Transverse target spin asymmetry in inclusive DIS with two-photon exchange

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We study the transverse target spin dependence of the cross section for inclusive electron-nucleon scattering with unpolarized beam. Such dependence is absent in the one-photon exchange approximation (Christ-Lee theorem) and arises only in higher orders of the QED expansion, from the interference of one-photon and absorptive two-photon exchange amplitudes as well as from real photon emission (bremsstrahlung). We demonstrate that the transverse spin-dependent two-photon exchange cross section is free of QED infrared and collinear divergences. We argue that in DIS kinematics the transverse spin dependence should be governed by a "partonlike" mechanism in which the two-photon exchange couples mainly to a single quark. We calculate the normal spin asymmetry in an approximation where the dominant contribution arises from quark-helicity flip due to interactions with nonperturbative vacuum fields (constituent quark picture) and is proportional to the quark transversity distribution in the nucleon. Such helicity-flip processes are not significantly Sudakov-suppressed if the infrared scale for gluon emission in the photon-quark subprocess is of the order of the chiral symmetry breaking scale, $\mu^2_{\text{chiral}} \gg \Lambda^2_{\text{QCD}}$. We estimate the asymmetry in the kinematics of the planned Jefferson Lab Hall A experiment to be of the order 10^{-4}, with different sign for proton and neutron. We also comment on the spin dependence in the limit of soft high-energy scattering.

I. INTRODUCTION

Transverse spin effects in deep-inelastic $eN/\mu N$ scattering (DIS) are presently a very active field of research, with many interesting developments in experiment and theory. Inclusive production with longitudinally polarized beams and transversely polarized targets measures the spin structure function g_2, which provides access to matrix elements of higher-twist operators describing quark-gluon correlations in the nucleon [1]. Another class of experiments measures the azimuthal distributions of identified hadrons in semi-inclusive production. The theoretical description of these observables relies on certain extensions of the usual collinear QCD expansion, which incorporate quark/hadron transverse momenta and give rise to a rich variety of distribution/fragmentation functions describing spin-orbit interactions of quarks. In analyzing the data one hopes to either learn about the spin-orbit interactions themselves or to use them to extract the quark transversity distributions in the nucleon.

A somewhat different transverse spin effect is the transverse target spin dependence of the cross section of inclusive DIS with unpolarized beam. Such dependence is absent in the $O(\alpha^2)$ cross section in the one-photon exchange approximation, being forbidden by the combination of P and T invariance and the Hermiticity of the electromagnetic current operator (Christ-Lee theorem) [2]. A target spin dependence appears at $O(\alpha^3)$ due to the interference of two-photon and one-photon exchange amplitudes, which can be understood qualitatively as the result of a non-Hermitian effective current induced by the imaginary part of the two-photon exchange amplitude.

Similar two-photon exchange effects were studied as corrections to the eN elastic scattering cross section [3–6], where they partly explain the discrepancy between the G_E/G_M ratio extracted using the Rosenbluth and polarization transfer methods [7,8]; they also play a role in parity-violating electron scattering [9].

The precision reached in eN scattering experiments with modern high-duty cycle accelerators allows one to contemplate accurate measurements of two-photon exchange observables. A Jefferson Lab Hall A experiment [10] plans to measure the transverse target spin asymmetry in inclusive DIS ($E_{\text{beam}} = 6$ GeV, $x = 0.1$–0.45, $Q^2 = 1$–3.5 GeV^2) at the level of a few times 10^{-4}, improving the sensitivity of the only previous measurement at SLAC [11] by 2 orders of magnitude (in the SLAC experiment, the asymmetry was found to be compatible with zero at the level of $\sim 3.5\%$). It is timely to estimate the expected asymmetry in this kinematics.

In this paper we study the transverse target spin dependence in inclusive DIS with unpolarized beam and its relation to the quark structure of the nucleon. This is a challenging problem, combining the complexity of higher-order QED radiative corrections with that of the QCD treatment of transverse spin-dependent deep-inelastic processes. We approach this problem in steps, establishing first some important general properties of the spin-dependent two-photon exchange cross section, then formulating a scheme of approximations which respects these general properties and allows us to estimate the expected asymmetry in DIS kinematics. In studying the general properties of the spin-dependent cross section we shall

employ both general principles [such as factorization of infrared (or IR) divergences, electromagnetic gauge invariance] and specific dynamical models which illustrate certain points.

First, we demonstrate that the transverse spin-dependent two-photon exchange cross section is free of QED IR divergences. On general grounds, it can be shown that the IR divergent terms take the form of a universal spin-independent factor multiplying the one-photon exchange cross section, which does not exhibit a transverse spin dependence. The IR finiteness can also be seen in the explicit expression for the two-photon exchange cross section for scattering from a spin-1/2 point particle. Furthermore, we show that the spin-dependent two-photon exchange cross section is free of QED collinear divergences, which appear in intermediate stages of the calculation for a composite target with off-shell constituents. Such singularities cancel as a consequence of electromagnetic gauge invariance. We illustrate this explicitly in a field-theoretical toy model of electron scattering from a spin-1/2 point particle dressed by a scalar field.

Second, we argue that the transverse spin-dependent cross section in DIS kinematics can be described in a "partonlike" picture, in which the two-photon exchange couples predominantly to a single quark, namely, the same quark which is hit in the interfering one-photon exchange process. Within this picture one then is dealing with two distinct contributions. In one the active quark helicity is conserved, but explicit interactions with the spectator system are required to bring about a nonzero result; this contribution is analogous to the twist-3 part of the spin structure function g_2. In the other, the quark helicity is flipped by interaction with nonperturbative vacuum fields (spontaneous breaking of chiral symmetry) and no interactions with the spectators are required; this contribution is proportional to the product of the quark transversity distribution in the nucleon and the amplitude for the quark helicity flip, which is of the order of a typical "constituent quark" mass. In DIS such processes, going through low-virtuality quarks whose chirality can be flipped by vacuum fields, are in principle Sudakov suppressed relative to those involving virtualities of the order Q^2. We show here that the Sudakov suppression is not very effective if the IR scale for gluon emission is of the order of the chiral symmetry breaking scale, $\mu_{\mathrm{chiral}}^2 \gg \Lambda_{\mathrm{QCD}}^2$, which appears natural from the point of view of the phenomenology of spontaneous chiral symmetry breaking in QCD. A specific realization of this scenario is the instanton vacuum model, in which the chiral symmetry breaking scale is given by the average instanton size $\mu_{\mathrm{chiral}}^2 \sim \rho^{-2} \approx$ $(600 \mathrm{~MeV})^2$ [12,13].

Third, in order to make a quantitative estimate we invoke the additional approximation of a "composite" nucleon, i.e., a weakly bound system of constituent quarks, in which the dominant contribution to the spin-dependent cross section comes from helicity flip at the quark level and can be calculated in terms of the quark transversity distribution in the nucleon. This approximation permits a fully self-consistent treatment of the two-photon exchange cross section, which maintains electromagnetic gauge invariance and is manifestly free of collinear divergences. It allows us to make a numerical estimate of the asymmetry in DIS kinematics and discuss its dependence on the kinematic variables.

We also comment on the behavior of the spin-dependent interference cross section in the limit of soft high-energy scattering (small energy and momentum transfer to the target), where one can make contact with general theorems about the high-energy behavior of QED amplitudes. Also, in this limit one can use the nonrelativistic approximation to describe the target excitation spectrum and see explicitly why scattering from a single quark dominates at larger momentum transfers. This provides a useful complement to the corresponding arguments fielded in DIS kinematics.

Two-photon exchange effects were extensively studied as corrections to the *eN* elastic scattering cross section [3–6]. The two-photon exchange effect in the transverse spin dependence investigated in this article is in many ways simpler than those corrections to the full cross section. In the transverse spin-dependent cross section, one is dealing with a pure higher-order QED observable, which is exactly zero in one-photon exchange approximation. More importantly, because the two-photon exchange in the transverse spin-dependent cross section is IR finite, no cancellations of IR divergences between two-photon exchange and real photon bremsstrahlung take place as in the spin-independent cross section. In fact, this circumstance makes it possible to discuss two-photon exchange as an "autonomous" physical effect in the first place.

This article is organized as follows. In Sec. II we define the transverse spin-dependent cross section and review the Christ-Lee theorem for the one-photon exchange contribution. In Sec. III we revisit in some detail the transverse spin-dependent cross section in the scattering from a point-like spin-1/2 particle, which explicitly shows the IR finiteness and allows us to estimate the effective photon virtualities in the two-photon exchange. In Sec. IV we demonstrate the absence of QED IR and collinear divergences in the transverse spin-dependent cross section on general grounds. In Sec. V we consider the transverse spin dependence in DIS in QCD. We present arguments in favor of dominance of scattering from a single quark, discuss the two contributions (quark helicity-conserving and quark helicity-flip), and the absence of significant Sudakov suppression of quark helicity-flip amplitudes. In Sec. VI we formulate the composite nucleon approximation, in which the quark helicity-flip contribution becomes dominant and can be calculated in a relativistic constituent quark model. In Sec. VII we present numerical results based on this approximation. In Sec. VIII we discuss the limit of soft

high-energy scattering. Our conclusions and perspectives for future studies are summarized in Sec. IX.

II. TRANSVERSE TARGET SPIN IN INCLUSIVE ELECTRON SCATTERING

We consider inclusive electron-nucleon scattering with unpolarized beam and polarized target,

$$e(l) + N(p) \rightarrow e(l') + X. \tag{1}$$

For parity-conserving interactions (strong, electromagnetic) the only allowed dependence of the cross section in the target rest frame on the target spin is through a term proportional to the true scalar

$$(S, l \times l'). \tag{2}$$

Here S denotes the target polarization vector, which is normalized according to $S^2 = 1$ for fully polarized target, and the vector $l \times l'$ is normal to the electron scattering plane. We write the differential cross section for scattering into a phase space element with given final electron momentum l' in the form

$$d\sigma = d\sigma_U + \frac{(S, l \times l')}{|l \times l'|} d\sigma_N. \tag{3}$$

The normal spin asymmetry of the differential cross section is then defined as

$$A_N \equiv \frac{d\sigma_N}{d\sigma_U}. \tag{4}$$

It can be interpreted as the asymmetry of the differential cross section for scattering to the "left" and "right" of a target polarized "upward" in the direction normal to the scattering plane, with otherwise identical kinematics,

$$A_N = \frac{d\sigma(\text{left}) - d\sigma(\text{right})}{d\sigma(\text{left}) + d\sigma(\text{right})}. \tag{5}$$

To describe transverse spin effects in DIS kinematics it is customary to define a coordinate system such that the momentum transfer

$$q \equiv l - l' \tag{6}$$

(i.e., the momentum of the virtual photon in one-photon exchange approximation) points in the negative z-direction, and the initial and final electron momenta lie in the xz-plane, with the average momentum pointing in the positive x-direction [see Fig. 1(a)]. In this frame the unit vector $l \times l'/|l \times l'|$ points in the negative y-direction, and the normal spin asymmetry coincides with the negative polarization asymmetry with respect to the target spin in the y-direction,

$$A_N = \frac{d\sigma(S_y = -1) - d\sigma(S_y = +1)}{d\sigma(S_y = -1) + d\sigma(S_y = +1)} \equiv -A_y. \tag{7}$$

It is clear that this definition applies not only to the target rest frame but also to the virtual photon-nucleon center-of-mass (CM) frame, in which the nucleon moves in the positive z direction.

The cross section for inclusive eN scattering with unpolarized beam is independent of the transverse target spin if the electromagnetic interaction is treated in one-photon exchange approximation (Christ-Lee theorem) [2]. In this approximation the cross section can be expressed in the well-known form [14]

$$d\sigma = \frac{e^4}{4(lp)Q^4} L^{\mu\nu} W_{\mu\nu} \frac{d^3 l'}{(2\pi)^3 2 l'^0}, \tag{8}$$

where e is the elementary charge and

$$Q^2 \equiv -q^2 = -(l' - l)^2 \tag{9}$$

the invariant momentum transfer. The leptonic tensor, $L_{\mu\nu}$, is symmetric for an unpolarized beam, $L_{\mu\nu} = L_{\nu\mu}$, and the contraction in Eq. (8) projects out the symmetric part of the hadronic tensor,

$$W_{\mu\nu} = \int d^4 x \, e^{i(qx)} \langle pS | J_\mu(x) J_\nu(0) | pS \rangle. \tag{10}$$

Using P and T invariance as well as the Hermiticity of the current operator, it can be shown that the symmetric part of
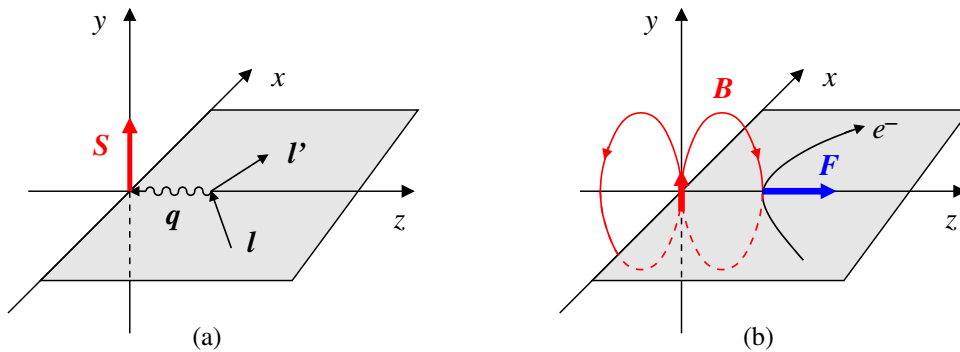

FIG. 1 (color online). (a) The coordinate system for describing the transverse spin dependence of inclusive DIS in the target rest frame. (b) Transverse target spin asymmetry in the scattering of an electron (charge $-e$) from a classical pointlike magnetic dipole. The asymmetry results from the Lorentz force, F, experienced by the charge moving in the magnetic field of the dipole, B.

the hadronic tensor remains unchanged under reversal of the target's transverse polarization, and the asymmetry (7) is zero. We shall see an explicit example of this general theorem in the cross section for a pointlike target in Sec. III.

Target spin dependence in P- and T-invariant inclusive scattering can arise only from higher-order electromagnetic interactions. At the order α^3, one can identify two distinct contributions to the cross section which give rise to a target spin dependence, see Fig. 2. One is the interference of one-photon and two-photon exchange amplitudes in the $ep \rightarrow e'X$ cross section [Fig. 2(a)]. This mechanism can, in a sense, be regarded as a non-Hermitian contribution to the current operator in the leading-order expression, arising from the imaginary part of the two-photon exchange contribution to the $ep \rightarrow e'X$ amplitude. The other contribution results from the interference of real photon radiation (bremsstrahlung) emitted by the electron and the interacting hadronic system [Fig. 2(b)]. An important point is that the two-photon exchange contribution to the spin-dependent cross section, Fig. 2(a), is free of QED IR divergences, as will be discussed in detail in Sec. IV. In the cross section spin difference and the asymmetry (7) two-photon exchange and real photon emission can thus be regarded as physically distinct contributions from the QED point of view and be discussed separately. This is in contrast to the two-photon exchange contributions to the cross section itself for given target spin (or the sum over target spins), where the IR divergences cancel only when two-photon exchange and real photon emission are added.

A nonzero target spin dependence of inclusive eN scattering could in principle arise if T invariance were violated explicitly in electroweak interactions. In fact, the SLAC experiment [11] measured the spin asymmetry with the

aim of testing T invariance of the ep interaction and found the asymmetry be consistent with zero at the level of 3.5%. Present understanding of the limits on the violation of fundamental symmetries suggests that P-conserving, T-violating effects in the standard model, which come as weak interaction corrections to P-violating effects, should lead to corrections to the DIS cross section of the order of at most $<10^{-8}$ [15]. These effects are significantly smaller than the asymmetry expected from two-photon exchange, $|A_N| \sim 10^{-4}$ (see below). The T-violating effects could in principle be separated from two-photon exchange by their different beam charge dependence [11]. However, electromagnetic effects at $O(\alpha^4)$, such as three-photon exchange and double two-photon exchange, would have the same spin and beam charge dependence as T-violation and exceed the latter by at least 2 orders of magnitude, making it practically impossible to probe explicit T-violation in this way.

III. TRANSVERSE SPIN DEPENDENCE FOR POINTLIKE TARGET

We begin our investigation of the transverse spin dependence by considering the scattering of an electron (charge $-e$) from a Dirac point particle of charge $+e$, referred to as "pointlike proton" in the following. While several calculations of the asymmetry in this model were performed long ago [16,17], it is worthwhile to revisit this problem for several reasons. First, the point particle calculation explicitly demonstrates the IR finiteness of the two-photon exchange contribution to the spin-dependent cross section, and allows us to investigate numerically the distribution of photon virtualities in the two-photon exchange graph. Second, the point particle result provides a crude—but manifestly self-consistent—estimate of the asymmetry, including real photon emission (which turns out not to contribute to the asymmetry in this case), and will serve as a reference point for more elaborate models including hadron structure. Third, we need the point particle result as an ingredient for the composite nucleon approximation in Sec. VI.

For a pointlike proton, the hadronic final state in inclusive ep scattering contains just the proton itself. Likewise, there are no excited hadronic intermediate states in higher-order processes. The Feynman diagrams contributing to the amplitude for inclusive ep scattering to order α^2 are shown in Fig. 3. Consider first the elastic scattering channel,

$$e(l) + p(p) \rightarrow e(l') + p(p'), \qquad (11)$$

the amplitude of which is given by the sum of diagrams (a)–(c). In this channel the sum over hadronic final states reduces to the sum over the final-state proton polarization states. The cross section for scattering from a transversely polarized proton is proportional to the squared modulus of the invariant amplitude, averaged (summed) over the initial (final) electron polarization, and summed
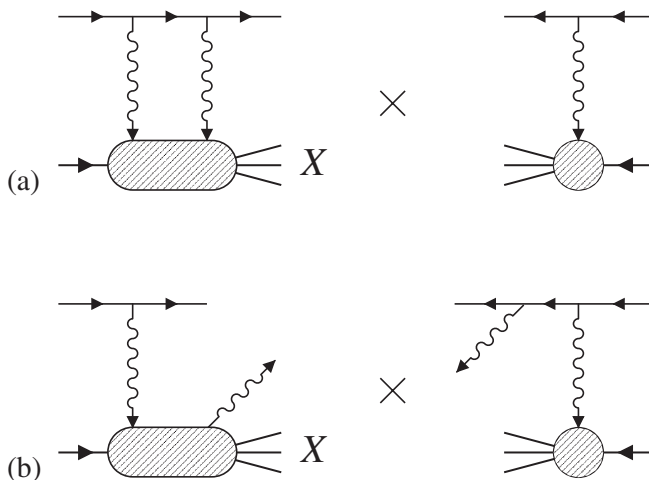

FIG. 2. QED processes contributing to the transverse target spin dependence of the inclusive eN cross section at $O(\alpha^3)$. (a) Interference of one-photon and two-photon exchange. (b) Interference of real photon emission (bremsstrahlung) by the electron and the hadronic system.
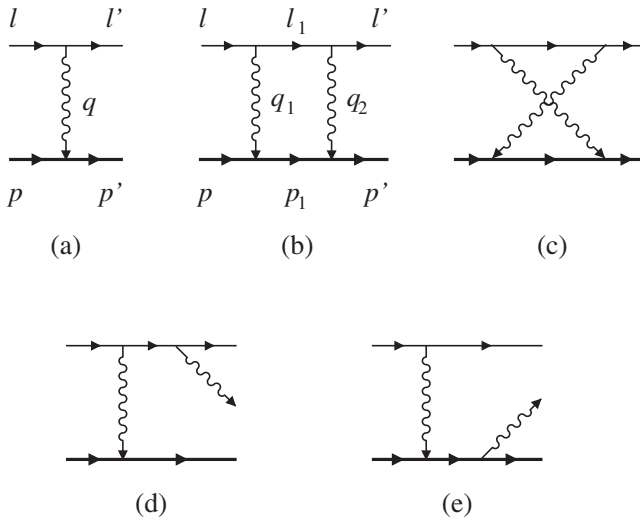
FIG. 3. Feynman diagrams contributing to the amplitude of ep scattering to order α^2. (a), (b), (c) Elastic scattering, $ep \rightarrow e'p$. The two-photon box diagram (b) gives rise to an imaginary part of the amplitude. (d), (e) Real photon emission.

over the final proton polarization. On general grounds, the dependence of this quantity on the polarization of the initial proton must be of the form (average/sum over electron and final proton polarizations is implied)

$$|\mathcal{M}_{ep \rightarrow e'p}|^2 = X_U - \frac{(SN)}{\sqrt{-N^2}} X_N, \qquad (12)$$

where S is the polarization 4-vector of the initial proton state,

$$N^\mu = -4\epsilon^{\mu\alpha\beta\gamma} l_\alpha l'_\beta p_\gamma \qquad (13)$$

the normal 4-vector characterizing the scattering process [18], and X_U and X_N are independent of the initial proton polarization. Noting that in the target rest frame $S^\mu = (0, \mathbf{S})$, and $N^\mu = (0, \mathbf{N})$ with $\mathbf{N} = 4M\mathbf{l} \times \mathbf{l'}$ normal to the scattering plane, we have

$$-\frac{(SN)}{\sqrt{-N^2}} = \frac{(\mathbf{S}, \mathbf{l} \times \mathbf{l'})}{|\mathbf{l} \times \mathbf{l'}|}, \qquad (14)$$

and the transverse target spin asymmetry (7) is given by

$$A_N = \frac{X_N}{X_U}. \qquad (15)$$

This representation allows us to calculate the asymmetry directly from the invariant amplitudes, without reference to a particular frame. We note that in the frame of Fig. 1(a), choosing the proton polarization to be along the y axis, the coefficients in Eq. (12) are given by

$$X_U = \tfrac{1}{2}[|\mathcal{M}(y-)|^2 + |\mathcal{M}(y+)|^2], \qquad (16)$$

$$X_N = \tfrac{1}{2}[|\mathcal{M}(y-)|^2 - |\mathcal{M}(y+)|^2], \qquad (17)$$

where $\mathcal{M}(y\pm) \equiv \mathcal{M}_{ep \rightarrow e'p}(y\pm)$ denotes the amplitude for scattering from a proton state with $S_y = \pm 1$, and the average/sum over the other polarizations is implicit. In this case expression (15) for the asymmetry reproduces the negative y spin asymmetry, Eq. (7). It is instructive to express the coefficients X_U and X_N also in terms of the amplitudes for scattering from a proton of given helicity. In a frame where the proton moves in the positive z direction, the helicity eigenstates $|\pm\rangle$ coincide with the eigenstates of S_z and are related to the S_y eigenstates by

$$|y\pm\rangle = \frac{|+\rangle \pm i|-\rangle}{\sqrt{2}}, \qquad (18)$$

and one obtains

$$X_U = \tfrac{1}{2}[|\mathcal{M}(-)|^2 + |\mathcal{M}(+)|^2], \qquad (19)$$

$$X_N = \mathrm{Im}[\mathcal{M}^*(-)\mathcal{M}(+)]. \qquad (20)$$

In the helicity basis the transverse spin dependence is related to the interference of helicity-flip and nonflip amplitudes in the cross section. In particular, it is seen from Eq. (20) that a spin dependence appears only if the helicity amplitudes develop an imaginary (absorptive) part.

In the approximation of zero electron mass, $m \rightarrow 0$, the electron helicity is conserved because of chiral invariance, and ep elastic scattering (11) is described by 3 independent helicity amplitudes. We parametrize the invariant amplitude as

$$\mathcal{M}_{ep \rightarrow e'p} = \bar{u}'\hat{P}u(2M\bar{U}'Uf_1 + \bar{U}'\hat{L}Uf_2)$$
$$+ \bar{u}'\hat{P}\gamma_5 u\bar{U}'\hat{L}\gamma_5 Uf_3, \qquad (21)$$

where $u \equiv u(l)$, $u' \equiv u(l')$ and $U \equiv U(p)$, $U' \equiv U(p')$ are the bispinors of the initial/final electron and proton, normalized as $\bar{u}u = \bar{u}'u' = 2m$, $\bar{U}U = \bar{U}'U' = 2M$, M is the proton mass,

$$L \equiv l + l', \qquad (22)$$

$$P \equiv p + p' \qquad (23)$$

are the sum of the initial and final electron/proton momenta, and we use the notation $\hat{P} \equiv P^\mu \gamma_\mu$. Here $f_1 - f_3$ are scalar functions of the kinematic invariants,

$$s \equiv (l + p)^2, \qquad (24)$$

$$t \equiv (l' - l)^2 = q^2; \qquad (25)$$

it is convenient to introduce also the crossing-symmetric variable

$$\nu \equiv (LP) = s - u = 2(s - M^2) + t. \qquad (26)$$

By straightforward calculation, using the standard expressions for the spin density matrices of the electron and proton spinors [18], one obtains the coefficients of the

squared modulus of the invariant amplitude, Eq. (12), as

$$
\begin{aligned}
X_U = [\nu^2 - t(t - 4M^2)]\{4M^2(-t + 4M^2)|f_1|^2 \\
+ (\nu^2 - t^2)|f_2|^2 + 8M^2\nu \, \mathrm{Re}(f_1^* f_2) \\
+ [\nu^2 - t(t - 4M^2)]|f_3|^2\},
\end{aligned}
\tag{27}
$$

$$
X_N = 4M[\nu^2 - t(t - 4M^2)]^{3/2}\sqrt{-t}\,\mathrm{Im}(f_1^* f_2).
\tag{28}
$$

Again, one sees from Eq. (28) that a spin dependence of the cross section appears only if the functions f_1 and f_2 develop an imaginary part.

In one-photon exchange approximation, the invariant amplitude for elastic ep scattering is given by the diagram of Fig. 3(a),

$$
\mathcal{M}^{(a)}_{ep \to e'p} = -\frac{e^2}{t} \bar{u}' \gamma^\mu u \, \bar{U}' \gamma_\mu U,
\tag{29}
$$

where the negative sign results from the different sign of the charges. The contribution to the functions $f_1 - f_3$ can easily be found by expanding the vector currents in the basis formed by the orthogonal 4-vectors L, q, N, and $P - (LP)L/L^2$, and using the relations between bilinear forms following from the three-gamma identities and the Dirac equation for the electron and proton spinors [14]. One obtains

$$
\left.\begin{matrix} f_1^{(a)} \\ f_2^{(a)} \\ f_3^{(a)} \end{matrix}\right\} = \frac{-e^2}{\nu^2 - t(t - 4M^2)} \times \begin{cases} 1, \\ \frac{\nu}{t}, \\ -1. \end{cases}
\tag{30}
$$

In this approximation the functions $f_1 - f_3$ are real, and the spin-dependent part of the squared modulus of the invariant amplitude (28) is zero,

$$
X_N^{(a)} = 0,
\tag{31}
$$

in accordance with the Christ-Lee theorem. The spin-independent part gives the usual expression for the squared modulus of the invariant amplitude in unpolarized ep elastic scattering,

$$
X_U^{(a)} = \frac{e^4}{t^2}[\nu^2 + t(t + 4M^2)].
\tag{32}
$$

A nonzero imaginary part of $f_1 - f_3$ arises at order α^2 from the two-photon exchange box diagram, Fig. 3(b). [The crossed-box diagram, Fig. 3(c), does not have an imaginary part in the physical region for ep scattering.] The contribution of diagram Fig. 3(b) to the invariant amplitude is given by the Feynman integral,

$$
\begin{aligned}
\mathcal{M}^{(b)}_{ep \to e'p} = -i \int \frac{d^4\Delta}{(2\pi)^4} \\
\times \frac{e^4 \bar{u}' \gamma^\mu \hat{l}_1 \gamma^\nu u \, \bar{U}' \gamma_\mu (\hat{p}_1 + M)\gamma_\nu U}{q_1^2 q_2^2 (l_1^2 + i0)(p_1^2 - M^2 + i0)},
\end{aligned}
\tag{33}
$$

where Δ represents a suitably chosen loop momentum, e.g.,

$$
q_{1,2} = q/2 \pm \Delta,
\tag{34}
$$

$$
l_1 = L/2 - \Delta,
\tag{35}
$$

$$
p_1 = P/2 + \Delta.
\tag{36}
$$

By projecting the numerator in Eq. (33) on the structures of Eq. (21), using the basis vectors described above, one can easily determine the corresponding contributions to the functions $f_1 - f_3$. Their imaginary part is then calculated by applying the Cutkosky rules, replacing the propagators of the intermediate particles by delta functions. We are interested only in the interference term, $\mathrm{Im}(f_1^* f_2)$, which governs the cross section spin difference, Eq. (28). Because the one-photon exchange amplitudes are real, the leading nonzero contribution to this term is

$$
\mathrm{Im}(f_1^* f_2) = f_1^{(a)} \mathrm{Im} f_2^{(b)} - f_2^{(a)} \mathrm{Im} f_1^{(b)}.
\tag{37}
$$

It is convenient to combine the functions in this way before performing the loop integral. In this way, one obtains a representation of the interference term as

$$
\mathrm{Im}(f_1^* f_2) = 2\pi^2 \int \frac{d^4\Delta}{(2\pi)^4} \delta(l_1^2)\delta(p_1^2 - M^2)\frac{\phi_A}{q_1^2 q_2^2},
\tag{38}
$$

where the integration is restricted over positive-energy intermediate states, $(l_1)^0, (p_1)^0 > 0$, and the numerator is given by

$$
\begin{aligned}
\phi_A = \frac{-e^6}{\nu^2 - t(t - 4M^2)} \Bigg\{ & \frac{1}{2t^2}(q^2 + q_1^2 - q_2^2)(q^2 - q_1^2 + q_2^2) \\
& + \frac{1}{4t}[(q_1 - q_2)^2 - q^2] \\
& + \frac{3(\nu - t) + 4M^2}{8t[\nu^2 - t(t - 4M^2)]}[(q_1 - q_2)^2 - q^2] \\
& \times [(q_1 - q_2)^2 + q^2] \Bigg\},
\end{aligned}
\tag{39}
$$

where $q^2 = t$. In simplifying Eq. (39) we have made use of the mass shell conditions $l_1^2 = 0$ and $p_1^2 = M^2$ implied by the delta functions. Equations (38) and (39) allow us to evaluate the spin-dependent cross section as an invariant integral.

An important observation is that the numerator (39) vanishes in the limits where the 4-momentum of one or the other photon in the two-photon exchange graph tends to zero,

$$
\phi_A \to 0 \quad \text{for} \begin{cases} q_1 \to 0, & q_2 \to q, \text{ or} \\ q_2 \to 0, & q_1 \to q. \end{cases}
\tag{40}
$$

This implies that the integral representing the spin-dependent interference cross section (38) is free of IR divergences. We shall see in Sec. IV that this property is

general and follows from the fact that the IR divergent terms have the form of a universal factor multiplying the one-photon exchange cross section, which does not exhibit a spin dependence. Note that the cancellation of IR divergences takes place only in the combination Eq. (37); the two-photon exchange contribution to the individual functions f_1 and f_2 (even their imaginary parts) is IR divergent.

The invariant integral in Eq. (38) can be evaluated in an arbitrary reference frame. A convenient way is to convert it to a phase space integral over the intermediate electron momentum, which can be evaluated in the ep center-of-mass (CM) frame using standard techniques. The relation of the CM momentum, l_{cm}, and scattering angle, θ_{cm}, to the invariants s and t is (we assume zero electron mass)

$$l_{cm} = \frac{s - M^2}{2\sqrt{s}}, \tag{41}$$

$$\sin^2(\theta_{cm}/2) = \frac{-st}{(s - M^2)^2}. \tag{42}$$

Evaluating the integral in this way, we obtain a simple result for the normal spin difference of the cross section,

$$\mathrm{Im}\,(f_1^* f_2) = \frac{-e^6}{256\pi l_{cm}^4 s^2 \sin^2\theta_{cm}}. \tag{43}$$

The normal spin asymmetry for the pointlike proton is then obtained by multiplying with the kinematic factor of Eq. (28), and dividing the result by the spin sum of the cross section, evaluated in one-photon exchange approximation, Eq. (32). In terms of the CM variables,

$$A_N = -\frac{2\alpha l_{cm}^2 M}{s^{3/2}} \frac{\sin^3(\theta_{cm}/2)\cos(\theta_{cm}/2)}{\cos^2(\theta_{cm}/2) + (2l_{cm}^2/s)\sin^4(\theta_{cm}/2)}. \tag{44}$$

Here $\alpha = e^2/(4\pi) = 1/137$ is the fine structure constant.

This result agrees with the one obtained earlier in Ref. [16]; see also Ref. [6]. In particular, in the high-energy limit, $s \gg M^2$, one has $l_{cm} \approx \sqrt{s}/2$, and Eq. (44) simplifies to

$$A_N = -\frac{\alpha M}{2\sqrt{s}} \frac{\sin^3(\theta_{cm}/2)\cos(\theta_{cm}/2)}{\cos^2(\theta_{cm}/2) + \frac{1}{2}\sin^4(\theta_{cm}/2)}, \tag{45}$$

which has its maximum at $\theta_{cm} = 2.18 = 125°$.

The sign of the normal spin asymmetry [Eq. (44)] is what one expects from the simple picture of an electron scattering from a pointlike magnetic dipole, see Fig. 1(b). In this picture the asymmetry is caused by the Lorentz force experienced by the charged particle moving in the magnetic field of the dipole, which in the scattering plane points in the direction opposite to the magnetic moment. As can be seen from Fig. 1(b), if the proton with magnetic moment $\boldsymbol{\mu}_p = e\mathbf{S}/(2M)$ is polarized upward, the electron with charge $-e$ is deflected to the right, leading to $A_N < 0$, cf. Eq. (5).

We can use the result of the pointlike proton approximation to make a rough order-of-magnitude estimate of the asymmetry expected in DIS experiments. Figure 4 shows the asymmetry for $s = 10$ and 20 GeV^2, corresponding approximately to the values reached in ep scattering at JLab with 6 and 12 GeV beam energy. The asymmetry is shown both as a function of the CM scattering angle, θ_{cm}, Eq. (42), and as a function of $|t| = -q^2 = Q^2$ itself. One sees that the asymmetry in this approximation is of the order of several times 10^{-4}. The maximum value of the asymmetry, as well as its position in θ_{cm}, depend only weakly on s. The change of the t-dependence with s mostly reflects the transformation from the kinematic variable θ_{cm} to t.

It is interesting to study the distribution of photon virtualities in the integral (38). This provides information about the effective range of the two-photon exchange interaction, which will be important for the calculation of
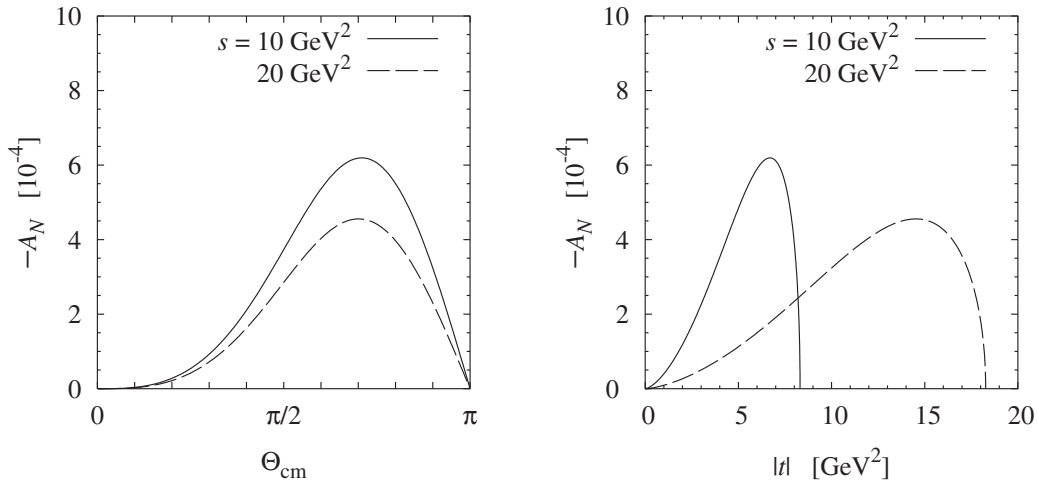

FIG. 4.   The normal spin asymmetry A_N (note the minus sign on the plot axis) in electron scattering from a pointlike proton, for $s = 10$ and 20 GeV^2, as a function of the CM scattering angle (left), and of $|t| = Q^2$ (right).

the asymmetry for a composite target in Sec. VI. It turns out that the distribution of photon virtualities in the integral (38) is governed by the scales $Q^2 = -t$ and s and does not involve any extraneous scales, as a result of the IR finiteness of the integral. One way to illustrate this is by evaluating the integral (38) with a nonzero "photon mass," replacing the photon propagators by

$$\frac{1}{q_{1,2}^2} \rightarrow \frac{1}{q_{1,2}^2 - \lambda^2}. \qquad (46)$$

The variation of the result with λ^2 gives an indication of the effective photon virtualities in the loop. Figure 5 shows the asymmetry for $s = 10$ GeV^2 as a function of the photon mass, λ^2, for several values of t, corresponding to large-angle scattering (θ_{cm} not close to 0 or π). One sees that the photon mass dependence is very smooth, confirming that no large contributions arise from the region $|q_1^2|, |q_2^2| \ll Q^2$. We observe that for $\lambda^2 \gtrsim 1$ GeV^2 the λ dependence is well described by the form

$$\mathrm{Im}\,(f_1^* f_2) \propto \frac{1}{(Q_{eff}^2 + \lambda^2)^2}, \qquad (47)$$

which would be the dependence if the virtualities in the integral were "frozen" at $-q_1^2 = -q_2^2 = Q_{eff}^2$. Extracting the value of Q_{eff}^2 from a fit to the numerical results we find $Q_{eff}^2 = (5.6, 5.8, 6.1)$ GeV^2 for $Q^2 = (2, 4, 6)$ GeV^2 in the given kinematics. Note, however, that the effective virtuality thus estimated depends strongly on the prescription; since the integrand of Eq. (38) is not positive definite, any definition of average is inherently ambiguous.

Another way of studying the distribution of photon virtualities in the asymmetry is to represent the integral (38) as an integral over one of the photon virtualities. This can be done using the fact that in the CM frame the
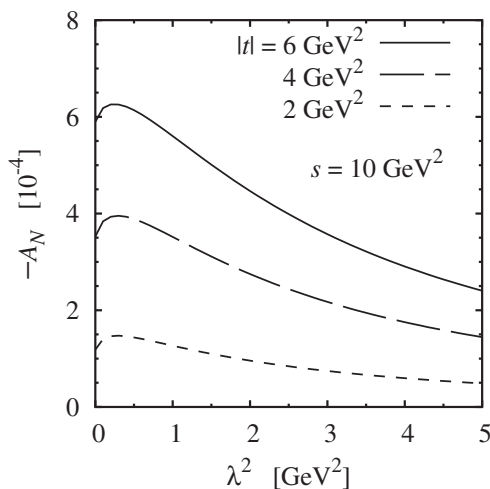

FIG. 5. The normal spin asymmetry A_N (note the minus sign on the plot axis) as a function of the photon mass, cf. Eqs. (38) and (46). The photon mass dependence gives an indication of the average virtualities in the two-photon box graph.

virtuality q_1^2 is directly related to the angle between the initial and intermediate electron momenta, $Q_1^2 \equiv -q_1^2 = 2l_{cm}^2[1 - \cos\theta(l_1, l)]$. Integrating over the corresponding azimuthal angle, one obtains a representation of the form

$$\mathrm{Im}\,(f_1^* f_2) = \int_0^{2l_{cm}^2} dQ_1^2 F(Q_1^2), \qquad (48)$$

where the integrand turns out to be a piecewise constant function,

$$F(Q_1^2) = \begin{cases} C_1 & 0 < Q_1^2 < Q^2, \\ C_2 & Q^2 < Q_1^2 < 2l_{cm}^2, \end{cases} \qquad (49)$$

in which $C_1 < 0$, $C_2 > 0$, with values depending on s and Q^2. One sees that the characteristic scales in the distribution of virtualities are Q^2 and $2l_{cm}^2 \sim s$. Numerical studies show that for large-angle scattering (θ_{cm} not close to 0 or π) the cancellation between the low and high virtuality regions is not precarious, and that the sign of the resulting integral is always determined by the high-virtuality contribution. This again proves that in the kinematics of large s and Q^2 the contribution from virtualities $Q_1^2 \ll Q^2$ does not significantly influence the result.

To complete our discussion of the transverse spin dependence of inclusive scattering from a point particle, we need to comment also on the real photon emission (bremsstrahlung) channel, $ep \rightarrow e'p\gamma$, the amplitudes for which are given by the diagrams of Figs. 3(d) and 3(e). The spin dependence of the cross section in this channel can be discussed along the lines of Eq. (12) *et seq.*, the only difference being that the sum over final states includes the integration over the relative momenta of the three-body final state and the sum over the photon polarizations. An expression analogous to Eq. (20) can be derived in terms of the helicity amplitudes; however, since diagrams (d) and (e) do not have an absorptive part (the intermediate particles are always off mass shell), the helicity amplitudes are real and no transverse spin dependence is obtained. In this sense the pointlike target provides a model for fully inclusive scattering; only the bremsstrahlung channel happens not to contribute to the transverse spin dependence. Note that this is specific to scattering from a point particle; for a target with internal excitations the Compton amplitude has an absorptive part and a non-zero interference cross section of the type of Fig. 2(b) can in principle arise. This contribution is IR finite (cf. the discussion in Sec. IV) and thus can be discussed separately from two-photon exchange.

IV. CANCELLATION OF INFRARED AND COLLINEAR DIVERGENCES

A new feature of two-photon exchange processes compared to one-photon exchange is that divergent terms can appear in the scattering amplitude, related to the vanishing of the virtualities of (at least) one of the photons. However, these divergent terms must cancel in the final result for

physical observables. The point particle calculation of Sec. III shows explicitly that IR divergences are absent in the spin-dependent part of the two-photon exchange cross section. We now want to demonstrate that this result is general and applies also to a target with internal structure; it follows from the general factorization property of IR singularities in QED. Furthermore, we analyze the collinear divergences which appear in the calculation of two-photon exchange corrections in models of hadron structure with off mass-shell constituents, and show that they cancel due to electromagnetic gauge invariance. These results will be used in our studies of the transverse spin dependence of the inclusive cross section in the presence of hadron structure below.

Consider the invariant amplitude for the two-photon transition to a (unspecified) hadronic final state, $eN \rightarrow e'X$, which enters in the transverse spin-dependent part of the inclusive eN cross section, see Fig. 6 [cf. Fig. 2(a)]. The absorptive part, in which the intermediate electron is on mass shell, can be represented as

$$\text{Im } \mathcal{M}_{ep \rightarrow e'X} = \int \frac{d^3 l_1}{2E_1(2\pi)^3} \frac{e^4 l^{\mu\nu} \text{ Im } T_{\mu\nu}}{q_1^2 q_2^2}, \quad (50)$$

where E_1 is the energy of the intermediate electron, and $q_{1,2}$ are the photon 4-momenta. In the numerator,

$$l_{\mu\nu} = \bar{u}(l') \gamma_\mu \hat{l}_1 \gamma_\nu u(l) \quad (51)$$

is the residue of the direct term of the electron virtual Compton amplitude (we neglect the electron mass), and $\text{Im } T_{\mu\nu}$ denotes the absorptive part of the virtual hadronic Compton amplitude for the $N \rightarrow X$ transition [19]. As a consequence of electromagnetic current conservation, the tensors satisfy the transversality conditions

$$q_{2\mu} l^{\mu\nu} = 0, \qquad l^{\mu\nu} q_{1\nu} = 0, \quad (52)$$

$$q_{2\mu} T^{\mu\nu} = 0, \qquad T^{\mu\nu} q_{1\nu} = 0. \quad (53)$$

The integral Eq. (50) can become divergent if either of the exchanged photon virtualities, q_1^2 or q_2^2, tends to zero in parts of the integration region. One distinguishes two types of such singularities:
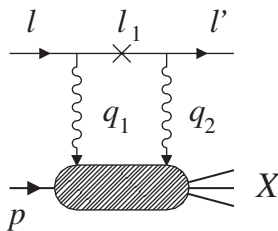

FIG. 6. The two-photon exchange amplitude giving rise to a transverse spin dependence of the $eN \rightarrow e'X$ inclusive cross section.

$$q_1 \rightarrow 0, \qquad q_2 \rightarrow q \qquad \text{"infrared,"}$$
$$q_1^2 \rightarrow 0 \quad \text{with } q_1 \neq 0 \qquad \text{"collinear,"}$$

and likewise for $q_1 \leftrightarrow q_2$. The mechanism for the cancellation of these singularities in physical observables is quite different in the two cases.

The cancellation of IR singularities is governed by the soft-photon theorem [20], which states that photons of wavelength $\lambda \gg R_{\text{hadron}}$ "see" only the charge and momenta of the initial and final particles in a reaction, not their polarization or the details of the reaction mechanism. Using the method of Refs. [21–23], the IR divergent contributions of individual two-photon exchange diagrams can be represented in the form of a divergent factor, depending only on the charges and momenta of the initial and final particles, multiplying the one-photon exchange amplitude for the process. Because this factor is spin independent, the IR divergent term in the spin-dependent cross section difference comes in the form of an overall factor multiplying the spin-dependent cross section difference in one-photon exchange approximation, which is zero on grounds of the Christ-Lee theorem. Note that the cross section for each individual target polarization does have IR divergent terms; they cancel only at the level of the cross section difference. This is exemplified by the point particle calculation of Sec. III, where one can verify that the two-photon exchange contribution to the absorptive parts of the individual amplitudes f_1 and f_2 are divergent, while the spin difference $\text{Im}(f_1^* f_2)$ is divergence-free. In summary, the reason why the cross section spin difference is IR finite is the spin independence of soft-photon contributions.

The cancellation of collinear singularities in the two-photon exchange contribution to inclusive eN scattering is due to the transversality of the electron and hadron Compton tensors (related to electromagnetic gauge invariance), and happens already at the level of the amplitude for given target spin. Physically, collinear singularities correspond to the emission of a finite-energy photon along the direction of the initial or final electron, which is assumed to be strictly massless here ($m = 0$). Consider the case that the photon with q_1 is emitted along the direction of the initial electron with 4-momentum l. The relevant integration region can be parametrized covariantly as

$$l_1 = zl, \quad (54)$$

$$q_1 = l - l_1 = (1 - z)l, \quad (55)$$

where z is the fraction of the momentum l carried by the intermediate electron. Obviously $q_1^2 = (1 - z)^2 l^2 = 0$ for massless electrons, and one encounters a divergence if values $z \neq 1$ are kinematically allowed, as is generally true in inelastic scattering (the case of elastic scattering will be discussed separately below). The only way a divergence can be avoided is if the numerator of the integral vanishes in the collinear limit, Eqs. (54) and (55).

Let us inspect the numerator of Eq. (50) in the collinear limit, Eqs. (54) and (55). Using the anticommutation relations for the gamma matrices and the Dirac equation for the initial electron spinor, the tensor Eq. (51) can be brought into the form

$$\lim_{l_1 \to zl} l_{\mu\nu} = \frac{2z}{1-z} j_\mu(l', l) q_{1\nu},$$
(56)

where

$$j_\mu(l', l) = \bar{u}(l') \gamma_\mu u(l)$$
(57)

is the matrix element of the electromagnetic current between the initial and final electron states. The contraction with the hadronic Compton tensor then gives zero by virtue of the transversality condition Eq. (53),

$$l^{\mu\nu} T_{\mu\nu} \propto j^\mu(l', l) T_{\mu\nu} q_1^\nu = 0.$$
(58)

A similar argument applies if the other photon momentum, q_2, becomes collinear to the final electron momentum, l'. In both collinear regions the numerator in the integrand of Eq. (50) tends to zero simultaneously with the denominator and the integral becomes convergent. In summary, the absence of collinear divergences in the two-photon contribution to inelastic eN scattering is directly related to the transversality of the hadronic Compton tensor. A similar observation was made earlier in Ref. [24] in applications to the single-spin asymmetry of elastic ep scattering induced by two-photon exchange.

The case of a pointlike target considered in Sec. III is somewhat special in the context of the above discussion. For elastic scattering from a point particle, the only way in which q_1^2 could vanish is if the 4-vector q_1 tends to zero, i.e., the only kinematically allowed value of z in Eqs. (54) and (55) is $z = 1$. In this case the collinear region is kinematically forbidden; the only singularities are IR divergences, which cancel by the mechanism described earlier.

The issue of collinear singularities becomes critical when one tries to incorporate effects of hadron structure in inelastic eN scattering with two-photon exchange. Specifically, in models where the two-photon exchange couples to hadronic constituents which are off mass shell, collinear divergences appear, which are canceled only by contributions involving explicitly the interactions between the constituents. This is because only the combination of off-shell and interaction effects maintains electromagnetic gauge invariance and transversality of the hadronic tensor. We note that a recent calculation of the transverse spin asymmetry in inclusive DIS in the parton model [25], which considered two-photon exchange with off-shell quarks without accompanying interaction effects, found a divergent result for the asymmetry [26]. The arguments presented above indicate that the reason for the divergence is the violation of electromagnetic gauge invariance in that approximation, and point out what needs to be done to obtain a meaningful finite result.

To illustrate the point, we consider the simple field-theoretical model of an electron scattering from a pointlike spin-1/2 particle (charge $+e$), coupled to a massive neutral scalar particle with a Lagrangian density

$$L_{\text{int}} = g \bar{\Psi} \Psi \phi.$$
(59)

This extension of the point particle calculation of Sec. III offers the simplest setting which allows one to study inelastic scattering with nontrivial target structure and includes both off-shell and interaction effects. We consider the scattering amplitude for the process in which a single scalar particle is produced in the final state. Its absorptive part is given by the sum of the cut Feynman diagrams of Fig. 7 [27]. It can be shown explicitly that the Compton tensor $T_{\mu\nu}$ defined by this model satisfies the transversality conditions, Eq. (53), provided that the full set of diagrams in Fig. 7 is included. Obviously, the separate contributions from each of the diagrams are not transverse.

Let us consider the diagram of Fig. 7(a), where both photons couple to the spinor particle after emission of the scalar. When taken alone, the contribution of this diagram to the hadronic Compton tensor $T_{\mu\nu}$ is not transverse, which is related to the fact that the intermediate-state spinor particle with momentum p_0 is off mass shell. The corresponding absorptive part reads

$$\text{Im} T_{\mu\nu} = g \pi \delta(p_1^2 - M^2) \bar{U}(p') \gamma_\mu (\hat{p}_1 + M) \gamma_\nu \frac{\hat{p}_0 + M}{p_0^2 - M^2} U(p),$$
(60)

where the delta function results from the Cutkosky cut. Since the spinor particle before the q_1 photon vertex is off
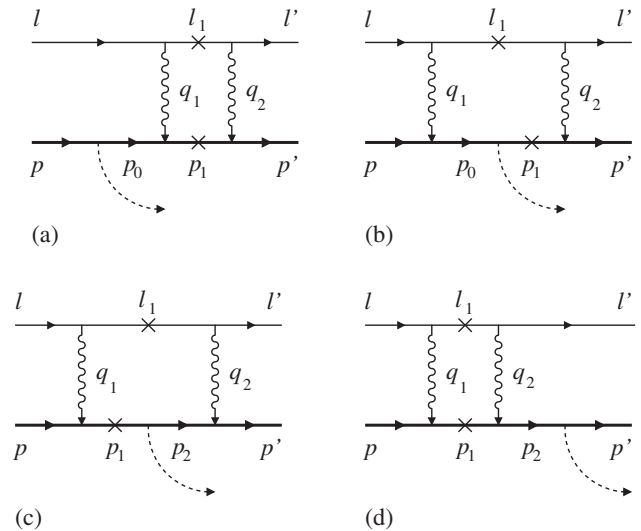

FIG. 7. The field-theoretical model for inelastic electron scattering with two-photon exchange. Thin solid lines denote the electron, thick solid lines the spinor particle (charge $+e$), dashed lines the massive neutral scalar particle. The crosses indicate that the intermediate particle is on mass-shell (Cutkosky cut).

mass shell, $p_0^2 \neq M^2$, and the spinor particle after the vertex is on mass shell, $p_1^2 = M^2$, the q_1 photon may have zero virtuality, $q_1^2 = 0$, while carrying a nonvanishing momentum. (For the q_2 photon both spinor particles at the vertex are on mass shell, and its virtuality can only go to zero if $q_2 \rightarrow 0$.) Consider now the collinear limit for the q_1 photon, Eqs. (54) and (55). In this case $q_1^2 \rightarrow 0$, and from $q_2 = q - (1 - z)l$ (where $q = l - l'$), one obtains $q_2^2 = zq^2$. The fraction z is kinematically fixed to be

$$z = (s_0 - M^2)/(s_0 - p_0^2), \quad (61)$$

where

$$s_0 \equiv (l + p_0)^2 = (l' + p')^2 \quad (62)$$

is the invariant of the q_1 exchange subprocess, and the off-shell momentum p_0 is reconstructed from 4-momentum conservation, $p_0 = p' - q = p' - l + l'$. Likewise, the 4-momentum p_1 is fixed as $p_1 = p' - q_2 = p' - zl + l'$. Calculating then the contraction of the electron and hadron Compton tensors in the collinear approximation, Eqs. (56) and (60), we find that it does not vanish in the collinear limit, and that the integral Eq. (50) is divergent. Using the fact that the delta function reduces the momentum integral to a two-dimensional one, the divergent part can conveniently be calculated by converting Eq. (50) to an angular integral in the CM frame of the final electron and the spinor particle (cf. Sec. III). Without the numerator, the divergent part is

$$\lim_{l_1 \rightarrow zl} \int \frac{d\Omega_1}{(q_1^2 - \lambda^2)(q_2^2 - \lambda^2)} = -\frac{\pi}{q^2 E_1^2} \ln\left(\frac{4E_1^2}{z\lambda^2}\right), \quad (63)$$

where

$$E_1 = \frac{s_0 - M^2}{2\sqrt{s_0}} \quad (64)$$

is the energy of the intermediate electron in the CM frame, and we have introduced a ''photon mass'' λ to regularize the singularity. Altogether, including the numerator factors, we obtain for the contribution to the $ep \rightarrow e'X$ amplitude from the collinear region in Fig. 7(a),

$$\mathrm{Im}\, \mathcal{M}_{ep \rightarrow e'X}^{(a)} = \frac{e^4 g}{32\pi q^2 E_1 \sqrt{s_0}} j_\mu(l', l)$$
$$\times \bar{U}(p')\gamma_\mu(\hat{p}_1 + M)U(p) \ln\left(\frac{4E_1^2}{z\lambda^2}\right), \quad (65)$$

where $p_1 = p' - zl + l'$. This expression is divergent in the physical limit, $\lambda \rightarrow 0$. Computing the interference cross section with the one-photon exchange amplitudes for the same ''spinor + scalar'' final state, it is straightforward to verify that Eq. (65) leads to a divergent result for the transverse spin-dependent cross section in this model, similar to what was obtained in Ref. [25].

Following the arguments present above, the divergence resulting from the ''off-shell'' diagram, Fig. 7(a), should be canceled by the contribution of the ''interaction'' diagram, Fig. 7(b), in which the spinor particle emits the scalar between the two photon couplings. Indeed, we find that Fig. 7(b), when calculated in the same collinear-photon approximation, produces an expression equal to Eq. (65) but opposite in sign, leading to exact cancellation of the divergence in the resulting amplitude. Similarly, the divergent term arising from the second exchanged photon being collinear to the final-state electron ($q_2^2 = 0$) cancels in the sum of contributions from the diagrams of Figs. 7(d) and 7(c). In summary, the field-theoretical model explicitly demonstrates that collinear divergences are absent if off-shell and interaction effects are treated consistently and electromagnetic gauge invariance is maintained by the approximations. This observation serves as a basis of our studies of the spin-dependent two-photon exchange cross section in QCD and a constituent quark model in Secs. V and VI.

The analysis presented here applies to collinear singularities arising from exchanges in which the virtuality of one of the photons tends to zero, while that of the other photon remains nonzero. In general, collinear divergences can also arise from the region in which both photon virtualities tend to zero simultaneously; this case corresponds to vanishing momentum of the intermediate electron in the CM frame. In our field-theoretical model such exchanges do not occur in the amplitudes for single-boson emission into the final state (i.e., in first order of the coupling constant), because in all diagrams of Fig. 7 at least one of the internal spinor particles attached to a photon vertex is on mass shell, making it impossible for that photon to have zero virtuality. They would, however, occur in higher-order amplitudes with multiple boson emission. Such exchanges would give rise to $\ln^2$-type singularities in the individual diagrams, which again cancel in the sum of all diagrams because of electromagnetic gauge invariance, as outlined above.

A comment is in order concerning the role of the electron mass in collinear singularities. The above expressions were derived for the case of zero electron mass, $m = 0$. If the electron mass is not neglected, the electron polarization vector s can have a component transverse to the direction of the initial electron, and a new kind of transverse spin dependence of the inclusive eN cross section appears, through a term proportional to

$$(s, l \times l'). \quad (66)$$

It corresponds to a beam spin asymmetry for electrons polarized in the direction normal to the scattering plane, while the target is unpolarized. For electrons polarized in this way, the electron virtual Compton tensor in the collinear limit is no longer proportional to the collinear photon momentum as in Eq. (56), and collinear photon exchange makes a nonzero contribution to the beam spin-dependent cross section, Ref. [24]. In this case, however,

the photon virtualities are limited by the (small) electron mass, and collinear photon exchange does not lead to a divergence but to a sizable finite contribution to the beam spin-dependent cross section, which is enhanced by logarithmic and double-logarithmic factors $\ln(Q^2/m^2)$ and $\ln^2(Q^2/m^2)$, as was shown in Ref. [24] (see also Refs. [28,29]).

V. TRANSVERSE SPIN DEPENDENCE IN DEEP-INELASTIC SCATTERING

The arguments of Sec. IV suggest that the two-photon exchange in the transverse spin-dependent cross section is free of QED IR and collinear divergences even when allowing for a nontrivial structure of the target and the hadronic final state. Based on these findings, we now want to discuss the transverse target spin dependence in DIS kinematics in QCD. We are not aiming for a full calculation of the two-photon exchange contribution in the collinear factorization scheme. Rather, we want to discuss the underlying assumptions and ingredients in such a calculation, and prepare the ground for a self-consistent approximate treatment of the problem.

Generally, in DIS kinematics we expect the dominant contribution to the target spin-dependent two-photon exchange cross section to arise from the amplitudes in which the two photons couple to a single quark, namely, the same quark as is hit in the interfering one-photon exchange amplitude, see Fig. 8(a). This follows from (a) the fact that the partonic final state in the two-photon exchange amplitude needs to be the same as in the interfering one-photon exchange amplitude, (b) that no large contributions arise from the soft regime of the two-photon exchange because of the IR finiteness of the asymmetry. More precisely, the only way in which a two-photon exchange coupling to different quarks could produce a final state similar to that of one-photon exchange in DIS would be if one of the photons were "hard" (with 4-momentum almost equal to q), and the other were "soft" (with longitudinal and transverse momentum in the target rest frame of the order of the soft interaction scale, say, the inverse nucleon radius, R_N^{-1}). The amplitude of such "hard-soft" configurations in the two-photon exchange is not enhanced compared to average configurations, thanks to the overall IR finiteness of the process. On the other hand, the phase space (integration volume) for such configurations is suppressed compared to those in which the two-photon exchange couples to the same quark and both photons have "average" 4-momenta of the order $q/2$. Thus, the two-photon coupling to the same quark should dominate. (A more explicit version of this argument will be presented in Sec. VIII for the case of soft high-energy scattering, using closure over nonrelativistic quark model states.) While this conclusion seems plausible, we presently have no way of proving it more rigorously, such as by way of a formal twist
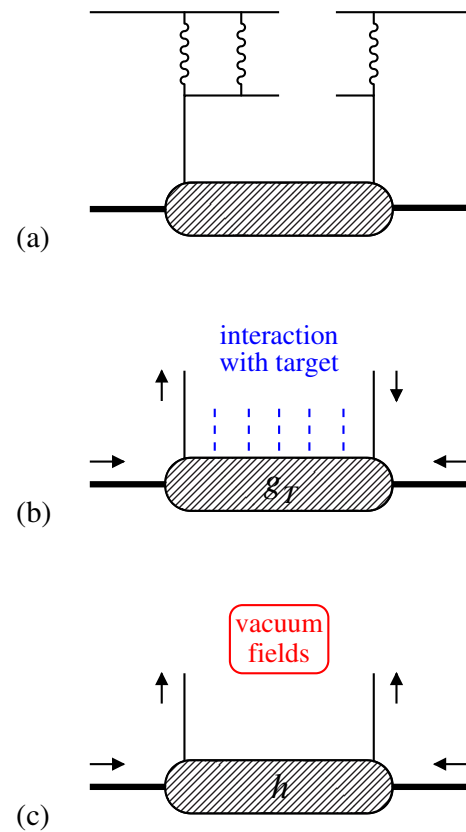

(a)

(b)

(c)

FIG. 8 (color online).   Transverse spin dependence of the DIS cross section in QCD. (a) Assumption of dominance of two-photon exchange with the same quark. (b) Quark helicity-conserving process involving interactions with the target remnants. (c) Quark helicity flip due to interaction with nonperturbative vacuum fields.

expansion as in one-photon DIS. We shall adopt it as a working assumption in the following.

It then follows that the transverse spin-dependent cross section can be described in a "partonlike" picture, in which the reaction happens predominantly with a single quark in the target. In this case one can easily see that one is dealing with two distinct contributions, defined by whether the quark helicity is conserved or flipped in the quark subprocess, see Figs. 8(b) and 8(c). [The hadron helicity is always flipped between the in and out state, as required by the transverse spin asymmetry, cf. Eq. (20).] Perturbative QCD interactions (gluon radiation) preserve the quark helicity and thus do not "mix" these contributions.

In the process of Fig. 8(b) the quark helicity is conserved in the quark subprocess. This contribution to the transverse spin-dependent cross section of unpolarized electron scattering is similar to that giving rise to the transverse spin structure function $g_T \equiv g_1 + g_2$ in longitudinally polarized electron scattering with a transversely polarized target. The latter is determined by the matrix element of the quark helicity-conserving (chirally even), transversely po-

larized twist-3 density $g_T(x)$, defined as ($i = 1, 2$ is a transverse index)

$$S^i g_{T,f}(x) = \frac{p^+}{M} \int \frac{dz^-}{8\pi} e^{ixp^+z^-/2}$$
$$\times \langle pS_T | \bar{\psi}_f(0)\gamma^i\gamma_5\psi_f(z)|pS_T\rangle_{z_\perp=0,z^+=0}, \quad (67)$$

where $z^\pm \equiv z^0 \pm z^3$ and $z_\perp$ are the usual light-cone vector components, $\bar{\psi}, \psi$ are the quark fields, f denotes the quark flavor, and we have omitted the gauge link in the light-ray operator for brevity. Indeed, the QCD calculation of the quark helicity-conserving two-photon exchange contribution would start from the "handbag graph" of Fig. 8(a), with the quark density given by Eq. (67) [25]. However, keeping only this graph is not a consistent approximation. On one hand, evaluating it with the initial and final quark on mass shell would give zero, as can be seen from the results of Sec. III, which show explicitly that the spin-dependent interference term for an on-shell point particle is proportional to the particle mass. On the other hand, allowing for finite virtuality of the initial and final quark leads to the appearance of collinear divergences [26], which are canceled only by graphs with explicit interactions of the active quark in the intermediate and final state, as was shown in detail in Sec. IV. The quark helicity-conserving contribution of Fig. 8(b) is thus of essentially "nonpartonic" character, requiring interaction of the active quark with the spectator system.

In the process of Fig. 8(c) the quark helicity is flipped in the course of the electron-quark scattering process. In short-distance processes such as DIS, the amplitude for quark helicity flip is usually thought to be of the order of the current quark mass, $m_f \sim$ few MeV, which is very small compared to typical hadronic mass scales. However, it is known that at larger distances the phenomenon of spontaneous chiral symmetry breaking sets in, and helicity-flip amplitudes of the order of a typical "constituent quark" mass, $M_q \approx 300$ MeV, are generated dynamically by interactions with nonperturbative vacuum fluctuations. We suggest here that this phenomenon plays an important role in the transverse spin-dependent eN cross section even in DIS kinematics. This perhaps somewhat surprising assertion is supported by the following arguments.

First, in QCD significant helicity-flip amplitudes should be present for quarks with virtualities smaller than some characteristic scale, μ^2_{chiral}, which is determined by the typical size of the nonperturbative field configurations instrumental in the spontaneous breaking of chiral symmetry. Quarks with virtualities $\gg \mu^2_{\text{chiral}}$ should experience only helicity-conserving perturbative interactions. This is explicitly seen in dynamical models of chiral symmetry breaking in QCD, such as the instanton vacuum or Dyson-Schwinger equations, which show a momentum-dependent dynamical quark mass which reduces to the current quark mass at virtualities $\gg \mu^2_{\text{chiral}}$ [in the instanton vacuum the

chiral symmetry breaking scale is determined by the average instanton size in the vacuum, $\mu^2_{\text{chiral}} \sim \rho^{-2} \approx (0.6 \text{ GeV})^2$]. Neglecting for the moment perturbative QCD radiation, the transverse spin dependence would be given by the imaginary part of the two-photon "box graph," in which the intermediate quark is on mass shell. It is precisely such low-virtuality quarks which experience large helicity-flip amplitudes due to chiral symmetry breaking [30].

Second, the previous argument can be generalized to account for the presence of perturbative QCD radiation. Generally, QCD radiation in DIS processes leads to a broad distribution of quark virtualities, extending up to the scale Q^2. The condition to propagate through a low-virtuality quark line (in order to enable a helicity flip) results in a suppression at the photon-quark vertices, measured by the Sudakov form factor. In the usual DIS cross section, which is given by the imaginary part of the quark Compton amplitude, this suppression is compensated by real gluon emissions. In the two-photon exchange process responsible for the transverse spin asymmetry, it is likely that this compensation happens only incompletely, and that a residual Sudakov suppression remains. To estimate the magnitude of this suppression, we consider the standard on-shell Sudakov form factor,

$$S(Q^2) = \exp\left(-\frac{\alpha_s C_F}{4\pi} \ln^2\frac{Q^2}{\mu^2}\right), \quad (68)$$

where $\alpha_s = 4\pi/[b\ln(Q^2/\Lambda^2_{\text{QCD}})]$ is the one-loop running coupling constant at the scale Q^2, with $b = 11 - (2/3)N_f$ and $\Lambda_{\text{QCD}} = 0.20$ GeV for $N_f = 3$, and $C_F = 4/3$. Furthermore, μ^2 denotes the IR cutoff for gluon emission. In the light of the above arguments about dynamical chiral symmetry breaking, it is natural to identify this cutoff with the chiral symmetry breaking scale,

$$\mu^2 \sim \mu^2_{\text{chiral}}. \quad (69)$$

Gluons of virtualities $k^2 < \mu^2$ are regarded as part of the nonperturbative vacuum fluctuations which lead to the spontaneous breaking of chiral symmetry and thus "contained" in the dynamical quark mass. Specifically, with the instanton vacuum value $\mu^2 = \rho^{-2} = 0.36$ GeV^2 we obtain

$$S(Q^2) = (0.89, 0.86, 0.83) \quad \text{for } Q^2 = (2, 3, 4) \text{ GeV}^2. \quad (70)$$

With this value of IR cutoff the Sudakov suppression of low-virtuality quark lines is not very substantial [31]. We conclude that a potentially sizable contribution to the transverse target spin dependence in inclusive DIS should come from the quark helicity-flip process of Fig. 8(c). If we chose instead the IR cutoff to be of the order

$$\mu^2 \sim \Lambda^2_{\text{QCD}}, \quad (71)$$

we would obtain a substantially larger Sudakov suppression. With $\mu^2 = \Lambda^2_{\rm QCD}(N_f = 3) = 0.04$ GeV^2 we would find

$$S(Q^2) = (0.56, 0.52, 0.5) \quad \text{for } Q^2 = (2, 3, 4) \text{ GeV}^2. \tag{72}$$

In this case amplitudes with quark helicity-flip would be significantly suppressed in QCD compared to the constituent quark model estimate. In the context of the present phenomenological discussion the choice of IR cutoff should in principle be regarded as an additional assumption; while it seems natural to choose it of the order of the chiral symmetry breaking scale, this could be rigorously justified only in an approximation scheme which treats the nonperturbative helicity-flipping fluctuations and perturbative gluon radiation in a unified framework.

In summary, we argue that a potentially sizable contribution to the transverse target spin dependence in inclusive DIS results from the quark helicity-flip process of the type Fig. 8(c). This contribution is of the order of a typical "constituent quark" mass, $M_q \approx 300$ MeV, multiplying the twist-2 quark transversity distribution, which is defined as (S denotes the nucleon polarization 4-vector)

$$h_f(x) = \int \frac{dz^-}{8\pi} e^{ixp^+z^-/2}$$
$$\times \langle pS_T | \bar{\psi}_f(0) \gamma^+ \gamma_5 \hat{S} \psi_f(z) | pS_T \rangle_{z_\perp=0, z^+=0}. \tag{73}$$

For a review of the properties of this distribution and its relation to other DIS observables, see e.g. Ref. [32].

It is interesting to compare the order-of-magnitude of the expected helicity-conserving and helicity-flip contributions to the spin-dependent cross section. While we can estimate the helicity-flip contribution in terms of the quark transversity distribution in the nucleon and the spin-dependent cross section for a pointlike constituent quark (see Sec. VI), we cannot presently calculate the helicity-conserving contribution in terms of $g_{T,f}$ and twist-3 quark-gluon operators in the nucleon. However, we can compare the ingredients, $g_{T,f}$ and $h_f(x)$, and try to guess the relative magnitude of the subprocess amplitudes in both contributions. Using the Wandzura-Wilczek relation for g_2 [33], which is valid in QCD up to terms proportional to twist-3 quark-gluon operators, we can express $g_{T,f}$, Eq. (67), as

$$g_{T,f}(x) = \int_x^1 \frac{dy}{y} g_f(y) + \text{quark-gluon}, \tag{74}$$

where g_f denotes the longitudinally polarized twist-2 quark density. The part given by matrix elements of twist-3 quark-gluon operators was measured in the SLAC E155 [34] and JLab Hall A [35] experiments and found to be small ($< 10^{-2}$), confirming theoretical predictions from the instanton vacuum model [36]. Neglecting it, we

can calculate $g_{T,f}$ in terms of the twist-2 polarized parton densities. Figure 9 shows $g_T(x)$ as estimated from Eq. (74), using the polarized parton densities of Ref. [37]. One sees that for $x \gtrsim 0.3$ the $g_{T,f}(x)$ are smaller than $g_f(x)$ at least by a factor of 2. A straightforward comparison between the helicity-flip and helicity-conserving contributions, assuming that the amplitudes of the quark subprocesses are otherwise comparable, is then

$$M_q h_f(x) \leftrightarrow M g_{T,f}(x), \tag{75}$$

where M_q is a "constituent quark" mass, measuring the generic strength of the quark helicity-flip amplitude due to nonperturbative vacuum fields. Since it is reasonable to assume that $h_f(x) \approx g_f(x)$, and M_q is not small (in the constituent quark model, $M_q \sim M/3$), we would conclude that the helicity-conserving contribution should be of the same order-of-magnitude as the helicity-flipping contributions. At least one can say that substantially different values for the two contributions could result only if the electron-quark subprocess amplitudes are very different in the two cases.

In fact, one can argue that the comparison of the two contributions as in Eq. (75) overestimates the helicity-conserving contribution. Namely, the electron-quark scattering amplitude for the helicity-conserving process is zero for on-shell, collinear quarks and requires nonzero virtuality. A more realistic comparison would thus be

$$M_q h_f(x) \leftrightarrow \frac{\langle \boldsymbol{k}_T^2 \rangle}{M} g_{T,f}(x), \tag{76}$$

where $\langle \boldsymbol{k}_T^2 \rangle$ denotes the average transverse momentum squared in the transverse momentum-dependent twist-3 distribution. Since one expects that $\langle \boldsymbol{k}_T^2 \rangle \sim$ (few 100 MeV)^2, the factor $\langle \boldsymbol{k}_T^2 \rangle/M^2$ further reduces the
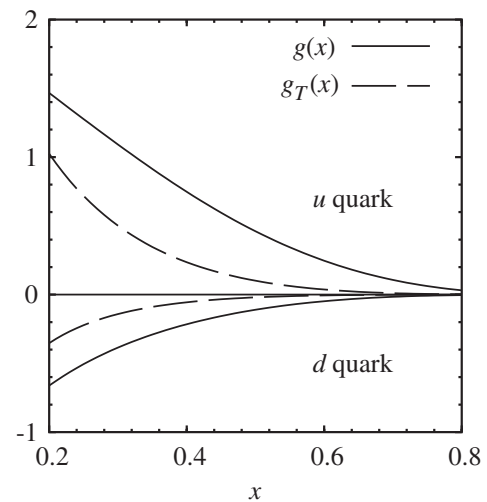

FIG. 9. The twist-3 transversely polarized quark distribution, $g_T(x)$, as estimated from Eq. (74), evaluated with the polarized parton densities of Ref. [37].

helicity-conserving compared to the helicity-flip contribution.

To summarize, we expect the dominant contribution to the transverse spin-dependent DIS cross section to come from amplitudes in which the two-photon exchange couples to a single quark. There are two distinct contributions to the transverse spin-dependent interference cross section in DIS, in which the quark helicity is either conserved or flipped in the electron-quark subprocess. Both contributions are "higher twist" in the sense that they involve dynamical effects not present in the leading-twist approximation (explicit spectator interactions or quark masses). We have argued that the quark helicity-flip contribution in QCD should be sizable if Sudakov suppression starts only at the chiral symmetry breaking scale, $\mu_{\rm chiral}^2 \gg \Lambda_{\rm QCD}^2$. Our order-of-magnitude estimates show that the quark helicity-conserving contribution is unlikely to dominate over the helicity-flip one.

VI. CONSTITUENT QUARK MODEL OF COMPOSITE NUCLEON

We now want to make a quantitative estimate of the transverse spin asymmetry of the DIS cross section which reflects the qualitative findings of our analysis of Sec. V. To this end, we employ a relativistic constituent quark model in the light-front formulation. This framework offers proper relativistic kinematics, while nevertheless maintaining close correspondence to the nonrelativistic description of bound states in the rest frame. The model describes DIS as elastic scattering from pointlike constituent quarks; it has a partonic limit, and the parton densities can be expressed as the longitudinal momentum densities of the light-front wave functions (from the QCD point of view these correspond to the parton densities at a low normalization point). Most importantly, through the constituent quark mass this model also generates nonzero quark helicity-flip amplitudes, which play an important role in the transverse spin asymmetry (see Sec. V); this aspect of constituent quark models was explored previously in relation to the high-Q^2 behavior of the proton form factor ratio QF_2/F_1 [38].

To arrive at a fully self-consistent scheme of approximations, we endow the constituent quark model with the additional dynamical assumption that the nucleon be a weakly bound state (composite). That is, we suppose that the quark transverse momenta, which are of the order of the inverse transverse size of the bound state, are parametrically small compared to the constituent quark mass,

$$\langle \boldsymbol{k}_T^2 \rangle \sim R_N^{-2} \ll M_q^2. \qquad (77)$$

This assumption permits several simplifications in the calculation of the transverse spin-dependent cross section from two-photon exchange. First, it suppresses two-photon exchange with different quarks (and other interference contributions involving different quarks) beyond the generic suppression discussed in Sec. V (see also Sec. VIII), and leaves the "partonlike" processes as the dominant ones. Second, among the "partonlike" processes it suppresses the quark helicity-conserving contribution of Fig. 8(b) (which is proportional to $\langle \boldsymbol{k}_T^2 \rangle$) and leaves the quark helicity-flip contribution of Fig. 8(c) (which is proportional to M_q) as the dominant one. Third, it allows us to consistently evaluate the latter in a relativistic impulse approximation with on-shell quarks, which exactly preserves electromagnetic gauge invariance and is free of collinear divergences.

The compositeness assumption (77) is not intended as a reflection of actual nucleon structure (in reality $|\boldsymbol{k}_T| \sim$ few 100 MeV in the constituent quark model), but as a theoretical idealization which allows us to calculate the two-photon exchange cross section in a self-consistent scheme. Compared to the usual one-photon exchange approximation for form factors and structure functions, in two-photon exchange processes one is dealing with several new effects (collinear divergences, exchanges with different constituents) which can qualitatively distort the results if not treated consistently. One therefore has to be prepared to make stronger assumptions about the structure of the bound state.

The technical implementation of the above ideas takes the form of a relativistic impulse approximation, in which the electron scatters elastically from a massive, on-shell constituent quark, see Fig. 10 [39]. In this approximation the squared modulus of the invariant amplitude, summed over all hadronic final states (corresponding to the cross section for inclusive eN scattering), is given by

$$\sum_X |\mathcal{M}_{eN \to e'X}|^2 = \sum_f \int_0^1 \frac{d\xi}{\xi} \int \frac{d^2 k_T}{(2\pi)^2}$$
$$\times {\rm tr}[\rho_f(\xi, \boldsymbol{k}_T | p)\Gamma_f(k, l, l')], \qquad (78)$$

where k is the 4-momentum of the active quark, with light-cone 3-momentum components $k^+ \equiv k^0 + k^3 = \xi p^+$ and $\boldsymbol{k}_T$, and energy $k^- = (\boldsymbol{k}_T^2 + M_q^2)/k^+$, corresponding to $k^2 = M_q^2$ (for simplicity we assume the constituent quark
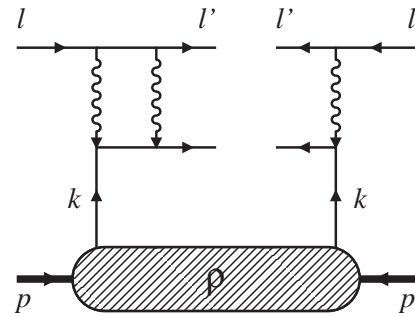

FIG. 10. Transverse spin-dependent two-photon exchange cross section in the constituent quark model with the composite nucleon approximation, Eq. (78).

mass M_q to be the same for both light quark flavors $f = u$, d). The matrix Γ_f represents the squared modulus of the invariant amplitude for elastic scattering of the electron from the on-shell quark with 4-momentum k,

$$|\mathcal{M}_{eq \to e'q}|^2 = \bar{U}_f(k)\Gamma_f(k, l, l')U_f(k), \qquad (79)$$

where U_f denotes the initial quark spinor, and the dependence on the initial quark helicity is contained in the spinors [the final quark helicity is summed over, and the initial/final electron helicities are averaged/summed over, as in Eq. (12)]. Furthermore, in Eq. (78) ρ_f denotes the density matrix of the active quark in the nucleon, depending on the quark momentum variables ξ and $\boldsymbol{k}_T$, as well as on the nucleon polarization. On grounds of Lorentz invariance, parity invariance, and the constraints imposed by the Dirac equation for the quark spinors, we can parametrize the density matrix for a quark in a transversely polarized nucleon as

$$\rho_f = \frac{\hat{k} + M_q}{2}[f_f(\xi, \boldsymbol{k}_T) - \gamma_5 \hat{a} h_f(\xi, \boldsymbol{k}_T)], \qquad (80)$$

where

$$a \equiv S - \frac{(Sk)k}{k^2}, \qquad (ak) = 0 \qquad (81)$$

is the component of the nucleon polarization vector orthogonal to the quark 4-momentum. The functions $f_f(\xi, \boldsymbol{k}_T)$ and $h_f(\xi, \boldsymbol{k}_T)$ in this parametrization are related to the quark unpolarized and transversity parton densities in this model by

$$\int \frac{d^2 k_T}{(2\pi)^2} f_f(\xi, \boldsymbol{k}_T) = f_f(\xi), \qquad (82)$$

$$\int \frac{d^2 k_T}{(2\pi)^2} h_f(\xi, \boldsymbol{k}_T) = h_f(\xi). \qquad (83)$$

By explicit calculation, one can verify that the transversity parton density thus obtained coincides with the one defined in terms of matrix elements of the quark light-ray operators, Eq. (73), if the quark fields there are identified with the massive constituent quarks of this model. As already mentioned, in the context of QCD these parton densities would correspond to a low normalization point of $\mu^2 \sim R_N^{-2}$.

When calculating DIS observables in the model defined by Eq. (78) *et seq.*, we work in a frame where the initial proton momentum and the momentum transfer are collinear and have components along the 3-direction: $\boldsymbol{q}_T$, $\boldsymbol{p}_T = 0$, with $p^+ > 0$. The mass-shell condition for the final-state quark, $(k + q)^2 = M_q^2$, then fixes the plus momentum fraction of the initial quark to be

$$\xi = x \frac{1 + \sqrt{1 + 4M_q^2/Q^2}}{1 + \sqrt{1 + 4x^2 M^2/Q^2}}, \qquad (84)$$

where

$$x \equiv \frac{Q^2}{2(pq)} \qquad (85)$$

is the usual Bjorken variable of the DIS process. Following the composite nucleon assumption, we neglect corrections of the order $\boldsymbol{k}_T^2/s$ in the quark momentum fraction but retain corrections due to the finite proton and quark masses. In this approximation the 4-momentum of the active quark can be expressed covariantly as

$$k = Ap + Bq + k_\perp, \qquad (86)$$

where $(pk_\perp), (qk_\perp) = 0$ and the scalars A and B are given by

$$A = \frac{\xi(\eta Q^2 + 2M_q^2)}{\eta Q^2 + M_q^2 + \xi^2 M^2}, \qquad (87)$$

$$B = \frac{\eta(M_q^2 - \xi^2 M^2)}{\eta Q^2 + M_q^2 + \xi^2 M^2}, \qquad (88)$$

with

$$\eta \equiv \frac{1}{2}\left(1 + \sqrt{1 + \frac{4M_q^2}{Q^2}}\right). \qquad (89)$$

The invariant of the electron-quark subprocess is then obtained as

$$s_{\text{sub}} \equiv 2(lk) = A(s - M^2) - BQ^2 + M_q^2, \qquad (90)$$

up to terms proportional to the quark transverse momentum which give corrections of the order $\langle \boldsymbol{k}_T^2 \rangle$.

The scheme of approximation defined by Eqs. (84)–(90) has several interesting properties. First, in the limit $Q^2 \to \infty$ we recover $\xi = x$ and $s_{\text{sub}} = xs$, as in the parton model. Second, in the limit of zero binding, if we consider the nucleon as an assembly of free quarks of mass M_q and neglect the binding forces between them, each quark should carry a fraction M_q/M of the nucleon's momentum. Indeed, for $x = M_q/M$ Eq. (84) gives $\xi = x = M_q/M$, and from Eqs. (87) and (88) one obtains $A = x$ and $B = 0$, showing that our approximations respect this limit. Third, our approximations are consistent with the overall kinematic boundaries of inclusive eN scattering. For a given eN CM energy ($s = 2E_{\text{beam}}M + M^2$ for fixed-target experiments) the minimum value of x attainable is

$$x_{\text{min}} = \frac{Q^2}{s - M^2 - Q^2 M^2/(s - M^2)}, \qquad (91)$$

corresponding to the maximum allowed energy loss of the electron, or a laboratory scattering angle of $\theta_{\text{lab}} = \pi$.

Conversely, for given x the maximum attainable value of Q^2 is

$$Q^2_{\max} = \frac{x(s - M^2)}{1 + xM^2/(s - M^2)}. \qquad (92)$$

With our choice of kinematic variables for the electron-quark subprocess, Eqs. (84)–(90), this overall kinematic boundary corresponds exactly to the maximum value of the CM scattering angle of the electron-quark subprocess, $\theta_{cm}(\text{electron} - \text{quark}) = \pi$, as one can show by explicit calculation. The reason for this coincidence is that the overall kinematic boundary corresponds to perfectly collinear kinematics (in the laboratory frame the electron bounces back with zero transverse momentum), which is correctly described in our approximation where transverse momenta are neglected.

The calculation of the normal spin asymmetry of the ep cross section in the composite nucleon approximation defined above is straightforward, and essentially amounts to evaluating the asymmetry for the pointlike target in the kinematics of the quark subprocess. The matrix Γ_f representing the squared amplitude of the quark-level subprocess, Eq. (79), is given by

$$\Gamma_f = \frac{1}{2M_q}\left(X_{U,f} + \frac{\hat{N}_{\text{sub}}\gamma_5}{\sqrt{-N^2_{\text{sub}}}}X_{N,f}\right), \qquad (93)$$

where $N^\mu_{\text{sub}} \equiv -4\epsilon^{\mu\alpha\beta\gamma}l_\alpha l'_\beta k_\gamma$ is the normal 4-vector of the electron-quark subprocess [cf. Eqs. (13) and (86)], and the functions $X_{U,f}$ and $X_{N,f}$ correspond to the results of Sec. III with $s \to s_{\text{sub}}$ [cf. Eq. (90)], $M \to M_q$, and target charge $e \to e_f e$, where e_f are the fractional quark charges. In our scheme of approximation, where the quark transverse momenta are neglected in kinematic factors, one has $(SN_{\text{sub}})/\sqrt{-N^2_{\text{sub}}} \approx (SN)/\sqrt{-N^2}$. The result for the transverse spin asymmetry in inclusive DIS in the composite nucleon approximation can then be expressed as

$$A_N(s, Q^2, x)_{\text{comp}} = R(\xi)A_N(s_{\text{sub}}, Q^2)_{M=M_q}, \qquad (94)$$

where ξ and s_{sub} are given by Eqs. (84) and (90), and A_N on the right-hand side is the asymmetry for a pointlike constituent quark of charge $+e$ and mass M_q [i.e., Eq. (44) with the mass M replaced by M_q], evaluated at the subprocess invariants s_{sub} and Q^2. The information about the quark structure of the target is contained in the structure factor

$$R(\xi) \equiv \frac{\sum_f e_f^3 h_f(\xi)}{\sum_f e_f^2 f_f(\xi)}, \qquad (95)$$

which is the ratio of the sums of quark transversity and unpolarized parton densities, weighted with the quark charges corresponding to the two-photon–one-photon in-

terference cross section (numerator) and the one-photon cross section (denominator). This ratio depends on the spin/flavor wave function of the quark bound state, as well as on the momentum distribution of the quarks. We shall discuss specific models for this ratio in Sec. VII.

Equation (94) was derived in the approximation of weak binding between the constituents, where the quark momentum distributions are concentrated around $\xi \sim M_q/M \sim 1/3$. It therefore should be applied only in the region around $x \sim 0.3$. In particular, for $x \to 1$ correlations between constituents in the wave function become important, and the picture of the composite nucleon is no longer applicable.

A cautionary remark is in order concerning the model dependence of the results presented here. The transverse spin-dependent DIS cross section involves not only the "good" (+) light-cone component of the current operator. This is seen e.g. in the study of the high-energy behavior of the asymmetry (see Sec. VIII), where it is noted that the leading high-energy contribution to the cross section, resulting from the + current components only, has no transverse spin dependence. Generally, in light-front quantization observables involving other than the good current component are more model dependent than those involving only the good component. In Ref. [40] this problem was addressed by eliminating the "bad" (−) component using gauge invariance, and applying a trick to the transverse component. The extension of this technique to the case of two-photon exchange processes is an interesting problem but beyond the scope of the present paper.

VII. NUMERICAL ESTIMATES

For a numerical estimate of the asymmetry we need to specify the spin/flavor wave function of the nucleon. Since Eq. (94) was derived for the idealized case of a composite nucleon (weak binding), the spin-flavor wave function needs to be modeled consistently with this approximation. We consider two simple models which meet this requirement.

(a) *SU*(6) *spin/flavor wave function.*—The simplest choice of wave function consistent with the composite nucleon assumption is the wave function of the nonrelativistic quark model. In this model, the probabilities $P_{f\sigma}$ for finding a quark in the proton wave function with flavor $f = u, d$ and spin projection $\sigma = +, -$ along the direction of the transverse proton spin, are

$$\begin{aligned} P_{u+} = \tfrac{5}{9}, & \qquad P_{u-} = \tfrac{1}{9}, \\ P_{d+} = \tfrac{1}{9}, & \qquad P_{d-} = \tfrac{2}{9}, \end{aligned} \qquad (96)$$

with $\sum_{f\sigma} P_{f\sigma} = 1$, see Ref. [41] and references therein. The probabilities for the neutron are obtained by interchanging $u \leftrightarrow d$. Neglecting the ef-

fect of spin on the quark momentum distributions, we obtain

$$R = \frac{e_u^3(P_{u+} - P_{u-}) + e_d^3(P_{d+} - P_{d-})}{e_u^2(P_{u+} + P_{u-}) + e_d^2(P_{d+} + P_{d-})} \quad (97)$$

$$= \begin{cases} \frac{11}{27} = 0.41 & \text{(proton)}, \\ -\frac{2}{9} = -0.22 & \text{(neutron)}. \end{cases} \quad (98)$$

The ratio of the neutron to the proton structure factors, and thus of the corresponding asymmetries, in this model is

$$\frac{R^n}{R^p} = -\frac{6}{11} = -0.55. \quad (99)$$

(b) *Transversity=helicity distributions.*—For a weakly bound nucleon we can neglect sea quarks and assume the valence quark transversity to be equal to the helicity distributions. It then becomes possible to evaluate the ratio (95) using phenomenological parametrizations for the unpolarized and helicity parton densities. With the parametrizations of Refs. [37,42] we find that for $Q^2 \sim$ few GeV^2 the ratio is practically independent of Q^2, and approximately constant in the region $0.2 < x < 0.5$, with values

$$R(x) \approx \begin{cases} 0.35 & \text{(proton)}, \\ -0.25 & \text{(neutron)}. \end{cases} \quad (100)$$

These values are close to the ones obtained with the $SU(6)$ wave function, Eq. (98).

It is interesting to note that with both models (a) and (b) the ratio of the proton and neutron asymmetries in the composite nucleon approximation is numerically not far from the ratio of the proton and neutron magnetic moments,

$$\frac{\mu^p}{\mu^n} = -1.46. \quad (101)$$

This is what one would expect from the simple classical picture of the normal spin asymmetry as being due to the scattering from the magnetic field generated by the target [see Fig. 1(b)].

For a numerical estimate of the asymmetry in the composite nucleon approximation, Eq. (94), we use a constituent quark mass $M_q = M/3$. Figure 11 shows the asymmetry for an electron-proton CM energy of $s = 10$ GeV^2 (corresponding approximately to the planned Jefferson Lab Hall A experiment [10] with 6 GeV beam energy), for $Q^2 = 2$ GeV^2, as a function of x. Note that for given s and Q^2, the minimum value of x which is kinematically attainable is given by Eq. (91). Comparison of Fig. 11 with Fig. 4 shows that the magnitude of the asym-

metry for the composite proton is reduced by a factor of ~ 4 compared to the pointlike proton approximation. This change results from a combination of various factors: the quark charges and polarizations in the structure factor Eq. (95), the change of the target mass $M \rightarrow M_q$, and the change of the effective CM energy $s \rightarrow s_{\text{sub}}$ [the latter effect partly compensates the change in the target mass, because the pointlike asymmetry Eq. (45) is proportional to $M/\sqrt{s}$]. Figure 11 shows the results obtained with assumptions (a) and (b) about the spin-flavor wave function of the target. One sees that the two models give comparable values of the asymmetry for both proton and neutron.
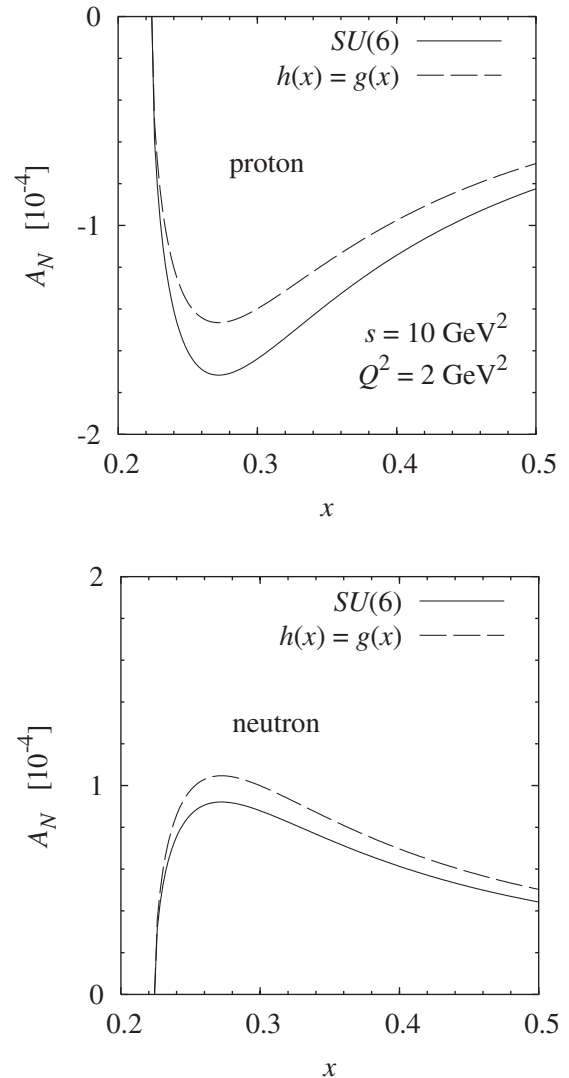

FIG. 11. The normal target spin asymmetry A_N in DIS kinematics in the composite nucleon approximation, Eq. (94), for both proton and neutron target, with different assumptions about the spin-flavor wave function: (a) $SU(6)$ symmetry, (b) transversity = helicity distributions. Note the different signs for proton and neutron. Shown is the asymmetry as a function of x, for $s = 10$ GeV^2 and $Q^2 = 2$ GeV^2. The values of x are kinematically restricted to $x > x_{\text{min}}$, Eq. (91).
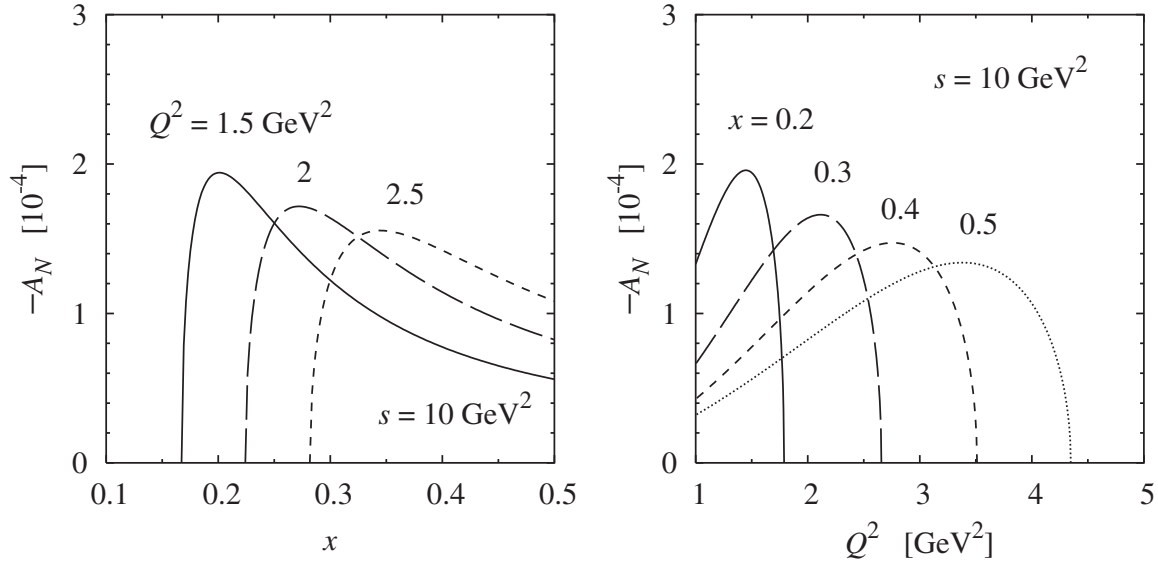
FIG. 12. The normal target spin asymmetry A_N (note the minus sign on the plot axis) for the proton in the composite nucleon approximation, Eq. (94), as a function of x and Q^2. Shown are the results for the $SU(6)$ spin/flavor wave function, with $s = 10$ GeV^2. Left: x-dependence for several values of Q^2 (indicated above the curves). The values of x are kinematically restricted to $x > x_{\min}$, Eq. (91). Right: Q^2-dependence for several values of x (indicated above the curves). The values of Q^2 are kinematically restricted to $Q^2 < Q^2_{\max}$, Eq. (92).

Of interest is also the deuteron target. Because of its isoscalar character, the structure factor (95) for the deuteron is

$$R^d(\xi) = \frac{e_u^3 + e_d^3}{e_u^2 + e_d^2} \frac{h^d(\xi)}{f^d(\xi)} = \frac{7}{15} \frac{h^d(\xi)}{f^d(\xi)}, \qquad (102)$$

where $h^d \equiv h_u^d = h_d^d$ and $f^d \equiv f_u^d = f_d^d$ are the isoscalar quark distributions in the deuteron. Approximating the latter by the average of proton and neutron distributions, and using isospin invariance, one has

$$h^d \equiv (h_u + h_d)/2, \qquad f^d \equiv (f_u + f_d)/2, \qquad (103)$$

where the distributions without a superscript refer to the proton. With the $SU(6)$ wave functions, cf. Eq. (96), one obtains

$$\frac{h^d}{f^d} = \frac{P_{u+} - P_{u-} + P_{d+} - P_{d-}}{P_{u+} + P_{u-} + P_{d+} + P_{d-}} = \frac{1}{3}, \qquad (104)$$

and thus

$$R^d = \frac{7}{45} \approx 0.16. \qquad (105)$$

The asymmetry for the deuteron has the same sign as for the proton, but its magnitude is reduced by a factor $21/55 \approx 0.38$. With the "transversity = helicity" approximation for the proton and neutron and the parametrizations [37,42], one obtains $h^d(\xi)/f^d(\xi) \approx 0.25$ at $\xi = 1/3$, close to the $SU(6)$ result.

It is interesting to study the dependence on the kinematic variables of the asymmetry obtained in the composite nucleon, Eq. (94). Figure 12 (left panel) shows the asymmetry as a function of x, for various fixed values of Q^2, and fixed s. One sees that the maximum value of the asymmetry decreases with increasing Q^2. This is because the
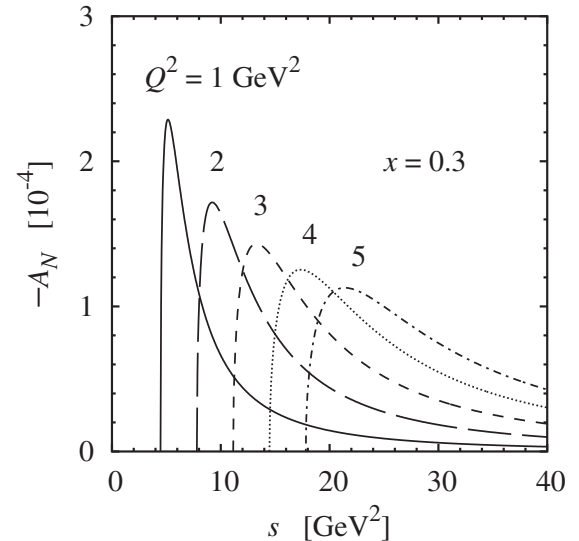

FIG. 13. The normal target spin asymmetry A_N (note the minus sign on the plot axis) for the proton in the composite nucleon approximation, as a function of the squared electron-proton CM energy, s, for $x = 0.3$ and various values of Q^2 (indicated above the curves). The asymptotic behavior at large s is $A_N \sim s^{-2}$, with the coefficient proportional to Q^3, cf. Eqs. (108) and (111).

magnitude of the asymmetry is inversely proportional to the invariant CM energy of the quark subprocess, $s_{sub}^{1/2}$ [cf. Eq. (45) with $s \to s_{sub}$], and s_{sub} is numerically close to Q^2 at the large subprocess scattering angle corresponding to the maximum value of the asymmetry. Figure 12 (right panel) shows the asymmetry as a function of Q^2, for various fixed values of x. For fixed s and x, the Q^2 range is kinematically restricted to values lower than Eq. (92).

Figure 13 shows the dependence of the asymmetry on the squared electron-nucleon CM energy, s, for fixed x and Q^2. This dependence could in principle be tested by comparing measurements at different beam energies, similar to the L/T separation of electroproduction cross sections. General considerations suggest that at large s the asymmetry vanishes as s^{-2}, cf. Eq. (108) in Sec. VIII. The asymmetry obtained in the composite nucleon approximation exhibits this behavior, see Fig. 13.

VIII. TRANSVERSE SPIN DEPENDENCE IN SOFT HIGH-ENERGY SCATTERING

In addition to DIS it is interesting to consider the transverse spin dependence in soft high-energy scattering, i.e., the limit of large scattering energy, but small energy and momentum transfer to the target,

$$s \gg \mu^2, \qquad Q^2, M_X^2 \sim \mu^2, \qquad (106)$$

where μ denotes a typical hadronic mass scale. In this limit one can analyze the two-photon exchange interference cross section with general methods for studying the high-energy behavior of QED amplitudes. Also in this limit, one can use closure over nonrelativistic quark model states to describe the inclusive final state and obtain a new perspective on the dominance of single-quark scattering at larger momentum transfers.

It is well known that in QED the large-s behavior of scattering amplitudes having the form of two blocks connected by t-channel photon exchange is determined only by the number of the exchanged photons and their polarization states [43]. The internal structure of the blocks, which themselves do not contain any large invariants, is important only insofar as it determines which polarization states can contribute. One can easily see that for an electron scattering from a pointlike target the so-called "nonsense" polarization, which gives the dominant contribution to the cross section, does not give rise to a transverse spin dependence. Indeed, the high-energy behavior of the asymmetry for the pointlike target, as follows from expanding Eq. (45) in the region of small CM angle, $\theta_{cm} = 2Q/\sqrt{s}$, is

$$A_N = \frac{\alpha M Q^3}{2 s^2} \qquad (s \gg Q^2, M^2), \qquad (107)$$

i.e., the asymmetry vanishes in the high-energy limit. Assuming the polarization structure of the blocks in the pointlike target case to be representative of the general case, we conclude that the s-dependence of the inclusive

asymmetry should be the same as in the point particle case,

$$A_N \sim s^{-2} \qquad (s \gg \mu^2; Q^2, M_X^2 \sim \mu^2). \qquad (108)$$

A general proof of this statement, adapting the methods of Ref. [43], we leave up to future work.

The Q^2 dependence of the asymmetry in the high-energy limit (106) can be studied using general arguments for soft scattering from a composite system based on the mean-field approximation of nuclear physics, see e.g. Refs. [44,45]. In the target rest frame, we consider the nucleon as a generic nonrelativistic bound state of massive quarks. For sufficiently small excitation energies, one can use closure over the nonrelativistic quark states to calculate the inclusive cross section. In this approach the electromagnetic coupling of the quark (with label i) is described by the operator $e_i \exp[-i(\boldsymbol{q}\boldsymbol{r}_i)]$, where $\boldsymbol{q}$ is the photon momentum and e_i and $\boldsymbol{r}_i$ the charge and position of the quark. Consider now the two contributions to the spin-dependent inclusive cross section shown in Fig. 14, where the momenta in the two-photon exchange amplitude are denoted by $\boldsymbol{q} - \boldsymbol{\Delta}$ and $\boldsymbol{\Delta}$. In the mean-field approximation, they are proportional, respectively, to

$$f^{(a)}(\boldsymbol{\Delta}) \propto \sum_i e_i^3, \qquad (109)$$

$$f^{(b)}(\boldsymbol{\Delta}) \propto \sum_{i \neq j} e_i^2 e_j F^2(\gamma \boldsymbol{\Delta}^2), \qquad (110)$$

where F denotes the elastic form factor of the ground state in mean-field approximation, and γ is a coefficient of order unity which results from the calculation of the recoil of the
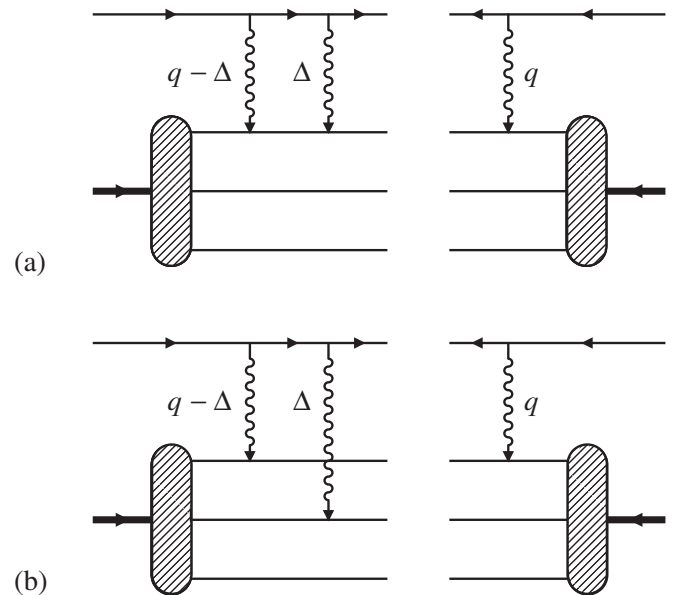

FIG. 14. Interference of one-photon and two-photon exchange in electron scattering from a bound state. (a) Two-photon exchange with the same constituent. (b) Two-photon exchange with different constituents.

spectator system [45]. Note that the form factor appears only in the contribution (b), where the two photons couple to different quarks, because here the momentum $\mathbf{\Delta}$ has to be routed through the nucleon wave function. To get the cross sections, the functions $f^{(a)}$ and $f^{(b)}$ are to be integrated over $\mathbf{\Delta}$, together with the photon propagators and the numerator factors accounting for the kinematic momentum dependence of the asymmetry. The IR finiteness of the spin-dependent cross section now guarantees that no large contributions arise from momenta $|\mathbf{\Delta}| \lesssim R_N^{-1}$ (R_N is the size of the bound state), because of the vanishing of the numerators. Thus, in the region of moderately large momentum transfers, $s \gg Q^2 \gg R_N^{-2}$, the relative magnitude of contributions (a) and (b) is essentially determined by the phase space available for the $\mathbf{\Delta}$-integral. In case (a) the integral extends up to $|\mathbf{\Delta}|^2 \sim Q^2$, while in case (b) it is limited to $|\mathbf{\Delta}|^2 \lesssim R_N^{-2}$ by the form factors. We conclude that in the region $s \gg Q^2 \gg R_N^{-2}$ the dominant contribution to the spin-dependent cross section comes from the coupling of the photons to the same quark; the contribution in which the two photons couple to different quarks is suppressed by a high power of $1/(R_N^2 Q^2)$, which depends on the detailed behavior of the bound-state form factor. Similar arguments apply to the other possible contributions to the interference cross section (not shown in Fig. 14) in which not all photons couple to the same quark.

The spin-independent cross section in one-photon exchange approximation is likewise dominated by the scattering of the two photons from the same quark; interference contributions are suppressed by the bound-state form factor. In this case the above reasoning just reproduces standard arguments for the approach to scaling in the nonrelativistic quark model. Combining the statements about the spin-dependent and spin-independent cross sections for a composite target, we conclude that the asymmetry (i.e., the ratio) should exhibit the same Q^2-dependence as the asymmetry for a point particle, Eq. (107),

$$A_N \sim Q^3 \qquad (s \gg Q^2 \gg R_N^{-2}). \qquad (111)$$

In summary, our arguments based on the mean-field approximation imply that the Q^2-dependence of the asymmetry at moderately large Q^2 is the minimal dependence dictated by kinematics (i.e., by the need to have transverse momentum transfer) but is not subject to any dynamical form factor suppression. We note that the asymmetry calculated in the constituent quark model with the composite nucleon approximation (see Sec. VI) shows the behavior described by Eqs. (108) and (111).

IX. SUMMARY AND OUTLOOK

The transverse spin dependence of the cross section of inclusive eN scattering, in spite of being a "simple" observable, is seen to give rise to many interesting questions of electrodynamics and strong interaction physics. Our treatment of these problems in large parts has been of exploratory nature. In the following we summarize our main conclusions and describe several problems deserving further study.

Concerning the electrodynamics aspects, we have pointed out that the transverse spin-dependent cross section due to two-photon exchange is free of IR divergences. No cancellation of IR divergences between two-photon exchange and real photon emission is required (as in the two-photon corrections to the spin-independent cross section), making the transverse spin-dependent cross section a clean two-photon exchange observable. However, real photon emission can still make a finite contribution to the spin dependence of $ep \rightarrow e'X$, which in practice cannot be separated from purely hadronic final states. To estimate this contribution is an interesting problem for further study.

Concerning the strong interaction aspects, we have argued that in DIS kinematics a sizable contribution to the transverse spin-dependent cross section results from quark helicity-flip processes made possible by the nonperturbative vacuum structure of QCD structure (chiral symmetry breaking). The key point is that such processes are not significantly Sudakov suppressed if the IR cutoff for gluon emission is of the order of the chiral symmetry breaking scale $\mu_{\text{chiral}}^2 \gg \Lambda_{\text{QCD}}^2$. While this seems natural in the context of the phenomenology of chiral symmetry breaking, we presently cannot offer rigorous arguments for the correctness of this choice.

We have presented qualitative arguments why the quark helicity-conserving contribution to the transverse spin-dependent cross section, related to $g_{T,f}$, is unlikely to dominate. A complete QCD calculation of this contribution in the collinear factorization approach, which maintains electromagnetic gauge invariance by including quark-gluon operators and avoids unphysical collinear divergences, is clearly an outstanding problem. The crucial question is whether the complete result will involve the Wandzura-Wilczek contribution to $g_{T,f}$ (which is given in terms of matrix elements of twist-2 operators), or whether only the twist-3 quark-gluon correlations will survive. In the former case the helicity-conserving contribution could be estimated in a model-independent way. In the latter case, it is likely to be very small, and the dominant contribution to the transverse spin-dependent cross section would most likely come from the quark helicity-flip process governed by the transversity distribution.

Our numerical estimates based on the constituent quark model suggest that the asymmetry in the kinematics of the planned Jefferson Lab Hall A experiment [10] is of the order a few times 10^{-4}, with different sign for proton and neutron. The predicted asymmetry for the proton is larger than for the neutron, suggesting that measurements with a transversely polarized proton target would be a useful complement to the planned measurements with ^{3}He.

[1] For a review, see J. P. Chen, A. Deur, and Z. E. Meziani, Mod. Phys. Lett. A **20**, 2745 (2005).

[2] N. Christ and T. D. Lee, Phys. Rev. **143**, 1310 (1966).

[3] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. Lett. **91**, 142304 (2003).

[4] P. A. M. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003).

[5] Y. C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaeghen, Phys. Rev. Lett. **93**, 122301 (2004).

[6] A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y. C. Chen, and M. Vanderhaeghen, Phys. Rev. D **72**, 013008 (2005).

[7] M. K. Jones *et al.* (Jefferson Lab Hall A Collaboration), Phys. Rev. Lett. **84**, 1398 (2000).

[8] O. Gayou *et al.* (Jefferson Lab Hall A Collaboration), Phys. Rev. Lett. **88**, 092301 (2002); arXiv:nucl-ex/0111010.

[9] A. V. Afanasev and C. E. Carlson, Phys. Rev. Lett. **94**, 212301 (2005).

[10] X. Jiang *et al.*, Target Normal Single-Spin Asymmetry in Inclusive DIS $n^\uparrow(e, e')$ with a Polarized ^{3}He Target, Jefferson Lab Hall A Experiment E-07-013. Proposal available at http://www.jlab.org/exp_prog/proposals/07/PR-07-013.pdf.

[11] S. Rock *et al.*, Phys. Rev. Lett. **24**, 748 (1970).

[12] D. Diakonov and V. Y. Petrov, Nucl. Phys. **B272**, 457 (1986).

[13] For a review, see D. Diakonov, Prog. Part. Nucl. Phys. **51**, 173 (2003); T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).

[14] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitayevskii, in *Course of Theoretical Physics*, Relativistic Quantum Theory (Pergamon, New York, 1973), Vol. IV.

[15] R. S. Conti and I. B. Khriplovich, Phys. Rev. Lett. **68**, 3262 (1992); I. B. Khriplovich, Nucl. Phys. **A663**, 147 (2000).

[16] A. O. Barut and C. Fronsdal, Phys. Rev. **120**, 1871 (1960).

[17] A. De Rujula, J. M. Kaplan, and E. De Rafael, Nucl. Phys. **B35**, 365 (1971).

[18] In our conventions $\epsilon^{0123} = 1$, $\mathrm{tr}[\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \gamma_5] = 4i\epsilon^{\alpha\beta\gamma\delta}$, and the spin density matrix of the initial proton is $\frac{1}{2}(\hat{p} + M)(1 - \gamma_5 \hat{S})$, cf. Ref. [14].

[19] The precise definition of the absorptive part of the hadronic Compton amplitude for the $N \to X$ transition in a nonperturbative context is not needed for the following. Later we shall evaluate Eq. (50) for a field-theoretical model of hadron structure, in which the absorptive part is given explicitly as the sum of Cutkosky cuts of Feynman diagrams.

[20] F. Low, Phys. Rev. **110**, 974 (1958).

[21] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) **13**, 379 (1961).

[22] Y. S. Tsai, Phys. Rev. **122**, 1898 (1961).

[23] R. N. Cahn and Y. S. Tsai, Phys. Rev. D **2**, 870 (1970).

[24] A. V. Afanasev and N. P. Merenkov, Phys. Rev. D **70**, 073002 (2004); Phys. Lett. B **599**, 48 (2004).

[25] A. Metz, M. Schlegel, and K. Goeke, Phys. Lett. B **643**, 319 (2006).

[26] In the analysis of Ref. [25] these divergences are referred to as infrared divergences. We adhere to the standard terminology of ''infrared'' and ''collinear'' divergences, as defined in Sec. IV.

[27] Since we are interested only in the absorptive part of the two-photon exchange amplitude, ''seagull'' terms with two photons coupling to the scalar at the same point do not contribute.

[28] M. Gorchtein, Phys. Rev. C **73**, 055201 (2006); **73**, 035213 (2006).

[29] D. Borisyuk and A. Kobushkin, Phys. Rev. C **73**, 045210 (2006).

[30] It is worth pointing out that our arguments for the role of nonperturbative helicity flip in the transverse spin dependence are not in conflict with the experience with QCD factorization for the usual DIS cross sections, where no nonperturbative quark mass is included. In the unpolarized cross sections (F_2, F_L) the dominant contribution comes from the helicity-conserving leading-twist contribution, and a nonperturbative quark mass would result only in a higher-twist correction of the order M_q^2/Q^2.

[31] Our simple estimate does not account for the fact that there is a certain mismatch of the virtualities of the photons which couple to the near on-shell quark in the two-photon and single-photon exchange amplitudes, cf. the discussion at the end of Sec. III. However, since the effective scale in the two-photon exchange, Q_{eff}^2, is of the same order of magnitude as Q^2, we expect that this mismatch would lead only to next-to-leading corrections in $\ln(Q^2/\mu^2)$.

[32] V. Barone, A. Drago, and P. G. Ratcliffe, Phys. Rep. **359**, 1 (2002).

[33] S. Wandzura and F. Wilczek, Phys. Lett. **72B**, 195 (1977).

[34] P. L. Anthony *et al.* (E155 Collaboration), Phys. Lett. B **553**, 18 (2003).

[35] X. Zheng *et al.* (Jefferson Lab Hall A Collaboration), Phys. Rev. C **70**, 065207 (2004).

[36] J. Balla, M. V. Polyakov, and C. Weiss, Nucl. Phys. **B510**, 327 (1998).

[37] M. Gluck, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D **63**, 094005 (2001).

[38] G. A. Miller and M. R. Frank, Phys. Rev. C **65**, 065205 (2002).
[39] G. A. Miller, Phys. Rev. C **69**, 052201 (2004).
[40] L. L. Frankfurt and M. I. Strikman, Phys. Rep. **76**, 215 (1981).
[41] F. E. Close, Nucl. Phys. **B80**, 269 (1974).
[42] M. Gluck, E. Reya, and A. Vogt, Eur. Phys. J. C **5**, 461 (1998).
[43] V. N. Gribov, L. N. Lipatov, and G. V. Frolov, Yad. Fiz. **12**, 994 (1970) [Sov. J. Nucl. Phys. **12**, 543 (1971)].
[44] S. Stein *et al.*, Phys. Rev. D **12**, 1884 (1975).
[45] L. Frankfurt, G. A. Miller, and M. Strikman, Phys. Rev. D **65**, 094015 (2002).