Diquarks and $\Lambda_b \to \Lambda_c$ weak decays

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In this work we investigate the weak $\Lambda_b \to \Lambda_c$ semileptonic and nonleptonic decays. The light-front quark model and diquark picture for heavy baryons are adopted to evaluate the $\Lambda_b \to \Lambda_c$ transition form factors. In the heavy quark limit we study the Isgur-Wise function. The transition form factors are obtained in the whole physical momentum regions. The numerical predictions on the branching ratios of nonleptonic decay modes $\Lambda_b \to \Lambda_c M$ and various polarization asymmetries are made. A comparison with other approaches is discussed.

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I. INTRODUCTION

The weak decays of Λ_b provide valuable information of the Cabibbo-Kobayashi-Maskawa (CKM) parameter V_{cb} and serve as an ideal laboratory to study nonperturbative QCD effects in the heavy baryon system. Recently, the DELPHI collaboration reported their measurement on the slope parameter ρ^2 in the Isgur-Wise function and the branching ratio of the exclusive semileptonic process $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ [1]. This experimental measurement reexcites great theoretical interests in semileptonic decays of Λ_b [2–7]. From PDG06 [8], signals of several nonleptonic processes, such as $\Lambda_b \to \Lambda_c \pi$, $\Lambda_c a_1$ (1260) have been observed. These processes are a good probe to test the factorization hypothesis which has been extensively explored in the B meson case [9]. The forthcoming LHCb project is expected to accumulate large samples of the b-flavor hadrons and offer an opportunity to study in detail the Λ_b decays. Thus, it probably is the time to investigate the Λ_b weak decays more systematically. In this work, we will concentrate on the $\Lambda_b \to \Lambda_c$ semileptonic and nonleptonic decays.

As in the B meson decays, the key for correct evaluation on the rate of the semileptonic decays is how to properly derive the hadronic matrix element which is parametrized by the $\Lambda_b \to \Lambda_c$ transition form factors. There are various approaches in the market to evaluate these form factors: the QCD sum rules [5], quark models [2,6,7,10], perturbative QCD approach [3,4] etc. In this work, we will study the heavy baryon form factors in the light-front quark model. The light-front quark model is a relativistic quark model based on the light-front QCD [11]. The basic ingredient is the hadron light-front wave function which is explicitly Lorentz invariant. The hadron spin is constructed using the Melosh rotation. The light-front approach has been widely applied to calculate various decay constants and form factors for the meson cases [12–16]. Different from the case discussed in [10] where the light-front quark model was also employed, we adopt the diquark picture for baryons. It has been known for a long time that two quarks in a color-antitriplet state attract each other and may form a correlated diquark [17]. Indeed, the diquark picture of baryons is considered to be appropriate for the low momentum transfer processes [18–21]. In the conventional quark model, the heavy baryon is composed of one heavy quark and two light quarks. The light spectator quarks participate only in the soft interactions as Λ_b transits into Λ_c; hence it is reasonable to employ the diquark picture for heavy baryons where the diquark serves as a whole spectator. Concretely, under the diquark approximation, Λ_b and Λ_c are of the one-heavy-quark-one-light-diquark (ud) structure which is analogous to the meson case and a considerable simplification in the calculations is expected. In fact, some nonperturbative interactions between the two light quarks can be effectively absorbed into the constituent diquark mass. In this phenomenological study, we use the rate of the semileptonic process $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ which will be accurately measured at LHCb and future ILC, to constrain the light scalar diquark mass.

For the nonleptonic two-body decays $\Lambda_b \to \Lambda_c M$ where M denote light mesons, the amplitude is factorized to a product of the meson decay constant and $\Lambda_b \to \Lambda_c$ transition form factors by the factorization assumption. Because there only a color-allowed diagram is involved, the factorization assumption is believed to be reliable in the B meson case [9]. However, the theoretical predictions on the nonleptonic two-body decays vary by a factor of 2–3 for various models. The main theoretical uncertainties arise from the model evaluations of the form factors. In order to reduce model dependence and obtain a more reliable prediction, we study the semileptonic decays and nonleptonic processes simultaneously. The present experimental data of the semileptonic decays (although the errors are still quite sizable) set constraints on the phenomenological parameters in the light-front approach. With these parameters, even though not very accurate yet, we evaluate the $\Lambda_b \to \Lambda_c$ form factors and make predictions on the widths of the semileptonic decay $\Lambda_b \to \Lambda_c + l\bar{\nu}$ and nonleptonic two-body decay $\Lambda_b \to \Lambda_c + M$ where M is a meson.

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We organize our paper as follows. In Sec. II, we formulate the form factors for the transition $\Lambda_b \to \Lambda_c$ in the light-front approach. We will show that in the heavy quark limit, the resultant form factors are related to one universal Isgur-Wise function. In Sec. III, the formulations of the decay ratios and the polarizations for the semileptonic and nonleptonic two-body decays are given. In Sec. IV, we present our numerical results and all relevant input parameters are given explicitly. We then compare our numerical results with the predictions by other approaches. Finally, Sec. V is devoted to conclusion and discussion.

II. $\Lambda_b \to \Lambda_c$ TRANSITION FORM FACTORS IN THE LIGHT-FRONT APPROACH

In the diquark picture, the heavy baryon $\Lambda_{b(c)}$ is composed of one heavy quark $b(c)$ and a light diquark $[ud]$. In order to form a color singlet hadron, the diquark $[ud]$ is in a color antitriplet. Because $\Lambda_{b(c)}$ is at the ground state, the diquark is in a 0^+ scalar state ($s=0$, $l=0$) and the orbital angular momentum between the diquark and the heavy quark is also zero, i.e. $L=l=0$.

A. Heavy baryon in the light-front approach

In the light-front approach, the heavy baryon Λ_Q with total momentum P, spin $S=1/2$, and scalar diquark can be written as

$$|\Lambda_Q(P,S,S_z)\rangle = \int\{d^3p_1\}\{d^3p_2\}2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2)\times\sum_{\lambda_1}\Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\lambda_1)C^{\alpha}_{\beta\gamma}F^{bc}\times|Q_\alpha(p_1,\lambda_1)[q^{\beta}_{1b}q^{\gamma}_{2c}](p_2)\rangle, \tag{1}$$

where Q represents b or c, $[q_1q_2]$ represents $[ud]$, λ denotes helicity, p_1, p_2 are the on-mass-shell light-front momenta defined by

$$\tilde{p}=(p^+,p_\perp),\qquad p_\perp=(p^1,p^2),\qquad p^-=\frac{m^2+p_\perp^2}{p^+}, \tag{2}$$

and

$$\begin{aligned}\{d^3p\} &\equiv \frac{dp^+d^2p_\perp}{2(2\pi)^3},\\ \delta^3(\tilde{p}) &= \delta(p^+)\delta^2(p_\perp),\\ |Q(p_1,\lambda_1)[q_1q_2](p_2)\rangle &= b^{\dagger}_{\lambda_1}(p_1)a^{\dagger}(p_2)|0\rangle,\\ [a(p'),a^{\dagger}(p)] &= 2(2\pi)^3\delta^3(\tilde{p}'-\tilde{p}),\\ \{d_{\lambda'}(p'),d^{\dagger}_{\lambda}(p)\} &= 2(2\pi)^3\delta^3(\tilde{p}'-\tilde{p})\delta_{\lambda'\lambda}.\end{aligned} \tag{3}$$

The coefficient $C^{\alpha}_{\beta\gamma}$ is a normalized color factor and F^{bc} is a normalized flavor coefficient,

$$C^{\alpha}_{\beta\gamma}F^{bc}C^{\alpha'}_{\beta'\gamma'}F^{b'c'}\langle Q_{\alpha'}(p'_1,\lambda'_1)[q^{\beta'}_{1b'}q^{\gamma'}_{2c'}](p'_2)|Q_\alpha(p_1,\lambda_1)\times[q^{\beta}_{1b}q^{\gamma}_{2c}](p_2)\rangle = 2^2(2\pi)^6\delta^3(\tilde{p}'_1-\tilde{p}_1)\delta^3(\tilde{p}'_2-\tilde{p}_2)\delta_{\lambda'_1\lambda_1}. \tag{4}$$

In order to describe the internal motion of the constituents, one needs to introduce intrinsic variables $(x_i,k_{i\perp})$ with $i=1,2$ through

$$\begin{aligned}&p_1^+=x_1P^+,\qquad p_2^+=x_2P^+,\qquad x_1+x_2=1,\\ &p_{1\perp}=x_1P_\perp+k_{1\perp},\qquad p_{2\perp}=x_2P_\perp+k_{2\perp},\\ &k_\perp=-k_{1\perp}=k_{2\perp},\end{aligned} \tag{5}$$

where x_i are the light-front momentum fractions satisfying $0<x_1,x_2<1$. The variables $(x_i,k_{i\perp})$ are independent of the total momentum of the hadron and thus are Lorentz-invariant variables. The invariant mass square M_0^2 is defined as

$$M_0^2=\frac{k_{1\perp}^2+m_1^2}{x_1}+\frac{k_{2\perp}^2+m_2^2}{x_2}. \tag{6}$$

The invariant mass M_0 is in general different from the hadron mass M which satisfies the physical mass-shell condition $M^2=P^2$. This is due to the fact that the baryon, heavy quark, and diquark cannot be on their mass shells simultaneously. We define the internal momenta as

$$k_i=(k_i^-,k_i^+,k_{i\perp})=(e_i-k_{iz},e_i+k_{iz},k_{i\perp})=\left(\frac{m_i^2+k_{i\perp}^2}{x_iM_0},x_iM_0,k_{i\perp}\right). \tag{7}$$

It is easy to obtain

$$\begin{aligned}M_0&=e_1+e_2,\\ e_i&=\frac{x_iM_0}{2}+\frac{m_i^2+k_{i\perp}^2}{2x_iM_0}=\sqrt{m_i^2+k_{i\perp}^2+k_{iz}^2},\\ k_{iz}&=\frac{x_iM_0}{2}-\frac{m_i^2+k_{i\perp}^2}{2x_iM_0},\end{aligned} \tag{8}$$

where e_i denotes the energy of the ith constituent. The momenta $k_{i\perp}$ and k_{iz} constitute a momentum vector $\vec{k}_i=(k_{i\perp},k_{iz})$ and correspond to the components in the transverse and z directions, respectively.

The momentum-space function Ψ^{SS_z} in Eq. (1) is expressed as

$$\Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\lambda_1)=\langle\lambda_1|\mathcal{R}_M^{\dagger}(x_1,k_{1\perp},m_1)|s_1\rangle\times\langle 00;\tfrac{1}{2}s_1|\tfrac{1}{2}S_z\rangle\phi(x,k_\perp), \tag{9}$$

where $\phi(x,k_\perp)$ is the light-front wave function which describes the momentum distribution of the constituents in the bound state with $x=x_2$, $k_\perp=k_{2\perp}$; $\langle 00;\frac{1}{2}s_1|\frac{1}{2}S_z\rangle$ is the corresponding Clebsch-Gordan coefficient with spin $s=s_z=0$ for the scalar diquark;

$\langle\lambda_1|\mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1)|s_1\rangle$ is the well-known Melosh transformation matrix element which transforms the conventional spin states in the instant form into the light-front helicity eigenstates,

$$\langle\lambda_1|\mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1)|s_1\rangle = \frac{\bar{u}(k_1, \lambda_1)u_D(k_1, s_1)}{2m_1} = \frac{(m_1 + x_1 M_0)\delta_{\lambda_1 s_1} + i\vec{\sigma}_{\lambda_1 s_1} \cdot \vec{k}_{1\perp} \times \vec{n}}{\sqrt{(m_1 + x_1 M_0)^2 + k_{1\perp}^2}}, \tag{10}$$

where $u_{(D)}$ denotes a Dirac spinor in the light-front (instant) form and $\vec{n} = (0, 0, 1)$ is a unit vector in the z direction. In practice, it is more convenient to use the covariant form for the Melosh transform matrix [12,15]

$$\langle\lambda_1|\mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1)|s_1\rangle\left\langle 00; \frac{1}{2}s_1 \,\middle|\, \frac{1}{2}S_z\right\rangle = \frac{1}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}}\bar{u}(p_1, \lambda_1)\Gamma u(\bar{P}, S_z), \tag{11}$$

where

$$\Gamma = 1, \qquad \bar{P} = p_1 + p_2 \tag{12}$$

for the scalar diquark. If the diquark is a vector which is usually supposed to be the case for the $\Sigma_{c(b)}$ baryon, the Melosh transform matrix should be modified.

The heavy baryon state is normalized as

$$\langle\Lambda(P', S', S_z')|\Lambda(P, S, S_z)\rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P})\delta_{S'S}\delta_{S_z'S_z}. \tag{13}$$

Thus, the light-front wave function satisfies the constraint

$$\int \frac{dx d^2 k_\perp}{2(2\pi^3)}|\phi(x, k_\perp)|^2 = 1. \tag{14}$$

In principle, the wave functions can be obtained by solving the light-front bound state equations. However, it is too hard to calculate them based on the first principle, so that instead, we utilize a phenomenological function, and the Gaussian form would be the most preferable one,

$$\phi(x, k_\perp) = N\sqrt{\frac{\partial k_{2z}}{\partial x_2}}\exp\left(\frac{-\vec{k}^2}{2\beta^2}\right), \tag{15}$$

with

$$N = 4\left(\frac{\pi}{\beta^2}\right)^{3/4}, \qquad \frac{\partial k_{2z}}{\partial x_2} = \frac{e_1 e_2}{x_1 x_2 M_0}, \tag{16}$$

where β determines the confinement scale of hadron. The phenomenological parameters in the light-front quark model are quark masses and the hadron wave function parameter β which should be prior determined before numerical computations can be carried out and we will do the job in the later subsections.

B. $\Lambda_b \to \Lambda_c$ transition form factors

The form factors for the weak transition $\Lambda_Q \to \Lambda_{Q'}$ are defined in the standard way as

$$\begin{aligned}\langle\Lambda_{Q'}(P', S', S_z')|\bar{Q}'\gamma_\mu(1 - \gamma_5)Q|\Lambda_Q(P, S, S_z)\rangle &= \bar{u}_{\Lambda_{Q'}}(P', S_z')\left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Lambda_Q}}f_2(q^2) + \frac{q_\mu}{M_{\Lambda_Q}}f_3(q^2)\right]u_{\Lambda_Q}(P, S_z) \\ &\quad - \bar{u}_{\Lambda_{Q'}}(P', S_z')\left[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Lambda_Q}}g_2(q^2) + \frac{q_\mu}{M_{\Lambda_Q}}g_3(q^2)\right] \\ &\quad \times \gamma_5 u_{\Lambda_{Q'}}(P, S_z),\end{aligned} \tag{17}$$

where $q \equiv P - P'$, Q and Q' denote b and c, respectively. The above formulation is the most general expression with only constraint of keeping the Lorentz invariance and parity conservation for strong interactions. There are six form factors f_i, g_i in total for the vector and axial-vector current $\bar{c}\gamma_\mu(1 - \gamma_5)b$ and all the information on the strong interaction is involved in them. Since $S = S' = 1/2$, we will be able to write $|\Lambda_Q(P, S, S_z')\rangle$ as $|\Lambda_Q(P, S_z)\rangle$ in the following formulations. A parametrization is more convenient for the heavy-to-heavy transitions, which is written in terms of the four-velocities as

$$\begin{aligned}\langle\Lambda_{Q'}(v', S_z')|\bar{Q}'\gamma_\mu(1 - \gamma_5)Q|\Lambda_Q(v, S_z)\rangle &= \bar{u}_{\Lambda_{Q'}}(v', S_z')[F_1(\omega)\gamma_\mu + F_2(\omega)v_\mu + F_3(\omega)v_\mu']u_{\Lambda_Q}(v, S_z) \\ &\quad - \bar{u}_{\Lambda_{Q'}}(v', S_z')[G_1(\omega)\gamma_\mu + G_2(\omega)v_\mu + G_3(\omega)v_\mu']\gamma_5 u_{\Lambda_Q}(v, S_z),\end{aligned} \tag{18}$$

where $v = \frac{P}{M_{\Lambda_Q}}$, $v' = \frac{P'}{M_{\Lambda_{Q'}}}$, and $\omega = v \cdot v'$. The relation between the two methods is

$$F_1 = f_1 - \frac{M_{\Lambda_Q} + M_{\Lambda_{Q'}}}{M_{\Lambda_Q}} f_2, \qquad F_2 = f_2 + f_3,$$
$$F_3 = \frac{M_{\Lambda_{Q'}}}{M_{\Lambda_Q}}(f_2 - f_3), \qquad G_1 = g_1 + \frac{M_{\Lambda_Q} - M_{\Lambda_{Q'}}}{M_{\Lambda_Q}} g_2,$$
$$G_2 = g_2 + g_3, \qquad G_3 = \frac{M_{\Lambda_{Q'}}}{M_{\Lambda_Q}}(g_2 - g_3). \tag{19}$$

The lowest order Feynman diagram for the $\Lambda_b \to \Lambda_c$ weak decay is depicted in Fig. 1. In [22], the light-front quark model for heavy pentaquark with one heavy quark and two light diquarks is presented. In our case, the heavy baryon $\Lambda_{b(c)}$ is composed of a heavy quark $b(c)$ and one diquark $[ud]$. Thus, most of our formulations are similar to that in [22] except there is only one diquark in our case.

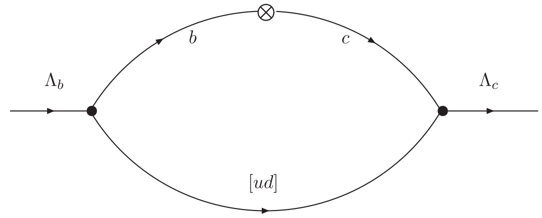

FIG. 1. Feynman diagram for $\Lambda_b \to \Lambda_c$ transitions, where $\otimes$ denotes $V - A$ current vertex.

Now, we are ready to calculate the weak transition matrix elements. Using the light-front quark model description of $|\Lambda_Q(P, S, S_z)\rangle$, we obtain

$$\langle \Lambda_{Q'}(P', S'_z)|\bar{Q}'\gamma^\mu(1-\gamma_5)Q|\Lambda_Q(P, S_z)\rangle = \int\{d^3p_2\}\frac{\phi^*_{\Lambda_{Q'}}(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{p_1^+ p_1'^+(p_1\cdot\bar{P} + m_1M_0)(p'_1\cdot\bar{P}' + m'_1M'_0)}} \times \bar{u}(\bar{P}', S'_z)\bar{\Gamma}'_{Lm}(\not{p}'_1 + m'_1)\gamma^\mu(1-\gamma_5)(\not{p}_1 + m_1)\Gamma u(\bar{P}, S_z), \tag{20}$$

where

$$\bar{\Gamma}' = \gamma_0\Gamma\gamma_0 = \Gamma = 1, \qquad m_1 = m_b, \qquad m'_1 = m_c, \qquad m_2 = m_{[\mathrm{ud}]}, \tag{21}$$

and P and P' denote the momenta of initial and final baryons, p_1, p'_1 are the momenta of b and c quarks, respectively. Because the diquark is a scalar, one does not need to deal with spinors which make computations more complex. In this framework, at each effective vertex, only the three-momentum rather than the four-momentum is conserved, hence $\tilde{p}_1 - \tilde{p}'_1 = \tilde{q}$ and $\tilde{p}_2 = \tilde{p}'_2$. From $\tilde{p}_2 = \tilde{p}'_2$, we have

$$x' = \frac{P^+}{P'^+}x, \qquad k'_\perp = k_\perp + x_2 q_\perp, \tag{22}$$

with $x = x_2$, $x' = x'_2$. Thus, Eq. (20) is rewritten as

$$\langle \Lambda_{Q'}(P', S'_z)|\bar{Q}'\gamma^\mu(1-\gamma_5)Q|\Lambda_Q(P, S_z)\rangle = \int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{x_1x'_1(p_1\cdot\bar{P} + m_1M_0)(p'_1\cdot\bar{P}' + m'_1M'_0)}} \times \bar{u}(\bar{P}', S'_z)(\not{p}'_1 + m'_1)\gamma^\mu(1-\gamma_5)(\not{p}_1 + m_1)u(\bar{P}, S_z). \tag{23}$$

Following [23], the Dirac and Pauli form factors can be derived from the helicity-conserving and helicity-flip matrix elements of the plus component of the vector current operators in the light-front framework. Analogously, the form factors corresponding to axial-vector current are obtained by the authors of [24]. In this work we choose the transverse frame where $q^+ = 0$, $q_\perp \neq 0$ which is similar to the treatment in [22]. We then have

$$f_1(q^2) = \frac{\langle \Lambda_{Q'}(P', \uparrow)|V^+|\Lambda_Q(P, \uparrow)\rangle}{2\sqrt{P^+P'^+}}, \qquad \frac{f_2(q^2)}{M_{\Lambda_Q}} = -\frac{\langle \Lambda_{Q'}(P', \uparrow)|V^+|\Lambda_Q(P, \downarrow)\rangle}{2q_{\perp L}\sqrt{P^+P'^+}},$$
$$g_1(q^2) = \frac{\langle \Lambda_{Q'}(P', \uparrow)|A^+|\Lambda_Q(P, \uparrow)\rangle}{2\sqrt{P^+P'^+}}, \qquad \frac{g_2(q^2)}{M_{\Lambda_Q}} = -\frac{\langle \Lambda_{Q'}(P', \uparrow)|A^+|\Lambda_Q(P, \downarrow)\rangle}{2q_{\perp L}\sqrt{P^+P'^+}}, \tag{24}$$

where $q_{\perp L} = q^1_\perp - iq^2_\perp$. The above relations can be written in a compact form as

$$\begin{aligned}
\langle \Lambda_{Q'}(P', S'_z)|V^+|\Lambda_Q(P, S_z)\rangle &= 2\sqrt{P^+P'^+}\left[f_1(q^2)\delta_{S'_zS_z} + \frac{f_2(q^2)}{M_{\Lambda_Q}}(\vec{\sigma}\cdot\vec{q}_\perp\sigma^3)_{S'_zS_z}\right],\\
\langle \Lambda_{Q'}(P', S'_z)|A^+|\Lambda_Q(P, S_z)\rangle &= 2\sqrt{P^+P'^+}\left[g_1(q^2)(\sigma^3)_{S'_zS_z} + \frac{g_2(q^2)}{M_{\Lambda_Q}}(\vec{\sigma}\cdot\vec{q}_\perp)_{S'_zS_z}\right].
\end{aligned} \tag{25}$$

It is noted that the form factors $f_3(q^2)$ and $g_3(q^2)$ cannot be extracted in terms of the above method for we have imposed the condition $q^+ = 0$. However, they do not contribute to the semileptonic decays $\Lambda_b \to \Lambda_c l\bar{\nu}_l$ and vanish in the heavy quark limit.

In order to extract $f_{1,2}(q^2)$ and $g_{1,2}(q^2)$ from Eq. (23), the following identities are necessary:

$$\begin{aligned}
\frac{1}{2}\sum_{S_z,S'_z} u(\bar{P}, S_z)\delta_{S'_zS_z}\bar{u}(\bar{P}', S'_z) &= \frac{1}{4\sqrt{P^+P'^+}}(\bar{\not{P}} + M_0)\gamma^+(\bar{\not{P}}' + M'_0),\\
\frac{1}{2}\sum_{S_z,S'_z} u(\bar{P}, S_z)(\sigma^3\sigma^i_\perp)_{S'_zS_z}\bar{u}(\bar{P}', S'_z) &= -\frac{1}{4\sqrt{P^+P'^+}}(\bar{\not{P}} + M_0)\sigma^{i+}(\bar{\not{P}}' + M'_0),\\
\frac{1}{2}\sum_{S_z,S'_z} u(\bar{P}, S_z)(\sigma^3)_{S'_zS_z}\bar{u}(\bar{P}', S'_z) &= \frac{1}{4\sqrt{P^+P'^+}}(\bar{\not{P}} + M_0)\gamma^+\gamma_5(\bar{\not{P}}' + M'_0),\\
\frac{1}{2}\sum_{S_z,S'_z} u(\bar{P}, S_z)(\sigma^i_\perp)_{S'_zS_z}\bar{u}(\bar{P}', S'_z) &= -\frac{1}{4\sqrt{P^+P'^+}}(\bar{\not{P}} + M_0)\sigma^{i+}\gamma_5(\bar{\not{P}}' + M'_0).
\end{aligned} \tag{26}$$

It should be noted that $u(\bar{P}, S_z)$ is not equal to $u(P, S_z)$, but the relation $\gamma^+u(\bar{P}, S_z) = \gamma^+u(P, S_z)$ is satisfied.

From Eqs. (23), (25), and (26), the transition form factors are obtained:

$$\begin{aligned}
f_1(q^2) &= \frac{1}{8P^+P'^+}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{x_1x'_1(p_1\cdot\bar{P} + m_1M_0)(p'_1\cdot\bar{P}' + m'_1M'_0)}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}} + M_0)\gamma^+(\bar{\not{P}}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)],\\
g_1(q^2) &= \frac{1}{8P^+P'^+}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{x_1x'_1(p_1\cdot\bar{P} + m_1M_0)(p'_1\cdot\bar{P}' + m'_1M'_0)}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}} + M_0)\gamma^+\gamma_5(\bar{\not{P}}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+\gamma_5(\not{p}_1 + m_1)],\\
\frac{f_2(q^2)}{M_{\Lambda_Q}} &= -\frac{1}{8P^+P'^+q^i_\perp}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{x_1x'_1(p_1\cdot\bar{P} + m_1M_0)(p'_1\cdot\bar{P}' + m'_1M'_0)}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}} + M_0)\sigma^{i+}(\bar{\not{P}}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)],\\
\frac{g_2(q^2)}{M_{\Lambda_Q}} &= \frac{1}{8P^+P'^+q^i_\perp}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x', k'_\perp)\phi_{\Lambda_Q}(x, k_\perp)}{2\sqrt{x_1x'_1(p_1\cdot\bar{P} + m_1M_0)(p'_1\cdot\bar{P}' + m'_1M'_0)}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}} + M_0)\sigma^{i+}\gamma_5(\bar{\not{P}}' + M'_0)(\not{p}_1 + m'_1)\gamma^+\gamma_5(\not{p}_1 + m_1)],
\end{aligned} \tag{27}$$

with $i = 1, 2$. The traces can be worked out straightforwardly:

$$\frac{1}{8P^+P'^+}\mathrm{Tr}[(\bar{\not{P}} + M_0)\gamma^+(\bar{\not{P}}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)] = -(p_1 - x_1\bar{P})\cdot(p'_1 - x'_1\bar{P}') + (x_1M_0 + m_1)(x'_1M'_0 + m'_1),$$

$$\frac{1}{8P^+P'^+}\mathrm{Tr}[(\bar{\not{P}} + M_0)\gamma^+\gamma_5(\bar{\not{P}}' + M'_0)(\not{p}'_1 + m'_1)\gamma^+\gamma_5(p_1 + m_1)] = (p_1 - x_1\bar{P})\cdot(p'_1 - x'_1\bar{P}') + (x_1M_0 + m_1)(x'_1M'_0 + m'_1),$$

and

$$\frac{1}{8P^+P'^+}\mathrm{Tr}[(\bar{\not P}+M_0)\sigma^{i+}(\bar{\not P}'+M_0')(\not p_1'+m_1')\gamma^+(\not p_1+m_1)]=(m_1'+x_1'\bar{M}_0')(p_{1\perp}^i-x_1\bar{P}_\perp^i)-(m_1+x_1M_0)(p_{1\perp}'^i-x_1'\bar{P}_\perp'^i),$$
$$\frac{1}{8P^+P'^+}\mathrm{Tr}[(\bar{\not P}+M_0)\sigma^{i+}\gamma_5(\bar{\not P}'+M_0')(\not p_1'+m_1')\gamma^+\gamma_5(\not p_1+m_1)]=(m_1'+x_1'\bar{M}_0')(p_{1\perp}^i-x_1\bar{P}_\perp^i)+(m_1+x_1M_0)(p_{1\perp}'^i-x_1'\bar{P}_\perp'^i). \tag{28}$$

Using $\bar{P}^{(\prime)}=p_1^{(\prime)}+p_2^{(\prime)}$ and other momentum relations, the products of momenta in Eqs. (27) and (28) are given in terms of the internal variables as

$$p_1\cdot\bar{P}=e_1M_0=\frac{m_1^2+x_1^2M_0^2+k_{1\perp}^2}{2x_1},\qquad p_1'\cdot\bar{P}'=e_1'M_0'=\frac{m_1'^2+x_1'^2M_0'^2+k_{1\perp}'^2}{2x_1'},$$
$$p_{1\perp}^{(\prime)i}-x_1\bar{P}_\perp^{(\prime)i}=k_{1\perp}^{(\prime)i},\qquad (p_1-x_1\bar{P})\cdot(p_1'-x_1'\bar{P}')=-k_{1\perp}\cdot k_{1\perp}'. \tag{29}$$

At last, we obtain the final expressions for the $\Lambda_Q\to\Lambda_{Q'}$ weak transition form factors

$$f_1(q^2)=\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x',k_\perp')\phi_{\Lambda_Q}(x,k_\perp)[k_{2\perp}\cdot k_{2\perp}'+(x_1M_0+m_1)(x_1'M_0'+m_1')]}{\sqrt{[(m_1+x_1M_0)^2+k_{2\perp}^2][(m_1'+x_1M_0')^2+k_{2\perp}'^2]}},$$
$$g_1(q^2)=\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x',k_\perp')\phi_{\Lambda_Q}(x,k_\perp)[-k_{2\perp}\cdot k_{2\perp}'+(x_1M_0+m_1)(x_1'M_0'+m_1')]}{\sqrt{[(m_1+x_1M_0)^2+k_{2\perp}^2][(m_1'+x_1M_0')^2+k_{2\perp}'^2]}},$$
$$\frac{f_2(q^2)}{M_{\Lambda_Q}}=\frac{1}{q_\perp^i}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x',k_\perp')\phi_{\Lambda_Q}(x,k_\perp)[(m_1+x_1M_0)k_{1\perp}'^i-(m_1'+x_1'M_0')k_{1\perp}^i]}{\sqrt{[(m_1+x_1M_0)^2+k_{2\perp}^2][(m_1'+x_1M_0')^2+k_{2\perp}'^2]}},$$
$$\frac{g_2(q^2)}{M_{\Lambda_Q}}=\frac{1}{q_\perp^i}\int\frac{dxd^2k_\perp}{2(2\pi)^3}\frac{\phi_{\Lambda_{Q'}}(x',k_\perp')\phi_{\Lambda_Q}(x,k_\perp)[(m_1+x_1M_0)k_{1\perp}'^i+(m_1'+x_1'M_0')k_{1\perp}^i]}{\sqrt{[(m_1+x_1M_0)^2+k_{2\perp}^2][(m_1'+x_1M_0')^2+k_{2\perp}'^2]}}. \tag{30}$$

C. The form factors in the heavy quark limit

It is well known that there is a nontrivial symmetry in QCD: the heavy quark symmetry in the infinite quark mass limit [25]. Since the masses of heavy quarks b and c are much larger than the strong interaction scale Λ_{QCD}, the spin of the heavy quark decouples from light quark and gluon degrees of freedoms, and an extra symmetry $SU_f(2)\otimes SU_s(2)$ is expected. This flavor and spin symmetry provides several model-independent relations for the heavy-to-heavy baryonic form factors: the six form factors f_i, g_i (i = 1, 2, 3) are related to a unique universal Isgur-Wise function $\zeta(v\cdot v')$. In the heavy quark limit, the heavy quark Q is described by a two-component spinor $Q_v=e^{im_Qv\cdot x}\frac{(1+\not v)}{2}Q$, where v is the velocity of the heavy baryon. The current $\bar{Q}'\gamma_\mu(1-\gamma_5)Q$ in the full theory is matched onto the current $\bar{Q}_{v'}'\gamma_\mu(1-\gamma_5)Q_v$ in the heavy quark effective theory. The baryon bound state and Dirac spinor field are replaced by

$$|\Lambda_Q(P,S_z)\rangle\to\sqrt{M_{\Lambda_Q}}|\Lambda_Q(v,S_z)\rangle,$$
$$u(\bar{P},S_z)\to\sqrt{m_Q}u(v,S_z). \tag{31}$$

The Isgur-Wise function which appears in the transition amplitude $\Lambda_Q\to\Lambda_{Q'}$ is defined as [26]

$$\langle\Lambda_{Q'}(v',S_z')|\bar{Q}_{v'}'\gamma^\mu(1-\gamma_5)Q_v|\Lambda_Q(v,S_z)\rangle$$
$$=\zeta(\omega)\bar{u}(v',S_z')\gamma_\mu(1-\gamma_5)u(v,S_z), \tag{32}$$

where $\omega\equiv v\cdot v'$. The heavy flavor symmetry implies that the Isgur-Wise function is normalized to be 1 at the zero-recoil point, $\zeta(1)=1$. The physical form factors are obtained as

$$f_1(q^2)=g_1(q^2)=\zeta(\omega),\qquad f_2=f_3=g_2=g_3=0, \tag{33}$$

where $q^2=M_{\Lambda_Q}^2+M_{\Lambda_{Q'}}^2-2M_{\Lambda_Q}M_{\Lambda_{Q'}}\omega$.

Since the momentum of Λ_Q is dominated by the momentum of the heavy quark Q, the momentum of the light spectator diquark x is of order $\Lambda_{\mathrm{QCD}}/m_Q$. The variable $X\equiv xm_b$ is of the order of Λ_{QCD}. In analog to the situation for heavy mesons, the wave function of Λ_Q should have a scaling behavior in the heavy quark limit [27],

$$\phi_{\Lambda_Q}(x,k_\perp)\to\sqrt{\frac{m_Q}{X}}\Phi(X,k_\perp), \tag{34}$$

where the factor $\sqrt{m_Q}$ is deliberately factorized out and the rest of $\phi_{\Lambda_Q}(x,k_\perp)$ is independent of m_Q, $\Phi(X,k_\perp)$ is normalized as

$$\int_0^\infty \frac{dX}{X} \int \frac{d^2k_\perp}{2(2\pi)^3} |\Phi(X, k_\perp)|^2 = 1. \tag{35}$$

For the on-shell diquark momentum p_2, we have $p_2^- = \frac{(p_{2\perp}^2 + m_2^2)}{p_2^+}$, and

$$v \cdot p_2 = \frac{m_2^2 + k_\perp^2 + X^2}{2X}. \tag{36}$$

Hence, the wave function $\Phi(X, k_\perp)$, which only depends on the velocity of the baryon, is the same for Λ_Q and $\Lambda_{Q'}$.

We now consider the transition form factors obtained in the previous section under the heavy quark limit. For the initial baryon, we have

$$\begin{gathered} M_{\Lambda_Q} \to m_Q, \qquad M_0 \to m_Q, \qquad e_1 \to m_Q, \\ e_2 \to v \cdot p_2, \qquad \vec{k}^2 \to (v \cdot p_2)^2 - m_2^2, \\ \not{p}_1 + m_1 \to m_Q(\not{v} + 1) \frac{e_1 e_2}{x_1 x_2 M_0} \to \frac{m_Q}{X}(v \cdot p_2), \end{gathered} \tag{37}$$

and

$$\Phi(X, k_\perp) = 4\sqrt{v \cdot p_2}\left(\frac{\pi}{\beta_\infty^2}\right)^{3/4} \exp\left(-\frac{(v \cdot p_2)^2 - m_2^2}{2\beta_\infty^2}\right). \tag{38}$$

The subscript in β_∞ represents the case of the heavy quark limit. Similar expressions can be obtained for the final baryon where a prime sign "′" would be attached to each variable.

The calculation of the Isgur-Wise function in the heavy quark limit becomes much simpler than that for f_i and g_i because they can be evaluated directly in the timelike region by choosing a reference frame where $q_\perp = 0$ [22]. The matrix element $\Lambda_Q \to \Lambda_{Q'}$ is

$$\begin{aligned} &\langle \Lambda_{Q'}(v', S_z')|\bar{Q}'_{v'}\gamma^\mu(1-\gamma_5)Q_v|\Lambda_Q(v, S_z)\rangle \\ &= \int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp)\Phi(X', k'_\perp)\bar{u}(v', S_z') \\ &\quad \times \gamma^\mu(1-\gamma_5)u(v, S_z), \end{aligned} \tag{39}$$

where $z \equiv X'/X$. By comparing the above equation with Eq. (32), we get

$$\zeta(\omega) = \int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp)\Phi(X', k'_\perp). \tag{40}$$

The obtained Isgur-Wise function $\zeta(\omega)$ is an overlapping integration of the initial and final wave functions and no spin information is left. The variable z is related to ω via

$$z \to z_\pm = \omega \pm \sqrt{\omega^2 - 1}, \qquad z_+ = \frac{1}{z_-}, \tag{41}$$

where $+(-)$ denotes the final baryon recoiling direction. There is a symmetry between z_+ and z_-. $\zeta(\omega)$ does not change when we replace z_+ by z_-, or vice versa. Equation (40) shows explicitly that $\zeta(\omega)$ depends only on the velocities of the initial and final baryons and is independent of the heavy quark masses.

The Isgur-Wise function can also be obtained from Eq. (20) by taking the heavy quark limit. It is not difficult to verify that $f_1(q^2) = g_1(q^2) = \zeta(\omega)$, $f_2 = g_2 = 0$ in leading order of $\Lambda_{\rm QCD}/m_Q$. The normalization of Isgur-Wise function at the zero-recoil point is guaranteed by our normalization condition for wave functions, Eq. (35). Its consistency with forms in the heavy quark limit implies the correctness of the light-front approach.

III. SEMILEPTONIC AND NONLEPTONIC DECAYS OF TRANSITION $\Lambda_b \to \Lambda_c$

The polarization effects in exclusive processes, such as $B \to \phi K^*$, offers nontrivial information about strong interaction, which is important to test different theoretical approaches. The decays of $\Lambda_b \to \Lambda_c$ indeed contain complex spin structures. In this section, we obtain formulations for the rates of semileptonic and nonleptonic processes. In this work, we concern only the exclusive decay modes.

A. Semileptonic decays of $\Lambda_b \to \Lambda_c l \bar{\nu}_l$

The transition amplitude of $\Lambda_b \to \Lambda_c$ contains several independent helicity components. The helicity amplitudes induced by the weak vector and axial-vector currents are described by $H^{V,A}_{\lambda',\lambda_W}$, where λ' and λ_W denote the helicities of the final baryon and the virtual W-boson, respectively. According to the definitions of the form factors for $\Lambda_b \to \Lambda_c$ given in Eq. (18), the helicity amplitudes are related to these form factors through the following expressions [28]:

$$\begin{aligned} H^V_{(1/2),0} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left((M_{\Lambda_b} + M_{\Lambda_c})f_1 - \frac{q^2}{M_{\Lambda_b}} f_2\right), \\ H^V_{(1/2),1} &= \sqrt{2Q_-}\left(-f_1 + \frac{M_{\Lambda_b} + M_{\Lambda_c}}{M_{\Lambda_b}} f_2\right), \\ H^A_{(1/2),0} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left((M_{\Lambda_b} - M_{\Lambda_c})g_1 + \frac{q^2}{M_{\Lambda_b}} g_2\right), \\ H^A_{(1/2),1} &= \sqrt{2Q_+}\left(-g_1 - \frac{M_{\Lambda_b} - M_{\Lambda_c}}{M_{\Lambda_b}} g_2\right), \end{aligned} \tag{42}$$

where $Q_\pm = 2(P \cdot P' \pm M_{\Lambda_b}M_{\Lambda_c}) = 2M_{\Lambda_b}M_{\Lambda_c}(\omega \pm 1)$. The amplitudes for the negative helicities are obtained in terms of the relation

$$H^{V,A}_{-\lambda'-\lambda_W} = \pm H^{V,A}_{\lambda',\lambda_W}, \tag{43}$$

where the upper (lower) sign corresponds to $V(A)$.

Because of the $V - A$ structure of the weak current, the helicity amplitudes are obtained as

$$H_{\lambda',\lambda_W} = H^V_{\lambda',\lambda_W} - H^A_{\lambda',\lambda_W}. \tag{44}$$

The helicities of the W-boson λ_W can be either 0 or 1, which correspond to the longitudinal and transverse polarizations, respectively. Following the definitions in literature, we decompose the decay width into a sum of the longitudinal and transverse parts according to the helicity states of the virtual W-boson. The differential decay rate of $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ is

$$\frac{d\Gamma}{d\omega} = \frac{d\Gamma_L}{d\omega} + \frac{d\Gamma_T}{d\omega}, \tag{45}$$

and the longitudinally (L) and transversely (T) polarized rates are respectively [28]

$$\frac{d\Gamma_L}{d\omega} = \frac{G_F^2|V_{cb}|^2}{(2\pi)^3}\frac{q^2 p_c M_{\Lambda_c}}{12M_{\Lambda_b}}[|H_{(1/2),0}|^2 + |H_{-(1/2),0}|^2],$$
$$\frac{d\Gamma_T}{d\omega} = \frac{G_F^2|V_{cb}|^2}{(2\pi)^3}\frac{q^2 p_c M_{\Lambda_c}}{12M_{\Lambda_b}}[|H_{(1/2),1}|^2 + |H_{-(1/2),-1}|^2], \tag{46}$$

where $p_c = M_{\Lambda_c}\sqrt{\omega^2 - 1}$ is the momentum of Λ_c in the reset frame of Λ_b. Integrating over the solid angle, we obtain the decay rate

$$\Gamma = \int_1^{\omega_{\max}} d\omega \frac{d\Gamma}{d\omega}, \tag{47}$$

where the upper bound of the integration $\omega_{\max} = \frac{1}{2}(\frac{M_{\Lambda_b}}{M_{\Lambda_c}} + \frac{M_{\Lambda_c}}{M_{\Lambda_b}})$ is the maximal recoil. In order to compare our results with those in the literature, we used the variable ω in the expression for the differential decay rate. In the heavy quark limit, the decay rate of $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ is simplified into

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2|V_{cb}|^2}{24\pi^3} M_{\Lambda_b}^5 \sqrt{\omega^2 - 1} r^3 \times (6\omega r^2 - 8\omega^2 r - 4r + 6\omega)\zeta(\omega)^2, \tag{48}$$

with $r = \frac{M_{\Lambda_c}}{M_{\Lambda_b}}$.

The polarization of the cascade decay $\Lambda_b \to \Lambda_c(\to p\pi) + W(\to l\nu)$ is expressed by various asymmetry parameters [7,28]. Among them, the integrated longitudinal and transverse asymmetries are defined by

$$a_L = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),0}|^2 - |H_{-(1/2),0}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),0}|^2 + |H_{-(1/2),0}|^2]},$$
$$a_T = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),1}|^2 - |H_{-(1/2),-1}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),1}|^2 + |H_{-(1/2),-1}|^2]}. \tag{49}$$

The ratio of the longitudinal to transverse decay rates R is defined by

$$R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),0}|^2 + |H_{-(1/2),0}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),1}|^2 + |H_{-(1/2),-1}|^2]}, \tag{50}$$

and the longitudinal Λ_c polarization asymmetry P_L is given as

$$P_L = \frac{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),0}|^2 - |H_{-(1/2),0}|^2 + |H_{(1/2),1}|^2 - |H_{-(1/2),-1}|^2]}{\int_1^{\omega_{\max}} d\omega q^2 p_c[|H_{(1/2),0}|^2 + |H_{-(1/2),0}|^2 + |H_{(1/2),1}|^2 + |H_{-(1/2),-1}|^2]} = \frac{a_T + Ra_L}{1 + R}. \tag{51}$$

B. Nonleptonic decay of $\Lambda_b \to \Lambda_c M$

Several exclusive nonleptonic decays of $\Lambda_b \to \Lambda_c + M$, where M is a meson, have been measured in recent experiments [8]. From the theoretical aspects, the nonleptonic decays are much more complicated than the semileptonic ones because of the strong interaction. Generally, the present theoretical framework is based on the factorization assumption, where the hadronic matrix element is factorized into a product of two matrix elements of single currents. One can be written as a decay constant while the other is expressed in terms of a few form factors according to the Lorentz structure of the current. For the weak decays of mesons, such a factorization approach is verified to work very well for the color-allowed processes and the nonfactorizable contributions are negligible. We have reason to believe that this would be valid for the baryon case, especially as the diquark picture is employed. The decays $\Lambda_b^0 \to \Lambda_c^+ M^-$ belong to this type. Thus, the study on these modes could be not only a test for the factorization hypothesis, but also a check of the consistency of the obtained form factors in the heavy bottomed baryon system.

For the nonleptonic decays $\Lambda_b^0 \to \Lambda_c^+ M^-$, the effective interaction at the quark level is $b \to c\bar{q}_1 q_2$. The relevant Hamiltonian is

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_1 q_2}^* (c_1 O_1 + c_2 O_2),$$
$$O_1 = (\bar{c}b)_{V-A}(\bar{q}_2 q_1)_{V-A}, \qquad O_2 = (\bar{q}_2 b)_{V-A}(\bar{c}q_1)_{V-A}, \tag{52}$$

where c_i denotes the short-distance Wilson coefficient, $V_{cb}(V_{q_1 q_2})$ is the CKM matrix elements, q_1 stands for u or c, and q_2 for d or s in the context. Then one needs to evaluate the hadronic matrix elements

$$\langle \Lambda_c M|\mathcal{H}_W|\Lambda_b\rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_1 q_2}^* \sum_{i=1,2} c_i \langle \Lambda_c M|O_i|\Lambda_b\rangle. \tag{53}$$

Under the factorization approximation, the hadronic matrix element is reduced to

$$\langle \Lambda_c M | O_i | \Lambda_b \rangle = \langle \Lambda_c | J_\mu | \Lambda_b \rangle \langle M | J'^\mu | 0 \rangle, \tag{54}$$

where $J(J')$ is the $V - A$ weak current. The first factor $\langle \Lambda_c | J_\mu | \Lambda_b \rangle$ is parametrized by six form factors as done in Eq. (17). The second factor defines the decay constants as follows:

$$\begin{aligned} \langle P(P) | A_\mu | 0 \rangle &= f_P P_\mu, \\ \langle S(P) | V_\mu | 0 \rangle &= f_S P_\mu, \\ \langle V(P, \epsilon) | V_\mu | 0 \rangle &= f_V M_V \epsilon^*_\mu, \\ \langle A(P, \epsilon) | A_\mu | 0 \rangle &= f_V M_A \epsilon^*_\mu, \end{aligned} \tag{55}$$

where $P(V)$ denotes a pseudoscalar (vector) meson, and $S(A)$ denotes a scalar (axial-vector) meson. In the definitions, we omit a factor $(-i)$ for the pseudoscalar meson decay constant.

In general, the transition amplitude of $\Lambda_b \to \Lambda_c M$ can be written as

$$\begin{aligned} \mathcal{M}(\Lambda_b \to \Lambda_c P) &= \bar{u}_{\Lambda_c}(A + B\gamma_5) u_{\Lambda_b}, \\ \mathcal{M}(\Lambda_b \to \Lambda_c V) &= \bar{u}_{\Lambda_c} \epsilon^{*\mu} [A_1 \gamma_\mu \gamma_5 + A_2 (p_{\Lambda_c})_\mu \gamma_5 \\ &\quad + B_1 \gamma_\mu + B_2 (p_{\Lambda_c})_\mu] u_{\Lambda_b}, \end{aligned} \tag{56}$$

where ϵ^μ is the polarization vector of the final vector or axial-vector mesons. Including the effective Wilson coefficient $a_1 = c_1 + c_2/N_c$, the decay amplitudes in the factorization approximation are [29,30]

$$\begin{aligned} A &= \lambda f_P (M_{\Lambda_b} - M_{\Lambda_c}) f_1(M^2), \\ B &= \lambda f_P (M_{\Lambda_b} + M_{\Lambda_c}) g_1(M^2), \\ A_1 &= -\lambda f_V M \left[g_1(M^2) + g_2(M^2) \frac{M_{\Lambda_b} - M_{\Lambda_c}}{M_{\Lambda_b}} \right], \\ A_2 &= -2\lambda f_V M \frac{g_2(M^2)}{M_{\Lambda_b}}, \\ B_1 &= \lambda f_V M \left[f_1(M^2) - f_2(M^2) \frac{M_{\Lambda_b} + M_{\Lambda_c}}{M_{\Lambda_b}} \right], \\ B_2 &= 2\lambda f_V M \frac{f_2(M^2)}{M_{\Lambda_b}}, \end{aligned} \tag{57}$$

where $\lambda = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{q_1 q_2} a_1$ and M is the meson mass. Replacing P, V by S and A in the above expressions, one can easily obtain similar expressions for scalar and axial-vector mesons.

The decay rates of $\Lambda_b \to \Lambda_c P(S)$ and up-down asymmetries are [30]

$$\begin{aligned} \Gamma &= \frac{p_c}{8\pi} \left[\frac{(M_{\Lambda_b} + M_{\Lambda_c})^2 - M^2}{M^2_{\Lambda_b}} |A|^2 \right. \\ &\quad \left. + \frac{(M_{\Lambda_b} - M_{\Lambda_c})^2 - M^2}{M^2_{\Lambda_b}} |B|^2 \right], \\ \alpha &= -\frac{2\kappa \mathrm{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2}, \end{aligned} \tag{58}$$

where p_c is the Λ_c momentum in the rest frame of Λ_b and $\kappa = \frac{p_c}{E_{\Lambda_c} + M_{\Lambda_c}}$. For $\Lambda_b \to \Lambda_c V(A)$ decays, the decay rates and up-down asymmetries are

$$\begin{aligned} \Gamma &= \frac{p_c (E_{\Lambda_c} + M_{\Lambda_c})}{8\pi M_{\Lambda_b}} \left[2(|S|^2 + |P_2|^2) \right. \\ &\quad \left. + \frac{E^2}{M^2} (|S + D|^2 + |P_1|^2) \right], \\ \alpha &= \frac{4M^2 \mathrm{Re}(S^* P_2) + 2E^2 \mathrm{Re}(S + D)^* P_1}{2M^2(|S|^2 + |P_2|^2) + E^2(|S + D|^2 + |P_1|^2)}, \end{aligned} \tag{59}$$

where E is energy of the vector (axial-vector) meson, and

$$\begin{aligned} S &= -A_1, \qquad P_1 = -\frac{p_c}{E} \left(\frac{M_{\Lambda_b} + M_{\Lambda_c}}{E_{\Lambda_c} + M_{\Lambda_c}} B_1 + B_2 \right), \\ P_2 &= \frac{p_c}{E_{\Lambda_c} + M_{\Lambda_c}} B_1, \\ D &= -\frac{p_c^2}{E(E_{\Lambda_c} + M_{\Lambda_c})} (A_1 - A_2). \end{aligned} \tag{60}$$

IV. NUMERICAL RESULTS

In this section, we will present our numerical results of the form factors for the transition $\Lambda_b \to \Lambda_c$. Then use them to predict the rates of the exclusive semileptonic $\Lambda_b \to \Lambda_c l \bar{\nu}_l$, and two-body nonleptonic processes, such as $\Lambda_b \to \Lambda_c M^-$, where $M = \pi, K, \rho, K^*, a_1$.

At first, we provide our input parameters in the light-front quark model. The baryon masses $M_{\Lambda_b} = 5.624$ GeV, $M_{\Lambda_c} = 2.285$ GeV are taken from [8]. The quark masses and the hadron wave function parameter β need to be specified. For the heavy quark masses, we take m_b and m_c from [15]. Following [22], the mass of a $[ud]$ diquark is assumed to be close to the constitute strange quark mass. In the literature, the mass of the constituent light scalar diquark $m_{[ud]}$ is rather arbitrary, for example, it is set as: 400 [22], 500 [21], 710 [7], and 650–800 MeV [31]. A recent result from lattice calculation gives the scalar diquark mass varies from 1190 to 696 MeV when a hopping parameter κ changes from 0.140 to 0.148 [32]. To reduce error and model dependence, we use the value of $BR(\Lambda_b \to \Lambda_c l \bar{\nu}_l) = 5.0^{+1.1}_{-0.8}(\mathrm{stat})^{+1.6}_{-1.2}(\mathrm{syst})\%$ measured by the DELPHI collaboration [1] to fix parameters. The present data favors diquark mass as $m_{[ud]} = 500$ MeV. All the input parameters are collected in Table I.

TABLE I. Quark mass and the parameter β (in units of GeV).

m_c	m_b	$m_{[ud]}$	$\beta_{c[ud]}$	$\beta_{b[ud]}$
1.3	4.4	0.50	0.35	0.40

A. $\Lambda_b \to \Lambda_c$ form factors and the Isgur-Wise function

Because the calculation of form factors is performed in the frame $q^+ = 0$ with $q^2 = -q_\perp^2 \le 0$, only the values of the form factors in the spacelike region can be obtained. The advantage of this choice is that the so-called Z-graph contribution arising from the nonvalence quarks vanishes. In this study, another advantage is that it simplifies the calculation of baryonic matrix elements. In order to obtain the physical form factors, an analytical extension from the spacelike region to the timelike region is required. The form factors in the spacelike region can be parametrized in a three-parameter form as

$$F(q^2) = \frac{F(0)}{(1 - \frac{q^2}{M_{\Lambda_b}^2})[1 - a(\frac{q^2}{M_{\Lambda_b}^2}) + b(\frac{q^2}{M_{\Lambda_b}^2})^2]}, \qquad (61)$$

where F represents the form factor $f_{1,2}$ and $g_{1,2}$. The parameters a, b, and $F(0)$ are fixed by performing a three-parameter fit to the form factors in the spacelike region which were obtained in previous sections. We then use these parameters to determine the physical form factors in the timelike region. The fitted values of a, b, and $F(0)$ for different form factors $f_{1,2}$ and $g_{1,2}$ are given in Table III. The q^2 dependence of the form factors is plotted in Fig. 2.

From Table II and Fig. 2, we find that the form factors f_1 and g_1 are nearly equal. At small recoil, i.e. large q^2 region, there is only a tiny difference between the two functions. Even at the maximal recoil point $q^2 = 0$, their difference is less than 3%. This can be understood by Eq. (30) where the difference between f_2 and g_2 is at the order of $\Lambda_{\rm QCD}^2/(M_{\Lambda_b}M_{\Lambda_c})$, a next-to-next-to-leading order in the $1/m_Q$ expansion. The form factor f_2 and g_2 are small comparing to f_1 and g_1. In practice, g_2 is approximately zero, and f_2 is about 20%–30% of f_1 and g_1. These conclusions are consistent with the results of [4]. From Table II, the parameter a for various form factors is close to 1 and the parameter b is small. The results suggest that the q^2-dependence of f_i and g_i approximately exhibits a dipole behavior $F(q^2) = \frac{F(0)}{(1-q^2/M_{\Lambda_b})^n}$ with $n = 2$.

In the heavy quark limit, the heavy baryons Λ_b and Λ_c have the same scale parameter β^∞ in their wave functions. We choose $\beta^\infty = 0.40$ GeV which is equal to the parameter in the Λ_b wave function. The Isgur-Wise function is usually parametrized by

$$\zeta(\omega) = 1 - \rho^2(\omega - 1) + \frac{\sigma^2}{2}(\omega - 1)^2 + \cdots, \qquad (62)$$

where $\rho^2 \equiv -\frac{d\zeta(\omega)}{d\omega}|_{\omega=1}$ is the slope parameter and $\sigma^2 \equiv \frac{d^2\zeta(\omega)}{d\omega^2}|_{\omega=1}$ is the curvature of the Isgur-Wise function. Our fitted values are

$$\rho^2 = 1.47, \qquad (63)$$

$$\sigma^2 = 1.90. \qquad (64)$$

The DELPHI collaboration reported their measurement on

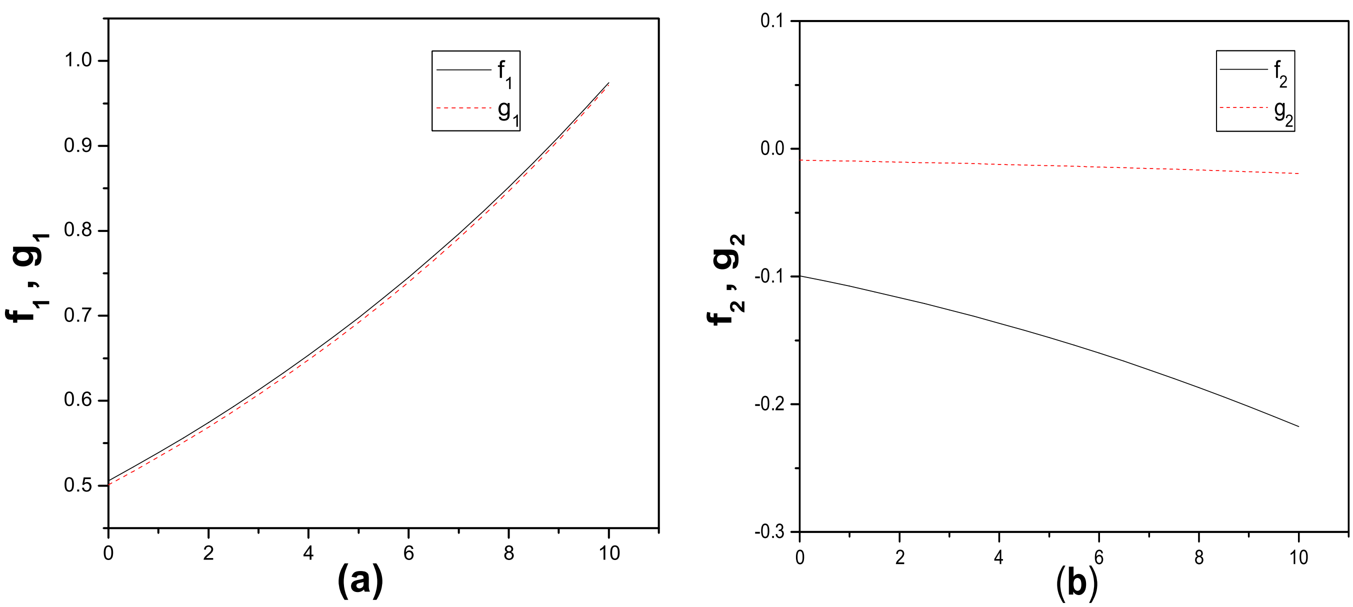

FIG. 2 (color online). (a) Form factors f_1 and g_1. (b) Form factors f_2 and g_2.

TABLE II. The $\Lambda_b \to \Lambda_c$ form factors given in the three-parameter form.

F	$F(0)$	a	b
f_1	0.505 68	1.00	0.75
f_2	−0.099 43	1.50	1.43
g_1	0.500 87	1.00	0.70
g_2	−0.008 89	1.50	1.45

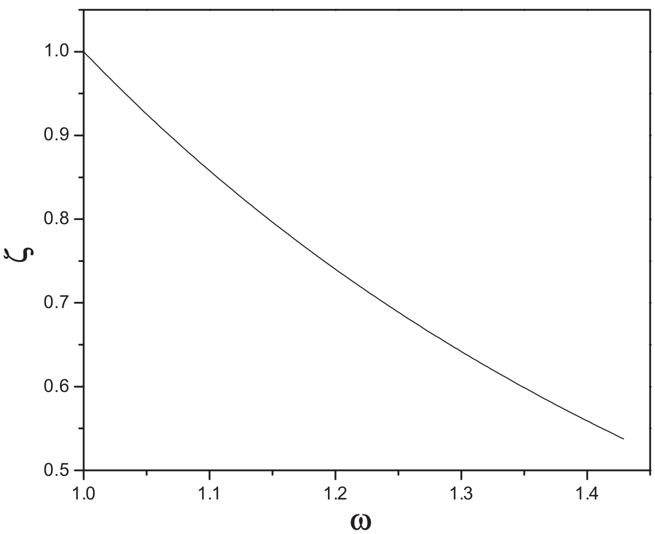

FIG. 3. The $\Lambda_b \to \Lambda_c$ Isgur-Wise function $\zeta(\omega)$ with diquark mass $m_{[ud]} = 500$ MeV.

the slope of the Isgur-Wise function $\zeta(\omega) = 1 - \rho^2(\omega - 1)$ as $\rho^2 = 2.03 \pm 0.46(\text{stat})^{+0.72}_{-1.00}(\text{syst})$ in the semileptonic decay $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ [1]. The recent theoretical calculations on the slope parameter ρ^2 are: $\rho^2 = 1.35 \pm 0.12$ in QCD sum rules [5]; $\rho^2 = 1.51$ in a relativistic quark model [7]. All the results are in agreement with the experiment data within the theoretical and experimental errors. The Isgur-Wise function in the total ω space is depicted in Fig. 3. The errors in the parameter β^∞ have only a minor effect which is consistent with the B meson case [33].

B. Semileptonic decay of $\Lambda_b \to \Lambda_c + l\bar{\nu}_l$

With the form factors and the Isgur-Wise function given in the above subsection, we are able to calculate the branching ratios and various asymmetries of $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ decay. Table III provides our numerical predictions. The results are presented for two cases: with taking the heavy quark limit and without taking the heavy quark limit. The ratio of longitudinal to transverse rates $R > 1$ implies that the longitudinal polarization dominates.

The significant difference for the transverse polarization asymmetry a_T in the two cases (with or without taking the heavy quark limit) implies that a_T is sensitive to the heavy quark symmetry breaking effects. Thus, measurement of a_T may be an ideal probe to test how well the heavy quark symmetry works in the weak decays of heavy baryons, not only for the rate estimate, but also other relevant measurable quantities such as a_T. Indeed, for the branching ratio and the Λ_c polarization asymmetry P_L, the deviation in the two cases is at the level of a few percents, thus the heavy quark limit provides a good approximation.

We also compare our results with the predictions by the relativistic quark model [7]. The two models result in nearly equal predictions for the longitudinal asymmetry a_L and the Λ_c polarization asymmetry P_L. This confirms the observation of [10] that these quantities are less model dependent.

C. Nonleptonic decays of $\Lambda_b \to \Lambda_c + M$

The nonleptonic decays $\Lambda_b \to \Lambda_c + M$ in the factorization approach have been studied in the previous section. Now, we present our numerical predictions on the decay rates and relevant measurable quantities. The CKM matrix elements take values [8]

$$V_{ud} = 0.9738, \qquad V_{us} = 0.2257, \qquad V_{cd} = 0.230, \qquad V_{cs} = 0.957, \qquad V_{cb} = 0.0416, \tag{65}$$

and the effective Wilson coefficient $a_1 = 1$. The meson decay constants are shown in Table IV.

The predictions for the branching ratios and up-down asymmetries are provided in Table V. The Tables VI and VII demonstrate comparisons of our result with that in other approaches. Some arguments are made in orders:

(1) For the processes with mesons π, ρ, D_s, D_s^*, a_1 being in the final states, the corresponding subprocesses are $b \to c\bar{u}d$ or $b \to c\bar{c}s$, which are the Cabibbo-favored processes. The decay ratios fall in the region 4×10^{-3} to 1×10^{-2}. They are the dominant decay modes which will be measured in the near future. For the processes with mesons K, K^*, D, D^* in the final states, the subprocesses are

TABLE III. The branching ratios and polarization asymmetries of $\Lambda_b \to \Lambda_c l \bar{\nu}_l$.

	Br	a_L	a_T	R	P_L
Within the heavy quark limit (this work)	6.2%	−0.926	−0.483	1.539	−0.751
Without the heavy quark limit (this work)	6.3%	−0.932	−0.601	1.466	−0.798
Within the heavy quark limit (in [7])	6.2%	−0.928	−0.483	1.59	−0.756
With $1/m_Q$ corrections (in [7])	6.9%	−0.940	−0.600	1.61	−0.810

TABLE IV. Meson decay constants f (in units of MeV) [15].

Meson	π	ρ	K	K^*	D	D^*	D_s	D_s^*	a_1
f	131	216	160	210	200	220	230	230	203

TABLE V. Branching ratios and up-down asymmetries of nonleptonic decays $\Lambda_b \to \Lambda_c M$ with the light diquark mass $m_{[ud]} = 500$ MeV.

	Within the heavy quark limit		Without the heavy quark limit	
	Br	α	Br	α
$\Lambda_b^0 \to \Lambda_c^+ \pi^-$	4.22×10^{-3}	-1	3.75×10^{-3}	-1
$\Lambda_b^0 \to \Lambda_c^+ \rho^-$	6.07×10^{-3}	-0.897	6.73×10^{-3}	-0.885
$\Lambda_b^0 \to \Lambda_c^+ K^-$	3.41×10^{-4}	-1	3.02×10^{-4}	-1
$\Lambda_b^0 \to \Lambda_c^+ K^{*-}$	3.15×10^{-4}	-0.865	3.50×10^{-4}	-0.857
$\Lambda_b^0 \to \Lambda_c^+ a_1^-$	5.84×10^{-3}	-0.758	6.49×10^{-3}	-0.760
$\Lambda_b^0 \to \Lambda_c^+ D_s^-$	1.18×10^{-2}	-0.984	1.14×10^{-2}	-0.982
$\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$	8.88×10^{-3}	-0.419	9.96×10^{-3}	-0.442
$\Lambda_b^0 \to \Lambda_c^+ D^-$	5.23×10^{-4}	-0.987	5.01×10^{-4}	-0.986
$\Lambda_b^0 \to \Lambda_c^+ D^{*-}$	4.61×10^{-4}	-0.459	5.12×10^{-4}	-0.481

TABLE VI. Branching ratios for nonleptonic decays $\Lambda_b \to \Lambda_c + M$ within different theoretical approaches (in units of 10^{-2}).

	This work	[30]	[34]	[35]	[36]	[37]
$\Lambda_b^0 \to \Lambda_c^+ \pi^-$	0.375	0.38	0.175	···	0.391	0.503
$\Lambda_b^0 \to \Lambda_c^+ \rho^-$	0.673	0.54	0.491	···	1.082	0.723
$\Lambda_b^0 \to \Lambda_c^+ K^-$	0.030	···	0.013	···	···	0.037
$\Lambda_b^0 \to \Lambda_c^+ K^{*-}$	0.035	···	0.027	···	···	0.037
$\Lambda_b^0 \to \Lambda_c^+ a_1^-$	0.649	···	0.532	···	···	···
$\Lambda_b^0 \to \Lambda_c^+ D_s^-$	1.140	1.1	0.770	2.23	1.291	···
$\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$	0.996	0.91	1.414	3.26	1.983	···
$\Lambda_b^0 \to \Lambda_c^+ D^-$	0.050	···	0.030	···	···	
$\Lambda_b^0 \to \Lambda_c^+ D^{*-}$	0.051	···	0.049	···	···	

TABLE VII. Up-down asymmetries for nonleptonic decays $\Lambda_b \to \Lambda_c M$ within different theoretical approaches.

	This work	[30]	[34]	[35]	[37]
$\Lambda_b^0 \to \Lambda_c^+ \pi^-$	-1	-0.99	-0.999	···	-1
$\Lambda_b^0 \to \Lambda_c^+ \rho^-$	-0.885	-0.88	-0.897	···	-0.885
$\Lambda_b^0 \to \Lambda_c^+ K^-$	-1	···	-1	···	-1
$\Lambda_b^0 \to \Lambda_c^+ K^{*-}$	-0.857	···	-0.865	···	0.885
$\Lambda_b^0 \to \Lambda_c^+ a_1^-$	-0.760	···	-0.758	···	···
$\Lambda_b^0 \to \Lambda_c^+ D_s^-$	-0.982	-0.99	-0.984	-0.98	···
$\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$	-0.442	-0.36	-0.419	-0.40	···
$\Lambda_b^0 \to \Lambda_c^+ D^-$	-0.986	···	-0.987	···	···
$\Lambda_b^0 \to \Lambda_c^+ D^{*-}$	-0.481	···	-0.459	···	···

$b \to c\bar{u}s$ or $b \to c\bar{c}d$ which are the Cabibbo-suppressed processes. Their decay ratios are of order $(3-5) \times 10^{-4}$.

(2) In the scheme adopted in this work, we obtain the ratio $\frac{BR(\Lambda_b^0 \to \Lambda_c^+ l^- \bar{\nu}_l)}{BR(\Lambda_b^0 \to \Lambda_c^+ \pi^-)}$ to be 16.8, and this theoretical prediction is consistent with the experimental measurement (a preliminary result) $\frac{BR(\Lambda_b^0 \to \Lambda_c^+ l^- \bar{\nu}_l)}{BR(\Lambda_b^0 \to \Lambda_c^+ \pi^-)} = 20.0 \pm 3.0(\text{stat}) \pm 1.2(\text{syst})$ [38].

(3) From Table VI, it is noted that the differences among the predictions on the branching ratios for nonleptonic decays by various theoretical approaches are obvious. It is hard to decide which model is closer to reality at present, because more precise data are still lacking. It may be more appropriate to employ the experimental data about the semileptonic decay as inputs to reduce the model dependence of the $\Lambda_b \to \Lambda_c$ transition form factors as we did in this work.

(4) All the up-down asymmetries α are negative, this result reflects the $V - A$ nature of the weak currents. Table VII shows that the numerical values of the up-down asymmetries α predicted by different approaches are nearly the same except for the process $\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$ where the difference is about 10%.

V. CONCLUSIONS

In this work, we investigate extensively the $\Lambda_b \to \Lambda_c$ transition form factors in the light-front approach and make predictions on the rates for the semileptonic decay $\Lambda_b \to \Lambda_c l \bar{\nu}_l$ and nonleptonic two-body decays

$\Lambda_b \to \Lambda_c + M$. In the light-front quark model, we adopt the diquark picture for the heavy baryons. It is believed that, for the heavy baryons which contain at least one heavy quark, the quark-diquark picture seems to work well, therefore one can employ it for evaluating the hadronic matrix elements of $\Lambda_b \to \Lambda_c$ transitions which are dominated by nonperturbative QCD effects. The light scalar diquark mass determined from the data on the semileptonic decay is about 500 MeV. Our numerical results show that the q^2-dependence of the momentum transfer of different form factors has a dipolelike behavior. The slope parameter of the universal Isgur-Wise function is found to be consistent with that obtained by fitting experimental data. The small difference for the branching ratio of the semileptonic decay with and without the heavy quark limit implies that the heavy quark symmetry is good in the heavy bottom baryon system. However, on the other aspect, the transverse polarization asymmetry is shown to be sensitive to the heavy quark symmetry breaking, and it is worth further and more accurate studies. Our results for the exclusive nonleptonic two-body decays $\Lambda_b \to \Lambda_c + M$ is modest among the predictions by other approaches. The semileptonic to nonleptonic $\Lambda_c^+ \pi^-$ decay ratio is well in accord with the experimental measurements. The nonleptonic decays, so far have not been accurately measured, and there are only upper bounds for some channels, so that it is still hard to judge the closeness of the present models to the physical reality yet. Fortunately, the LHCb will run and a remarkable amount of data on Λ_b production and decay will be accumulated in the future LHCb, then one may be able to verify the different models.

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