretations of $Y(4360)$ further, the $D_1\bar{D}$ and $D_{s1}\bar{D}_s$ threshold effects especially deserve considering seriously [42].

The Lattice QCD simulations predict that lightest charmonium hybrid ($c\bar{c}g$) is about 4.4 GeV [43–45]. It is obvious that $Y(4360)$ mass is very near to 4.4 GeV, so we can not completely exclude the possibility of $Y(4360)$ as a 1^{--} charmonium hybrid, although $Y(4260)$ is already assumed to be a good candidate of 1^{--} charmonium hybrid [46–48]. Supposing that $Y(4360)$ is a charmonium hybrid, its main decay modes should be D_2^*D, $D_0^*D^*$, D_1D, $D_1'D$ according to the famous "$S+P$" selection rule in hybrid decay, and the DD, D^*D^*, D^*D^* modes should be highly suppressed. Consequently measuring the DD, D^*D, D^*D^*, D_2^*D, $D_0^*D^*$, D_1D, $D_1'D$ modes are critical in distinguishing the canonical charmonium from the charmonium hybrid interpretation.

$\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\chi_0(2^3P_0)$, and $\chi_1(2^3P_1)$ are the main $Y(4360)$ $E1$ transition modes as a 2^3D_1 $c\bar{c}$ state, which possibly may be used to produce $X(3940)$ and $Y(3940)$, since they are expected to belong 2^3P_J multiplet. The strong suppression of $Y(4660)$ $E1$ transitions to n^3P_J ($n = 1, 2, 3$) multiplet is predicted. The $M1$ transition of $Y(4360)$ and $Y(4660)$ should to too weak to be observed.

We have discussed the open-charm strong decays of $Y(4360)$ and $Y(4660)$ in the flux-tube model in detail. Both $Y(4360)$ and $Y(4660)$ are predicted to dominantly decay into DD, D^*D, D^*D^*, the partial width of some $S+P$ final states can be comparable with that of DD, D^*D, or D^*D^* for certain parameters. Measuring the ratios of the amplitudes in the D^*D^* final state will show whether $Y(4360)$ and $Y(4660)$ are consistent with the charmonium assignments made in this work. If $Y(4360)$ is a pure 3^3D_1 charmonium, M_{F2} amplitude is predicted to be largest and $M_{P2}/M_{P0} = -2/\sqrt{5}$. For $Y(4660)$ as a pure 5^3S_1 $c\bar{c}$ state, we predict that the amplitude M_{F2} is zero and $M_{P2}/M_{P0} = -2/\sqrt{5}$. Similarly the $D_2^*D^*$ amplitude ratios in $Y(4660)$ decay can test whether $Y(4660)$ is an S-wave or D-wave $c\bar{c}$ state, although the branching ratio of $D_2^*D^*$ is predicted to be small. Provided that $Y(4660)$ is a 5^3S_1 $c\bar{c}$ state, we have $M_{D1}:M_{D2}:M_{D3} = 1:\sqrt{5/3}:-4\sqrt{7/3}$, and the amplitudes M_{S1} and M_{G3} are zero, which is nonzero for a D-wave state decay. The above results are generally correct for S-wave or D-wave initial state decay. The careful measurement of these relative branching ratios would play a critical role in understanding $Y(4360)$ and $Y(4660)$.

The Belle and *BABAR* Collaboration have measured the exclusive $e^+e^- \to DD$, $e^+e^- \to DD^*$, and $e^+e^- \to D^*D^*$ cross section using initial state radiation [49–51], and the shapes of the cross sections are similar. There is a peak in the $Y(4660)$ region, however, no structure is clearly observed near the position of $Y(4360)$ so far. Therefore $Y(4660)$ as a 5^3S_1 $c\bar{c}$ state is consistent with current experimental data, however, $Y(4360)$ may be a state beyond the quark model. Since the DD_1 threshold is 4291.4 MeV ($M_D + M_{D_1} \approx 4291.4$ MeV), which is close to $Y(4260)$, a possible way of reconciling $Y(4260)$ and $Y(4360)$ is that $Y(4260)$ is mainly the S-wave DD_1 threshold effect and $Y(4360)$ is a charmonium hybrid. The relevant work is in progress [52].

The confirmations and more experimental studies on $Y(4360)$ and $Y(4660)$ at BES and CLEO are expected. Careful study of $Y(4360)$ and $Y(4660)$ will greatly shed light on the charmonium spectroscopy.

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