PHYSICAL REVIEW D **77,** 014031 (2008)

Bottom baryons

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(Received 2 October 2007; published 28 January 2008)

Recently the CDF and D0 Collaborations observed several bottom baryons. In this work we perform a systematic study of the masses of bottom baryons up to $1/m_Q$ in the framework of heavy quark effective field theory using the QCD sum rule approach. The extracted chromo-magnetic splitting between the bottom baryon heavy doublet agrees well with the experimental data.

DOI: 10.1103/PhysRevD.77.014031 PACS numbers: 12.38.Lg, 14.20.Mr

I. INTRODUCTION

Recently the CDF Collaboration observed four bottom baryons $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ [1,2]. The D0 Collaboration announced the observation of Ξ_b [3], which was confirmed by the CDF Collaboration later [4,5]. Very recently, *BABAR* Collaboration reported the observation of Ω_c^* with the mass splitting $m_{\Omega_c^*} - m_{\Omega_c} = 70.8 \pm 1.0 \pm 1.1$ MeV [6]. We collect the masses of these recently observed bottom baryons in Table I.

The heavy hadron containing a single heavy quark is particularly interesting. The light degrees of freedom (quarks and gluons) circle around the nearly static heavy quark. Such a system behaves as the QCD analogue of the familiar hydrogen bounded by electromagnetic interaction. The heavy quark expansion provides a systematic tool for heavy hadrons. When the heavy quark mass $m_Q \to \infty$, the angular momentum of the light degree of freedom is a good quantum number. Therefore, heavy hadrons form doublets. For example, Ω_b and Ω_b^* will be degenerate in the heavy quark limit. Their mass splitting is caused by the chromo-magnetic interaction at the order $\mathcal{O}(1/m_Q)$, which can be taken into account systematically in the framework of heavy quark effective field theory (HQET).

In the past two decades, various phenomenological models have been used to study heavy baryon masses [7–12]. Capstick and Isgur studied the heavy baryon system in a relativized quark potential model [7]. Roncaglia *et al.* predicted the masses of baryons containing one or two heavy quarks using the Feynman-Hellmann theorem and semiempirical mass formulas [8]. Jenkins studied heavy baryon masses using a combined expansion of $1/m_Q$ and $1/N_c$ [9]. Mathur *et al.* predicted the masses of charmed and bottom baryons from lattice QCD [10]. Ebert *et al.* calculated the masses of heavy baryons with the light-diquark approximation [11]. Using the relativistic Faddeev approach, Gerasyuta and Ivanov calculated the masses of the S-wave charmed baryons [13]. Later, Gerasyuta and Matskevich studied the charmed (**70**, **1**$^-$) baryon multiplet using the same approach [14]. Stimulated by recent experimental progress, there have been several theoretical papers on the masses of Σ_b, Σ_b^*, and Ξ_b using the hyperfine interaction in the quark model [15–19]. Recently the strong decays of heavy baryons were investigated systematically using the $^3P_0$ model in Ref. [20].

QCD sum rule is a useful nonperturbative method in hadron physics [21], which has been applied to study heavy baryon masses previously [12,22–33]. The mass sum rules of $\Lambda_{c,b}$ and $\Sigma_{c,b}$ were obtained in full QCD in Refs. [12,22,23]. The mass sum rules of Σ_Q and Λ_Q in the leading order of HQET have been discussed in Refs. [24–26]. Dai *et al.* calculated the $1/m_Q$ correction to the mass sum rules of Λ_Q and $\Sigma_Q^{(*)}$ in HQET [27]. Later the mass sum rules of Λ_Q and $\Sigma_Q^{(*)}$ were reanalyzed in Ref. [28]. The mass sum rules of orbitally excited heavy baryons in the leading order of HQET were discussed in Refs. [29,30] while the $1/m_Q$ correction was considered in Ref. [31]. Recently Wang studied the mass sum rule of $\Omega_{c,b}^*$ [32] while Durães and Nielsen studied the mass sum rule of $\Xi_{c,b}$ using the full QCD Lagrangian [33].

In order to extract the chromo-magnetic splitting between the bottom baryon doublets reliably, we derive the mass sum rules up to the order of $1/m_Q$ in the heavy quark effective field theory in this work. We perform a systematic study of the masses of Ξ_b, Ξ_b', Ξ_b^*, Ω_b, and Ω_b^* through the inclusion of the strange quark mass correction. The result-

TABLE I. The masses of bottom baryons recently observed by the CDF and D0 Collaborations.

	mass (MeV)	Experiment
Σ_b^+	$5808^{+2.0}_{-2.3}$(stat) ± 1.7(syst)	
Σ_b^-	$5816^{+1.0}_{-1.0}$(stat) ± 1.7(syst)	
Σ_b^{*+}	$5829^{+1.6}_{-1.8}$(stat) ± 1.7(syst)	CDF [1,2]
Σ_b^{*-}	$5837^{+2.1}_{-1.9}$(stat) ± 1.7(syst)	
	5774 ± 11(stat) ± 15(syst)	D0 [3]
Ξ_b^-	5793 ± 2.5(stat) ± 1.7(syst)	CDF [4,5]

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ing chromo-magnetic mass splitting agrees well with the available experimental data. As a cross-check, we reproduce the mass sum rules of Λ_b, Σ_b, and Σ_b^* which have been derived in literature previously. As a by-product, we extend the same formalism to the case of charmed baryons while keeping in mind that the heavy quark expansion does not work well for the charmed hadrons.

This paper is organized as follows. We present the formulation of the leading order QCD sum rules in HQET for bottom baryons in Sec. II. The following section is about the $1/m_Q$ correction. The numerical analysis and a short discussion are presented in Sec. IV.

II. QCD SUM RULES FOR HEAVY BARYONS

We first introduce our notations for the heavy baryons. Inside a heavy baryon there is one heavy quark and two light quarks (u, d, or s). It belongs to either the symmetric $\mathbf{6_F}$ or antisymmetric $\bar{\mathbf{3}}_\mathbf{F}$ flavor representation (see Fig. 1). For the S-wave heavy baryons, the total flavor-spin wave function of the two light quarks must be symmetric since their color wave function is antisymmetric. Hence the spin of the two light quarks is either $S = 1$ for $\mathbf{6_F}$ or $S = 0$ for $\bar{\mathbf{3}}_\mathbf{F}$. The angular momentum and parity of the S-wave heavy baryons are $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$ for $\mathbf{6_F}$ and $J^P = \frac{1}{2}^+$ for $\bar{\mathbf{3}}_\mathbf{F}$. The names of S-wave heavy baryons are listed in Fig. 1, where we use $*$ to denote $\frac{3}{2}^+$ baryons and the $\prime$ to denote the $J^P = \frac{1}{2}^+$ baryons in the $\mathbf{6_F}$ representation. In this work, we use B to denote the heavy baryons with $\frac{1}{2}^+$ in $\bar{\mathbf{3}}_\mathbf{F}$ and B' and B^* to denote those states with $\frac{1}{2}^+$ and $\frac{3}{2}^+$ in $\mathbf{6_F}$.

We will study heavy baryon masses in HQET using the QCD sum rule approach. HQET plays an important role in the investigation of the heavy hadron properties [34]. In the limit of $m_Q \to \infty$, the heavy quark field $Q(x)$ in full QCD can be decomposed into its small and large components

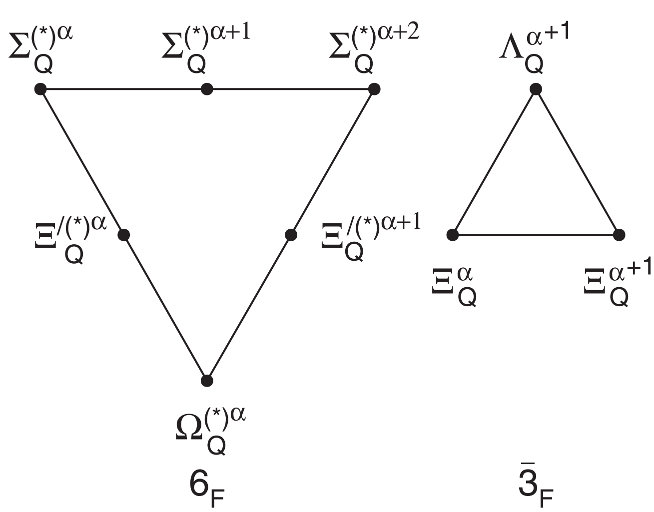

FIG. 1. The SU(3) flavor multiplets of heavy baryons. Here α, $\alpha + 1$, $\alpha + 2$ denote the charges of heavy baryons.

$$Q(x) = e^{-im_Q v\cdot x}[H_v(x) + h_v(x)], \tag{1}$$

where v^μ is the velocity of the heavy baryon. Accordingly the heavy quark field $h_v(x)$ reads

$$h_v(x) = e^{im_Q v\cdot x}\frac{1+\not{v}}{2}Q(x), \tag{2}$$

$$H_v(x) = e^{im_Q v\cdot x}\frac{1-\not{v}}{2}Q(x). \tag{3}$$

The Lagrangian in HQET reads

$$\mathcal{L}_{\rm HQET} = \bar{h}_v iv\cdot D h_v + \frac{1}{2m_Q}\bar{h}_v(iD_\perp)^2 h_v - C_{\rm mag}\frac{g}{4m_Q}\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v. \tag{4}$$

The second and third term in the above Lagrangian corresponds to the kinetic and chromo-magnetic corrections at the order of $1/m_Q$. Here $D_\perp^\mu = D^\mu - v^\mu v\cdot D$ and $D^\mu = \partial^\mu + igA^\mu$. $C_{\rm mag}(\mu)$ is a renormalization coefficient $C_{\rm mag}(\mu) = (\alpha_s(m_Q)/\alpha_s(\mu))^{3/\beta_0}[1 + \frac{13\alpha_s}{6\pi}]$, where $\beta_0 = 11 - 2n_f/3$ and n_f are the number of quark flavors [34].

In order to derive the mass sum rules of B, B', and B^*, we use the following interpolating currents for the heavy baryons with $J^P = \frac{1}{2}^+$ in $\mathbf{6_F}$,

$$J_{B'}(x) = \epsilon_{abc}[q_1^{aT}(x)C\gamma_\mu q_2^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x), \tag{5}$$

$$\bar{J}_{B'}(x) = -\epsilon_{abc}\bar{h}_v^c(x)\gamma_5\gamma_t^\mu[\bar{q}_2^b(x)\gamma_\mu C\bar{q}_1^{aT}(x)]. \tag{6}$$

For the heavy baryons with $J^P = \frac{3}{2}^+$ in $\mathbf{6_F}$,

$$J_{B^*}^\mu(x) = \epsilon_{abc}[q_1^{aT}(x)C\gamma_\nu q_2^b(x)]\left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right)h_v^c(x), \tag{7}$$

$$\bar{J}_{B^*}^\mu(x) = \epsilon_{abc}\bar{h}_v^c(x)\left(-g_t^{\mu\nu} + \frac{1}{3}\gamma_t^\nu\gamma_t^\mu\right)[\bar{q}_2^b(x)\gamma_\nu C\bar{q}_1^{aT}(x)]. \tag{8}$$

For the heavy baryons with $J^P = \frac{1}{2}^+$ in $\bar{\mathbf{3}}_\mathbf{F}$

$$J_B(x) = \epsilon_{abc}[q_1^{aT}(x)C\gamma_5 q_2^b(x)]h_v^c(x), \tag{9}$$

$$\bar{J}_B(x) = -\epsilon_{abc}\bar{h}_v^c(x)[\bar{q}_2^b(x)\gamma_5 C\bar{q}_1^{aT}(x)]. \tag{10}$$

Here a, b, and c are color indices, $q_i(x)$ denotes up, down, and strange quark fields. T is the transpose matrix, and C is the charge conjugate matrix. $g_t^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$, $\gamma_t^\mu = \gamma^\mu - \not{v}v^\mu$.

The overlapping amplitudes of the interpolating currents with B, B', and B^* are defined as

$$\langle 0|J_B|B\rangle = f_B u_B, \tag{11}$$

$$\langle 0|J_{B'}|B'\rangle = f_{B'}u_{B'}, \tag{12}$$

$$\langle 0|J^{\mu}_{B^*}|B^*\rangle = \frac{1}{\sqrt{3}} f_{B^*} u^{\mu}_{B^*}, \tag{13}$$

where $u^{\mu}_{B^*}$ is the Rarita-Schwinger spinor in HQET. $f_{B'} = f_{B^*}$ due to heavy quark symmetry.

The binding energy $\bar{\Lambda}_i$ is defined as the mass difference between the heavy baryon and heavy quark when $m_Q \to \infty$. In order to extract $\bar{\Lambda}_i$, we consider the following correlation function:

$$i \int d^4x e^{iq\cdot x} \langle 0|T\{J_{B^{(\prime)}}(x) \bar{J}_{B^{(\prime)}}(0)\}|0\rangle = \frac{1+\not{v}}{2} \Pi_{B^{(\prime)}}(\omega), \tag{14}$$

with $\omega = v \cdot q$.

The dispersion relation for $\Pi(\omega)$ is

$$\Pi(\omega) = \int \frac{\rho(\omega')}{\omega' - \omega - i\epsilon} d\omega', \tag{15}$$

where $\rho(\omega)$ denotes the spectral density in the limit of $m_Q \to \infty$. At the phenomenological level,

$$\Pi(\omega) = \frac{f_i^2}{\bar{\Lambda}_i - \omega} + \text{Continuum}. \tag{16}$$

Making the Borel transformation with variable ω, we obtain

$$f_i^2 e^{-\bar{\Lambda}_i/T} = \int_0^{\omega_0} \rho(\omega) e^{-\omega/T} d\omega, \tag{17}$$

where we have invoked the quark-hadron duality assumption and approximated the continuum above ω_0 with the perturbative contribution at the quark-gluon level. The mass sum rules of B, B', and B^* are

$$\begin{aligned} f_B^2 e^{-\bar{\Lambda}_B/T} = &\int_0^{\omega_B} \bigg[\frac{\omega^5}{20\pi^4} - \frac{(m_{q_1}^2 + m_{q_2}^2 - m_{q_1} m_{q_2})\omega^3}{4\pi^4} + \frac{\langle g^2 GG\rangle \omega}{128\pi^4} + \frac{m_{q_2}\langle \bar{q}_2 q_2\rangle + m_{q_1}\langle \bar{q}_1 q_1\rangle}{4\pi^2}\omega \\ &- \frac{2m_{q_2}\langle \bar{q}_1 q_1\rangle + 2m_{q_1}\langle \bar{q}_2 q_2\rangle}{4\pi^2} \bigg] e^{-\omega/T} d\omega - \frac{m_{q_1}\langle g_c \bar{q}_2 \sigma G q_2\rangle + m_{q_2}\langle g_c \bar{q}_1 \sigma G q_1\rangle}{32\pi^2} \\ &+ \frac{m_{q_1}\langle g_c \bar{q}_1 \sigma G q_1\rangle + m_{q_2}\langle g_c \bar{q}_2 \sigma G q_2\rangle}{12\cdot 32\pi^2} + \frac{\langle \bar{q}_1 q_1\rangle\langle \bar{q}_2 q_2\rangle}{6} + \frac{\langle \bar{q}_1 q_1\rangle\langle g_c \bar{q}_2 \sigma G q_2\rangle + \langle \bar{q}_2 q_2\rangle\langle g_c \bar{q}_1 \sigma G q_1\rangle}{96T^2}, \end{aligned} \tag{18}$$

$$\begin{aligned} f_{B'}^2 e^{-\bar{\Lambda}_{B'}/T} = &\int_0^{\omega_{B'}} \bigg[\frac{3\omega^5}{20\pi^4} + \frac{(3m_{q_1} m_{q_2} - 3m_{q_1}^2 - 3m_{q_2}^2)\omega^3}{4\pi^4} - \frac{\langle g^2 GG\rangle \omega}{128\pi^4} - \frac{6m_{q_1}\langle \bar{q}_2 q_2\rangle + 6m_{q_2}\langle \bar{q}_1 q_1\rangle}{4\pi^2}\omega \\ &+ \frac{3m_{q_1}\langle \bar{q}_1 q_1\rangle + 3m_{q_2}\langle \bar{q}_2 q_2\rangle}{4\pi^2}\omega \bigg] e^{-\omega/T} d\omega + \frac{\langle \bar{q}_1 q_1\rangle\langle \bar{q}_2 q_2\rangle}{2} - \frac{3m_{q_1}\langle g_c \bar{q}_2 \sigma G q_2\rangle + 3m_{q_2}\langle g_c \bar{q}_1 \sigma G q_1\rangle}{32\pi^2} \\ &+ \frac{5m_{q_1}\langle g_c \bar{q}_1 \sigma G q_1\rangle + 5m_{q_2}\langle g_c \bar{q}_2 \sigma G q_2\rangle}{128\pi^2} + \frac{\langle \bar{q}_2 q_2\rangle\langle g_c \bar{q}_1 \sigma G q_1\rangle + \langle \bar{q}_1 q_1\rangle\langle g_c \bar{q}_2 \sigma G q_2\rangle}{32T^2}. \end{aligned} \tag{19}$$

The mass sum rule of B^* is same as that of B' at the leading order of HQET. In the above equations, $\langle \bar{q}_i q_i\rangle$ is the quark condensates, $\langle g^2 GG\rangle$ is the gluon condensate, and $\langle g\bar{q}_i \sigma G q_i\rangle$ is the quark-gluon mixed condensate. The above sum rules have been derived in the massless light-quark limit in Refs. [24–27]. Up and down quark mass correction is tiny for heavy baryons Λ_b, Σ_b, and Σ_b^*. In this work we have included the finite quark mass correction which is important for heavy baryons Ξ_b, Ξ_b', Ξ_b^*, Ω_b, and Ω_b^*.

The binding energy $\bar{\Lambda}_i$ can be extracted using the following formula:

$$\bar{\Lambda}_i = \frac{T^2 d\mathbb{R}_i}{\mathbb{R}_i dT}, \tag{20}$$

where $\mathbb{R}_i$ denotes the right-hand part in the above sum rules.

III. $1/m_Q$ CORRECTION

In order to calculate the $1/m_Q$ correction, we insert the heavy baryon eigenstate of the Hamiltonian up to the order $\mathcal{O}(1/m_Q)$ into the correlation function

$$i \int d^4x e^{iq\cdot x} \langle 0|T[J_i(x) \bar{J}_i(0)]|0\rangle. \tag{21}$$

Its pole contribution is

$$\begin{aligned} \Pi(\omega) &= \frac{(f+\delta f)^2}{(\bar{\Lambda} + \delta m) - \omega} \\ &= \frac{f^2}{\bar{\Lambda} - \omega} - \frac{f^2 \delta m}{(\bar{\Lambda} - \omega)^2} + \frac{2f\delta f}{\bar{\Lambda} - \omega}, \end{aligned} \tag{22}$$

where both δm and δf are $\mathcal{O}(1/m_Q)$.

We consider the three-point correlation function

$$\frac{1+\not{v}}{2}\delta^O\Pi(\omega,\omega') = i^2\int d^4z d^4y e^{ip\cdot z} \times e^{ip'\cdot y}\langle 0|T[J_i(z)O(x)\bar{J}(y)]|0\rangle, \quad (23)$$

where operators $O=\mathcal{K}$ and $\mathcal{S}$ correspond to the kinetic energy and chromo-magnetic interaction in Eq. (4). The double dispersion relation for $\delta^O\Pi(\omega,\omega')$ reads

$$\delta^O\Pi(\omega,\omega') = \int_0^\infty ds\int_0^\infty ds'\frac{\rho^O(s,s')}{(s-\omega)(s'-\omega')}. \quad (24)$$

At the hadronic level,

$$\delta^{\mathcal{K}}\Pi(\omega,\omega') = \frac{f^2\mathcal{K}_i}{(\bar{\Lambda}-\omega)(\bar{\Lambda}-\omega')}+\cdots, \quad (25)$$

$$\delta^{\mathcal{S}}\Pi(\omega,\omega') = \frac{f^2\mathcal{S}_i}{(\bar{\Lambda}-\omega)(\bar{\Lambda}-\omega')}+\cdots \quad (26)$$

with

$$\mathcal{K}_i = \frac{1}{2m_Q}\langle B_i|\bar{h}_v(iD_\perp)^2h_v|B_i\rangle, \quad (27)$$

$$\mathcal{S}_i = -\frac{1}{4m_Q}\langle B_i|\bar{h}_v g\sigma_\mu G^{\mu\nu}h_v|B_i\rangle. \quad (28)$$

After setting $\omega=\omega'$ in Eqs. (25) and (26) and comparing them with Eq. (22), we can extract δm

$$\delta m_i = -(\mathcal{K}_i + C_{\rm mag}\mathcal{S}_i). \quad (29)$$

Here the renormalization coefficient $C_{\rm mag}$ for bottom baryons is $C_{\rm mag}\approx 0.8$ [29].

We calculate the diagrams listed in Fig. 2 to derive $\delta^O\Pi(\omega,\omega')$. After invoking double Borel transformation to Eq. (24), we obtain the spectral density $\rho^O(s,s')$. Then we redefine the integration variable

$$s_+ = \frac{s+s'}{2}, \quad (30)$$

$$s_- = \frac{s-s'}{2}. \quad (31)$$

Now the integral in Eq. (24) is changed as

$$\int_0^\infty ds\int_0^\infty ds'\ldots = 2\int_0^\infty ds_+\int_{-s_+}^{+s_+}ds_-\ldots. \quad (32)$$

In the subtraction of the continuum contribution, quark-hadron duality is assumed for the integration variable s_+ [35].

For $B(\frac{1}{2}^+)$ in $\bar{\mathbf{3}}_{\mathbf{F}}$, the $1/m_Q$ correction comes from the kinetic term only.

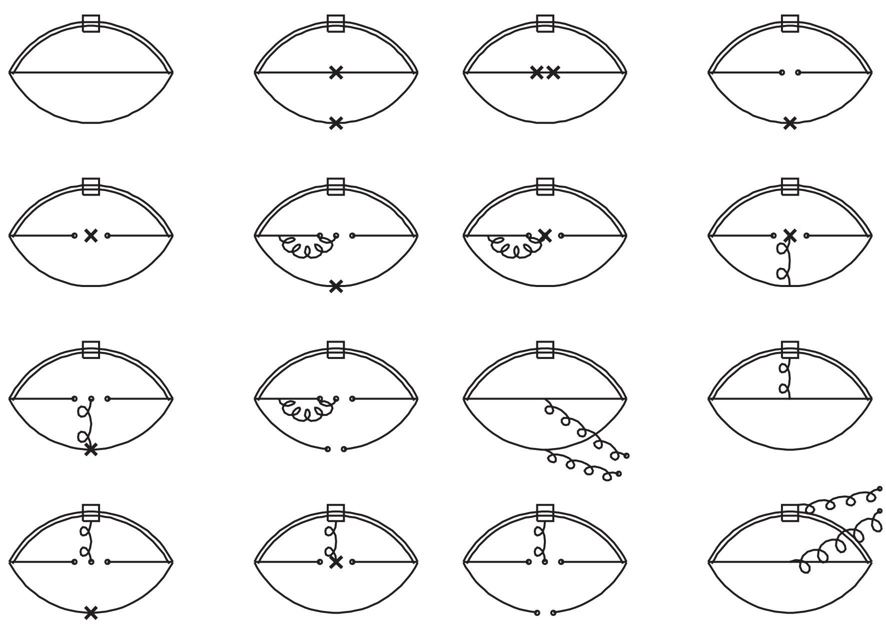

FIG. 2. The diagrams for the $1/m_Q$ corrections. Here the current quark mass correction is denoted by the cross. The first 11 diagrams correspond to the kinetic corrections and the last five diagrams are chromo-magnetic corrections. White squares denote the operators of $1/m_Q$.

$$\mathcal{K}_B = -\frac{e^{\bar{\Lambda}_B/T}}{m_Q f_B^2}\bigg\{\int_0^{\omega_B}\bigg[\frac{54\omega^7}{7!\pi^4} - \frac{9\omega^5}{5!\pi^4}(m_{q_1}^2 + m_{q_2}^2 - m_{q_1}m_{q_2}) + \frac{3\langle g^2GG\rangle\omega^3}{128\cdot 3!\pi^4} + \frac{3\omega^3}{4\cdot 3!\pi^2}(m_{q_1}\langle\bar{q}_1q_1\rangle + m_{q_2}\langle\bar{q}_2q_2\rangle - 2m_{q_2}\langle\bar{q}_1q_1\rangle - 2m_{q_1}\langle\bar{q}_2q_2\rangle) - \frac{3\omega}{128\pi^2}(m_{q_1}\langle g_c\bar{q}_1\sigma Gq_1\rangle + m_{q_2}\langle g_c\bar{q}_2\sigma Gq_2\rangle) + \frac{3\omega}{32\pi^2}(m_{q_1}\langle g_c\bar{q}_2\sigma Gq_2\rangle + m_{q_2}\langle g_c\bar{q}_1\sigma Gq_1\rangle)\bigg]e^{-\omega/T}d\omega - \frac{1}{32}[\langle\bar{q}_1q_1\rangle\langle g_c\bar{q}_2\sigma Gq_2\rangle + \langle\bar{q}_2q_2\rangle\langle g_c\bar{q}_1\sigma Gq_1\rangle]\bigg\}, \quad (33)$$

$$\mathcal{S}_B = 0. \quad (34)$$

Here $\mathcal{S}_B = 0$ is consistent with the simple expectation in the constituent quark model that the chromo-magnetic interaction $\langle S_Q \cdot j_l\rangle = 0$ since $j_l = 0$ for $B(\frac{1}{2}^+)$ in $\bar{\mathbf{3}}_\mathbf{F}$.

For $B'(\frac{1}{2}^+)$ in $\mathbf{6}_\mathbf{F}$, the $1/m_Q$ corrections are

$$\mathcal{K}_{B'} = -\frac{e^{\bar{\Lambda}_{B'}/T}}{m_Q f_{B'}^2}\bigg\{\int_0^{\omega_{B'}}\bigg[\frac{18\cdot 11\omega^7}{7!\pi^4} - \frac{9\omega^5}{5!\pi^4}(4m_{q_1}^2 + 4m_{q_2}^2 - 3m_{q_1}m_{q_2}) - \frac{\langle g^2GG\rangle\omega^3}{128\cdot 3!\pi^4} + \frac{3\omega^3}{4\cdot 3!\pi^2}(5m_{q_1}\langle\bar{q}_1q_1\rangle + 5m_{q_2}\langle\bar{q}_2q_2\rangle - 6m_{q_2}\langle\bar{q}_1q_1\rangle - 6m_{q_1}\langle\bar{q}_2q_2\rangle) + \frac{11\omega}{128\cdot 4\pi^2}(m_{q_1}\langle g_c\bar{q}_1\sigma Gq_1\rangle + m_{q_2}\langle g_c\bar{q}_2\sigma Gq_2\rangle)\bigg]e^{-\omega/T}d\omega - \frac{3}{32}[\langle\bar{q}_1q_1\rangle\langle g_c\bar{q}_2\sigma Gq_2\rangle + \langle\bar{q}_2q_2\rangle\langle g_c\bar{q}_1\sigma Gq_1\rangle]\bigg\}. \quad (35)$$

$$\mathcal{S}_{B'} = \frac{e^{\bar{\Lambda}_{B'}/T}}{m_Q f_{B'}^2}\bigg\{\int_0^{\omega_{B'}}\bigg[\frac{2g_c^2\omega^7}{105\pi^6} + \frac{\langle g^2GG\rangle\omega^3}{16\cdot 3!\pi^4} - \frac{\omega}{32\pi^2}(m_{q_1}\langle g_c\bar{q}_1\sigma Gq_1\rangle + m_{q_2}\langle g_c\bar{q}_2\sigma Gq_2\rangle - 2m_{q_2}\langle g_c\bar{q}_1\sigma Gq_1\rangle - 2m_{q_1}\langle g_c\bar{q}_2\sigma Gq_2\rangle)\bigg]e^{-\omega/T}d\omega - \frac{1}{48}[\langle\bar{q}_1q_1\rangle\langle g_c\bar{q}_2\sigma Gq_2\rangle + \langle\bar{q}_2q_2\rangle\langle g_c\bar{q}_1\sigma Gq_1\rangle]\bigg\}. \quad (36)$$

Through explicit calculation, we obtain

$$\mathcal{K}_{B^*} = \mathcal{K}_{B'}, \quad (37)$$

$$\mathcal{S}_{B^*} = -\mathcal{S}_{B'}/2, \quad (38)$$

$$m_{B^*} - m_{B'} = \frac{3}{2}\mathcal{S}_{B'}, \quad (39)$$

which are consistent with the heavy quark symmetry.

IV. RESULTS AND DISCUSSION

In our numerical analysis, we use [36–42]

$$\langle\bar{q}q\rangle = -(0.240\ \text{GeV})^3, \quad \langle\bar{s}s\rangle = (0.8\pm 0.1)\times\langle\bar{q}q\rangle, \quad \langle g_s\bar{q}\sigma Gq\rangle = -M_0^2\times\langle\bar{q}q\rangle, \quad M_0^2 = (0.8\pm 0.2)\ \text{GeV}^2,$$
$$\langle g_s^2G^2\rangle = (0.48\pm 0.14)\ \text{GeV}^4, \quad m_u = m_d = 5.3\ \text{MeV}, \quad m_s = 125\ \text{MeV}, \quad m_c = 1.25\pm 0.09\ \text{GeV},$$
$$m_b = 4.8\ \text{GeV}, \quad \alpha_s(m_c) = 0.328, \quad \alpha_s(m_b) = 0.189.$$

The values of the u, d, s and charm quark masses correspond to the $\overline{MS}$ scheme at a scale $\mu \approx 2$ GeV and $\mu = \bar{m}_c$ respectively [42]. The b quark mass is obtained from the $\Upsilon(1S)$ mass [42,43].

Since the energy gap between the S-wave heavy baryons and their radial/orbital excitations is around 500 MeV, the continuum contribution can be subtracted quite cleanly. We require that the high-order power corrections be less than 30% of the perturbative term to ensure the convergence of the operator product expansion. This condition yields the minimum value for the working region of the Borel parameter. In this work, we choose the working region as $0.4 < T < 0.6$ GeV.

In Fig. 3–5, we give the dependence of $\bar{\Lambda}$, $\mathcal{K}_i$, $\mathcal{S}_i$ and mass splitting $m_{B_b^*} - m_{B_b'}$ on T and ω_c for Σ_b, Ξ_b', Ω_b. The variation of a sum rule with both T and ω_i contributes to the errors of the extracted value, together with the truncation of the operator product expansion and the uncertainty

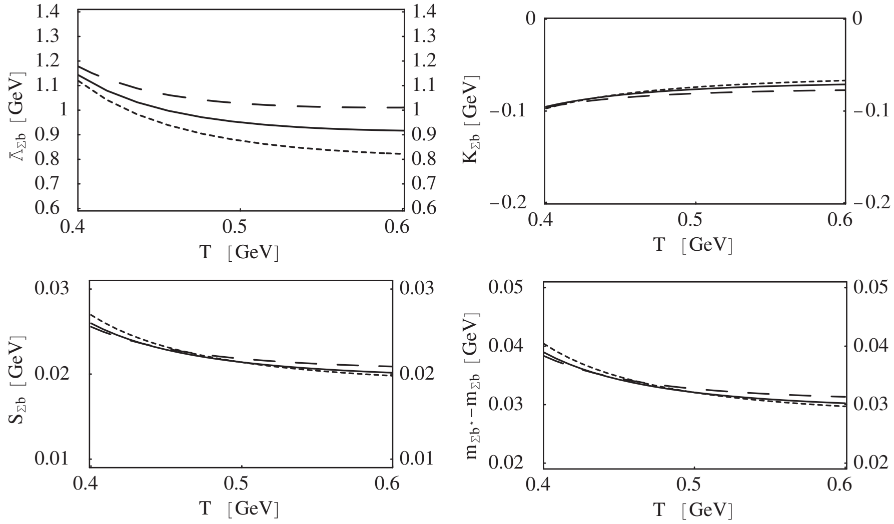

FIG. 3. The dependences of $\bar{\Lambda}_{\Sigma_b}$, $\mathcal{K}_{\Sigma_b}$, $\mathcal{S}_{\Sigma_b}$, and the mass splitting $m_{\Sigma_b^*} - m_{\Sigma_b}$ on T. Here the dotted, solid, and dashed lines correspond to the threshold value $\omega_{\Sigma_b} = 1.2, 1.3, 1.4$ GeV, respectively.

of vacuum condensate values. We collect the extracted $\bar{\Lambda}$, $\mathcal{K}_i$, $\mathcal{S}_i$ and mass splitting $m_{B_c^*} - m_{B_c'}$ in Table II.

The masses of bottom baryons from the present work are presented in Table III. It is well known that the heavy quark expansion does not work very well for the charmed baryons since the charm quark is not heavy enough to ensure the good convergence of $1/m_Q$ expansion. For example, the chromo-magnetic splitting between Ω_c^* and Ω_c from our work is around 133 MeV, which is much larger than the experimental value 67.4 MeV. However, we still choose to present the masses of S-wave charmed baryons also in Table III simply for the sake of comparison with experimental data.

In our calculation, we adopt the phenomenological spectral function by the classical and simple ansatz of a single resonance pole plus the perturbative continuum. The systematic uncertainty of hadron parameters obtained with such an approximation was discussed recently in Ref. [44]. We have not considered the next-to-leading order α_s corrections, which may also result in large contribution and uncertainty as indicated by the study of the α_s corrections in the light-quark baryon system in Ref. [45].

In short summary, inspired by recent experimental observation of charmed and bottom baryons [1–6], we have investigated the masses of heavy baryons systematically using the QCD sum rule approach in HQET. The chromo-

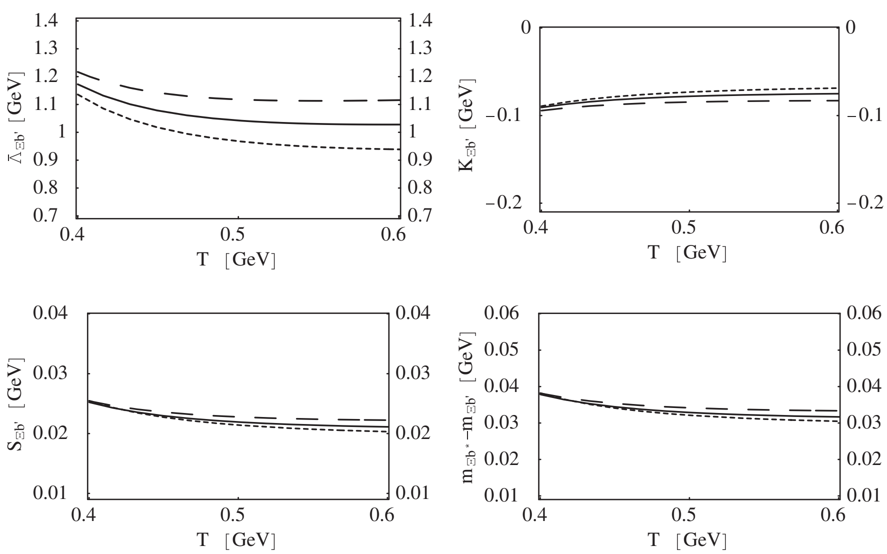

FIG. 4. The dependences of $\bar{\Lambda}_{\Xi_b'}$, $\mathcal{K}_{\Xi_b'}$, $\mathcal{S}_{\Xi_b'}$, and the mass splitting $m_{\Xi_b^*} - m_{\Xi_b'}$ on T. The dotted, solid, and dashed lines correspond to $\omega_{\Xi_b'} = 1.3, 1.4, 1.5$ GeV, respectively.

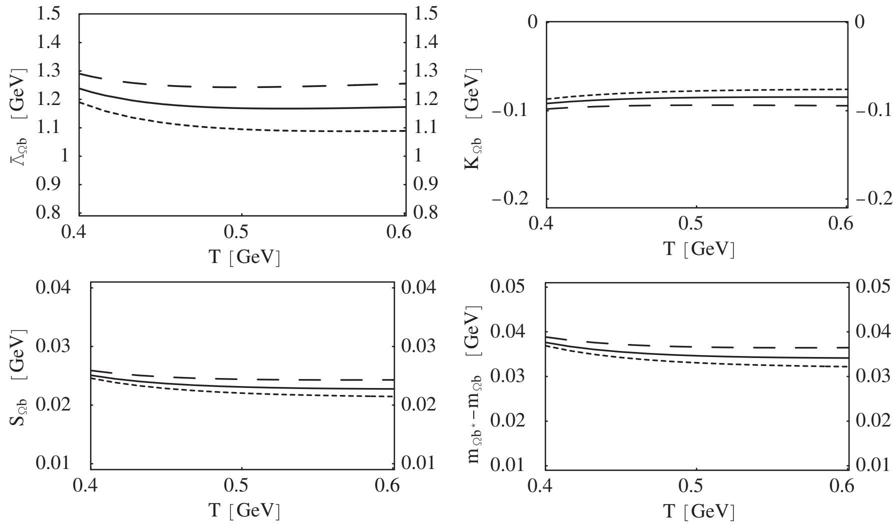

FIG. 5. The dependences of $\bar{\Lambda}_{\Omega_b}$, $\mathcal{K}_{\Omega_b}$, $\mathcal{S}_{\Omega_b}$, and the mass splitting $m_{\Omega_b^*} - m_{\Omega_b}$ on T. The dotted, solid, and dashed lines correspond to $\omega_{\Omega_b} = 1.45$, 1.55, 1.65 GeV, respectively.

TABLE II. The central values in this table are extracted at $T = 0.5$ GeV, $\omega_i = 1.3$ GeV for $\Sigma_b^{(*)}$, $\omega_i = 1.4$ GeV for $\Xi_b^{\prime(*)}$, $\omega_i = 1.55$ GeV for $\Omega_b^{(*)}$, $\omega_i = 1.1$ GeV for Λ_b, and $\omega_i = 1.25$ GeV for Ξ_b (in MeV).

	Σ_b	Ξ_b'	Ω_b^0	Λ_b	Ξ_b
$\bar{\Lambda}$	950^{+78}_{-74}	1042^{+76}_{-74}	1169 ± 74	773^{+68}_{-59}	908^{+72}_{-67}
δm	59^{+4}_{-2}	60^{+6}_{-4}	67^{+7}_{-3}	65^{+2}_{-1}	72 ± 1
mass splitting	$m_{\Sigma_b^*} - m_{\Sigma_b}$	$m_{\Xi_b^*} - m_{\Xi_b'}$	$m_{\Omega_b^*} - m_{\Omega_b}$	-	-
this work	26 ± 1	26 ± 1	28^{+8}_{-2}	-	-
experiment [1,2]	21	-	-	-	-

TABLE III. Masses of the heavy baryons from the present work and other approaches and the comparison with experimental data (in MeV).

Baryon	$I(J^P)$	Ours	Ref. [7]	Ref. [8]	Ref. [9]	Ref. [10]	Ref. [11]	Ref. [28,32]	EXP [2–4,6,42]
Σ_c	$1(\frac{1}{2}^+)$	2411^{+93}_{-81}	2440	2453		2452	2439	2470	2454.02(0.18)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2508^{+97}_{-91}		2580	2580.8	2599	2578		2575.7(3.1)
Ω_c	$0(\frac{1}{2}^+)$	2657^{+102}_{-99}		2710		2678	2698		2697.5(2.6)
Σ_c^*	$1(\frac{3}{2}^+)$	2534^{+96}_{-81}	2495	2520		2538	2518	2590	2518.4(0.6)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2634^{+102}_{-94}		2650		2680	2654		2646.6(1.4)
Ω_c^*	$0(\frac{3}{2}^+)$	2790^{+109}_{-105}		2770	2760.5	2752	2768	2790	~2768
Λ_c	$0(\frac{1}{2}^+)$	2271^{+67}_{-49}	2265	2285		2290	2297		2286.46(0.14)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2432^{+79}_{-68}		2468		2473	2481		2467.9(0.4)
Σ_b	$1(\frac{1}{2}^+)$	5809^{+82}_{-76}	5795	5820	5824.2	5847	5805	5790	5808
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5903^{+81}_{-79}		5950	5950.9	5936	5937		
Ω_b	$0(\frac{1}{2}^+)$	6036 ± 81		6060	6068.7	6040	6065		
Σ_b^*	$1(\frac{3}{2}^+)$	5835^{+82}_{-77}	5805	5850	5840.0	5871	5834	5820	5829
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5929^{+83}_{-79}		5980	5966.1	5959	5963		
Ω_b^*	$0(\frac{3}{2}^+)$	6063^{+83}_{-82}		6090	6083.2	6060	6088	6000	
Λ_b	$0(\frac{1}{2}^+)$	5637^{+68}_{-56}	5585	5620		5672	5622		5624(9)
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5780^{+73}_{-68}		5810	5805.7	5788	5812		5774 5793

magnetic splitting of the bottom baryon doublet from the present work agrees well with the recent experimental data. Recently $\Xi_b^{(*)}$ was observed by the CDF Collaboration [1,2]. Our results are also consistent with their experimental value. Our prediction of the masses of Ξ_b', Ξ_b^*, Ω_b, and Ω_b^* can be tested through the future discovery of these interesting states at Tevatron at Fermi Lab.

ACKNOWLEDGMENTS

X. L. thanks W. Wei for useful discussion. This project was supported by the National Natural Science Foundation of China under Grants No. 10625521, 10705001, and 10721063, Key Grant Project of Chinese Ministry of Education No. 305001, and the China Postdoctoral Science foundation No. 20060400376. H. X. C. is grateful to Monkasho for supporting his stay at Research Center for Nuclear Physics where this work is done. A. H. is supported in part by the Grant for Scientific Research Contract No. 19540297 from the Ministry of Education, Culture, Science and Technology, Japan.

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