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Nonet symmetry in η, η' and $B \to K\eta$, $K\eta'$ decays

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The nonet symmetry scheme seems to describe rather well the masses and $\eta - \eta'$ mixing angle of the ground state pseudoscalar mesons. It is expected that nonet symmetry should also be valid for the matrix elements of the pseudoscalar density operators which play an important role in charmless two-body B decays with η or η' in the final state. Starting from the divergences of the $SU(3)$ octet and singlet axial vector currents, we show that nonet symmetry for the pseudoscalar mass term implies nonet symmetry for the pseudoscalar density operators. In this nonet symmetry scheme, we find that the branching ratio $B \to PP$, PV with η in the final state agrees well with data, while those with η' are underestimated, but by increasing the $B \to \eta'$ form factor by 40%–50%, one could explain the tree-dominated $B^- \to \pi^- \eta'$ and $B^- \to \rho^- \eta'$ measured branching ratios. With this increased form factor and with only a moderate annihilation contribution, we are able to obtain 62×10^{-6} for the penguin-dominated $B^- \to K^- \eta'$ branching ratios, quite close to the measured value. This supports the predicted value for the $B \to \eta'$ form factor in PQCD and light-cone sum rules approach. A possible increase by 15% of $\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle$ for η_0 would bring the predicted $B^- \to K^- \eta'$ branching ratio to 69.375×10^{-6}, very close to experiment.

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I. INTRODUCTION

Unlike the low-lying vector mesons where the flavor diagonal 1^- $q\bar{q}$ states are eigenstate because of the OZI selection rule, the 0^- pseudoscalar $q\bar{q}$ state can mix with each other. Since QCD interactions through the exchange of gluons are flavor-independent, one expects the wave function for the pseudoscalar nonet also flavor-independent in the limit of vanishsing current quark mass ($m_q \to 0$, $q = u, d, s$) and the η and η' can be described as two linear combinations of the $q\bar{q}$ state, the $SU(3)$ singlet η_0 and the $SU(3)$ octet η_8 which mix with each other through a small $SU(3)$ symmetry breaking mixing parameter. In fact, with m_u and $m_d \ll m_s$, $m_s \ll \Lambda_{\rm QCD}$, and because of the $U(1)$ QCD-anomaly, the η_0 mass is much larger compared to the η_8 mass, the $\eta - \eta'$ mixing angle is $O(m_s/\Lambda_{\rm QCD})$ so that the physical η and η' are almost pure η_8 and η_0 eigenstate, respectively, in contrast with the ideal mixing for the 1^- low-lying vector meson states. Another feature of the 0^- $q\bar{q}$ nonet is that, because of the spontaneous breakdown of $SU(3) \times SU(3)$ symmetry, the octet mesons are massless Goldstone bosons in the limit of vanishing current quark mass. This simplifies considerably the description of the ground state pseudoscalar meson system. As shown in [1], a rather accurate description of the mass and mixing angle in the $\eta - \eta'$ system is obtained by adding a $U(1)$ QCD-anomaly term for the η_0 mass in the nonet pseudoscalar meson pseudoscalar meson mass matrix. This mass matrix is generated by the quark mass term and is the leading term in the large N_c expansion while higher order terms in the chiral Lagrangian [2] is $O(1/N_c)$ and is thus suppressed in the large N_c limit. This justifies the nonet symmetry mass term for the pseudoscalar mass matrix. Vice-versa, from the nonet symmetry value for the off-diagonal mass term $\langle \eta_0|H_{\rm SB}|\eta_8\rangle$, where $H_{\rm SB} = m_s\bar{s}s + m_u\bar{u}u + m_d\bar{d}d$ one would get a mixing angle $\theta = -18°$ in good agreement with a value $\theta \approx -(22 \pm 3)°$ in [1], or $\theta \approx -(18.4 \pm 2)°$ in [3] and a similar value $\theta \approx -(17 - 20)°$ [4] obtained from the two-photon decay width of η and η'. However, if we use the Gell-Mann-Okubo (GMO) mass formula for the octet mass m_8^2, we would have, in terms of the $\eta - \eta'$ mixing angle θ

$$m_\eta^2 = m_8^2 - \tan\theta^2(m_{\eta'}^2 - m_8^2) \tag{1}$$

which, for $\theta = -18°$ gives $m_\eta = 483$ MeV, about 60 MeV below experiment. Thus the $\eta - \eta'$ mixing which contributes to L_7 in [2] has driven the m_η below the GMO value by 63 MeV. This is also the case with a nonet mass matrix in the quark basis [5,6] which has a large $\eta - \eta'$ mixing and an upper bound for the η mass far below experiment. The higher order terms L_4, L_5, L_6, L_8 and chiral logarithms obtained in Ref. [2] shift m_η upward by a similar amount with the result that the η mass is very close to the GMO value, in agreement with experiment. Similar result is also obtained in [6] more recently. Thus, nonet symmetry seems to be a good approximation for the 0^- nonet mass term. One could then go further and try to see if the matrix elements of the pseudoscalar density local operator e.g. $\bar{s}i\gamma_5 s$ could also satisfies nonet symmetry. This will allow a simple calculation of the penguin matrix elements in the charmless two-body decays of B meson with η or η' in the final states. In this paper we will use the divergence equation for the octet and singlet axial vector current to show that nonet symmetry scheme for the mass term implies nonet symmetry for the pseudoscalar density $\bar{q}i\gamma_5 q$ for η and η'. The basic idea is to include in the matrix elements of the axial vector current and its divergence the $\eta_{0,8}$ pole contribution which will add the mixing mass term $\langle\eta_0|H_{\rm SB}|\eta_8\rangle$ to the divergence equation and allows us to obtain the nonet symmetry expression for the matrix element of the pseudoscalar density operators between the vacuum and $\eta_{0,8}$. In the next section we will first derive a divergence equation for the $\bar{u}\gamma_\mu\gamma_5 u$ and $\bar{s}\gamma_\mu\gamma_5 s$ axial vector current, in the presence of the $SU(3)\times SU(3)$-breaking $H_{\rm SB}$ current quark mass term. Section III is an analysis of $B^-\to P\eta$ and $B^-\to P\eta'$, $P=K^-,\pi^-$ in QCD factorization (QCDF) with nonet symmetry for the pseudoscalar density and $B\to\eta$ and $B\to\eta'$ transition form factors. We find that the branching ratio for modes with η in the final state agrees well with data, while those with η' in the final state are underestimated. We then increase the $B\to\eta'$ form factor by 40%–50%, to bring the tree-dominated $B^-\to\pi^-\eta'$ and $B^-\to\rho^-\eta'$ to the measured values. The increased form factor is then used to obtain a branching ratio close to data for the penguin-dominated $B^-\to K^-\eta'$ decay.

II. PSEUDOSCALAR DENSITY MATRIX ELEMENT AND NONET SYMMETRY

Let $|\eta_0\rangle$, $|\eta_8\rangle$ be the $SU(3)$ singlet and octet eigenstate of the $I=0$, pseudoscalar nonet in the absence of the $SU(3)$ symmetry breaking quark mass term $H_{\rm SB}$, in terms of the flavor diagonal $q\bar{q}$ component

$$|\eta_0\rangle = (|u\bar{u} + d\bar{d} + s\bar{s}\rangle)/\sqrt{3}, \qquad |\eta_8\rangle = (|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle)/\sqrt{6}. \tag{2}$$

Consider now the matrix element of the axial vector current matrix element $\bar{u}\gamma_\mu\gamma_5 u$ and $\bar{s}\gamma_\mu\gamma_5 s$ between the vacuum and η_0 and η_8:

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u|\eta_0\rangle = if_u p_\mu/\sqrt{3}, \qquad \langle 0|\bar{u}\gamma_\mu\gamma_5 u|\eta_8\rangle = if_u p_\mu/\sqrt{6}. \tag{3}$$

and

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta_0\rangle = if_s p_\mu/\sqrt{3}, \qquad \langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta_8\rangle = -2if_s p_\mu/\sqrt{6}. \tag{4}$$

where f_u and f_s are defined as the decay constant of $u\bar{u}$ and $s\bar{s}$ state, respectively. Except for the momentum dependence factor p_μ, the above axial vector current matrix elements depend on the same f_u and f_s according to nonet symmetry scheme with identical $q\bar{q}$ spatial wave function in η_0 and η_8 [1], but f_s could be different from f_u by an $SU(3)$ breaking s-quark mass term. The octet $A_{8\mu}$ and singlet $A_{0\mu}$ axial vector current matrix elements between the vacuum and η_8, η_0 are then given by

$$\langle 0|A_{\mu 8}|\eta_8\rangle = \frac{(f_u + f_d + 4f_s)}{6}p_\mu, \qquad \langle 0|A_{\mu 0}|\eta_0\rangle = \frac{(f_u + f_d + f_s)}{3}p_\mu. \tag{5}$$

(p_μ is the 4-momentum of η_0 and η_8. Similar, for other members of the $SU(3)$ octet, we have f_π and f_K for $\pi^+ = u\bar{d}$, $K^+ = u\bar{s}$ meson, respectively. Assuming each s- quark contributes to the decay constant a symmetry breaking term ϵ, to first order in ϵ, (rewriting $f_{q\bar{q}} = f_q$), we have [7]

$$f_\pi = f_{u\bar{d}} \approx f_u, \qquad f_K = f_{u\bar{s}} = (1+\epsilon)f_{u\bar{d}}, \qquad f_s = (1+2\epsilon)f_u \approx (1+\epsilon)f_K. \tag{6}$$

The usual way to obtain the pseudoscalar density matrix elements is to take the divergence of the axial vector current between the vacuum and the pseudoscalar meson octet. For example, taking the matrix elements of $\bar{u}i\gamma_5 d$, $\bar{u}i\gamma_5 s$, $(\bar{u}i\gamma_5 u - (\bar{d}i\gamma_5 d)$ between the vacuum and π^+, K^+, π^0, respectively, we have

$$f_\pi B_0(m_u + m_d) = (m_u + m_d)\langle 0|\bar{u}i\gamma_5 d|u\bar{d}\rangle, \qquad f_K B_0(m_u + m_s) = (m_u + m_s)\langle 0|\bar{u}i\gamma_5 s|u\bar{s}\rangle. \tag{7}$$

and for π^0

$$f_u B_0(m_u + m_d) = (m_u + m_d)\langle 0|\bar{u}i\gamma_5 u|u\bar{u}\rangle. \tag{8}$$

where the π and K meson masses are the usual expressions in terms of B_0 and the current quark mass [1,2]. The expression for π^0 is obtained by putting [8]

$$\langle 0|\bar{u}i\gamma_5 u|\pi^0\rangle = -\langle 0|\bar{d}i\gamma_5 d|\pi^0\rangle. \tag{9}$$

Apart from the difference in f_π and f_K, we see that the above pseudoscalar density matrix element in Eqs. (7) and (8) satisfies $SU(3)$ symmetry. We will see below that to have nonet symmetry for the pseudoscalar density matrix element between the vacuum and $\eta_{0,8}$, the pole term in the divergence equation must be included. We now consider the divergence of the $I=0$ $A_{{\rm n}\mu}$ and $A_{{\rm s}\mu}$ axial vector current

$$A_{{\rm n}\mu} = (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d), \qquad A_{{\rm s}\mu} = \bar{s}\gamma_\mu\gamma_5 s. \tag{10}$$

The divergence is given by

$$\partial A_{\rm n} = 2(m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d) + 2\frac{\alpha_s}{4\pi}G\tilde{G}, \qquad \partial A_{\rm s} = 2m_s\bar{s}i\gamma_5 s + \frac{\alpha_s}{4\pi}G\tilde{G}. \tag{11}$$

The matrix elements of $\partial A_{\rm n}$ and $\partial A_{\rm s}$ between the vacuum and $\eta_{0,8}$ are given by

$$\langle 0|\partial A_{\rm n}|\eta_0\rangle = 2m_u\langle 0|\bar{u}i\gamma_5 u|\eta_0\rangle + 2m_d\langle 0|\bar{d}i\gamma_5 d|\eta_0\rangle, +2\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_0\rangle, \tag{12}$$

$$\langle 0|\partial A_{\rm s}|\eta_0\rangle = 2m_s\langle 0|\bar{s}i\gamma_5 s|\eta_0\rangle + \langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_0\rangle. \tag{13}$$

and for η_8

$$\langle 0|\partial A_{\rm n}|\eta_8\rangle = 2m_u\langle 0|\bar{u}i\gamma_5 u|\eta_8\rangle + 2m_d\langle 0|\bar{d}i\gamma_5 d|\eta_8\rangle, +2\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_8\rangle, \tag{14}$$

$$\langle 0|\partial A_{\rm s}|\eta_8\rangle = 2m_s\langle 0|\bar{s}i\gamma_5 s|\eta_8\rangle + \langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_8\rangle. \tag{15}$$

In the limit $m_u = m_d = 0$, since the l.h.s of Eq. (14) is $f_u m_8^2$, the matrix element $2\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_8\rangle$ on the r.h.s is $O(m_8^2)$ and is given by the η_0 pole contribution. We now evaluate Eqs. (12)–(15) with the pole terms included using the nonet symmetry expressions for $m_{0,8}^2$ and m_{08}^2 [1]

$$\begin{aligned} m_8^2 &= B_0\tfrac{2}{3}(2m_s + \hat{m}),\\ m_0^2 &= \bar{m}_0^2 + B_0\tfrac{2}{3}(m_s + 2\hat{m}),\\ m_{08}^2 &= B_0\tfrac{2}{3}\sqrt{2}(-m_s + \hat{m}). \end{aligned} \tag{16}$$

in standard notation [2] ($\hat{m} = (m_u + m_d)/2$). At the η_0 and η_8 mass, $p^2 = m_0^2$ and $p^2 = m_8^2$ in the l.h.s of Eqs. (12)–(15) respectively. As mentioned above, since $m_{u,d} \ll m_s$, $SU(3)$ is broken and the $\eta_{0,8}$ pole will contribute to both the 1.h.s and r.h.s of Eqs. (12)–(15). The pole terms on the r.h.s come from the QCD-anomaly matrix element $\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_0\rangle$ and $\langle 0|\frac{\alpha_s}{4\pi}G\tilde{G}|\eta_8\rangle$ induced by $SU(3)$-breaking $\eta_0 - \eta_8$ mixing mass term m_{08}^2. The presence of the $\eta_{0,8}$ pole term is important, since its contribution is the same order as the current-quark mass terms in $m_{0,8}^2$. Indeed had we dropped the $\eta_{0,8}$ pole term we would run into contradiction with the divergence equation. To obtain the pseudoscalar density matrix elements, let us bring the p^2-dependence pole term in the l.h.s to the r.h.s of Eqs. (12)–(15). Putting $f_u = f_d$ and $\langle 0|\bar{u}i\gamma_5 u|\eta_{0,8}\rangle = \langle 0|\bar{d}i\gamma_5 d|\eta_{0,8}\rangle$, we find, for η_0

$$f_u\frac{1}{\sqrt{3}}(\bar{m}_0^2 + B_0\frac{2}{3}(m_s + 2\hat{m})) = f_u\frac{1}{\sqrt{3}}\bar{m}_0^2 - f_u\frac{1}{\sqrt{6}}\bigg(B_0\frac{2\sqrt{2}}{3}(\hat{m} - m_s)\bigg) + 2\frac{1}{\sqrt{3}}\hat{m}\langle 0|\bar{u}i\gamma_5 u|u\bar{u}\rangle, \tag{17}$$

$$f_s\frac{1}{\sqrt{3}}\bigg(\bar{m}_0^2 + B_0\frac{2}{3}(m_s + 2\hat{m})\bigg) = f_s\frac{1}{\sqrt{3}}\bar{m}_0^2 - f_s\frac{2}{\sqrt{6}}B_0\frac{2\sqrt{2}}{3}(\hat{m} - m_s) + 2\frac{1}{\sqrt{3}}m_s\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle. \tag{18}$$

and. similarly, for η_8

$$f_u\frac{1}{\sqrt{6}}\bigg(B_0\frac{2}{3}(2m_s + \hat{m})\bigg) = -f_u\frac{1}{\sqrt{3}}B_0\frac{2\sqrt{2}}{3}(\hat{m} - m_s) + 2\frac{1}{\sqrt{6}}\hat{m}\langle 0|\bar{u}i\gamma_5 u|u\bar{u}\rangle, \tag{19}$$

$$-f_s\frac{2}{\sqrt{6}}\bigg(B_0\frac{2}{3}(2m_s + \hat{m})\bigg) = -f_s\frac{1}{\sqrt{3}}B_0\frac{2\sqrt{2}}{3}(\hat{m} - m_s) - 2\frac{2}{\sqrt{6}}m_s\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle. \tag{20}$$

Comparing the l.h.s and the r.h.s of Eqs. (17) and (18), we get the pseudoscalar density matrix element for η_0

$$\langle 0|\bar{u}i\gamma_5 u|u\bar{u}\rangle = B_0 f_u, \tag{21}$$

$$\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle = B_0 f_s. \tag{22}$$

Similarly, by comparing l.h.s and the r.h.s of Eqs. (19) and (20), we get the same expression for the pseudoscalar density matrix element, but in η_8.

We have shown that, by including the η_0 and η_8 pole in the divergence equations, and by using the nonet symmetry expressions for the current quark mass contributions to the η_0 and η_8 mass, the pseudoscalar density operators matrix elements between η_0 and η_8 can be obtained by nonet symmetry and quark counting rule. Like the matrix elements $\langle 0|\bar{u}i\gamma_5 d|\pi^+\rangle$, $\langle 0|\bar{u}i\gamma_5 u|\pi^0\rangle$, and $\langle 0|\bar{u}i\gamma_5 s|K^+\rangle$, they are given by the parameter B_0 and the decay constant involved. Experimentally, from the known value of the $\eta - \eta'$ mixing angle, $\theta = (-20 \pm 2)^\circ$, one has $m_{08}^2 = -(0.81 \pm 0.05)m_K^2$ to be compared with the nonet symmetry value of $m_{08}^2 \simeq -0.90m_K^2$ [1], we expect nonet symmetry for the pseudoscalar density matrix elements in $\eta - \eta'$ valid to this accuracy. Since the octet m_8^2 mass gets about 15% increase from higher order terms L_4, L_5, L_6, L_8 and chiral logarithms [2], Eqs. (19) and (20) show that $\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle$ in η will be increased by a similar amount. Note that the r.h.s of Eqs. (19) and (20) gets this increase from higher order terms in the pole and other terms. Higher order $SU(3)$ breaking contribution to the singlet m_0^2 mass is not known, but if we assume a similar 15% increase from the nonet value in Eq. (16), $\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle$ in η_0 will also be increased by a similar amount. This could be another source of enhancement for the $B \to K\eta'$ branching ratio, as found below. We note that it might be possible to obtain the pseudoscalar density matrix elements in Eqs. (17)–(20) using the known values of $m_{0,8}^2$ and m_{08}^2, but because the precise dependence on the quark mass is not known and because of the experimental errors involved, the physical interpretation of the result will be lost. We would like to stress that in our derivation, the anomaly contribution to the η_0 mass has been included in the divergence equation, thus the enhancement factor for $\langle 0|\bar{s}i\gamma_5 s|\eta_0\rangle$ suggested in

[9] would have the origin elsewhere. With the pseudoscalar density matrix elements given above and nonet symmetry for the $B \to \eta$, η' transition form factors, we shall now compute the $B^- \to K^-\eta$, $K^-\eta'$ and $B^- \to \pi^-\eta$, $\pi^-\eta'$ decay branching ratios in QCDF.

III. $B^- \to K^-(\eta, \eta')$ AND $B^- \to \pi^-(\eta, \eta')$ DECAY IN QCD FACTORIZATION

The $B \to M_1M_2$ decay amplitude in QCDF is given by [10,11]:

$$\begin{aligned} \mathcal{A}(B \to M_1M_2) &= \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V_{ps}^* \Big(-\sum_{i=1}^{10} a_i^p \langle M_1M_2|O_i|B\rangle_H \\ &\quad + \sum_i^{10} f_B f_{M_1} f_{M_2} b_i \Big), \end{aligned} \tag{23}$$

where the QCD coefficients a_i^p contain the vertex corrections, penguin corrections, and hard spectator scattering contributions, the hadronic matrix elements $\langle M_1M_2|O_i|B\rangle_H$ of the tree and penguin operators O_i are given by factorization model [12,13], b_i are annihilation contributions. The values for a_i^p, $p = u, c$, computed from the expressions in [10,11] at the renormalization scale $\mu = m_b$, with $m_b = 4.2$ GeV are

$$\begin{aligned} a_4^c &= -0.033 - 0.013i + 0.0009\rho_H, \\ a_4^u &= -0.027 - 0.017i + 0.0009\rho_H, \\ a_6^c &= -0.045 - 0.003i, \\ a_6^u &= -0.042 - 0.013i, \\ a_8^c &= -0.0004 - 0.0001i, \\ a_8^u &= 0.0004 - 0.0001i, \\ a_{10}^c &= -0.0011 - 0.0001i - 0.0006\rho_H, \\ a_{10}^u &= -0.0011 + 0.0006i - 0.0006\rho_H. \end{aligned} \tag{24}$$

for $i = 4, 6, 8, 10$. For other coefficients, $a_i^u = a_i^p = a_i$

$$\begin{aligned} a_1 &= 1.02 + 0.015i - 0.012\rho_H, \\ a_2 &= 0.156 - 0.089i + 0.074\rho_H, \\ a_3 &= 0.0025 + 0.0030i - 0.0024\rho_H, \\ a_5 &= -0.0016 - 0.0034i + 0.0029\rho_H, \\ a_7 &= -0.00003 - 0.00004i - 0.00003\rho_H, \\ a_9 &= -0.009 - 0.0001i + 0.0001\rho_H. \end{aligned} \tag{25}$$

where the complex parameter $\rho_H \exp(i\phi_H)$ represents the endpoint singularity contribution in the hard-scattering corrections $X_H = (1 + \rho_H \exp(i\phi_H)) \ln(\frac{m_B}{\Lambda_h})$ [10,11] (we have put the phase $\phi_H = 0$ in the above expressions).

For the annihilation terms, we have

$$\begin{aligned} b_2 &= -0.0038 - 0.0065\rho_A - 0.0018\rho_A^2, \\ b_3 &= -0.0065 - 0.0150\rho_A - 0.0085\rho_A^2, \\ b_3^{ew} &= -0.00011 - 0.00015\rho_A + 0.000003\rho_A^2. \end{aligned} \tag{26}$$

where b_i are evaluated with the factor $f_B f_{M_1} f_{M_2}$ included and ρ_A, like ρ_H, appears in the divergent annihilation term $X_A = (1 + \rho_A \exp(i\phi_A)) \ln(\frac{m_B}{\Lambda_h})$.

For the CKM matrix elements, since the inclusive and exclusive data on $|V_{ub}|$ differ by a large amount and the higher inclusive data exceeds the unitarity limit for $R_b = |V_{ud}V_{ub}^*|/|V_{cd}V_{cb}^*|$ with the current value $\sin(2\beta) = 0.687 \pm 0.032$ [14], we shall determine $|V_{ub}|$ from the more precise $|V_{cb}|$ data. We have [15]

$$|V_{ub}| = \frac{|V_{cb}V_{cd}^*|}{|V_{ud}^*|} |\sin\beta| \sqrt{1 + \frac{\cos^2\alpha}{\sin^2\alpha}}. \tag{27}$$

With $\alpha = (99^{+13}_{-9})°$ [14] and $|V_{cb}| = (41.78 \pm 0.30 \pm 0.08) \times 10^{-3}$ [16], we find

$$|V_{ub}| = 3.60 \times 10^{-3}. \tag{28}$$

in good agreement with the exclusive data in the range $|V_{ub}| = 3.33 - 3.51$ [16]. The recent measurements of the $B_s - \bar{B}_s$ mixing also allow the extraction of $|V_{td}|$ from $B_d - \bar{B}_d$ mixing data. The current determination [17] gives $|V_{td}/V_{ts}| = (0.208^{+0.008}_{-0.006})$ which in turn can be used to determined the angle γ from the unitarity relation [15]

$$|V_{td}| = \frac{|V_{cb}V_{cd}^*|}{|V_{tb}^*|} |\sin\gamma| \sqrt{1 + \frac{\cos^2\alpha}{\sin^2\alpha}}. \tag{29}$$

with $|V_{tb}| = 1$, we find $\gamma = 66°$ which implies an angle $\alpha = 91.8°$, in good agreement with the value found in the current UT-fit value of $(88 \pm 16)°$ [18]. In the following in our B decay calculations, we shall use the unitarity triangle values for $|V_{ub}|$ and γ. For other hadronic parameters we use the values in Table 1 of [11] and take $m_s(2\text{ GeV}) = 80$ MeV, $f_u = f_\pi$, $f_s = f_\pi(1 + 2(\frac{f_K}{f_\pi} - 1))$. For the $B \to \pi$ and $B \to K$ transition form factor, we use the current light-cone sum rules central value [19]

$$F_0^{B\pi}(0) = 0.258, \qquad F_0^{BK}(0) = 0.33 \tag{30}$$

With $\eta - \eta'$ mixing angle $\theta = -20°$, we have

$$\begin{aligned} |\eta\rangle &= (0.58(|u\bar{u}\rangle + |d\bar{d}\rangle) - 0.57|s\bar{s}\rangle), \\ |\eta'\rangle &= (0.40(|u\bar{u}\rangle + |d\bar{d}\rangle) + 0.82|s\bar{s}\rangle). \end{aligned} \tag{31}$$

From Eq. (31), we find

$$F^{B\eta}(0) = 0.58F_0^{B\pi}(0), \qquad F^{B\eta'}(0) = 0.40F_0^{B\pi}(0). \tag{32}$$

The $B \to K(\eta', \eta)$ decay amplitude can now be obtained from the factorization formula for the hadronic matrix elements in Eq. (23) with the pseudoscalar density matrix element obtained in Eq. (22) and the form factors given above. We have

$$\langle 0|\bar{s}i\gamma_5 s|\eta\rangle = C_\eta B_0 f_s, \qquad \langle 0|\bar{s}i\gamma_5 s|\eta'\rangle = C_{\eta'} B_0 f_s. \tag{33}$$

where $B_0 = m_K^2/(m_s + \hat{m})$ and $C_\eta = -0.57$, $C_{\eta'} = 0.82$, the fraction of $s\bar{s}$ state in η and η' respectively. This contributes to the O_6 matrix element a term $f_s r_\chi^K$, with $r_\chi^K = 2m_K^2/(m_b + \hat{m})(m_s + \hat{m})$, similar to that in $\bar{B}^0 \to K^-\pi^+$ decay, except that in $B^- \to K^-\eta$ and $B^- \to K^-\eta'$, the O_6 matrix element is enhanced by a factor f_s/f_K. In this way, the decay amplitude in units of GeV are

$$\begin{aligned} A(B^- \to \pi^-\pi^0) &= (0.110 + 0.204i) \times 10^{-7}, \\ A(\bar{B}^0 \to K^-\pi^+) &= -(0.368 + 0.004i) \\ &\quad \times 10^{-7}(F_0^{B\to\pi}(0)/0.258) \\ &\quad - (0.090 + 0.002i) \times 10^{-7}. \end{aligned} \tag{34}$$

from which the branching ratios are, with $\rho_H = 0$, $\rho_A = 0.6$ (only the central values for the relevant parameters are used in the calculations)

$$\begin{aligned} \mathcal{B}(B^- \to \pi^-\pi^0) &= 5.050 \times 10^{-6}, \\ \mathcal{B}(\bar{B}^0 \to K^-\pi^+) &= 18.249 \times 10^{-6}. \end{aligned} \tag{35}$$

in good agreement with the current measured branching ratios [20]

$$\begin{aligned} \mathcal{B}(B^- \to \pi^-\pi^0) &= (5.7 \pm 0.4) \times 10^{-6}, \\ \mathcal{B}(\bar{B}^0 \to K^-\pi^+) &= (19.04 \pm 0.6) \times 10^{-6}. \end{aligned} \tag{36}$$

We note a sizable annihilation contribution, given by the last term in Eq. (34), is needed to produce a large $\mathcal{B}(\bar{B}^0 \to K^-\pi^+)$. This is not surprising since annihilation contribution is also needed to explain the large branching ratios of $B^+ \to \pi^+ K^{*0}$ and $B^0 \to K^-\rho^+$ decay [21]. Our result also shows that the values 0.258 for $F_0^{B\pi}(0)$ and 0.33 for $F_0^{BK}(0)$ given above are reasonable. We will use these values in the calculation of the decay modes with η and η'. We find

$$\begin{aligned} A(B^- \to K^-\eta) &= -(0.283 + 0.032i) \\ &\quad \times 10^{-7}(F_0^{B\to\eta}(0)/0.150) \\ &\quad + (0.317 + 0.080i) \\ &\quad \times 10^{-7}(F_0^{B\to K}(0)/0.33) \\ &\quad + (0.015 + 0.0004i) \times 10^{-7}. \end{aligned} \tag{37}$$

$$\begin{aligned} A(B^- \to K^-\eta') &= -(0.192 + 0.022i) \\ &\quad \times 10^{-7}(F_0^{B\to\eta'}(0)/0.104) \\ &\quad - (0.425 + 0.039i) \\ &\quad \times 10^{-7}(F_0^{B\to K}(0)/0.33) \\ &\quad - (0.111 + 0.003i) \times 10^{-7}. \end{aligned} \tag{38}$$

where the last term in Eqs. (37) and (38) are the annihilation contributions ($\rho_H = 0$, $\rho_A = 0.6$). The predicted branching ratios are then

$$\begin{aligned} \mathcal{B}(B^- \to K^-\eta) &= 0.431 \times 10^{-6}, \\ \mathcal{B}(B^- \to K^-\eta') &= 48.263 \times 10^{-6}. \end{aligned} \tag{39}$$

to be compared with the current experimental values [20]

$$\begin{aligned} \mathcal{B}(B^- \to K^-\eta) &= (2.2 \pm 0.3) \times 10^{-6}, \\ \mathcal{B}(B^- \to K^-\eta') &= (69.7^{+2.8}_{-2.7}) \times 10^{-6}. \end{aligned} \tag{40}$$

We see that the $\mathcal{B}(B^- \to K^-\eta')$ is underestimated by about 30%, while the $\mathcal{B}(B^- \to K^-\eta)$ is very much suppressed, but because of large cancellation in the $B^- \to K^-\eta$ amplitude due to the negative $s\bar{s}$ amplitude in the η meson wave function, a precise prediction for $\mathcal{B}(B^- \to K^-\eta)$ is more difficult. For $B^- \to K^-\eta'$, since b_3 contributes both to $\bar{B}^0 \to K^-\pi^+$ and $B^- \to K^-\eta'$ decays, it is difficult to adjust the annihilation term for $B^- \to K^-\eta'$ without overestimating the $\bar{B}^0 \to K^-\pi^+$ branching ratio. Another possibility is to increase the form factor $F_0^{B\to\eta'}(0)$ from the nonet symmetry value to bring the predicted value closer to data. That this is the case can be seen by looking at the $B^- \to \pi^-\eta'$ decays. We have

$$\begin{aligned} A(B^- \to \pi^-\eta) &= (0.119 + 0.147i) \\ &\quad \times 10^{-7}(F_0^{B\to\eta}(0)/0.150) \\ &\quad - (0.002 - 0.003i) \\ &\quad \times 10^{-7}(F_0^{B\to\pi}(0)/0.258) \\ &\quad - (0.004 - 0.003i) \times 10^{-7}. \end{aligned} \tag{41}$$

$$\begin{aligned} A(B^- \to \pi^-\eta') &= (0.081 + 0.100i) \\ &\quad \times 10^{-7}(F_0^{B\to\eta'}(0)/0.104) \\ &\quad + (0.008 - 0.002i) \\ &\quad \times 10^{-7}(F_0^{B\to\pi}(0)/0.258) \\ &\quad + (0.033 - 0.021i) \times 10^{-7}. \end{aligned} \tag{42}$$

(the last term in the above amplitudes is the annihilation contributions). This gives

$$\begin{aligned} \mathcal{B}(B^- \to \pi^-\eta) &= 3.388 \times 10^{-6}, \\ \mathcal{B}(B^- \to \pi^-\eta') &= 1.910 \times 10^{-6}. \end{aligned} \tag{43}$$

comparing with the current measured branching ratios [20]:

$$\begin{aligned} \mathcal{B}(B^- \to \pi^-\eta) &= (4.4 \pm 0.4) \times 10^{-6}, \\ \mathcal{B}(B^- \to \pi^-\eta') &= (2.6^{+0.6}_{-0.5}) \times 10^{-6}. \end{aligned} \tag{44}$$

we see that the predicted $\mathcal{B}(B^- \to \pi^-\eta)$ agrees more or less with experiment, considering theoretical uncertainties in the CKM parameters and in the $B \to \pi$ and $B \to K$ form factors. while $\mathcal{B}(B^- \to \pi^- \eta')$ is below the *BABAR* value of $(4.0 \pm 0.8 \pm 0.4) \times 10^{-6}$ [20]. Existing QCDF calculations [22] also underestimate $\mathcal{B}(B^- \to \rho^- \eta')$ by a factor of ≈ 2 as seen from the recent data [20] which gives

$$\mathcal{B}(B^- \to \rho^- \eta) = (5.4 \pm 1.2) \times 10^{-6}, \quad \mathcal{B}(B^- \to \rho^- \eta') = (9.1^{+3.7}_{-2.8}) \times 10^{-6}. \tag{45}$$

Since the above tree-dominated decays with η, η' in the final state are more sensitive to the $F^{B\to\eta}$ and $F^{B\to\eta'}$ form factor, by increasing the $F^{B\to\eta'}$ form factor by 40%–50% from the nonet symmetry value, one could bring $\mathcal{B}(B^- \to \pi^- \eta')$, $\mathcal{B}(B^- \to \rho^- \eta')$, and $\mathcal{B}(B^- \to K^- \eta')$ closer to the measured branching ratios. For example, by taking $F_0^{B\to\eta'}(0) = 0.156$, one gets

$$\mathcal{B}(B^- \to \pi^- \eta') = 3.888 \times 10^{-6}, \quad \mathcal{B}(B^- \to K^- \eta') = 61.837 \times 10^{-6}. \tag{46}$$

which largely improves the prediction for $\mathcal{B}(B^- \to K^- \eta')$ but the predicted $\mathcal{B}(B^- \to \pi^- \eta')$ slightly exceeds the HFAG new average, though consistent with the *BABAR* value for this mode. We note also the predicted $\mathcal{B}(B^- \to \rho^- \eta')$ in [22] approaches the measured value with the increased form factor $F_0^{B\to\eta'}(0) = 0.156$. As mentioned earlier, additional sources of enhancement of $\mathcal{B}(B^- \to K^- \eta')$ could come from a possible higher order $SU(3)$ breaking effects in the matrix element $\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle$ for η_0. Assuming a 15% increase of this matrix element from its nonet value, we would have $\mathcal{B}(B^- \to K^- \eta') = 69.375 \times 10^{-6}$, very close to the measured value.

V. CONCLUSION

We have shown that nonet symmetry for the pseudoscalar meson mass term implies nonet symmetry for the pseudoscalar density matrix element. We then use nonet symmetry for the pseudoscalar density matrix element and the $B \to \eta$, $B \to \eta'$ form factors to compute two-body charmless B decays with η, η' in the final state. The discrepancy with experiment for tree-dominated decays with η' in the final state indicates that the $F_0^{B\to\eta'}(0)$ form factors should be bigger than the nonet symmetry value by 40%–50%. This value, together with a moderate annihilation contribution found in $\bar{B}^0 \to K^- \pi^+$ decay, produces a $B^- \to K^- \eta'$ branching ratio close to data. Our value for the $F_0^{B\to\eta'}(0)$ form factor supports the current calculations in PQCD and light-cone sum rules approach [23,24]. A possible increase by 15% of the pseudoscalar density matrix element $\langle 0|\bar{s}i\gamma_5 s|s\bar{s}\rangle$ for η_0 would bring the predicted $B^- \to K^- \eta'$ branching ratio very close to experiment.

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