# $b \to s \nu \bar{\nu}$  decay in the MSSM: Implication of  $b \to s \gamma$  at large  $\tan \beta$

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The decay  $b \rightarrow s \nu \bar{\nu}$  is discussed in the minimal supersymmetric standard model with general flavor mixing for squarks, at large tan $\beta$ . In this case, in addition to the chargino loop contributions which were analyzed in previous studies, tan $\beta$ -enhanced contributions from the gluino and charged Higgs boson loops might become sizable compared with the standard model contribution, at least in principle. However, it is demonstrated that the experimental bounds on the new physics contributions to the radiative decay  $b \rightarrow$  $s\gamma$  should strongly constrain these contributions to  $b \rightarrow s\nu\bar{\nu}$ , especially on the gluino contribution. We also briefly comment on a possible constraint from the  $B_s \to \mu^+ \mu^-$  decay.

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## **I. INTRODUCTION**

Recently, there has been significant experimental improvements in the measurements of flavor-changing neutral current (FCNC) processes of *B* mesons at *B* factories and Tevatron. For the  $b \rightarrow s$  transition, experimental data for  $b \rightarrow s\gamma$  and  $b \rightarrow s\ell^+\ell^-$  ( $l = e, \mu$ ) decays,  $B_s - \bar{B}_s$ oscillation, and  $B_s \rightarrow \mu^+ \mu^-$  decay have already started to constrain possible contributions from new physics beyond the standard model.

<span id="page-0-0"></span>Here we focus our attention to one of the  $b \rightarrow s$  processes, the decay into neutrino pairs [\[1,](#page-6-0)[2](#page-6-1)],

$$
b \to s \nu \bar{\nu}.\tag{1}
$$

It is known that the decays of the *B* mesons induced by the partonic process [\(1](#page-0-0)), especially the inclusive branching ratio  $BR(\bar{B} \to X_s \nu \bar{\nu})$ , have small theoretical uncertainty due to the absence of photonic penguin and strong suppression of light quark contributions. On the other hand, experimental search of the decay ([1\)](#page-0-0) is a hard task. At present, only the upper bounds are known for both inclusive [\[3](#page-6-2)] and exclusive [[4\]](#page-6-3) branching ratios, at 90% C.L.,

$$
\sum_{\nu} \text{Br}(\bar{B} \to X_s \nu \bar{\nu}) < 6.4 \times 10^{-4},
$$
\n
$$
\sum_{\nu} \text{Br}(B^+ \to K^+ \nu \bar{\nu}) < 1.4 \times 10^{-5},
$$
\n
$$
\sum_{\nu} \text{Br}(\bar{B}^0 \to K_s^0 \nu \bar{\nu}) < 1.6 \times 10^{-4},
$$
\n
$$
\sum_{\nu} \text{Br}(\bar{B}^0 \to K^{*0} \nu \bar{\nu}) < 3.4 \times 10^{-4},
$$
\n
$$
\sum_{\nu} \text{Br}(B^+ \to K^{*+} \nu \bar{\nu}) < 1.4 \times 10^{-4},
$$
\n(2)

which are still 1 order of magnitude larger than the standard model predictions for the inclusive [[5](#page-7-0)] and exclusive [\[6\]](#page-7-1) modes,

<span id="page-0-1"></span>
$$
\sum_{\nu} \text{Br}(\bar{B} \to X_s \nu \bar{\nu})_{\text{SM}} = (3.7 \pm 0.2) \times 10^{-5},
$$
\n
$$
\sum_{\nu} \text{Br}(\bar{B} \to K \nu \bar{\nu})_{\text{SM}} = (3.8^{+1.2}_{-0.6}) \times 10^{-6},
$$
\n
$$
\sum_{\nu} \text{Br}(\bar{B} \to K^* \nu \bar{\nu})_{\text{SM}} = (1.3^{+0.4}_{-0.3}) \times 10^{-5}.
$$
\n(3)

Future upgrades of the *B* factories [[7](#page-7-2)] will extend the search region for the exclusive decays. For example,  $Br(B^+ \rightarrow K^+ \nu \bar{\nu})$  around the level of the standard model prediction [\(3](#page-0-1)) is expected to be measured at the precision of 20% with integrated luminosity 50–100  $ab^{-1}$ . On the other hand, a future  $e^+e^-$  collider running on the *Z*-boson resonance (GIGA-Z) has a potential [[8](#page-7-3)] to produce very large number of  $Z \rightarrow b\bar{b}$  events, and possibility to greatly improve previous studies of the inclusive modes [[3](#page-6-2)] at the LEP I, to measure the inclusive branching ratio.

In this paper, we consider the decay  $(1)$  $(1)$  in the framework of the minimal supersymmetric standard model (MSSM) [\[9\]](#page-7-4) with general flavor mixing of squarks, and study the contributions of new particles, namely, the supersymmetric (SUSY) particles and Higgs bosons. In cases where the value of  $tan \beta$ , the ratio of the vacuum expectation values of two Higgs boson doublets in the MSSM, is not much larger than unity, it is shown  $[10,11]$  $[10,11]$  $[10,11]$  $[10,11]$  $[10,11]$  that the chargino-squark loops give main part of the new physics contributions to the decay, and may become sizable when large flavor mixing is present in the left-right mixing part of the up-type squark mass matrix. Note that this is also the case for the SUSY contributions to the related decays  $K \to \pi \nu \bar{\nu}$  [\[12,](#page-7-7)[13\]](#page-7-8).

At large tan $\beta$ , say similar to or larger than  $m_t/m_b \sim 40$ , the MSSM loop contributions other than charginos might become also important, at least in principle. For example, gluino-squark loop contributions are generated by tan $\beta$ -enhanced large left-right mixing of down-type squarks. When, in addition, sizable mixing between down-type squarks in the second and third generations are present, gluino contribution might become sizable. It is also possible that, as explained later, charged Higgs bosons might give sizable loop contributions due to the

flavor-changing effective Higgs-quark couplings, generated by  $O(\tan \beta)$  SUSY loop corrections, as pointed out in Ref. [\[14\]](#page-7-9) for the  $K \to \pi \nu \bar{\nu}$  decays. However, parameters in the SUSY and Higgs sectors should receive stringent constraints from existing measurements of the FCNC processes, which might suppress possible magnitudes of their contributions to  $b \rightarrow s \nu \bar{\nu}$ . In this paper, we will present a rough estimate of the possible constraints from the decay  $b \rightarrow s\gamma$ , by showing correlations between the new physics contributions to the Wilson coefficients for  $b \rightarrow s \nu \bar{\nu}$  and those for  $b \rightarrow s\gamma$ , for each SUSY/Higgs sector separately. We will also comment on the implication of the  $B_s \rightarrow$  $\mu^+ \mu^-$  decay to the Higgs boson contributions.

The paper is organized as follows. In Sec. II, we present basic formulas for the analysis of the  $b \rightarrow s \nu \bar{\nu}$  decay in the MSSM. In Sec. III, numerical results for the new physics contributions in the MSSM to  $b \rightarrow s \nu \bar{\nu}$  are presented as correlations with those to  $b \rightarrow s\gamma$  for each new physics sector. An additional constraint from the  $B_s \to \mu^+ \mu^$ decay on the Higgs boson contributions is briefly commented on in Sec. IV. Finally, a conclusion is given in Sec. V.

## **II.**  $b \rightarrow s \nu \bar{\nu}$  **DECAY IN THE MSSM**

<span id="page-1-0"></span>The  $b \rightarrow s \nu \bar{\nu}$  decay is described by the effective Hamiltonian, in the notation of Ref. [[6\]](#page-7-1),

$$
H_{\rm eff} = -\frac{4G_F}{\sqrt{2}} K_{ts}^* K_{tb} [C_\nu \mathcal{O}_L + C_\nu' \mathcal{O}_R], \tag{4}
$$

where  $K_{ij}$  is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Here the relevant operators are

$$
O_L = \frac{\alpha}{2\pi} (\bar{s}_L \gamma^\mu b_L)(\bar{\nu}_L \gamma_\mu \nu_L), \tag{5}
$$

$$
O_R = \frac{\alpha}{2\pi} (\bar{s}_R \gamma^\mu b_R)(\bar{\nu}_L \gamma_\mu \nu_L).
$$
 (6)

<span id="page-1-4"></span>The inclusive branching ratio is then expressed in terms of the Wilson coefficients  $(C_v, C'_v)$  in Eq. [\(4\)](#page-1-0) as [\[11\]](#page-7-6)

$$
\sum_{\nu} \text{Br}(\bar{B} \to X_s \nu \bar{\nu}) \sim \frac{N_{\nu} \alpha^2}{4\pi^2} \text{Br}(\bar{B} \to X_c e \bar{\nu}_e) \frac{|K_{tb} K_{ts}^*|^2}{|K_{cb}|^2} \times (|C_{\nu}|^2 + |C_{\nu}'|^2), \tag{7}
$$

up to the QCD corrections and  $O(m_c^2/m_b^2)$  corrections to the semileptonic decays  $\bar{B} \to X_c e \bar{\nu}_e$ . Interference between  $C<sub>v</sub>$  and  $C'<sub>v</sub>$  appears in the branching ratios of the exclusive modes  $\bar{B} \rightarrow (K \nu \bar{\nu}, K^* \nu \bar{\nu}, \cdots)$  [\[2](#page-6-1),[6](#page-7-1)]. Note that, in the massless quark limit,  $(C_v, C'_v)$  are independent of the renormalization scale in QCD.

In the MSSM, the interaction ([4](#page-1-0)) is generated by the *Z*-boson penguin and box diagrams. The standard model particles only contribute to  $C_{\nu}$ , giving at the leading order in QCD [[1](#page-6-0)[,2](#page-6-1)[,6,](#page-7-1)[10,](#page-7-5)[15\]](#page-7-10),

$$
C_{\nu, \text{SM}} = -\frac{1}{\sin^2 \theta_W} \frac{x}{8(x-1)^2} [x^2 + x - 2 + 3(x-2) \log x],
$$
\n(8)

where  $x = m_t^2/m_W^2$ . Numerically,  $C_{\nu,SM}$  is about  $-6.8$  for  $m_t = 171 \text{ GeV}.$ 

New particles in the MSSM, namely, the SUSY particles and Higgs bosons, may contribute to both  $C_{\nu}$  and  $C_{\nu}$ ,

$$
C_{\nu} = C_{\nu, \text{SM}} + C_{\nu}(\text{new}), \qquad C_{\nu}' = C_{\nu}'(\text{new}),
$$
  
\n
$$
C_{\nu}^{(')}(\text{new}) = C_{\nu, \tilde{g}}^{(')} + C_{\nu, \tilde{\chi}^{\pm}}^{(')} + C_{\nu, \tilde{\chi}^{0}}^{(')} + C_{\nu, H^{\pm}}^{(')}.
$$
\n(9)

 $C_{\nu}^{(')}$  (new) consists of the contributions of the gluino  $\tilde{g}$ —down-type squark loops, chargino  $\tilde{\chi}^{\pm}$  -up-type squark loops, neutralino  $\tilde{\chi}^0$ —down-type squark loops, and charged Higgs boson  $H^{\pm}$ —top quark loops. Below we list the analytic forms of these one-loop contributions for each sector:

<span id="page-1-3"></span><span id="page-1-2"></span>
$$
C_{\nu,\tilde{g}} = -\frac{4g_s^2}{3e^2 K_{ts}^* K_{tb}} (\Gamma_{DL}^{\dagger})_{2i} (\Gamma_{DR})_{ik} (\Gamma_{DR}^{\dagger})_{kj}
$$

$$
\times (\Gamma_{DL})_{j3} C_{24} (\tilde{d}_i, \tilde{d}_j, \tilde{g}), \qquad (10)
$$

$$
C'_{\nu,\tilde{g}} = \frac{4g_s^2}{3e^2 K_{ts}^* K_{tb}} (\Gamma^{\dagger}_{DR})_{2i} (\Gamma^{\dagger}_{DL})_{ik} (\Gamma^{\dagger}_{DL})_{kj}
$$
  
×  $(\Gamma_{DR})_{j3} C_{24} (\tilde{d}_i, \tilde{d}_j, \tilde{g}),$  (11)

$$
C_{\nu,\tilde{\chi}^{\pm}} = \frac{a_{ik2}^{C*} a_{jl3}^C}{2e^2 K_{is}^* K_{tb}} \left[ -\delta_{kl} (\Gamma_{UL})_{i\gamma} (\Gamma_{UL}^{\dagger})_{\gamma j} C_{24} (\tilde{u}_i, \tilde{u}_j, \tilde{\chi}_k^{\pm}) \right. \\
\left. + \delta_{ij} V_{k1}^* V_{l1} \left\{ C_{24} (\tilde{u}_i, \tilde{\chi}_k^{\pm}, \tilde{\chi}_l^{\pm}) - \frac{1}{4} \right\} \\
\left. - \frac{1}{2} \delta_{ij} U_{k1} U_{l1}^* m_{\tilde{\chi}_k^{\pm}} m_{\tilde{\chi}_l^{\pm}} C_0 (\tilde{u}_i, \tilde{\chi}_k^{\pm}, \tilde{\chi}_l^{\pm}) \right] \\
\left. + \frac{a_{ik2}^{C*} a_{il3}^{C*} m_W^2}{2e^2 K_{ts}^* K_{tb}} U_{k1} U_{l1}^* m_{\tilde{\chi}_k^{\pm}} m_{\tilde{\chi}_l^{\pm}} D_0 (\tilde{u}_i, \tilde{\chi}_k^{\pm}, \tilde{\chi}_l^{\pm}, \tilde{l}^-) \right), \tag{12}
$$

<span id="page-1-1"></span>
$$
C'_{\nu,\tilde{\chi}^{\pm}} = \frac{b_{ik2}^{C*}b_{jl3}^{C}}{2e^{2}K_{ts}^{*}K_{tb}} \bigg[ \delta_{kl}(\Gamma_{UR})_{i\gamma}(\Gamma_{UR}^{\dagger})_{\gamma j} C_{24}(\tilde{u}_{i}, \tilde{u}_{j}, \tilde{\chi}_{k}^{\pm}) + \delta_{ij}U_{k1}U_{l1}^{*} \bigg\{ C_{24}(\tilde{u}_{i}, \tilde{\chi}_{k}^{\pm}, \tilde{\chi}_{l}^{\pm}) - \frac{1}{4} \bigg\} - \frac{1}{2} \delta_{ij}V_{k1}^{*}V_{l1}m_{\tilde{\chi}_{k}^{\pm}}m_{\tilde{\chi}_{l}^{\pm}} C_{0}(\tilde{u}_{i}, \tilde{\chi}_{k}^{\pm}, \tilde{\chi}_{l}^{\pm}) \bigg] - \frac{b_{ik2}^{C*}b_{il3}^{C}m_{W}^{2}}{e^{2}K_{ts}^{*}K_{tb}} U_{k1}U_{l1}^{*}D_{27}(\tilde{u}_{i}, \tilde{\chi}_{k}^{\pm}, \tilde{\chi}_{l}^{\pm}, \tilde{l}^{-}), \qquad (13)
$$

<span id="page-2-2"></span>
$$
C_{\nu,\tilde{\chi}^{0}} = \frac{a_{ik2}^{N*} a_{j13}^{N}}{2e^{2} K_{ts}^{*} K_{tb}} \left[ -\delta_{kl} (\Gamma_{DR})_{i\gamma} (\Gamma_{DR}^{\dagger})_{\gamma j} C_{24} (\tilde{d}_{i}, \tilde{d}_{j}, \tilde{\chi}_{k}^{0}) \right. \\
\left. + \delta_{ij} (N_{k3}^{*} N_{l3} - N_{k4}^{*} N_{l4}) \left\{ C_{24} (\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}) - \frac{1}{4} \right\} \\
+ \frac{1}{2} \delta_{ij} (N_{k3} N_{l3}^{*} - N_{k4} N_{l4}^{*}) m_{\tilde{\chi}_{k}^{0}} m_{\tilde{\chi}_{l}^{0}} C_{0} (\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}) \right] \\
+ \frac{a_{ik2}^{N*} a_{il3}^{N} m_{W}^{2}}{2e^{2} K_{ts}^{*} K_{tb}} \left[ \tilde{N}_{k}^{*} \tilde{N}_{l} D_{27} (\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}, \tilde{\nu}) \right. \\
\left. + \frac{1}{2} \tilde{N}_{k} \tilde{N}_{l}^{*} m_{\tilde{\chi}_{k}^{0}} m_{\tilde{\chi}_{l}^{0}} D_{0} (\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}, \tilde{\nu}) \right], \tag{14}
$$

<span id="page-2-6"></span>
$$
C'_{\nu,\tilde{\chi}^{0}} = \frac{b_{ik2}^{N*}b_{j13}^{N}}{2e^{2}K_{ts}^{*}K_{tb}} \bigg[ \delta_{kl}(\Gamma_{DL})_{i\gamma}(\Gamma_{DL}^{\dagger})_{\gamma j} C_{24}(\tilde{d}_{i}, \tilde{d}_{j}, \tilde{\chi}_{k}^{0}) - \delta_{ij}(N_{k3}N_{l3}^{*} - N_{k4}N_{l4}^{*}) \bigg\{ C_{24}(\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}) - \frac{1}{4} \bigg\} - \frac{1}{2} \delta_{ij}(N_{k3}^{*}N_{l3} - N_{k4}^{*}N_{l4}) m_{\tilde{\chi}_{k}^{0}} m_{\tilde{\chi}_{l}^{0}} C_{0}(\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}) - \frac{b_{ik2}^{N*}b_{i13}^{N}m_{W}^{2}}{2e^{2}K_{ts}^{*}K_{tb}} \bigg[ \tilde{N}_{k} \tilde{N}_{l}^{*} D_{27}(\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}, \tilde{\nu}) + \frac{1}{2} \tilde{N}_{k}^{*} \tilde{N}_{l} m_{\tilde{\chi}_{k}^{0}} m_{\tilde{\chi}_{l}^{0}} D_{0}(\tilde{d}_{i}, \tilde{\chi}_{k}^{0}, \tilde{\chi}_{l}^{0}, \tilde{\nu}) \bigg], \qquad (15)
$$

<span id="page-2-1"></span>
$$
C_{\nu,H^{\pm}} = \frac{h_t^2 \cos^2 \beta}{4e^2} \frac{x_{tH}}{(x_{tH} - 1)^2} (1 - x_{tH} + \log x_{tH}), \quad (16)
$$

$$
C'_{\nu,H^{\pm}} = -\frac{(\hat{Y}_d)_{2\alpha} K_{t\alpha}^* K_{t\beta} (\hat{Y}_d)_{3\beta}^* \sin^2 \beta}{4e^2 K_{ts}^* K_{tb}} \frac{x_{tH}}{(x_{tH} - 1)^2} \times (1 - x_{tH} + \log x_{tH}),
$$
\n(17)

<span id="page-2-0"></span>where  $x_{tH} = m_t^2/m_{H^{\pm}}^2$ ,  $h_t = g_2 m_t/(\sqrt{2}m_W \sin\beta)$ . We assume flavor degeneracy in the lepton and slepton sectors. The formulas  $(10)$ – $(17)$  $(17)$  $(17)$  are derived from previous studies of the  $b \rightarrow s \nu \bar{\nu}$  decays in the MSSM [\[10](#page-7-5)[,11\]](#page-7-6), as well as related works on the  $K \rightarrow \pi \nu \bar{\nu}$  decays [[12](#page-7-7)[–14\]](#page-7-9).  $C_{0,24}(a, b, c) \equiv C_{0,24}(m_a^2, m_b^2, m_c^2)$  and  $D_{0,27}(a, b, c, d) \equiv$  $D_{0,27}(m_a^2, m_b^2, m_c^2, m_d^2)$  are the three-point functions for the *Z*-penguin diagrams and four-point functions for the box diagrams, respectively [[16\]](#page-7-11), in the convention of Ref.  $[17]$ . Ultraviolet divergence of  $C_{24}$  cancels out in the formulas  $(10)$ – $(15)$  $(15)$  $(15)$ . We ignore the masses of  $(u, d, c)$ quarks, and include those of  $(s, b)$  only when they are multiplied by  $tan \beta$ . In this approximation, the neutral Higgs boson contributions to  $C_{\nu}^{(\prime)}$  vanish.

The couplings and mixing matrices in Eqs.  $(10)$  $(10)$  $(10)$ – $(17)$  are given as follows: The squark mixing matrices  $(\Gamma_{QL}, \Gamma_{QR})$  $(Q = U, D)$  give relations between the mass eigenstates  $\tilde{q}_i = (\tilde{u}_i, \tilde{d}_i)$  (*i* = 1–6) to the gauge eigenstates in the "super-CKM" basis  $(\tilde{q}_{L\alpha}, \tilde{q}_{R\alpha})$  ( $\alpha = 1-3$ ), which are related to the mass eigenbasis of the quarks  $q_{\alpha} = [u_{\alpha}]$  $(u, c, t), d_{\alpha} = (d, s, b)$  by SUSY transformation, as

<span id="page-2-4"></span>
$$
\tilde{q}_{L\alpha} = (\Gamma^{\dagger}_{QL})_{\alpha j} \tilde{q}_{j}, \qquad \tilde{q}_{R\alpha} = (\Gamma^{\dagger}_{QR})_{\alpha j} \tilde{q}_{j}.
$$
 (18)

These matrices are determined to diagonalize the  $6 \times 6$ mass matrices of squarks in the super-CKM basis,

<span id="page-2-3"></span>
$$
M_{\tilde{q}}^{2} = \begin{pmatrix} M_{\tilde{Q}LL}^{2} & (M_{\tilde{Q}RL}^{2})^{\dagger} \\ M_{\tilde{Q}RL}^{2} & M_{\tilde{Q}RR}^{2} \end{pmatrix},
$$
\n
$$
(M_{\tilde{Q}LL}^{2})_{\alpha\beta} = (m_{\tilde{Q}LL}^{2})_{\alpha\beta} + (m_{Q}^{(0)})^{\dagger} (m_{Q}^{(0)})
$$
\n
$$
+ \delta_{\alpha\beta} (I_{3q_{L}} - e_{q}\sin^{2}\theta_{W}) m_{Z}^{2} \cos 2\beta,
$$
\n
$$
(M_{\tilde{Q}RR}^{2})_{\alpha\beta} = (m_{\tilde{Q}RR}^{2})_{\alpha\beta} + (m_{Q}^{(0)}) (m_{Q}^{(0)})^{\dagger}
$$
\n
$$
+ \delta_{\alpha\beta} e_{q}\sin^{2}\theta_{W} m_{Z}^{2} \cos 2\beta,
$$
\n
$$
(M_{\tilde{U}RL}^{2})_{\alpha\beta} = (m_{\tilde{U}RL}^{2})_{\alpha\beta} - m_{U}^{(0)} \mu^{*} \cot \beta,
$$
\n
$$
(M_{\tilde{D}RL}^{2})_{\alpha\beta} = (m_{\tilde{D}RL}^{2})_{\alpha\beta} - m_{D}^{(0)} \mu^{*} \tan \beta.
$$
\n(19)

In Eq. [\(19\)](#page-2-3), off-diagonal elements of the soft SUSY breaking mass matrices  $(m_{\tilde{Q}LL,RR,RL}^2)$  induce flavor mixings which are not constrained by the CKM matrix in general, and may cause potentially large FCNC.  $(m_Q^{(0)})_{\alpha\beta}$  are the "bare" mass matrices of the quarks. For the up-type squarks, it is just the running mass matrix  $(m_U^{(0)})_{\alpha\beta} =$  $(m_U)_{\alpha\beta} = \text{diag}(m_u, m_c, m_t) \sim \text{diag}(0, 0, m_t)$  in the standard model. For the down-type squarks, in contrast,  $(m_D^{(0)})_{\alpha\beta}$  may substantially deviate from the standard model mass matrix  $(m_D)_{\alpha\beta} = \text{diag}(m_d, m_s, m_b)$ , as explained later. The quark-squark-chargino and quarksquark-neutralino couplings  $(a_{ik\alpha}^C, b_{ik\alpha}^C, a_{ik\alpha}^N, b_{ik\alpha}^N)$  are then given in terms of the mixing matrices for squarks [\(18\)](#page-2-4), for charginos  $(V, U)$ , and for neutralinos  $N$  [[18](#page-7-13)], as

<span id="page-2-5"></span>
$$
a_{ik\alpha}^{C} = g_2(\Gamma_{UL})_{i\beta} V_{k1}^* K_{\beta\alpha} - h_t(\Gamma_{UR})_{i3} V_{k2}^* K_{i\alpha},
$$
  
\n
$$
b_{ik\alpha}^{C} = -(\Gamma_{UL})_{i\beta} U_{k2} K_{\beta\gamma} (\hat{Y}_d)_{\alpha\gamma}^*,
$$
  
\n
$$
a_{ik\alpha}^{N} = \sqrt{2} \left( -\frac{g_2}{2} N_{k2}^* + \frac{g_Y}{6} N_{k1}^* \right) (\Gamma_{DL})_{i\alpha} + (\hat{Y}_d)_{\beta\alpha} N_{k3}^*(\Gamma_{DR})_{i\beta},
$$
  
\n
$$
b_{ik\alpha}^{N} = \frac{\sqrt{2}g_Y}{3} N_{k1}(\Gamma_{DR})_{i\alpha} + (\hat{Y}_d)_{\alpha\beta}^* N_{k3}(\Gamma_{DL})_{i\beta},
$$
\n(20)

Finally,  $\tilde{N}_k = N_{k2} - \tan^2 \theta_W N_{k1}$  in Eqs. [\(14\)](#page-1-1) and [\(15\)](#page-2-2) denote the neutrino-sneutrino-neutralino couplings.

We need some explanation for  $(\hat{Y}_d)_{\alpha\beta}$ , the bare Yukawa coupling matrix for down-type quarks. We start from the effective Lagrangian for the couplings of  $d_{iR}$  to the Higgs boson doublets  $(H_D, H_U)$  in the MSSM, after integrating out the SUSY particles,

$$
\mathcal{L}_{\text{eff}} = -(\hat{Y}_d)_{ij} \bar{d}_{ik} (d_{jL} H_D^0 - K_{kj}^* u_{kL} H_D^-) \n- (\Delta Y_d)_{ij} \bar{d}_{ik} (d_{jL} H_U^{0*} + K_{kj}^* u_{kL} H_U^-) + (\text{H.c}).
$$
\n(21)

The couplings  $(\Delta Y_d)_{ij}$  are forbidden at the tree-level by supersymmetry, but induced by SUSY particle loops with soft SUSY breaking. The running mass matrix in the standard model  $(m_D)_{\alpha\beta} = \text{diag}(m_d, m_s, m_b)$  is then given by

<span id="page-3-0"></span>
$$
(m_D)_{\alpha\beta} = \frac{\sqrt{2}m_W}{g_2} \cos\beta [\hat{Y}_d + \tan\beta \Delta Y_d]_{\alpha\beta},
$$
  

$$
\equiv [m_D^{(0)} + \delta m_D]_{\alpha\beta}.
$$
 (22)

Although the loop-generated  $\Delta Y_d$  is suppressed relative to the tree-level coupling  $\hat{Y}_d$ , its contribution to  $m_D$ ,  $\delta m_D$ , is enhanced by  $tan \beta$ , as seen in Eq. ([22](#page-3-0)), and may become numerically comparable to the tree-level part  $m_D^{(0)} \propto \hat{Y}_D$  at large  $tan \beta$  [\[19](#page-7-14)]. On the other hand, the couplings of  $(d_{iR}, \tilde{d}_{iR})$  to heavier Higgs bosons  $(H^0, A^0, H^{\pm})$  and Higgsinos  $H_D$  are determined by  $\hat{Y}_d$ , as shown in Eqs.  $(17)$  $(17)$  $(17)$  and  $(20)$ , without tan $\beta$ -enhanced contributions from  $\Delta Y_d$ . As a consequence, at large tan $\beta$ , these couplings may significantly deviate from the tree-level values [\[20\]](#page-7-15) given in terms of  $(m_D)_{\alpha\beta}$  and, since  $\Delta Y_d$  is not flavor diagonal in general, include flavor-mixing parts not determined by the CKM matrix, even in the super-CKM basis. The bare quark mass matrix  $m_D^{(0)}$  should be also used in the mass matrix ([19](#page-2-3)) of the down-type squarks, which also receives no contributions from  $\Delta Y_d$ . The correction [\(22\)](#page-3-0) therefore affects the masses and mixing matrices  $(\Gamma_{DL}, \Gamma_{DR})$  of the down-type squarks, generating additional flavor mixing for squarks. These  $tan \beta$ -enhanced corrections to the down-type quarks and squarks are often comparable to the tree-level contributions in the MSSM at large tan $\beta$ , and should be included in a realistic analysis of processes involving these particles [[20,](#page-7-15)[21](#page-7-16)].

Now we turn to the behavior of the SUSY and Higgs contributions [\(10](#page-2-0))–([17](#page-2-1)) to  $(C_v, C'_v)$ . The main part of these contributions comes from the *Z* penguin diagrams through effective  $Z_{\mu} \bar{s}_L \gamma^{\mu} b_L$  and  $Z_{\mu} \bar{s}_R \gamma^{\mu} b_R$  vertices. Appearance of these vertices needs both the mixing between the second and third generations of quarks/squarks, and the  $SU(2)$   $\times$  $U(1)$  gauge symmetry breaking in the loops. For small or moderate values of  $tan \beta$ , the largest SU(2) breaking in the loops are provided by the top quark and squarks. As a consequence,  $C_{\nu,H^{\pm}}$  ([16\)](#page-2-6) and  $C_{\nu,\tilde{\gamma}^{\pm}}$  [\(12\)](#page-1-2) are relevant. The former, however, is suppressed by  $1/\tan^2\beta$  and only relevant for tan $\beta \sim 1$ , which is disfavored by an experimental lower limit on the mass of the lightest Higgs boson. Therefore, only the latter,  $C_{\nu, \tilde{\chi}^{\pm}}$ , is left as a potentially important SUSY contribution to  $b \rightarrow s \nu \bar{\nu}$ . Previous studies have shown [[5](#page-7-0)[,6](#page-7-1)[,11\]](#page-7-6) that  $C_{\nu, \tilde{\chi}^{\pm}}$  is enhanced by large  $M_{\tilde{U}RL}^2$ , especially by its flavor-mixing parts. Similar behavior is observed for the chargino contributions to the  $K \to \pi \nu \bar{\nu}$ decays [[13](#page-7-8)].

At large tan $\beta$ , however, other contributions to  $b \rightarrow s \nu \bar{\nu}$ have the possibility to become sizable by the following reasons: First, the SU(2)-breaking left-right mixing of the down-type squarks  $(M_{\tilde{D}RL}^2)$  increases as tan $\beta$  and may enhance the gluino contribution. Second, off-diagonal parts of the effective Yukawa coupling  $\hat{Y}_d$  in Eq. [\(22\)](#page-3-0) induce the flavor-changing couplings of the down-type quarks, which are enhanced by  $tan \beta$  and not necessarily suppressed by the corresponding CKM matrix elements or quark masses. Especially, the element  $(\hat{Y}_d)_{23}$ , induced by flavor mixing in  $M_{\tilde{D}RR}^2$ , might give large Yukawa couplings of  $s_R$  and enhance  $C'_{\nu,H^{\pm}}$ . This is similar to the case of  $K \rightarrow$  $\pi \nu \bar{\nu}$  at large tan $\beta$  [[14](#page-7-9)], where loop-induced couplings  $((\hat{Y}_d)_{13}, (\hat{Y}_d)_{23})$  give large effective  $\bar{s}_R d_R Z$  coupling. Therefore, the gluino  $(10)$  and  $(11)$  $(11)$  and charged Higgs boson ([17\)](#page-2-1) contributions must be considered in the analysis of  $b \rightarrow s \nu \bar{\nu}$  at large tan $\beta$ .

# **III. SUSY AND HIGGS CONTRIBUTIONS TO**  $b \rightarrow s \nu \bar{\nu}$  AND CORRELATION WITH  $b \rightarrow s \gamma$

We present numerical results for the new physics con-tributions ([10](#page-2-0))–[\(17\)](#page-2-1) to the  $b \rightarrow s \nu \bar{\nu}$  decay in the MSSM. We concentrate on the cases with large  $tan \beta$ , which were not considered in previous studies.

In the estimation of possible magnitudes of the new physics contributions [\(10\)](#page-2-0)–([17](#page-2-1)) to  $b \rightarrow s \nu \bar{\nu}$ , we need to take into account the constraints on SUSY and Higgs parameters from other FCNC processes. In this section, we consider the implication of the constraints from the radiative decay  $b \rightarrow s\gamma$ . This constraint is expected to be crucial since the  $SU(2) \times U(1)$  breaking and flavor mixing between quarks/squarks in the second and third generations, which are necessary to enhance the contributions to  $b \rightarrow s \nu \bar{\nu}$ , may also give large contributions to  $b \rightarrow s \gamma$ . Another reason to focus on  $b \rightarrow s\gamma$  is the rather good agreement between experimental data [[22](#page-7-17)] and the standard model prediction [\[23\]](#page-7-18) of the inclusive branching ratio  $Br(\bar{B} \to X_s \gamma)$ . Indeed, the decay  $b \to s \gamma$  in the MSSM has been shown  $[10,21,24-29]$  $[10,21,24-29]$  $[10,21,24-29]$  $[10,21,24-29]$  $[10,21,24-29]$  $[10,21,24-29]$  $[10,21,24-29]$  to give strong constraints on the Higgs and SUSY parameters. It should also be noted that the SUSY contributions to  $b \rightarrow s\gamma$  are enhanced by  $tan \beta$  [[25](#page-7-21),[26](#page-7-22)].

Here we do not attempt precise calculation of the experimental constraints from  $b \rightarrow s\gamma$ . Instead, we present a very rough estimation of the expected constraints in terms of the Wilson coefficients  $(C_7, C_7')(\mu)$  for  $b \to s\gamma$ , defined as

$$
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} K_{ts}^* K_{tb} (C_7(\mu) \mathcal{O}_7(\mu) + C_7'(\mu) \mathcal{O}_7'(\mu)),
$$
  
\n
$$
\mathcal{O}_7 = \frac{e}{16\pi^2} m_b(\mu) (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu},
$$
  
\n
$$
\mathcal{O}_7' = \frac{e}{16\pi^2} m_b(\mu) (\bar{s}_R \sigma^{\mu\nu} b_L) F_{\mu\nu}.
$$
\n(23)

Below we show the correlations between  $C_v^{(')}$  (new), Eqs. ([10](#page-2-0))–[\(17\)](#page-2-1), and new physics contributions to  $C_7^{(l)}$ ,  $C_7^{(l)}$ 7

(new), for each sector of new physics, namely, the gluinosquark, chargino-squark, and charged Higgs boson-top quark loop contributions, varying squarks mixing parameters which are relevant to  $b \rightarrow s \nu \bar{\nu}$ . For simplicity, we assume the flavor structures of the soft SUSY breaking terms in the squark mass matrices [\(19\)](#page-2-3) as

<span id="page-4-1"></span>
$$
m_{\tilde{Q}XX}^2 = M_{\tilde{Q}}^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & (\delta_{XX}^q)_{23} \\ 0 & (\delta_{XX}^q)_{23} & 1 \end{pmatrix}
$$
  
(XX = LL, RR), (24)

$$
m_{\tilde{U}RL}^2 = m_t \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & (A_u)_{32} & (A_u)_{33} \end{pmatrix},
$$
 (25)

<span id="page-4-0"></span>Since *CP* violation is not essential for the analysis in this paper, all SUSY and Higgs parameters, including those in Eqs. [\(24\)](#page-4-0) and ([25](#page-4-1)) are set to be real. We also set  $m_{\tilde{D}RL}^2 = 0$ in Eq. ([19](#page-2-3)) since its contribution to  $M_{\tilde{D}RL}^2$  is, when the vacuum stability bounds [\[30\]](#page-7-23) is applied,  $O(m_bM_{\tilde{O}})$  and subdominant compared to the second term  $m_D^{(0)} \mu^*$  tan $\beta =$  $O(m_b \tan \beta M_{\tilde{Q}})$ . Note that the condition ([24](#page-4-0)) for  $m_{\tilde{Q}LL}^2$  may be imposed only either  $\tilde{Q} = \tilde{U}$  or  $\tilde{Q} = \tilde{D}$ , due to the SU(2) symmetry  $(m_{\tilde{U}LL}^2)_{\alpha\beta} = K_{\alpha\gamma}(m_{\tilde{D}LL}^2)_{\gamma\delta} K_{\beta\delta}^*$ .

We calculate the new physics contributions to  $C_{\nu}^{(')}$  and  $C_7^{(')}$  at the leading one-loop order (see Refs. [\[10,](#page-7-5)[25](#page-7-21)–[27](#page-7-24)] for the formulas of  $C_7^{(')}$ ), but improved by including the tan $\beta$ -enhanced corrections to the quark/squark Yukawa couplings from Eq. [\(22\)](#page-3-0) and, for  $C_7^{(')}$ , also from the proper vertex corrections<sup>1</sup> to the  $u_{iR}$  couplings to  $(H^0, A^0, H^{\pm})$ [\[28](#page-7-25)[,29\]](#page-7-20), in the effective Lagrangian formalism [[28](#page-7-25)]. In these formulas, we use the running quark masses and  $\alpha_s$ at the renormalization scale  $\mu = M_{\tilde{Q}}$ , calculated from  $m_t$ (pole) = 171 GeV,  $m_b(m_b) = 4.2$  GeV,  $m_s$ (2 GeV) = 95 MeV,  $m_q$ (others) = 0 and  $\alpha_s$ ( $m_Z$ ) = 0.12, which give  $C_{\nu}^{(')}(\mu)$  and  $C_{7}^{(')}(\mu)$  at the renormalization scale  $\mu = M_{\tilde{Q}}$ . For SUSY and Higgs parameters, we fix the following parameters:  $\tan\beta = 50$ ,  $M_{\tilde{Q}} = 500 \text{ GeV}, \quad m_{\tilde{g}} =$ 500 GeV,  $M_2 = 300$  GeV,  $M_1 = 150$  GeV, while varying other parameters. We also impose the bounds  $m_{\tilde{Y}^{\pm}}$ 100 GeV and  $m_{\tilde{q}} > 250$  GeV, suggested by experimental search limits for SUSY particles.

For each sector of the new physics, rough estimates of the bounds on the contributions to  $(C_v, C'_v)$  are obtained by requiring that the magnitudes of  $(C_7, C'_7)$  (new) should be smaller than the standard model contribution  $C_{7,SM}(\mu \sim$  $m_W$ ) ~ -0.2.

### **A. Gluino contributions**

The gluino-squark contributions  $C_{\nu, \tilde{g}}^{(')}$  are induced by the flavor and left-right mixing of the down-type squarks. In Fig. [1,](#page-4-2) the gluino contribution  $C_{\nu, \tilde{g}}$  is shown as a correlation with  $C_{7,\tilde{g}}$ , for parameter scan over  $(\delta_{LL}^d)_{23} =$  $[-0.3, 0.3], \quad (\delta_R^d$  $_{RR}^{d}$ <sub>23</sub> = [-0.3, 0.3], and  $\mu$  =  $[-550, 550]$  GeV.  $(A_u)_{33}$  and  $(A_u)_{32}$  are set to 0. Correlation between  $C'_{\nu, \tilde{g}}$  and  $C'_{7, sg}$  for the same parameters is obtained from Fig. [1](#page-4-2) by changing the sign of the horizontal axis. Large  $|C_{\nu,\tilde{g}}|$  is obtained for large negative  $\mu$ and large  $(\delta_{LL,RR})_{23}$ , which cause large  $\tilde{b}_R - \tilde{s}_L$  mixing. It is seen that  $|C_{\nu,\tilde{\ell}}|$  can be larger than 1, which gives about 30% correction to the standard model prediction of the decay width [\(7\)](#page-1-4). However, by requiring  $|C_{7,\tilde{g}}|$  <  $|C_{7,SM}(\mu_W)| \sim 0.2$ , magnitudes of  $C_{\nu, \tilde{g}}$  are constrained to be much smaller than  $C_{\nu,SM} \sim -6.8$ . Therefore, without very precise cancellation between new physics contributions to  $b \rightarrow s\gamma$ , gluino contributions to  $b \rightarrow s\nu\bar{\nu}$  should be completely negligible, even for  $tan \beta \gg 1$ , to satisfy the bound from  $b \rightarrow s\gamma$ .

Here we briefly comment on the neutralino contributions  $C_{\nu,\tilde{\chi}^0}^{(')}$ , Eqs. [\(14\)](#page-1-1) and [\(15\)](#page-2-2). Similar to the gluino contributions,  $C_{\nu,\tilde{\chi}^0}^{(')}$  are induced by the  $\tilde{b} - \tilde{s}$  mixing in the loops. However, due to small couplings, these contributions are much smaller than the gluino contributions  $C_{\nu, \tilde{g}}^{(l)}$  for most parameter regions and therefore are not discussed here.

### **B. Chargino contributions**

The chargino-squark loop contributions  $C_{\nu, \tilde{\chi}^{\pm}}$ , Eq. ([12\)](#page-1-2), have been studied in previous works [\[6,](#page-7-1)[11\]](#page-7-6) at small or moderate value of  $tan \beta$ . In these works, it has been shown that they might give sizable contributions, larger than the

<span id="page-4-2"></span>

FIG. 1. Correlation between  $C_{\nu,c\tilde{g}}$  and  $C_{\tilde{g}}$ . Parameters are  $\tan\beta = 50$ ,  $\mu = [-550, 550] \text{ GeV}$ ,  $(\delta_{LL,RR}^d)_{23} = [-0.3, 0.3]$ . Other parameters are given in the text. Horizontal lines indicate the region  $|C_{7,\tilde{g}}|$  < 0.2.

<sup>&</sup>lt;sup>1</sup>These vertex corrections also appear in  $C_{\nu,H^{\pm}}$ . However, we ignored the corrections in Eq. [\(16\)](#page-2-6), since  $C_{\nu,H^{\pm}}$  itself is strongly suppressed by  $1/\tan^2\beta$  and numerically negligible.

uncertainty of the standard model predictions ([3\)](#page-0-1), for large flavor-mixing element of  $m_{\tilde{U}RL}^2$  in Eq. ([25](#page-4-1)), especially its  $({\tilde{t}}_R, {\tilde{c}}_L)$ -mixing element  $(m_{\tilde{U}RL}^2)_{32} \sim (A_u)_{32} m_t$ .

Figure [2](#page-5-0) shows the correlation between  $C_{\nu,\tilde{\chi}^{\pm}}$  and  $C_{7,\tilde{\chi}^{\pm}}^{(l)}$ , for varying parameters over  $(A_u)_{33} = [-1500, 1500]$  GeV,  $(A_u)_{32} = [-1500, 1500] \text{ GeV}, \text{ and } (\delta_{LL}^u)_{23} = [-0.3, 0.3].$ Other parameters are fixed at  $\mu = 500$  GeV,  $m_{\tilde{l}_L}^{\perp} =$ 400 GeV, and  $(\delta_{RR}^u)_{23} = 0$ . For these parameters,  $C_{\nu,\tilde{\chi}^{\pm}}^{\tilde{\ell}}$ is negligibly small  $( $0.02$ ) and not shown here. As is the$ case of the gluino contributions, SUSY parameters which give large  $C_{\nu,\tilde{\chi}^{\pm}}$  tend to also give large  $C^{(')}_{7,\tilde{\chi}^{\pm}}$ . The resulting constraint on  $C_{\nu,\tilde{\chi}^{\pm}}$  gets tighter as tan $\beta$  increases, since  $C_{7,\tilde{\chi}^{\pm}}^{(')}$  are enhanced by tan $\beta$  while  $C_{\nu,\tilde{\chi}^{\pm}}$  is not. Nevertheless, the correlation is not so strong as in the gluino sector, as seen in Fig. [2.](#page-5-0) This is due to the different dependences of  $C_{\nu,\tilde{\chi}^{\pm}}$  and  $C_{7,\tilde{\chi}^{\pm}}^{(')}$  on 2 A-term elements,  $(A_u)_{33}$  and  $(A_u)_{32}$  $(A_u)_{32}$  $(A_u)_{32}$  in Eq. ([25](#page-4-1)). In fact, as seen in Fig. 2, we may have  $|C_{\nu,\tilde{\chi}^{\pm}}| > 1$  while keeping  $|C_{7,\tilde{\chi}^{\pm}}^{(\prime)}| < 0.2$ . An even larger value of  $C_{\nu, \tilde{Y}^{\pm}}$  might be possible by careful choice of the SUSY parameters. The resulting deviations of the decay widths from the standard model predictions [\(3\)](#page-0-1) could be proved at future *B* factories, if the theoretical uncertainties of the exclusive widths in Eq.  $(3)$  $(3)$  $(3)$ , mainly coming from the meson form factors, are reduced. However, one must note that the large chargino contribution is realized by the fine tuning between SUSY parameters, especially  $(A_u)_{33}$  and  $(A_u)_{32}$ , to realize small  $C^{(')}_{7,\tilde{\chi}^{\pm}}$ .

## **C. Charged Higgs boson contributions**

As discussed in the previous section, only  $C'_{\nu,H^{\pm}}$ , Eq.  $(17)$ , is relevant at large tan $\beta$ . This contribution comes

<span id="page-5-0"></span>

FIG. 2. Correlation between  $C_{\nu, \tilde{\chi}^{\pm}}$  and  $C_{7,\tilde{\chi}^{\pm}}$  for parameters  $(A_u)_{33} = [-1500, 1500] \text{ GeV}, \quad (\hat{A}_u)_{32} = [-1500, 1500] \text{ GeV},$ and  $(\delta_{LL}^u)_{23} = [-0.3, 0.3]$ . The points with  $|C'_{7,\tilde{\chi}^{\pm}}|$  smaller (larger) than  $|C_{7,SM}| \sim 0.2$  are denoted by dots (crosses). Other parameters are set as in the text.

<span id="page-5-1"></span>

FIG. 3. Correlation between  $C_{\nu,H^{\pm}}$  and  $C_{7,H^{\pm}}^{(')}$  at tan $\beta = 50$ ,  $(\delta^d_{LL,RR})_{23} = [-0.3, 0.3]$ , and  $m_{H^{\pm}} = [400, 100]$  GeV. Other parameters are given in the text.

from the  $H^{-} \bar{s}_R t_L$  coupling  $\sim (\hat{Y}_d)_{2\alpha} K_{t\alpha}^* \sim (\hat{Y}_d)_{23}$ , which is generated by the flavor mixing involving  $\tilde{s}_R$  through the  $tan \beta$ -enhanced loop corrections [\(22\)](#page-3-0). In Fig. [3](#page-5-1), we show the correlations between  $C'_{\nu,H^{\pm}}$  and  $C^{(')}_{7,H^{\pm}}$  at  $\mu =$  $-500 \text{ GeV}, (A_u)_{33} = 0 \text{ GeV}, (A_u)_{32} = 0 \text{ GeV}, (\delta_{LL}^d)_{23} =$  $[-0.3, 0.3], \quad (\delta_{RR}^d)_{23} = [-0.3, 0.3], \quad \text{and} \quad m_{H^{\pm}} =$ [400, 1000] GeV. In contrast to the gluino and chargino contributions to  $b \to s\gamma$ , the main parts of  $C_{7,H^{\pm}}^{(')}$  are not enhanced by  $tan \beta$ . Moreover, the correlation between  $C'_{\nu,H^{\pm}}$  and  $C_{7,H^{\pm}}$  is severely affected by different parameter dependences of two generation-mixing  $H^{\pm}$  couplings: the effective  $\bar{s}_R t_L H^-$  coupling  $\sim (\hat{Y}_d)_{23}$  in  $C'_{\nu,H^{\pm}}$  and  $C'_{7,H^{\pm}}$ , and *O*(tan $\beta$ ) proper vertex corrections to the  $\bar{s}_L t_R H^$ coupling  $[28,29]$  $[28,29]$  in  $C_{7,H^{\pm}}$ . As a result, similar to the case of chargino contributions, there is the possibility to have sizable  $C'_{\nu,H^{\pm}}$  while keeping  $C^{(')}_{7,H^{\pm}}$  small.

# IV. CONSTRAINT FROM  $B_s \to \mu^+ \mu^-$  ON THE  $H^{\pm}$ **CONTRIBUTION**

In addition to  $b \rightarrow s\gamma$ , several other  $b \rightarrow s$  FCNC processes have been measured in recent experiments. Since most of these measurements show rather good consistency with the standard model predictions, they should give additional constraints on the SUSY and Higgs parameters, and their contributions to  $b \rightarrow s \nu \bar{\nu}$ . For example, measurements of the  $B_s - \bar{B}_s$  oscillation [[31](#page-7-26)] impose constraints on the  $\tilde{b} - \tilde{s}$  mixing, especially on  $(\delta_{LL}^d)_{23}$  and  $(\delta_{RR}^d)_{23}$  [[32\]](#page-7-27). Here we just show a case of these constraints: implication of the upper limit of the branching ratio for  $B_s \to \mu^+ \mu^$ on the  $H^{\pm}$  contributions  $C'_{\nu,H^{\pm}}$ , at large tan $\beta$ .

As seen in Eq. ([17](#page-2-1)), large value of  $C'_{\nu,H^{\pm}}$  is obtained for parameters which give large effective  $H^{-} \bar{s}_R t_L$  Yukawa coupling  $\sim (\hat{Y}_d)_{23}$ . As discussed in Sec. II, the parameter

 $(\hat{Y}_d)_{23}$  also gives the flavor-changing  $(H^0, A^0)\bar{s}_Rb_L$  couplings of the heavier neutral Higgs bosons  $(H^0, A^0)$ . On the other hand, at large  $tan \beta$ , this coupling gives "tree-level" contributions to the  $B_s \rightarrow \mu^+ \mu^-$  decay by the Higgs penguin diagrams [[21](#page-7-16),[33](#page-7-28)], which are often much larger than the standard model contributions by orders of magnitude. Requiring that these Higgs penguin contributions do not saturate the experimental upper bound  $Br(B<sub>s</sub> \rightarrow$  $\mu^+ \mu^ < 10^{-7}$  at 95% C.L. [[34](#page-7-29)], and neglecting mass difference between  $(H^0, A^0)$ , the condition

<span id="page-6-4"></span>
$$
|(\hat{Y}_d)_{32}|^2 + |(\hat{Y}_d)_{23}|^2 < 0.2\cos^2\beta (m_A/500 \text{ GeV})^4, \quad (26)
$$

is imposed on the  $b - s$  mixing Yukawa couplings  $((\hat{Y}_d)_{32}, (\hat{Y}_d)_{23})$  at the renormalization scale  $\mu_b \sim m_b$ . In the approximation of neglecting the QCD running between  $\mu_b$  and  $M_{\tilde{Q}}$ , and also the *O*(tan*β*)-enhanced correction to the  $\bar{t}_L b_R H^+$  coupling  $\sim (\hat{Y}_d)_{33}$ , Eq. ([26](#page-6-4)) implies the bound  $|C'_{\nu,H^{\pm}}|$  < 0.15 for tan $\beta = 50$  and  $m_A < 1000$  GeV, which is completely negligible compared to  $C_{\nu, SM}$ . We expect that this strong constraint still holds when the more rigorous estimation of  $B_s \to \mu^+ \mu^-$  is adopted.

# **V. CONCLUSION**

We have studied the flavor-changing decay  $b \rightarrow s \nu \bar{\nu}$  in the MSSM at large  $tan \beta$  and with general flavor mixing of squarks. This case is interesting since the gluino and  $H^{\pm}$ loops, which are negligible at moderate value of  $tan \beta$  and with minimal flavor violation for squarks, are enhanced and might give contributions to this decay, comparable to the standard model and chargino loop contributions. This is due to the tan $\beta$ -enhanced  $SU(2) \times U(1)$  gauge symmetry breaking and flavor mixing in the down-type squark sector, and loop-generated effective flavor-changing couplings of the charged Higgs boson to quarks and squarks. However, the contributions to  $b \rightarrow s \nu \bar{\nu}$  by new physics should be constrained by experimental data for other  $b \rightarrow s$ processes.

In this paper, we have focused our attention on the constraints from the radiative decay  $b \rightarrow s\gamma$ , since both of the  $b \rightarrow s \nu \bar{\nu}$  and  $b \rightarrow s \gamma$  decays are enhanced by the  $SU(2) \times U(1)$  symmetry breaking and flavor mixing between the second and third generations of the quarks/

squarks in the loops. As a very rough estimation of the constraints by  $b \rightarrow s\gamma$ , we have calculated the correlations between new physics contributions to the Wilson coefficients  $C_v^{(')}$  and  $C_7^{(')}$  for the  $b \rightarrow s \nu \bar{\nu}$  and  $b \rightarrow s \gamma$  decays, respectively, for each new physics sector: gluino-squark, chargino-squark, and charged Higgs-quark loops. Calculation has been done at the leading order, but including tan $\beta$ -enhanced corrections to the quark Yukawa couplings in the loops. It has been demonstrated that the requirement that the new physics contributions  $C_7^{(')}$  (new) for each sector are smaller than  $C_{7,SM}$  strongly constrains the new physics contributions  $C_{\nu}^{(')}$  (new). Especially, the gluino contributions  $C_{\nu, \tilde{g}}^{(')}$  are suppressed much below  $C_{\nu, \text{SM}}$  due to their strong correlation with  $C_{7,\tilde{g}}^{(')}$ . In contrast, although the constraints by  $C_7^{(')}$  are also tight for chargino and charged Higgs boson contributions, there still remains a possibility that their contributions to  $C_{\nu}^{(')}$  become sizable,  $O(10)\%$ , while keeping contributions to  $C_7^{(')}$  below  $C_{7,SM}$ .

As an example of the constraints by other  $b \rightarrow s$  processes, we have also considered the Higgs penguin contributions to the decay  $B_s \to \mu^+ \mu^-$ , which might become much larger than the standard model contribution at large  $tan \beta$ . It has been shown that the present experimental upper bound of the decay ratio may impose strong constraints on  $C'_{\nu,H^{\pm}}$ , suppressing it much below  $C_{\nu,SM}$ .

For a more realistic analysis of  $b \rightarrow s \nu \bar{\nu}$  in the MSSM and estimation of the new physics contributions, we need to scan over wider parameter space, including correlations between different contributions to  $C_{\nu}^{(')}$ , main parts of the QCD corrections and hadronic effects, and constraints from other flavor-changing processes using more precise formulas of the new physics contributions. We leave such studies for future works.

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