# Are there approximate relations among transverse momentum dependent distribution functions?

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(Received 8 October 2007; published 24 January 2008)

Certain *exact* relations among transverse momentum dependent parton distribution functions due to QCD equations of motion turn into *approximate* ones upon the neglect of pure twist-3 terms. On the basis of available data from HERMES, we test the practical usefulness of one such "Wandzura-Wilczek-type approximation," namely, of that connecting $h_{1L}^{\perp(1)a}(x)$ to $h_L^a(x)$, and discuss how it can be further tested by future CLAS and COMPASS data.

DOI: 10.1103/PhysRevD.77.014023 PACS numbers: 13.88.+e, 13.60.−r, 13.85.Ni, 13.85.Qk

## I. INTRODUCTION

Semi-inclusive deep inelastic lepton nucleon scattering (SIDIS), hadron production in electron-positron annihilations and the Drell-Yan process [1–14], allow one to access information on transverse momentum dependent (TMD) parton distribution functions (pdf) and fragmentation functions [15]. In order to be sensitive to "intrinsic" transverse parton momenta it is necessary to measure adequate transverse momenta in the final state, e.g. in SIDIS the transverse momenta of produced hadrons with respect to the virtual photon. Some data on such processes are available [16–32], and at least in the case of twist-2 observables factorization applies [33–35].

Eight twist-2 and 16 twist-3 TMD pdfs describe the nucleon structure in these processes, namely [36,37],

$$\underbrace{f_1^a,\, f_{1T}^{\perp a},\, g_{1L}^a,\, g_{1T}^a,\, h_{1T}^a,\, h_{1L}^{\perp a},\, h_{1T}^{\perp a},\, h_1^{\perp a}}_{\text{twist}-2},\, \underbrace{e^a,\, g_T^a,\, h_L^a,\, \ldots}_{\text{twist}-3} \tag{1}$$

which are functions of $x$ and $\mathbf{p}_T^2$. (The dots denote 13 further twist-3 TMD pdfs. The renormalization scale dependence is not indicated for brevity.) Integrating over transverse momenta one is left with six independent "collinear" pdfs [38,39]

$$\underbrace{f_1^a(x),\, g_1^a(x),\, h_1^a(x)}_{\text{twist}-2},\, \underbrace{e^a(x),\, g_T^a(x),\, h_L^a(x)}_{\text{twist}-3}, \tag{2}$$

where the relations hold $j(x) = \int d^2\mathbf{p}_T j(x, \mathbf{p}_T^2)$ for $j = f_1^a, e^a, g_T^a, h_L^a$ while $g_1^a(x) = \int d^2\mathbf{p}_T g_{1L}^a(x, \mathbf{p}_T^2)$ and $h_1^a(x) = \int d^2\mathbf{p}_T\{h_{1T}^a(x, \mathbf{p}_T^2) + \mathbf{p}_T^2/(2M_N^2) h_{1T}^{\perp a}(x, \mathbf{p}_T^2)\}$.

In view of the proliferation of novel functions in (1) one may ask whether some of the unknown TMD pdfs could be related to (possibly better) known ones. Since all structures in (1) are independent [36], any such relations can only be approximate.

Candidates for such *approximate* relations can be obtained as follows. From QCD equations of motion (eom), one obtains among others the following *exact* relations [7]:

$$g_{1T}^{\perp(1)a}(x) \overset{\text{eom}}{=} x g_T^a(x) - x\tilde{g}_T^a(x), \tag{3}$$

$$-2h_{1L}^{\perp(1)a}(x) \overset{\text{eom}}{=} x h_L^a(x) - x\tilde{h}_L^a(x), \tag{4}$$

with the transverse moments defined as ($g_{1T}^{\perp(1)}$ analog)

$$h_{1L}^{\perp(1)a}(x) \equiv \int d^2\mathbf{p}_T \frac{\mathbf{p}_T^2}{2M_N^2} h_{1L}^{\perp a}(x, \mathbf{p}_T^2), \tag{5}$$

and with $\tilde{g}_T^a(x)$, $\tilde{h}_L^a(x)$ denoting pure twist-3 "interaction dependent" terms due to quark-gluon-quark correlations (and current quark mass terms). In the next step, we recall the relations among the collinear pdfs (2) [39–41]

$$g_T^a(x) = \int_x^1 \frac{dy}{y} g_1^a(y) + \tilde{g}_T^{\prime a}(x), \tag{6}$$

$$h_L^a(x) = 2x \int_x^1 \frac{dy}{y^2} h_1^a(y) + \tilde{h}_L^{\prime a}(x), \tag{7}$$

where $\tilde{g}_T^{\prime a}(x)$, $\tilde{h}_L^{\prime a}(x)$ also denote pure twist-3 (and mass) terms [42,43], though different ones than in (3) and (4). Equations (6) and (7) isolate "pure twist-3 terms" in the "twist-3" pdfs $g_T^a(x)$, $h_L^a(x)$. This is because in (2) the underlying "working definition" of twist [44] (a pdf is "twist $t$" if its contribution to the cross section is suppressed, in addition to kinematic factors, by $1/Q^{t-2}$ with $Q$ the hard scale in the process) differs from the strict definition of twist (mass dimension of the operator minus its spin).

The remarkable observation is that $\tilde{g}_T^{\prime a}(x)$ is consistent with zero within error bars [45–49] and to a good accuracy

$$g_T^a(x) \overset{\text{WW}}{\approx} \int_x^1 \frac{dy}{y} g_1^a(y) \quad (\text{exp. observation}) \tag{8}$$

which is the "Wandzura-Wilczek (WW) approximation."

Lattice QCD [50,51] and the instanton model of the QCD vacuum [52] support this observation. Interestingly the latter predicts also $\tilde{h}_L^{\prime a}(x)$ to be small [53], such that

 

$$h_L^a(x) \approx 2x \int_x^1 \frac{\mathrm{d}y}{y^2} h_1^a(y) \qquad \text{(prediction)}. \tag{9}$$

On the basis of this positive experimental and (or) theoretical experience with the smallness of pure twist-3 (and mass) terms one may suspect that the analog terms in the relations (3) and (4) could also be negligible. If true one would have valuable WW-type approximations

$$g_{1T}^{\perp(1)a}(x) \overset{!?}{\approx} x \int_x^1 \frac{\mathrm{d}y}{y} g_1^a(y), \tag{10}$$

$$h_{1L}^{\perp(1)a}(x) \overset{!?}{\approx} -x^2 \int_x^1 \frac{\mathrm{d}y}{y^2} h_1^a(y), \tag{11}$$

that could be satisfied with an accuracy comparable to that of (8). This remains to be tested in experiment.

An immediate application (or test) for the relations (10) and (11) is provided by the following single/double spin asymmetries (SSA/DSA) in SIDIS:

$$A_{UL}^{\sin 2\phi} \propto \sum_a e_a^2 h_{1L}^{\perp(1)a} H_1^{\perp a}, \tag{12}$$

$$A_{LT}^{\cos(\phi-\phi_S)} \propto \sum_a e_a^2 g_{1T}^{\perp(1)a} D_1^a, \tag{13}$$

where the first index $U$ (or $L$) means that the leptons are unpolarized (or longitudinally polarized), the second $L$ (or $T$) indicates the longitudinal (or transverse) polarization of the nucleon, and $\phi$ ($\phi_S$) denotes the azimuthal angle of the produced hadron $h$ (angle of the target polarization vector $S$) with respect to the axis defined by the virtual photon, see Fig. 2. The superscripts $\sin 2\phi$ or $\cos(\phi - \phi_S)$ mean that the spin asymmetries were weighted correspondingly in order to isolate the contributions responsible for the particular azimuthal distributions.

In (12) $H_1^{\perp a}$ denotes the Collins fragmentation function [3–5] on which data from SIDIS [21–24] on the SSA

$$A_{UT}^{\sin(\phi+\phi_S)} \propto \sum_a e_a^2 h_1^a H_1^{\perp a} \tag{14}$$

and from $e^+e^-$ annihilations [28,29] give rise to a first but already consistent picture of $H_1^\perp$ [54–56]. The $D_1^a$ in (13) is the unpolarized fragmentation function which enters, of course, also the respective denominators in the asymmetries (12)–(14) proportional to $\sum_a e_a^2 f_1^a D_1^a$.

Final HERMES [17–19], preliminary CLAS [25] data on (12) and preliminary COMPASS data [32] on (13) are available. So first tests of the WW-type approximations (10) and (11) are now or soon will be possible.

In this paper we shall present a test of the approximation (11). Under the assumption that this approximation *works*, we shall see that it yields results for the SSA (12) compatible with HERMES data [17–19]. From another point of view our work provides a first independent cross check from SIDIS for the emerging picture of $H_1^\perp$ [54–56]. The SSA (12) was recently studied in [57].

A test of the approximation (10) was suggested in [58] along the lines of the study of the SSA (13) discussed previously also in [59].

Among the eight structure functions in SIDIS described in terms of twist-2 pdfs and fragmentation functions [37], the SSAs (12) and (13) are the only ones for which WW-type approximations could be of use. Exact eom-relations exist, in fact, for all eight twist-2 pdfs in (1). But the relations (3) and (4) are special in the sense that they connect the respective TMD pdfs, namely, $g_{1T}^\perp$ and $h_{1L}^\perp$, to collinear twist-3 pdfs, namely, $g_T$ and $h_L$. Those in turn are related to twist-2 pdfs, $g_1$ and $h_1$, by means of (experimentally established or theoretically predicted) WW-approximations (8) and (9).

Experiments may or may not confirm that the WW-type approximations (10) and (11) work.

What would it mean if (10) and (11) were found to be satisfied to within a very good accuracy? First, that would be of practical use for understanding and interpreting the first data [17–32]. Second, it would call for theoretical explanations why pure twist-3 terms should be small. (Only for the smallness of the collinear pure twist-3 terms in (8) and (9) lattice QCD [50,51] and/or instanton vacuum [52,53] provide explanations.)

What would it mean if (10) and (11) were found to work poorly? This scenario would be equally interesting. In fact, all eight pdfs in (1) are independent structures, and any of them contain different types of information on the internal structure of the nucleon. The measurement of the complete set of all 18 structure functions available in SIDIS [6] is therefore indispensable for our aim to learn more about the nucleon structure.

One type of information accessible in this way concerns effects related to the orbital motion of quarks, and, in particular, correlations of spin and transverse momentum of quarks which are dominated by valence quarks and hence play a more important role at large $x$. E.g. it was shown that spin-orbit correlations may lead to significant contributions to partonic momentum and helicity distributions [60] in the large-$x$ limit. Spin-orbit correlations are presumably of similar importance for transversity, and crucial for $h_{1L}^\perp$, which describes transversely polarized quarks in a longitudinally polarized nucleon, and is a measure for the correlation of the transverse spin and the transverse momentum of quarks in a longitudinally polarized nucleon.

This paper is organized as follows. In Sec. II we estimate $h_{1L}^\perp$ by means of the WW-type approximation (11) using various models for $h_1$, and discuss model-independent features of these estimates. In Sec. III we introduce SIDIS notations and definitions. In Sec. IV we evaluate the SSA (12) in the WW-type approximation (11) and compare the results to available HERMES data [17–19].

In Secs. V and VI we discuss what can be learned from future measurements at CLAS, and COMPASS. Section VII contains the conclusions.

## II. W-TYPE APPROXIMATION FOR $h_{1L}^{\perp}$

In order to model $h_{1L}^{\perp(1)a}(x)$ by means of the WW-type approximation (11) one inevitably has to use, in addition, models for the transversity pdf. Figure 1(a) shows four different models: saturation of the Soffer bound [61] at the low initial scale of the leading-order parameterizations [62,63] (choosing $h_1^u > 0$ and $h_1^d < 0$), the chiral quark-soliton model ($\chi$QSM) [64], the nonrelativistic model assumption $h_1^a(x) = g_1^a(x)$ at the low scale of the parameterization [63], and the hypercentral model [65]. All curves in Fig. 1 are leading order evolved to 2.5 GeV$^2$ which is a relevant scale in experiment, see below.

These (and many other [66,67]) models agree that $h_1^u(x) > 0$ and $h_1^d(x) < 0$ with $|h_1^d(x)| < h_1^u(x)$, though the predictions differ concerning the magnitudes, see Fig. 1(a). Models in which antiquark distribution functions can be computed, e.g. [64], predict that the transversity antiquark pdfs are far smaller than the quark ones.

Let us therefore establish first a robust feature of the relation (11), namely, the ratio $h_{1L}^{\perp(1)q}(x)/h_1^q(x)$ exhibits little dependence on the transversity model, see Fig. 1(b).

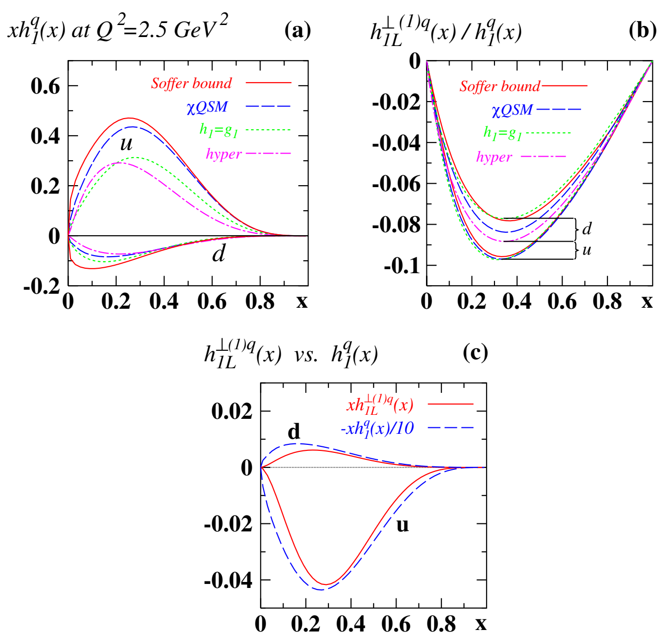


FIG. 1 (color online). (a) Transversity, $xh_1^q(x)$, vs $x$, from various models. (b) The ratio $h_{1L}^{\perp(1)q}(x)/h_1^q(x)$ vs $x$ in various models, with $h_{1L}^{\perp}$ estimated by means of the WW-type approximation (11). (c) $xh_{1L}^{\perp(1)q}(x)$ vs $x$ from the WW-type approximation (11) and $h_1^a(x)$ from $\chi$QSM [64], in comparison with $(-\frac{1}{10})xh_1^q(x)$ from that model. All results here refer to a scale of 2.5 GeV$^2$.

A "universal" behavior of this ratio at large $x$ is not surprising. By inspecting (11) for large $x$ one finds

$$\lim_{\text{large } x} \frac{h_{1L}^{\perp(1)a}(x)}{h_1^a(x)} \sim (1-x), \tag{15}$$

which agrees with general results from large-$x$ counting rules [68]. This is also true for (10). That the WW-type approximations respect the relative large-$x$ behavior of the involved pdfs can intuitively be understood by considering that multi-parton-correlations are likely to vanish faster at large $x$ than twist-2 terms.

Also a universal small-$x$ behavior of the ratio can be understood from Eq. (11), namely, for $h_1^a(x) \sim x^\alpha$ at small $x$ one obtains

$$\lim_{\text{small } x} \frac{h_{1L}^{\perp(1)a}(x)}{h_1^a(x)} \sim \begin{cases} x & \text{for } \alpha < 1, \\ x\log x & \text{for } \alpha = 1, \end{cases} \tag{16}$$

while for $\alpha > 1$ the ratio is proportional to $x^{2-\alpha}$ if the (in that case well defined) "negative Mellin moment of transversity" $\int_0^1 dx x^{-2} h_1^a(x)$ is nonzero, and proportional to $x$ else; i.e. the ratio tends to zero with $x \to 0$ for $\alpha < 2$ which is the case for all models in Fig. 1.[1]

Nevertheless it is interesting to observe that the ratio is rather robust also at intermediate $x$. For the hypercentral model [65] the ratio is flavor-independent, since there $h_1^u(x) = -4h_1^d(x)$ holds trivially due to the imposed $SU(2)_{\text{spin}} \times SU(2)_{\text{flavour}}$ spin-flavor-symmetry. In the other models, however, one observes departures from that, see Fig. 1(b).

As a common feature we finally observe

$$\left| \frac{h_{1L}^{\perp(1)a}(x)}{h_1^a(x)} \right| \lesssim 0.1. \tag{17}$$

In the following we will use the $\chi$QSM, see Fig. 1(c), which has several advantages. First, it is a faithful field theoretic model of the nucleon [72,73] that describes the twist-2 pdfs $f_1^a(x)$ and $g_1^a(x)$ within 10%–30% accuracy [74]. Second, this model is derived from the instanton vacuum model [75,76] which predicts that the "collinear WW-type approximation" (9) works well [53]. Third, below we will use $h_1^a(x)$ from the $\chi$QSM in combination with information on the Collins effect from the analysis [55] where this model was used. This helps to minimize the model-dependence in our study. But we shall see that our conclusions do not depend on the choice of model.

[1] Notice that all curves in Fig. 1 are results of leading-order evolution [69] starting from low scales—ranging from 0.079 GeV$^2$ for [65], till 0.36 GeV$^2$ for $\chi$QSM [64]. Next-to-leading-order evolution [70] and Regge asymptotics [71] predict a behavior $h_1^a(x) \sim \mathcal{O}(x^0)$ for $x \to 0$.

## III. $A_{UL}^{\sin 2\phi}$ AT HERMES

Let us denote the momenta of the target, incoming and outgoing lepton by $P$, $l$ and $l'$ and introduce $s = (P + l)^2$, the four-momentum transfer $q = l - l'$ with $Q^2 = -q^2$ and $W^2 = (P + q)^2$. Then $y = Pq/Pl$ and

$$x = \frac{Q^2}{2Pq}, \qquad z = \frac{PP_h}{Pq}, \qquad \cos\theta_\gamma = 1 - \frac{2M_N^2 x(1-y)}{sy}, \tag{18}$$

where $\theta_\gamma$ denotes the angle between target polarization vector and momentum $\mathbf{q}$ of the virtual photon $\gamma^*$, see Fig. 2, and $M_N$ is the nucleon mass. The component of the momentum of the produced hadron transverse with respect to $\gamma^*$ is denoted by $\mathbf{P}_{h\perp}$ and $P_{h\perp} = |\mathbf{P}_{h\perp}|$.

In the HERMES experiment $A_{UL}^{\sin 2\phi}$ was measured on proton for pion production [17,18] and on deuteron target for pion and kaon production [19] in the kinematic range

$$1\ \text{GeV}^2 < Q^2 < 15\ \text{GeV}^2, \qquad W > 2\ \text{GeV},$$
$$0.023 < x < 0.4, \qquad 0.2 < y < 0.85, \qquad 0.2 < z < 0.7. \tag{19}$$

The momenta of produced hadrons were subject to somehow different cuts: 4.5 GeV $< |\mathbf{P}_h| <$ 13.5 GeV in [17,18] vs 2 GeV $< |\mathbf{P}_h| <$ 15 GeV in [19]. The resolution cut $P_{h\perp} > 50$ MeV was applied throughout [17–19]. This results in the following mean values:

$$\langle x\rangle = 0.09, \qquad \langle y\rangle = 0.53, \qquad \langle z\rangle = 0.38,$$
$$\langle Q^2\rangle = 2.4\ \text{GeV}^2, \qquad \langle P_{h\perp}\rangle = 0.4\ \text{GeV}, \tag{20}$$
$$\langle Q\rangle = 1.55\ \text{GeV}, \qquad \langle\cos\theta_\gamma\rangle = 0.98.$$

In the experiment the SSA was defined as

$$A_{UL}^{\sin 2\phi} = \frac{\sum_i \sin(2\phi_i)(N_i^{\leftrightarrows} - N_i^{\rightrightarrows})}{\sum_i \frac{1}{2}(N_i^{\leftrightarrows} + N_i^{\rightrightarrows})} \tag{21}$$

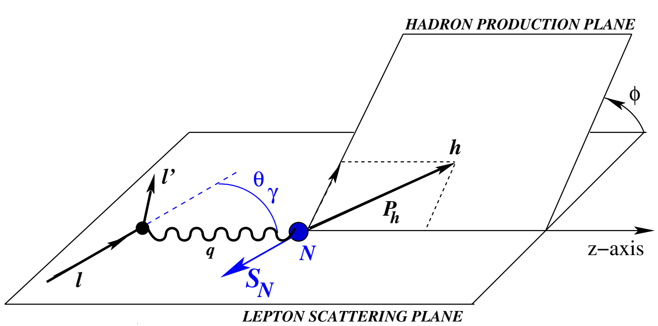


FIG. 2 (color online). Kinematics of the SIDIS process $lN \to l'hX$ and the definitions of azimuthal angles in the lab frame. Here the target polarization is antiparallel to the beam (i.e. $\phi_S = \pi$).

where $N_i^{\leftrightarrows}$ ($N_i^{\rightrightarrows}$) denotes the number of events $i$ with target polarization antiparallel (parallel) to the beam.

## IV. $A_{UL}^{\sin 2\phi}$ IN WW-TYPE APPROXIMATION

The expression for the SSA is given by [7]

$$A_{UL}^{\sin 2\phi}(x) = \frac{\int \mathrm{d}y[\cos\theta_\gamma(1-y)/Q^4] F_{UL}^{\sin 2\phi}}{\int \mathrm{d}y[(1-y+\frac{1}{2}y^2)/Q^4] F_{UU,T}} \tag{22}$$

where in the notation of [37] the denominator is given by

$$F_{UU,T}(x) = \sum_a e_a^2 x f_1^a(x)\langle D_1^a\rangle. \tag{23}$$

Since our purpose is to test the relation (11), we focus on the $x$-dependence of the SSA, and denote here and in the following averages over $z$ within the cuts (19) by $\langle\ldots\rangle = \int \mathrm{d}z(\ldots)$.

The tree-level expression [7] for the structure function $F_{UL}^{\sin 2\phi}$ is given in terms of an integral which convolutes transverse parton momenta in the distribution and the fragmentation function (we neglect soft factors [34,35])

$$F_{UL}^{\sin 2\phi}(x,z) = \int \mathrm{d}^2\mathbf{p}_T \int \mathrm{d}^2\mathbf{K}_T \delta^{(2)}(z\mathbf{p}_T + \mathbf{K}_T - \mathbf{P}_{h\perp}) \times \left[\frac{2(\mathbf{e}_h\mathbf{p}_T)(\mathbf{e}_h\mathbf{K}_T) - (\mathbf{p}_T\mathbf{K}_T)}{M_N m_h}\right] \times \sum_a e_a^2 x h_{1L}^{\perp a}(x,\mathbf{p}_T^2)\frac{H_1^{\perp a}(z,\mathbf{K}_T^2)}{z}, \tag{24}$$

where $\mathbf{e}_h = \mathbf{P}_{h\perp}/P_{h\perp}$ and $m_h$ denotes the mass of the produced hadron.

Had the events in the numerator of (21) been weighted by $P_{h\perp}^2/(M_N m_h)$ in addition to $\sin(2\phi)$, the convolution integral could be solved in a model-independent way with the result given in terms of the transverse moment (5) of $h_{1L}^\perp$ and an analog moment for $H_1^\perp$ [8]. Including such an additional weight makes data analysis more difficult due to acceptance effects. Omitting it, however, forces one to resort to models.

We shall assume the distributions of transverse parton momenta to be Gaussian (and the respective widths $\langle\mathbf{p}_{h_{1L}}^2\rangle$ and $\langle\mathbf{K}_{H_1}^2\rangle$ to be flavor and $x$- or $z$-independent):

$$h_{1L}^{\perp a}(x,\mathbf{p}_T^2) \equiv h_{1L}^{\perp a}(x)\frac{\exp(-\mathbf{p}_T^2/\langle\mathbf{p}_{h_{1L}}^2\rangle)}{\pi\langle\mathbf{p}_{h_{1L}}^2\rangle},$$
$$H_1^{\perp a}(z,\mathbf{K}_T^2) \equiv H_1^{\perp a}(z)\frac{\exp(-\mathbf{K}_T^2/\langle\mathbf{K}_{H_1}^2\rangle)}{\pi\langle\mathbf{K}_{H_1}^2\rangle}. \tag{25}$$

The normalizations are such that one obtains for the unpolarized functions $f_1^a(x) = \int \mathrm{d}^2\mathbf{p}_T f_1^a(x,\mathbf{p}_T)$ and $D_1^a(z) = \int \mathrm{d}^2\mathbf{K}_T D_1^a(z,\mathbf{K}_T)$ with analog Ansätze.

The Gauss Ansatz satisfactorily describes data on many hard reactions [77], provided the transverse momenta are much smaller than the hard scale of the process, i.e.

$\langle P_{h\perp}\rangle \ll \langle Q\rangle$ which is the case at HERMES, see (20). In fact, the $z$-dependence of $\langle P_{h\perp}\rangle$ at HERMES [19] is well described in the Gauss Ansatz [78].

Of course, one has to keep in mind that (25) is a crude approximation, and it is not clear whether it works also for polarized distribution and fragmentation functions. Moreover, since also unintegrated forms of (3) and (4) hold, this Ansatz cannot be equally valid for all pdfs.

What is convenient for our purposes is that (25) allows one to solve the convolution integral (24). We obtain

$$F_{UL}^{\sin 2\phi}(x) = \sum_a e_a^2 x h_{1L}^{\perp(1)a}(x)\langle C_{\rm Gauss} H_1^{\perp(1/2)a}\rangle. \qquad (26)$$

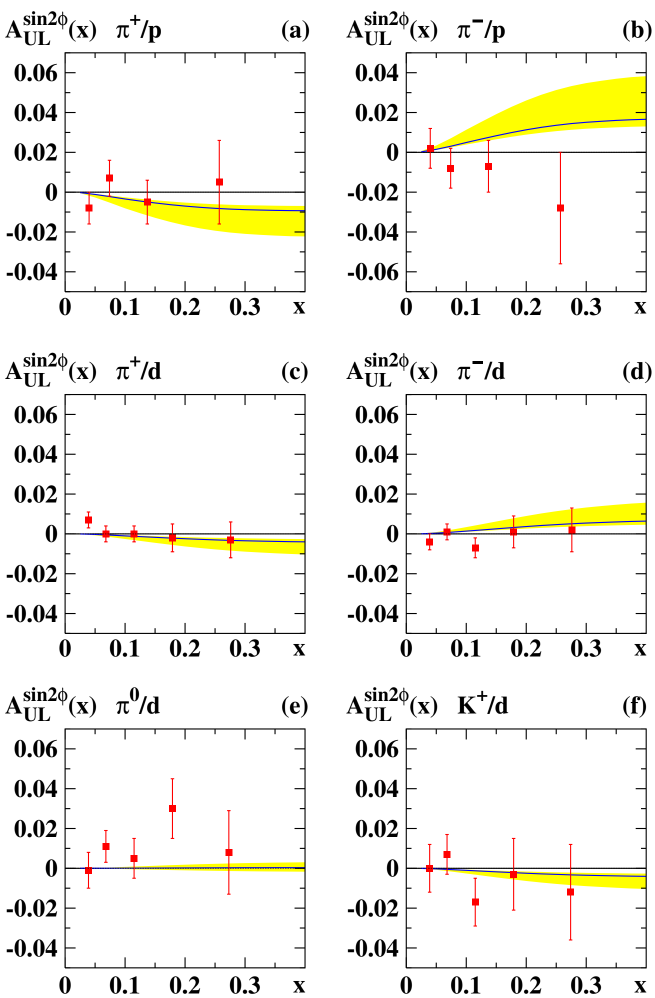


FIG. 3 (color online). Longitudinal target SSA $A_{UL}^{\sin 2\phi}$ as a function of $x$. The proton (a, b) and deuterium (c–f) target data are from HERMES [17,19]. The theoretical curves are obtained using information on the Collins fragmentation function from [55,56], predictions from the instanton vacuum model and chiral quark-soliton model for $h_L^a(x)$ and $h_1^a(x)$ [53,64], and—this is crucial in our context—assuming the validity of the WW-type approximation (11). The shaded error bands are due to the uncertainties in (27) and (28), see Appendix A for details.

The 1/2-transverse-moment $H_1^{\perp(1/2)a}(z)$ and $C_{\rm Gauss}(z)$, which is also a function of the Gauss model parameters, are defined in App. A. On the basis of the information on the Collins effect from the analyses [54–56] we estimate

$$\langle C_{\rm Gauss} H_1^{\perp(1/2){\rm fav}}\rangle \approx (0.035 \pm 0.008) \times (2.2^{+2.1}_{-0.1}), \qquad (27)$$

$$\langle C_{\rm Gauss} H_1^{\perp(1/2){\rm unf}}\rangle \approx -(0.038 \pm 0.007) \times (2.2^{+2.1}_{-0.1}). \qquad (28)$$

The first factors, with errors due to statistical accuracy of the (preliminary) HERMES data [23], are from [55]. The second factors are due to the transverse momentum dependence of the Collins function; their sizeable uncertainties reflect that the latter is presently poorly constrained by data [56]. See Appendix A for details.

The errors in (27) and (28) are estimated conservatively, such that deviations from our predictions for the SSA should be attributed alone to the failure of (11).

For the estimate of $h_{1L}^{\perp(1)a}(x)$ by means of (11) we use predictions of the chiral quark-soliton model for $h_1^a(x)$ [64] as shown in Fig. 1(c), see Sec. II.

Our results shown in Figs. 3(a)–3(e) for pion production from proton and deuteron targets are consistent with the HERMES data [17–19], and do not exclude that (11) is a useful approximation.

Figure 3(f) shows also the SSA for $K^+$ production. Also here our result is compatible with data [19]; however, in this case one tests in addition assumptions on the kaon Collins effect, see Appendix B.

It is clear that using other transversity models to estimate $h_{1L}^{\perp}$, one would arrive at the same conclusions, though at quantitatively somewhat different estimates. The spread of predictions for transversity from the various models in Fig. 1(a) gives roughly some flavor on the spread of estimates for $A_{UL}^{\sin 2\phi}$ from those models.

## V. $A_{UL}^{\sin 2\phi}$ AT CLAS

One may roughly expect $|A_{UL}^{\sin 2\phi}| \lesssim \frac{1}{5}|A_{UT}^{\sin(\phi+\phi_S)}|$ on the basis of the approximation (11), see Appendix A. Thus, $A_{UL}^{\sin 2\phi}$ could be far more difficult to measure than the transverse target Collins effect SSA. Therefore what is needed is a high luminosity experiment sensitive to the region $0.2 \lesssim x \lesssim 0.5$, where the suppression of $h_{1L}^{\perp(1)a}$ with respect to $h_1^a(x)$ is less pronounced.

Higher statistics at CLAS at Jefferson Lab, due to 2 orders of magnitude higher luminosity, provides access to much larger $x$ and larger $z$ than HERMES and COMPASS. Large $z$ may also enhance the SSA due to the Collins function $H_1^{\perp(1/2)a}(z) \propto z D_1^a(z)$, as observed in [55]. This makes CLAS an ideal experiment for studies of this SSA, in particular, and spin-orbit correlations in general.

Comparison of the various data sets will also allow one to draw valuable conclusions on the energy dependence of the process, possible power-corrections, etc.

The preliminary data from CLAS [25] have shown nonzero SSAs for charged pions, and a compatible with zero within error bars result for $\pi^0$. Within our approach it is possible to understand the results for $\pi^+$ and $\pi^0$; however, we obtain for $\pi^-$ an opposite sign compared to the data. In view of this observation, it is worth looking again at Fig. 3(b) which shows HERMES data on the $\pi^-$-SSA. Does Fig. 3(b) hint at an incompatibility? Charged pions and, in particular, the $\pi^-$ (the latter simply because it has the lowest production rate in DIS) may have significant higher twist contributions, in particular, from exclusive vector mesons and semi-exclusive pion production at large $z$.

New data expected from CLAS with $E_{\rm beam} = 6$ GeV [79] will increase the existing statistics by about an order of magnitude and more importantly provide comparable to $\pi^+$ sample of $\pi^0$ events. The neutral pion sample is not expected to have any significant contribution from exclusive vector mesons, neither is it expected to have significant higher twist corrections due to semi-exclusive production of pions with large $z$ [80], where the separation between target and current fragmentation is more pronounced.

Higher statistics of upcoming CLAS runs at 6 [79] and 12 GeV [81] will provide access also to higher values of $Q^2$ where contributions from exclusive and semi-exclusive processes are more suppressed.

JLab upgrade to 12 GeV will allow to run at an order of magnitude higher luminosities than current CLAS, providing a comprehensive set of single and double spin asymmetries covering a wide range in $x$ and $z$. That will allow detailed studies of kinematic dependences of target SSAs and clarify the situation.

## VI. $A_{UL}^{\sin 2\phi}$ AT COMPASS

COMPASS has taken data with longitudinally (and transversely) polarized deuterium and proton targets which are being analyzed. The 160 GeV muon beam available at COMPASS allows one to extend the measurements of $A_{UL}^{\sin 2\phi}$ and other SSAs into the small $x$-region. By combining all data for $Q^2 > 1$ GeV$^2$, the average $\langle Q^2\rangle$ at COMPASS is a bit higher than that at HERMES. Therefore, the curves in Figs. 3(a)–3(f) show roughly our predictions for COMPASS.

From (16) and (17) one may expect $A_{UL}^{\sin 2\phi}$ to be substantially smaller, especially at small $x$, than the transverse target SSA $A_{UT}^{\sin(\phi+\phi_S)}$ found compatible with zero in the COMPASS deuterium target experiment [22,24,30].

It will be interesting to see whether these predictions will be confirmed by COMPASS.

## VII. CONCLUSIONS

The longitudinal SSA [17–20] were subject to intensive, early studies [82–86] that were based on assumptions concerning the flavor dependence of $H_1^\perp$ [87–89] that turned out not to be supported by data on the Collins effect from SIDIS with transverse target polarization [21–24] and $e^+e^-$-annihilations [28,29]. These data give rise to a new, consistent picture of $H_1^\perp$ [54–56] which invites reanalyses of longitudinal SSAs.

In this work we did this for $A_{UL}^{\sin 2\phi} \propto \sum_a e_a^2 h_{1L}^{\perp(1)a} H_1^{\perp a}$ from the particular point of view of the question of whether there are useful, approximate relations among different TMD pdfs. In fact, QCD equations of motion relate the pdf entering this SSA to $h_L^a(x)$ and certain pure twist-3 (and quark mass) terms. Neglecting such terms yields an approximation for $h_{1L}^{\perp(1)a}$ similar in spirit to the WW-approximation for $g_T^a(x)$ that is supported by data.

Our study reveals that data do not exclude the possibility that such WW-type approximations work. As a byproduct we observe that data on the two SSAs due to Collins effect, $A_{UL}^{\sin 2\phi}$ and $A_{UT}^{\sin(\phi+\phi_S)}$, are compatible.

In Ref. [58] predictions for $A_{LT}^{\cos(\phi-\phi_S)} \propto \sum_a e_a^2 g_{1T}^{(1)a} D_1^a$ were made assuming the validity of a WW-type approximation for the relevant pdf. Comparing these predictions to preliminary COMPASS data [32] one arrives at the same conclusion. Also here data do not exclude the possibility that the WW-type approximation works.

In order to make more definite statements precise measurements of these SSAs are necessary, preferably in the region around $x \sim 0.3$ where the SSAs are largest. An order of magnitude more data on target SSA expected from the upcoming CLAS run [79] will certainly improve our current understanding of this and other SSAs and shed light on spin-orbit correlations.

The value of a precise $A_{UL}^{\sin 2\phi}$ should not be underestimated. This SSA is in any case an independent source of information on the Collins effect. An experimental confirmation of the utility of the WW-type approximation (11), however, would mean that it is possible to extract information on transversity, via (11), from a longitudinally polarized target.

## ACKNOWLEDGMENTS

We thank M. Anselmino, B. Pasquini, A. Prokudin, and I. Savin for valuable discussions. The work is partially supported by BMBF (Verbundforschung), German-Russian collaboration (DFG-RFFI) under Contract No. 436 RUS 113/881/0 and is part of the European Integrated Infrastructure Initiative Hadron Physics project under Contract No. RII3-CT-2004-506078. A. E. is also supported by the Grants RFBR 06-02-16215 and 07-02-91557, RF MSE RNP.2.2.2.2.6546 (MIREA), and by the Heisenberg-Landau Program of JINR. The work was supported in part by DOE Contract No. DE-AC05-06OR23177, under which Jefferson Science Associates, LLC, operates the Jefferson Lab. T. T. is supported by the Cusanus-Werk.

## APPENDIX A: PION COLLINS EFFECT

Within the Gauss model one can, of course, rewrite the expression for the SSA (12) in many ways. However, we are interested in exploring the approximation (11) and wish to introduce the transverse moment (5) of $h_{1L}^{\perp a}$ which in the Gauss model is given by

$$h_{1L}^{\perp(1)a}(x) \overset{\text{Gauss}}{=} \frac{\langle \mathbf{p}_{h_{1L}}^2 \rangle}{2M_N^2} h_{1L}^{\perp a}(x). \tag{A1}$$

In order to use information on the Collins function from the analysis of HERMES data [23] in Ref. [55] (the reasons why here this is preferable are explained in Sec. IV) we introduce the (1/2)-transverse moment of $H_1^\perp$ which is defined as and given in Gauss model by

$$\begin{aligned} H_1^{\perp(1/2)a}(z) &\equiv \int \mathrm{d}^2\mathbf{K}_T \frac{|\mathbf{K}_T|}{2zm_\pi} H_1^{\perp a}(z, \mathbf{K}_T) \\ &\overset{\text{Gauss}}{=} \frac{\sqrt{\pi}\langle \mathbf{K}_{H_1}^2\rangle^{1/2}}{4m_\pi z} H_1^{\perp a}(z). \end{aligned} \tag{A2}$$

With the above definitions the numerator of $A_{UL}^{\sin 2\phi}$ is given by (26) with the function $C_{\text{Gauss}}$ defined as

$$C_{\text{Gauss}}(z) = \frac{8zM_N}{(\pi\langle \mathbf{K}_{H_1}^2\rangle)^{1/2}} \frac{1}{1 + z^2\langle \mathbf{p}_{h_{1L}}^2\rangle / \langle \mathbf{K}_{H_1}^2\rangle}. \tag{A3}$$

In [55] the following information on the Collins effect was obtained from HERMES data [23] on the SSA (14):

$$\langle 2B_{\text{Gauss}} H_1^{\perp(1/2)\text{fav}} \rangle = (3.5 \pm 0.8)\%, \tag{A4}$$

$$\langle 2B_{\text{Gauss}} H_1^{\perp(1/2)\text{unf}} \rangle = -(3.8 \pm 0.7)\%, \tag{A5}$$

with

$$B_{\text{Gauss}} = \frac{1}{\sqrt{1 + z^2\langle \mathbf{p}_{h_1}^2\rangle / \langle \mathbf{K}_{H_1}^2\rangle}}, \tag{A6}$$

where $\langle \mathbf{p}_{h_1}^2 \rangle$ is the Gaussian width of the transversity pdf.

In order to use the results (A4) and (A5) we approximate

$$\langle C_{\text{Gauss}} H_1^{\perp(1/2)a} \rangle \approx \frac{4\langle z\rangle M_N}{(\pi\langle \mathbf{K}_{H_1}^2\rangle)^{1/2}} \underbrace{\left\langle \frac{2H_1^{\perp(1/2)a}}{1 + z^2\langle \mathbf{p}_{h_{1L}}^2\rangle / \langle \mathbf{K}_{H_1}^2\rangle} \right\rangle}_{\approx \langle 2B_{\text{Gauss}} H_1^{\perp(1/2)a}\rangle}. \tag{A7}$$

For $\langle \mathbf{K}_{H_1}^2 \rangle$ we use results from [56] where the Collins function was also assumed to exhibit a Gaussian $k_T$-dependence. In the notation of [56] one has

$$\frac{1}{\langle \mathbf{K}_{H_1}^2\rangle} = \frac{1}{\langle \mathbf{K}_{D_1}^2\rangle} + \frac{1}{M^2} \tag{A8}$$

where the width of the unpolarized fragmentation function was fixed from a study of data on the Cahn effect [90] $\langle \mathbf{K}_{D_1}^2 \rangle = 0.20\ \text{GeV}^2$. The parameter $M$ was fitted to data from SIDIS and $e^+e^-$-annihilations (neglecting evolution effects) to be $M^2 = (0.70 \pm 0.65)\ \text{GeV}^2$ [56]. This yields for the first factor in Eq. (A7)

$$\frac{4M_N\langle z\rangle}{(\pi\langle \mathbf{K}_{H_1}^2\rangle)^{1/2}} \simeq 2.2^{+2.1}_{-0.1}. \tag{A9}$$

Using for $f_1^a(x)$ and $D_1^a(z)$ the LO parameterizations [62,91] at $Q^2 = 2.5\ \text{GeV}^2$ gives the results in Fig. 3.

A remark concerning the error estimates in Fig. 3 is in order. Strictly speaking the errors in (A4), (A5), and (A9) are not independent but correlated which we disregard. This means that the errors in Fig. 3 are somewhat overestimated. In view of the approximations we make, however, this is not undesired, as it helps to estimate the errors more conservatively. With such more conservative error estimates we are on the safe side from the point of view of testing the WW-type approximation (11). In fact, a deviation of our results from data would then presumably be due to a failure of the approximation (11).

We notice the following rough estimate. From (17) and the mean value in (A9) one may estimate roughly

$$|A_{UL}^{\sin 2\phi}| \lesssim \tfrac{1}{5}|A_{UT}^{\sin(\phi+\phi_S)}|, \tag{A10}$$

as other factors in the two SSAs are either the same or of similar magnitude.

## APPENDIX B: KAON COLLINS EFFECT

We also wish to estimate the SSA for $K^+$. For that we notice that, since pions and kaons are both Goldstone bosons of chiral symmetry breaking, one has in the chiral limit

$$\lim_{m_K\to 0} \frac{H_1^{\perp(1/2)a/K}}{D_1^{a/K}} = \lim_{m_\pi\to 0} \frac{H_1^{\perp(1/2)a/\pi}}{D_1^{a/\pi}}. \tag{B1}$$

This implies that in the real world with explicit chiral symmetry breaking, i.e. for nonzero pion- and kaon-masses $m_\pi$ and $m_K$, one may assume the following relations to hold approximately

$$\begin{gathered} \frac{H_1^{\perp(1/2)\bar{s}/K^+}}{D_1^{\bar{s}/K^+}} \approx \frac{H_1^{\perp(1/2)u/K^+}}{D_1^{u/K^+}} \approx \frac{H_1^{\perp(1/2)u/\pi^+}}{D_1^{u/\pi^+}}, \\ \frac{H_1^{\perp(1/2)\text{unf}/K^+}}{D_1^{\text{unf}/K^+}} \approx \frac{H_1^{\perp(1/2)\text{unf}/\pi^+}}{D_1^{\text{unf}/\pi^+}}, \end{gathered} \tag{B2}$$

where it is understood that the fragmentation of $d$- and $\bar{u}$-flavor into $K^+$ is unfavored. The estimate (B2) relies on the assumption that "the way from the chiral limit to the real world situation'' proceeds quantitatively in a similar way for both polarization dependent and independent quantities. (Notice that the unpolarized ''favored'' $\bar{s}$- and $u$-flavor fragmentations into $K^+$ are actually different—with the latter being smaller than the former [92]. In the view of the precision of data, however, the effects of strangeness can be neglected due to the smallness of the corresponding pdfs. For example, the chiral quark-soliton model predicts a negligible strangeness contribution to transversity (more precisely: to the tensor charge) [93].)

On the basis of (B1) we estimate

$$\langle 2B_{\rm Gauss}H_1^{\perp(1/2)u/K^+}\rangle \approx (1.0 \pm 0.2)\%, \tag{B3}$$

$$\langle 2B_{\rm Gauss}H_1^{\perp(1/2){\rm unf}/K^+}\rangle \approx -(1.0 \pm 0.2)\%. \tag{B4}$$

From (B3) and (B4) we obtain after similar approximations as in Appendix A the result in Fig. 3(f).


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