Electromagnetic couplings of elementary vector particles

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On the basis of the three fundamental principles of (i) Poincaré symmetry of space-time, (ii) electromagnetic gauge symmetry, and (iii) unitarity, we construct an universal Lagrangian for the electromagnetic interactions of elementary vector particles, i.e., massive spin-1 particles transforming in the $(\frac{1}{2}, \frac{1}{2})$ representation space of the homogeneous Lorentz group. We make the point that the first two symmetries alone do not fix the electromagnetic couplings uniquely but solely prescribe a general Lagrangian depending on two free parameters, here denoted by ξ and g. The first one defines the electric-dipole and the magnetic-quadrupole moments of the vector particle, while the second determines its magnetic-dipole and electric-quadrupole moments. In order to fix the parameters one needs an additional physical input suited for the implementation of the third principle. As such, one chooses Compton scattering off a vector target and requires the cross section to respect the unitarity bounds in the high-energy limit. As a result, we obtain the universal $g = 2$ and $\xi = 0$ values which completely characterize the electromagnetic couplings of the considered elementary vector field at tree level. The nature of this vector particle, Abelian versus non-Abelian, does not affect this structure. Merely, a partition of the $g = 2$ value into non-Abelian, g_{na}, and Abelian, $g_a = 2 - g_{na}$, contributions occurs for non-Abelian fields with the size of g_{na} being determined by the specific non-Abelian group appearing in the theory of interest, be it the standard model or any other theory.

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I. INTRODUCTION

In the forthcoming years, energies ranging from several hundred GeV to a few TeVs are expected to become accessible at the particle accelerators, a progress which will facilitate testing various fundamental theoretical concepts. In particular, it is quite possible that some of the elementary high-spin particles predicted by supersymmetric- or excited-lepton theories could be observed either as gauge fields to some still unknown non-Abelian groups or as matter fields. Additional effects may or may not come from the more recently developed respective theories of large extra dimensions, noncommutative space-time, etc. In view of the theoretical uncertainties it appears quite important indeed to single out the impact of the first principles underlying the space-time on the properties of the elementary high-spin fields and in first place on their electromagnetic properties. As an example one may think of the value of the gyromagnetic factor, g. In recent time, voicing universality of the $g = 2$ value for particles of any spin becomes stronger (see Ref. [1] and references therein for a recent review). An indirect indication in favor of $g = 2$ is provided already by the Drell-Hearn-Gerasimov sum rule [2] (generalized by Weinberg [3] to any spin) which assigns to strong interactions the anomalous magnetic moment of the nucleon in terms of the $(g - 2)e/2m$ difference. Ferrara, Porrati, and Telegdi made the point [4] that unitarity of the amplitude of Compton scattering off a target of *any spin s* demands for the universal value of $g = 2$. Specifically for spin-$3/2$, they showed that $g = 2$ also allows to avoid the pathology of acausal propagation [5] of such particles within an electromagnetic environment as suffered by the Rarita-Schwinger formalism. However, the resolution of this so-called Velo-Zwanziger problem was obtained at the cost of the introduction of *nonminimal* electromagnetic couplings. In contrast to the Ferrara, Porrati, and Telegdi approach, in the recently proposed covariant projector formalism of Ref. [6], spin-$3/2$ causal propagation and $g = 2$ were achieved by means of a Lagrangian containing *only minimal* couplings but of *second order* in the momenta. The latter formalism treats high-spins as appropriate sectors of finite-dimensional multispin valued homogeneous Lorentz group (HLG) representations that behave as invariant eigensubspaces of the two Casimir operators of the Poincaré group, the squared four-momentum, p^2, and the squared Pauli-Lubanski vector, $\mathcal{W}^2$. It is the goal of the present study to apply the covariant projector formalism to fields residing in the $(\frac{1}{2}, \frac{1}{2})$ irreducible representation of the HLG, the massive vector fields, and explore consequences on their electromagnetic couplings.

The paper is organized as follows. In the following section we briefly review the covariant projector framework of Ref. [6] with the emphasis on the $(\frac{1}{2}, \frac{1}{2})$ representation of the HLG and work out the electromagnetic interactions. In Sec. III we calculate the cross section for Compton scattering off a vector target. In Sec. IV we discuss our results within the light of the Abelian and non-Abelian contributions to the tree-level electromagnetic couplings of arbitrary gauge fields. The paper ends with brief conclusions and has one Appendix.

II. DESCRIPTION OF VECTOR PARTICLES

A. General remarks on vector fields

Vector fields, V_μ, have been previously studied by several authors with the emphasis on their electromagnetic properties. Recently, it was pointed out in [7] that $g = 2$ is required to avoid appearance of divergent $\mathcal{O}(\omega^{-1})$ terms in the radiative decay interferences for polarized vector mesons, with ω standing for the photon energy. On the other side, Proca's theory goes with a fixed $g = 1$ value, and it is not very clear how to reconcile it with $g = 2$ except for the W boson, the gauge particle of the electroweak $SU(2)_L \times U(1)_Y$ group.

The construction of the interacting $(W^+W^-\gamma)$ Lagrangian is quite intricate indeed. The minimally gauged Proca Lagrangian is complemented by a Lagrangian of the same Proca form but based on the non-Abelian field tensor [1]. The contribution of $g = 1$ of the former is then enhanced precisely by the required one unit through the latter after $SU(2)_L \times U(1)_Y/U_{em}(1)$ spontaneous symmetry breaking, to give $g = 2$ (see Ref. [8] for a textbook presentation). Here, $U_{em}(1)$ stands for the electromagnetic gauge group. In this manner, the gyromagnetic ratio of the W boson is equally partitioned into Abelian and non-Abelian contributions. Such a symmetrical partition is not likely to be universal, although in the special case of the W boson some unification theories seem to preserve it [9]. However, for different vector gauge bosons, the new non-Abelian theories throughout may provide larger or lesser non-Abelian contributions to g. Within this context, it is desirable to have a scheme for the description of vector fields that goes beyond Proca's formalism and allows to end up with a $g = 2$ for any vector particle irrespective of its nature, Abelian or non-Abelian.

In the present work we derive such a scheme and prove that the electromagnetic couplings at tree level of *any* massive elementary vector particle are completely fixed by the three fundamental principles of (i) Poincaré symmetry of space-time, (ii) $U(1)_{em}$ gauge symmetry, and (iii) unitarity. Modifications to this picture can arise only at one-loop level due to electromagnetic corrections or diagrams involving interactions with other fields.

We first make the point that the Proca framework is incomplete in observing that the Proca Lagrangian neglects viable terms containing anticommutators, $[p_\mu, p_\nu]$, of the four-momenta, which do not contribute to the free equation of motion at all, but affect the electromagnetic moments in the gauged one when they become proportional to the electromagnetic field tensor, $F_{\mu\nu}$, according to $[\pi_\mu, \pi_\nu] = ieF_{\mu\nu}$ with $\pi_\mu = p_\mu + eA_\mu$. We will show below that the unique $g = 1$ value in the Proca theory upon $U(1)_{em}$ gauging appears precisely as an artifact of the mentioned shortcoming of the free Proca Lagrangian. This shortcoming has been avoided within the framework of the covariant projector formalism recently suggested in Ref. [6]. Within this context, exploring the electromagnetic properties of vector particles within the latter scheme is worthwhile.

In the following we shall obtain a general Lagrangian for a vector particle whose interaction with an electromagnetic field is consistent with Poincaré symmetry as implemented by the covariant projection formalism of Ref. [6] and $U(1)_{em}$ gauge principle. In contrast to Proca's framework, we shall encounter not one but infinitely many equivalent free particle theories which, upon gauging, begin differing through their predictions on the values of the multipole moments, only one of which corresponds to physical reality. In order to fix these values, one then needs additional physical input. As such we consider Compton scattering off a vector target and demand finite total cross section in the high-energy limit in order to respect the unitarity bounds. Taking this path allows us to completely fix the electromagnetic couplings of any elementary vector particle at tree level. In fact, there is no freedom left anymore in the Lagrangian designed to account for all possible terms containing $[p_\mu, p_\nu]$ anticommutators, terms that are notoriously missed by the Proca theory.

B. The covariant projector formalism

In Ref. [6], a formalism was proposed which describes fields of mass m and spin s from a given finite-dimensional and multispin valued representation of the homogeneous Lorentz group in terms of simultaneous projection over the eigensubspaces of the two Casimir operators of the Poincaré group, the squared four-momentum, p^2, and the squared Pauli-Lubanski vector $\mathcal{W}^2$. In particular, in the case of representations containing two different spin-values differing in one unit, say, s and $(s-1)$, the free particle equation obtained in this way reads

$$-\frac{p^2}{m^2}\frac{1}{2s}\left[\frac{\mathcal{W}^2}{p^2} + s(s-1)\mathbb{1}\right]_{AB}\Psi_B^{(m,s)} = \Psi_A^{(m,s)}, \qquad (1)$$

where capital Latin letters $A, B, C, \ldots$ specify the HLG representation of interest. The general expression for the $\mathcal{W}^2$ operator can be found in [6] and reads

$$(\mathcal{W}_\lambda \mathcal{W}^\lambda)_{AB} = \frac{1}{4}\epsilon_{\lambda\rho\sigma\mu}(M^{\rho\sigma})_{AC}p^\mu \epsilon^\lambda_{\tau\xi\nu}(M^{\tau\xi})_{CB}p^\nu \equiv T_{AB\mu\nu}p^\mu p^\nu. \qquad (2)$$

From now onward we shall introduce as a new notation the tensor $\tilde{\Gamma}_{AB\mu\nu}$ according to

$$\tilde{\Gamma}_{AB\mu\nu} = -\frac{1}{2s}(T_{AB\mu\nu} + s(s-1)\delta_{AB}g_{\mu\nu}). \quad (3)$$

Notice, that for the $(\frac{1}{2}, \frac{1}{2})$ representation space, the capital Latin indices coincide with the Lorentz indices. A straightforward calculation (see Appendix I in [6]) yields

$$\tilde{\Gamma}_{\alpha\beta\mu\nu} = g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}. \quad (4)$$

Using this tensor, the free equation of motion for a vector particle becomes

$$[\tilde{\Gamma}_{\alpha\beta\mu\nu}\partial^\mu\partial^\nu + m^2 g_{\mu\nu}]V^\nu = 0, \quad (5)$$

where V^ν denotes the vector field. The tensor in Eq. (4) can be decomposed into its symmetric and antisymmetric parts as

$$\tilde{\Gamma}_{\alpha\beta\mu\nu} = \tilde{\Gamma}^S_{\alpha\beta\mu\nu} + \tilde{\Gamma}^A_{\alpha\beta\mu\nu}, \quad (6)$$

with

$$\tilde{\Gamma}^S_{\alpha\beta\mu\nu} = \frac{1}{2}(\tilde{\Gamma}_{\alpha\beta\mu\nu} + \tilde{\Gamma}_{\alpha\beta\nu\mu}) = g_{\alpha\beta}g_{\mu\nu} - \frac{1}{2}(g_{\alpha\nu}g_{\beta\mu} + g_{\alpha\mu}g_{\beta\nu}), \quad (7)$$

$$\tilde{\Gamma}^A_{\alpha\beta\mu\nu} = \frac{1}{2}(\tilde{\Gamma}_{\alpha\beta\mu\nu} - \tilde{\Gamma}_{\alpha\beta\nu\mu}) = \frac{1}{2}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}). \quad (8)$$

As discussed in [6] and also evident from Eq. (5), the covariant mass and spin projector in Eq. (1) fixes uniquely only the part of the tensor $\Gamma_{\alpha\beta\mu\nu}$ that is symmetric in the indices (μ, ν). Equation (5) is indisputably insensitive to the antisymmetric part which acquires relevance exclusively upon gauging when $[p_\mu, p_\nu]$ become proportional to the electromagnetic field tensor, $F_{\mu\nu}$, according to $[\pi_\mu, \pi_\nu] = ieF_{\mu\nu}$ with $\pi_\mu = p_\mu + eA_\mu$. Indeed, it is precisely the $\tilde{\Gamma}^A_{\alpha\beta\mu\nu}$ term which triggers the interactions with multipoles higher than the electric charge. A complete formalism is required to account for the most general form of the antisymmetric tensor.

C. General Lagrangian for an elementary vector particle in an electromagnetic background

In the vector case under investigation, the most general tensor $\Gamma^A_{\alpha\beta\mu\nu}$ has to be constructed from the metric and the Levi-Civita tensors and is given by

$$\Gamma^A_{\alpha\beta\mu\nu} = \left(g - \frac{1}{2}\right)(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}) + \xi\varepsilon_{\alpha\beta\mu\nu}, \quad (9)$$

where g and ξ are free parameters, so far. In what follows we shall replace $\tilde{\Gamma}^A_{\alpha\beta\mu\nu}$ in Eq. (8) by $\Gamma^A_{\alpha\beta\mu\nu}$ from the last equation. As a result, the most general tensor compatible with the covariant mass and spin projector in Eq. (1) becomes

$$\Gamma_{\alpha\beta\mu\nu} = g_{\alpha\beta}g_{\mu\nu} + (g-1)g_{\alpha\mu}g_{\beta\nu} - g g_{\alpha\nu}g_{\beta\mu} + \xi\varepsilon_{\alpha\beta\mu\nu}. \quad (10)$$

The corresponding gauged equation of motion is then obtained as

$$[\Gamma_{\alpha\beta\mu\nu}D^\mu D^\nu + m^2 g_{\mu\nu}]V^\nu = 0. \quad (11)$$

It can be derived from the following Lagrangian:

$$\mathcal{L} = -(D^\mu V^\alpha)^\dagger \Gamma_{\alpha\beta\mu\nu} D^\nu V^\beta + m^2 V^\alpha V_\alpha, \quad (12)$$

where $D^\mu = \partial^\mu - ieA^\mu$ and $(-e)$ is the charge of the vector particle. The hermiticity of the Lagrangian requires the couplings g, ξ to be real. Although the projection over the eigensubspaces of the Casimir operators of the Poincaré group studied here is well defined for massive particles only, the free Lagrangian,

$$\mathcal{L}_{\text{free}} = -(\partial^\mu V^\alpha)^\dagger \Gamma_{\alpha\beta\mu\nu}\partial^\nu V^\beta + m^2 V^\alpha V_\alpha, \quad (13)$$

possesses a smooth massless limit. In this limit the free Lagrangian reveals as a symmetry the invariance under the $U_V(1)$ gauge transformations

$$V_\alpha \to V_\alpha + \partial_\alpha \Lambda. \quad (14)$$

The mass term can now be generated through the conventional Higgs mechanism [6] in reference to this symmetry. A straightforward calculation yields the following interacting Lagrangian:

$$\mathcal{L}_{\text{int}} = -ie[(V^\alpha)^\dagger \Gamma_{\alpha\beta\mu\nu}\partial^\nu V^\beta - (\partial^\nu V^\alpha)^\dagger \Gamma_{\alpha\beta\nu\mu} V^\beta]A^\mu + e^2 (V^\alpha)^\dagger \Gamma_{\alpha\beta\mu\nu} V^\beta A^\mu A^\nu. \quad (15)$$

The respective $V^\beta(p)V^\alpha(p')A^\mu(k)$, and $V^\alpha(p')V^\beta(p)\times A^\mu(k)A^\nu(k')$ vertex functions extracted from Eq. (15) read

$$V_{\alpha\beta\mu} = ie(\Gamma_{\alpha\beta\mu\nu}p^\nu - \Gamma_{\alpha\beta\nu\mu}p'^\nu),$$
$$V_{\alpha\beta\mu\nu} = -ie^2(\Gamma_{\alpha\beta\mu\nu} + \Gamma_{\alpha\beta\nu\mu}), \quad (16)$$

with all incoming particles. Explicitly,

$$V_{\alpha\beta\mu} = ie(g_{\alpha\beta}(p-p')_\mu - g_{\alpha\mu}[gk+p]_\beta + g_{\beta\mu}[p'+gk]_\alpha + \xi\varepsilon_{\alpha\beta\mu\nu}(p+p')^\nu). \quad (17)$$

This vertex describes the electromagnetic interactions of a particle with magnetic (electric) dipole moment μ ($\tilde{\mu}$) and quadrupole electric (magnetic) moment $Q(\tilde{Q})$ given by (see, e.g., Ref. [10])

$$\mu = \frac{ge}{2m}, \qquad Q = -\frac{(g-1)e}{m^2},$$
$$\tilde{\mu} = \frac{\xi e}{2m}, \qquad \tilde{Q} = -\frac{\xi e}{m^2}. \quad (18)$$

It satisfies the Ward identity

$$(p+p')^\mu V_{\alpha\beta\mu} = -ie[\Delta^{-1}_{\alpha\beta}(p') - \Delta^{-1}_{\alpha\beta}(p)], \quad (19)$$

where $\Delta_{\alpha\beta}(p)$ is the propagator of the massive vector particle which in the unitary gauge (with respect to the gauge freedom in the massless case, see [6]) we are using here reads

$$\Delta_{\alpha\beta}(p) = \frac{-g_{\alpha\beta} + \frac{p_\alpha p_\beta}{m^2}}{p^2 - m^2 + i\varepsilon}. \quad (20)$$

Notice that Poincaré and gauge invariance alone allow the vector particle to carry any arbitrary magnetic- and electric-dipole moments, which then enter the definition of the electric- and magnetic-quadrupole moments, respectively. As long as the electric-dipole and the magnetic-quadrupole moments are CP violating, the ξ value is expected to be rather small. Nonetheless, we will keep this term for the sake of completeness of our Poincaré covariant projector and will fix it from unitarity arguments together with g. The Proca theory corresponds instead to a *fixed unphysical* $g = 1$ value and, in being incomplete, as mentioned in the introduction, fails to predict a quadrupole-electric moment.

A Lagrangian for vector particles containing g as a free parameter has earlier been considered by Corben and Schwinger [11] and used later by Lee and Yang [12]. In contrast to our approach, Poincaré invariance is not made manifest in the Corben-Schwinger paper, but is somehow hidden in the restriction of all derivatives to second order, and $(\frac{1}{2}, \frac{1}{2})$ to spin-1. In our formalism the restriction of derivatives to second order is dictated by the squared Pauli-Lubanski operator in Eq. (2) around which the Poincaré projector is constructed. Compared to [11], the advantage of our scheme lies in its generality. Consciously putting first principles at work sheds light on the path for obtaining the most general Lagrangian for a particle of spin s transforming in a specific representation of the HLG. Moreover, in the next section we will add another fundamental principle to the first two, namely, unitarity, which will allow us to completely fix the electromagnetic couplings of an elementary vector particle at tree level.

Our interacting Lagrangian with the unspecified value for the gyromagnetic ratio and the electric-dipole moment (and the related quadrupoles as shown above) appeared as a consequence of the fact that Poincaré invariance provides not one but infinitely many equivalent free particle Lagrangians according to Eq. (12) in combination with Eq. (10). These Lagrangians become distinguishable only upon gauging precisely through the different values for the respective gyromagnetic ratio, and the electric-dipole moment predicted by them. Obviously, only one of the $g(\xi)$ possible values corresponds to physical reality. In order to fix these values, one needs additional physical information. In the present work we shall demand finite total cross section of Compton scattering off a vector target in the high-energy limit and determine g and ξ accordingly.

III. COMPTON SCATTERING OFF A VECTOR TARGET

The differential cross section for $\gamma(k, \epsilon)V(p, \zeta) \rightarrow \gamma(k', \epsilon')V(p', \zeta')$ in the laboratory frame is given as

$$\frac{d\sigma}{d\Omega} = \frac{1}{4(4\pi)^2}|\bar{\mathcal{M}}|^2 \frac{1}{(m + \omega(1 - \cos\theta))^2}, \quad (21)$$

where ω stands for the energy of the incoming photon. The invariant amplitude can be written as

$$\mathcal{M} = \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_c, \quad (22)$$

where $\mathcal{M}_s$, $\mathcal{M}_u$, and $\mathcal{M}_c$ denote in turn the contributions of the s- and u- channel exchange, and the contact term. The explicit forms of these amplitudes are

$$\mathcal{M}_s = [V_{\sigma\beta\mu}(p, -p - k)\Delta^{\sigma\rho}(p + k)V_{\alpha\rho\nu}(p' + k', -p')] \times \zeta^\beta \epsilon^\mu \zeta'^\alpha \epsilon'^\nu, \quad (23)$$

$$\mathcal{M}_u = [V_{\sigma\beta\nu}(p, -p + k')\Delta^{\sigma\rho}(p - k')V_{\alpha\rho\mu}(p' - k', -p')] \times \zeta^\beta \epsilon^\mu \zeta'^\alpha \epsilon'^\nu, \quad (24)$$

$$\mathcal{M}_c = V_{\alpha\nu\beta\mu}\zeta^\beta \epsilon^\mu \zeta^\alpha \epsilon^\nu. \quad (25)$$

As a check, replacing ϵ^μ by k^μ and using the Ward identity we obtain

$$\mathcal{M}_s(\epsilon^\mu \rightarrow k^\mu) = e^2(\Gamma_{\alpha\beta\nu\mu}(p + k)^\mu + \Gamma_{\alpha\beta\mu\nu}p'^\mu)\zeta^\beta \zeta'^\alpha \epsilon'^\nu, \quad (26)$$

$$\mathcal{M}_u(\epsilon^\mu \rightarrow k^\mu) = -e^2(\Gamma_{\alpha\beta\nu\mu}p^\mu + \Gamma_{\alpha\beta\mu\nu}(p' - k)^\mu)\zeta^\beta \zeta'^\alpha \epsilon'^\nu, \quad (27)$$

$$\mathcal{M}_c(\epsilon^\mu \rightarrow k^\mu) = V_{\alpha\nu\beta\mu}\zeta^\beta \zeta'^\alpha k^\mu \epsilon'^\nu. \quad (28)$$

Upon summing up the three contributions one sees that gauge invariance is satisfied

$$\mathcal{M}(\epsilon \rightarrow k) = 0. \quad (29)$$

A similar calculation for the outgoing photon confirms once again gauge invariance to be satisfied. Using the conditions $k \cdot \varepsilon = k' \cdot \varepsilon' = p \cdot \zeta = p' \cdot \zeta' = 0$, we calculated $\mathcal{M}$ explicitly in Eq. (A1) in the Appendix. Inspection of the latter expression shows that the divergent terms in the high-energy limit come from the $1/m^2$ terms which are proportional to $(g - 2)$, ξ, their product and their second power, thus, they vanish for $g = 2$ and $\xi = 0$. Another and perhaps easier way to see that such cancellation occurs only for the mentioned values is to calculate the cross section. A straightforward calculation of the squared amplitude yields Eq. (A4) from the Appendix. The latter expression shows that in the classical limit, $\eta \rightarrow 0$, the differential cross section is independent of g and ξ, as it should be,

$$\left.\frac{d\sigma(g, \xi)}{d\Omega}\right|_{\eta\rightarrow 0} = \frac{r_0^2}{2}(1 + x^2), \quad (30)$$

and the total cross section coincides with the Thompson result,

$$\sigma(g,\xi)|_{\eta\to 0} = \frac{8\pi}{3} r_0^2 \equiv \sigma_T. \tag{31}$$

More interesting is the high-energy limit, $\eta \gg 1$, in which case we find

$$\frac{d\sigma(g,\xi)}{d\Omega}\bigg|_{\eta\gg 1} = \frac{r_0^2}{96(-1+x)^2}[80 + g^4(21+(-8+x)x) + 8g^3(-10+(-1+x)x) + 88\xi^2 + 21\xi^4 - 8x\xi^4 + x^2(-4+\xi^2)^2 + 2g^2(4(17 + x(4+x)) + (21+(-8+x)x)\xi^2) + 8g(-4(4+x+x^2) + (-10+(-1+x)x)\xi^2)]. \tag{32}$$

In general, for arbitrary g and ξ, the angular distribution of the emitted photon is sharply peaked in forward direction and the total cross section diverges violating the unitarity bounds [4,13,14]. In order to check the values of g and ξ avoiding this ultraviolet catastrophe we integrate the differential cross section in Eq. (32) from $x = -1 + \epsilon$ to $x = 1 - \epsilon$, with $\epsilon \to 0$ to obtain

$$\sigma(g,\xi)|_{\eta\gg 1} = \frac{8\pi r_0^2}{3}\frac{1}{128}\bigg[2(1-\epsilon)(g^2+4g-4+\xi^2)^2 + 2((g-2)^2+\xi^2)(7g^2-12g+12 + 7\xi^2)\bigg(\frac{1}{\epsilon-2}+\frac{1}{\epsilon}\bigg) + 2((g-2)^2+\xi^2) \times (3g^2+8g-4+3\xi^2)\log\bigg(\frac{2}{\epsilon}-1\bigg)\bigg]. \tag{33}$$

The latter equation makes manifest that the only values of g and ξ preventing the violation of unitarity at high energies are indeed $g = 2$ and $\xi = 0$, respectively.

Using the first principles of (i) the covariant projection on the mass-m and spin-1 eigensubspace of the Casimir operators of the Poincaré group in the $(\frac{1}{2},\frac{1}{2})$ representation space of the HLG in combination with the most general form of the antisymmetric part of the corresponding tensor (not fixed by the projection), (ii) $U(1)_{em}$ gauge principle, and (iii) unitarity in the high-energy limit, we were able to uniquely fix the electromagnetic couplings of *any* elementary vector particle at tree level. Any massive spin-1 particle described by means of a four-vector field must have a magnetic-dipole moment of $\mu = e/m$, an electric-quadrupole moment of $Q = -e/m^2$, and vanishing electric-dipole and magnetic-quadrupole moments at tree level. This is the prime result of this work.

These are precisely the parameter values that enter the description of the electromagnetic properties of the W boson in the standard model. However, our results, being based on first principles, are valid for any elementary spin-1 particle described by a four-vector field. Notice that in our derivation no assumptions have been made about other interactions of the vector particle and its Abelian or non-Abelian nature. In the following section we discuss our results in the context of non-Abelian gauge theories. Before this, and for the sake of completeness, we present our results for Compton scattering off any elementary massive vector particle in terms of the dimensionless variable η. The full angular distribution for the case $g = 2$, $\xi = 0$, is

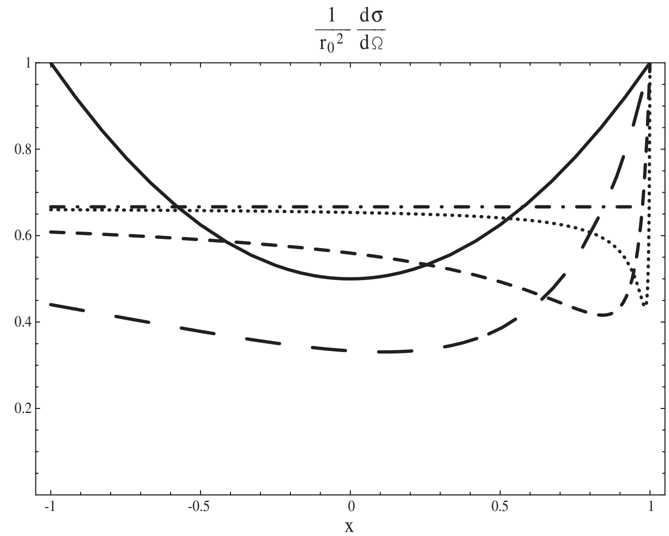

FIG. 1. Differential cross section normalized to the classical squared radius as a function of $x = \cos\theta$ for different values of the energy of the incident photon in the laboratory frame: $\eta = 0$ (thick line), $\eta = 1$ (long dashed line), $\eta = 10$ (short-dashed line), $\eta = 100$ (dotted line), and $\eta = \infty$ (dot-dashed line), where $\eta = \omega/m$.

$$\frac{d\sigma(2,0)}{d\Omega} = \frac{r_0^2}{6(1+\eta(1-x))^4}[3 + 6\eta + 11\eta^2 + 8\eta^3 + 4\eta^4 + x^4\eta^2(3+4\eta^2) - 2x^3\eta(3+3\eta + 4\eta^2 + 8\eta^3) - 2x\eta(3+11\eta+12\eta^2 + 8\eta^3) + x^2(3+6\eta+14\eta^2+24\eta^3 + 24\eta^4)]. \tag{34}$$

At high energies we obtain it flat according to

$$\frac{d\sigma(2,0)}{d\Omega}\bigg|_{\eta\gg 1} = \frac{2}{3}r_0^2, \tag{35}$$

and the total cross section coincides with the Thompson one. Integrating Eq. (34) we obtain the total cross section as

$$\sigma(2,0) = \frac{8\pi r_0^2}{3}\frac{2\eta(9+54\eta+129\eta^2+168\eta^3+140\eta^4+48\eta^5) - 3(1+2\eta)^3(3+3\eta+4\eta^2)\log(1+2\eta)}{12\eta^3(1+2\eta)^3} \tag{36}$$

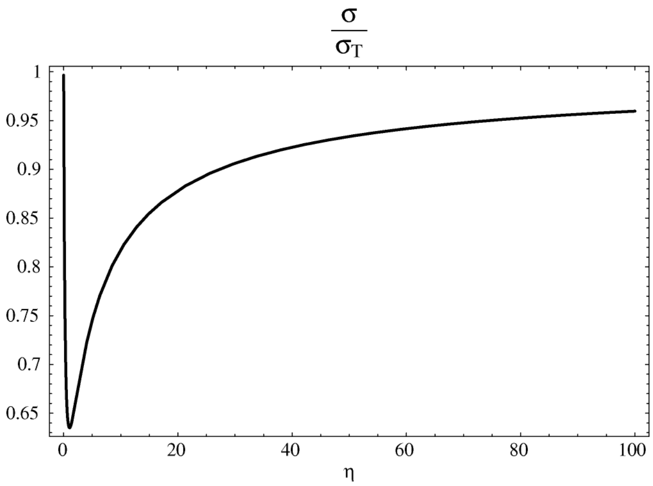

FIG. 2. Total cross section normalized to the Thompson value as a function of the energy of the incident photon in the laboratory frame, where $\eta = \omega/m$.

In Fig. 1 we display the differential cross section as a function of $x = \cos\theta$ for different values of the energy of the incident photon. Starting from the classical angular distribution the radiation slightly peaks in forward direction at intermediate energies, but this effect is rapidly damped and the angular distribution becomes flat at high energies. In Fig. 2 we show the total cross section as a function of the energy of the incoming photon. It decreases from the Thompson value at $\eta = 0$ as the energy increases in the low-energy region, reaches its minimum at $\eta = 1$, and from there onward it smoothly rises approaching again the Thompson value in the high-energy limit, $\eta \gg 1$.

IV. PARTITION OF $g = 2$ INTO ABELIAN AND NON-ABELIAN CONTRIBUTIONS FOR NON-ABELIAN VECTOR GAUGE PARTICLES

With the $g = 2$ value for an elementary massive vector particle, the covariant projector formalism exploited here has predicted for a second time a universal gyromagnetic ratio for a high-spin particle. Earlier, the same number was obtained for spin-3/2 in Ref. [6]. There, and upon prohibiting spin-3/2 to spin-1/2 conversion, the electromagnetically gauged spin-3/2 Lagrangian was obtained to depend on g alone. The demand for causal spin-3/2 propagation within an electromagnetic environment restricted then g to $g = 2$. In this way the so called Velo-Zwanziger problem of superluminal propagation of spin-3/2 fields within an electromagnetic environment as suffered by the Rarita-Schwinger formalism [5] was resolved and the solution related to the value of the gyromagnetic factor. Also in this case, the nature of the particle, Abelian versus non-Abelian, was irrelevant to the solution. On the other side, it is well known that non-Abelian gauge theories contain "nonminimal" electromagnetic interactions (see [4] and references therein) which contribute to the net electromagnetic couplings of gauge bosons. In order to understand our results within the latter context let us consider the case of the electroweak $W^{\pm}$ bosons for which the partition of its gyromagnetic ratio into Abelian and non-Abelian contributions is well known [1]. In that respect it is important to recall that the photon field, A, is not among the four gauge bosons, $\mathbf{W} = \{W^{\pm}, W^3\}$, and B, of the $SU(2)_L \times U_Y(1)$ group but partakes both B and W^3 according to

$$\begin{aligned} W^3_\mu &= \cos\theta_W Z^0_\mu + \sin\theta_W A_\mu, \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z^0_\mu, \end{aligned} \tag{37}$$

in standard notations. As long as the Abelian field B belongs to $U(1)_Y$, while the non-Abelian isovector field $\mathbf{W}$ is associated with $SU(2)_L$, the Abelian contribution to the electromagnetic interactions of the W take their origin from $U(1)_Y$ gauging while the non-Abelian ones arise from the $SU(2)_L$ gauging, both in combination with $e = g_W \sin\theta_W$ (g_W stands for the universal electroweak coupling). The three physical massive gauge bosons $W^{\pm}$ and Z^0 emerge only after spontaneous $SU(2)_L \times U(1)_Y/U_{em}(1)$ breaking, and it is only at this level that one can identify their corresponding electromagnetic interactions. What one observes is a partition of the gyromagnetic ratio in two sectors, the non-Abelian one, $g_{na} = 1$, as provided by the non-Abelian field tensor, and the Abelian one, $g_a = 2 - g_{na}$, coming from the Abelian $U(1)_Y$ gauging. The $g_a = 1$ value required by the specifics of the electroweak gauge group coincides by chance with the one provided by Proca theory, so that using Proca's Lagrangian in the standard model is of no harm. However, for any other gauge group, that provides a $g_{na} \neq 1$, this concept will necessarily collapse. For a general non-Abelian theory, based on a group, call it G, and different from the electroweak one, $U(1)_{em}$ will manifest itself only after the spontaneous $G/U_{em}(1)$ breaking at low energies. Concerning physics beyond the standard model both aspects are completely unknown, so far. Apparently, the respective g_{na} value will depend both on the $U(1)_{em}$ embedding in G and on the details of the spontaneous symmetry breaking and can be lesser or bigger than 1. It is obvious that Proca's theory is not applicable to this case. Instead, one can make use of the Lagrangian with the free g parameter as defined by Eqs. (10) and (12) and employ it, this time at the level *before* the spontaneous symmetry breaking. Fixing g_a to $g_a = 2 - g_{na}$ guarantees $g = g_a + g_{na} = 2$ for any needed partition of the net gyromagnetic ratio at the final stage.

What after all should be abundantly clear is that whatever the unknown group G, the $U_{em}(1)$ embedding in it, or the mechanisms for the spontaneous symmetry breaking might be, with the Lagrangian defined by Eqs. (10) and (12) one can always end up with a net gyromagnetic ratio of $g_a + g_{na} = 2$ for a vector gauge boson. At any rate, the three first principles mentioned above must be respected by the final form of the interaction of the vector particle with the electromagnetic field and the respective electromagnetic properties concluded here will always hold valid.

V. CONCLUSIONS AND PERSPECTIVES

In this work we studied the structure of the Lagrangian of an elementary vector particle (massive particle transforming in the $(\frac{1}{2}, \frac{1}{2})$ representation of the HLG) interacting with an electromagnetic field. The Lagrangian's derivation was based on the three fundamental principles of (i) Poincaré invariance of space-time, (ii) $U(1)_{em}$ gauge symmetry of electromagnetism, and (iii) unitarity bounds for the Compton scattering cross section. The first two principles lead to a general Lagrangian depending on two free parameters, g and ξ, both required in the definition of the four electromagnetic multipoles characterizing a vector particle. Requiring the total cross section for Compton scattering off a vector target to respect the unitarity bounds in the high-energy limit allows to fix the free parameters to $g = 2$, $\xi = 0$ and thereby to determine the tree-level electromagnetic properties of *any* vector particle, be it Abelian or non-Abelian. It must have a magnetic-dipole moment of $\mu = e/m$ (a gyromagnetic ratio of $g = 2$), an electric-quadrupole moment of $Q = -e/m^2$, and vanishing electric-dipole and magnetic-quadrupole moments at tree level. Modifications to this picture can arise only at one-loop level either through higher-order electromagnetic effects, or, through electromagnetic corrections induced by interactions with other particles. For gauge vector bosons the electromagnetic couplings are partitioned into non-Abelian (g_{na}) and Abelian (g_a) contributions obeying the restriction, $g = g_a + g_{na} = 2$. The specific respective g_a and g_{na} values depend on the gauge group G, the embedding of $U(1)_{em}$ in it, and the details of the spontaneous symmetry breaking $G \longrightarrow U_{em}(1)$ at low energies.

The results obtained here are valid for elementary vector particles, i.e., massive spin-1 particles transforming in the $(\frac{1}{2}, \frac{1}{2})$ representation of the HLG as is the case of the W boson. This is certainly not the only possible HLG representation for the description of spin-1 though the one of the widest spread, and it would be interesting to check validity of the concepts presented here for spin-1 fields transforming in other representations such as the totally antisymmetric second-rank tensor, $(1, 0) \oplus (0, 1)$, (considered in [15]), or the totally symmetric one, (1, 1) (considered, among others, in [16]). Although we expect the calculation to evolve similarly to the one presented here, the new problems need to be worked out anew, a task that is beyond the scope of the present study. Finally, another challenge for future research would be to explore within the context of the covariant projector formalism the link between $g = 2$ and the renormalizability of an effective field theory as found in Ref. [17].

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APPENDIX

The explicit expression for the invariant amplitude in the Compton scattering off a vector target is a bit cumbersome and given by

$$
\begin{aligned}
\mathcal{M} = {}& -e^2\Big\{2(\zeta\cdot\zeta')\Big[\frac{(p\cdot\varepsilon)(p'\cdot\varepsilon')}{p\cdot k} - \frac{(p\cdot\varepsilon')(p'\cdot\varepsilon)}{p\cdot k'} - \varepsilon\cdot\varepsilon'\Big] - g\Big[(\varepsilon'\cdot[\zeta,\zeta']\cdot k')\Big(\frac{p\cdot\varepsilon}{p\cdot k} - \frac{p'\cdot\varepsilon}{p\cdot k'}\Big) \\
& - (\varepsilon\cdot[\zeta',\zeta]\cdot k)\Big(\frac{p\cdot\varepsilon'}{p\cdot k'} - \frac{p'\cdot\varepsilon'}{p\cdot k}\Big)\Big] - 2\xi\Big[\frac{p\cdot\varepsilon\langle\zeta\zeta'\varepsilon'k'\rangle + p'\cdot\varepsilon'\langle\zeta'\zeta\varepsilon k\rangle}{p\cdot k} + \frac{p\cdot\varepsilon'\langle\zeta\zeta'\varepsilon k\rangle + p'\cdot\varepsilon\langle\zeta'\zeta\varepsilon'k'\rangle}{p\cdot k'}\Big] \\
& + \frac{g\xi}{2}\Big[\frac{1}{p\cdot k}(k\cdot\zeta\langle\varepsilon\zeta'\varepsilon'k'\rangle - \varepsilon\cdot\zeta\langle k\zeta'\varepsilon'k'\rangle + k'\cdot\zeta'\langle\varepsilon'\zeta\varepsilon k\rangle - \zeta'\cdot\varepsilon'\langle k'\zeta\varepsilon k\rangle) - \frac{1}{p\cdot k'}(k'\cdot\zeta\langle\varepsilon'\zeta'\varepsilon k\rangle - \varepsilon'\cdot\zeta\langle k'\zeta'\varepsilon k\rangle \\
& + k\cdot\zeta'\langle\varepsilon\zeta\varepsilon'k'\rangle - \zeta'\cdot\varepsilon\langle k\zeta\varepsilon'k'\rangle)\Big] + \frac{g^2}{2}\Big[\frac{1}{p\cdot k}[k\cdot\zeta(\varepsilon'\cdot[\varepsilon,\zeta']\cdot k') - \varepsilon\cdot\zeta(k\cdot[\varepsilon',k']\cdot\zeta')] - \frac{1}{p\cdot k'}[k'\cdot\zeta(\varepsilon\cdot[\varepsilon',\zeta']\cdot k) \\
& - \varepsilon'\cdot\zeta(k'\cdot[\varepsilon,k]\cdot\zeta')]\Big] + \frac{\xi^2}{2}\Big[\frac{1}{p\cdot k}\langle\mu\zeta\varepsilon'k'\rangle\langle\mu\zeta'\varepsilon k\rangle - \frac{1}{p\cdot k'}\langle\mu\zeta\varepsilon'k'\rangle\langle\mu\zeta'\varepsilon k\rangle\Big]\Big\} + \frac{e^2}{2m^2}\Big\{(g-2)\xi\Big[\frac{1}{p\cdot k'}[(\varepsilon\cdot[p,\zeta] \\
& \cdot k)\langle p'\zeta'\varepsilon'k'\rangle + (\varepsilon'\cdot[p',\zeta']\cdot k')\langle p\zeta\varepsilon k\rangle] - \frac{1}{p\cdot k'}[(\varepsilon'\cdot[p,\zeta]\cdot k')\langle p'\zeta'\varepsilon k\rangle + (\varepsilon\cdot[p',\zeta']\cdot k)\langle p\zeta\varepsilon'k'\rangle]\Big] \\
& + (g-2)^2\Big[\frac{1}{p\cdot k}(\varepsilon'\cdot[p',\zeta']\cdot k')(\varepsilon'\cdot[p',\zeta']\cdot k') - \frac{1}{p\cdot k'}(\varepsilon\cdot[p',\zeta']\cdot k)(\varepsilon\cdot[p',\zeta']\cdot k)\Big] \\
& + \xi^2\Big[\frac{1}{p\cdot k}\langle p\zeta\varepsilon k\rangle\langle p'\zeta'\varepsilon'k'\rangle - \frac{1}{p\cdot k'}\langle p\zeta\varepsilon'k'\rangle\langle p'\zeta'\varepsilon k\rangle\Big]\Big\}. \qquad \text{(A1)}
\end{aligned}
$$

Here we used Holstein's notation [1]

$$S \cdot [Q, R] \cdot T = S \cdot QR \cdot T - S \cdot RQ \cdot T, \tag{A2}$$

and defined

$$\langle \alpha ABC \rangle \langle \alpha A' B' C' \rangle = \epsilon_{\alpha\beta\mu\nu} A^{\beta} B^{\mu} C^{\nu} \epsilon^{\alpha}{}_{\sigma\rho\gamma} A'^{\beta} B'^{\mu} C'^{\nu}. \tag{A3}$$

The resulting cross section is then obtained as

$$\begin{aligned} \frac{d\sigma(g,\xi)}{d\Omega} = {}& \frac{r_0^2}{96(1+\eta(1-x))^4}\{48 + 96\eta + x^4\eta^2(48 + \eta^2(-4 + 4g + g^2 + \xi^2)^2) + 2\eta^3(80 - 48g + 3g^4 + 24\xi^2 \\ & + 3\xi^4 + g^2(8 + 6\xi^2)) + 2\eta^2(104 - 48g + 3g^4 + 24\xi^2 + 3\xi^4 + g^2(8 + 6\xi^2)) + \eta^4(80 - 80g^3 + 21g^4 \\ & + 88\xi^2 + 21\xi^4 - 16g(8 + 5\xi^2) + 2g^2(68 + 21\xi^2)) - 2x^3\eta[48 + 48\eta - \eta^2(-16 + 48g + g^4 + 8\xi^2 + \xi^4 \\ & + 2g^2(-20 + \xi^2)) + \eta^3(16 + 12g^3 + 5g^4 - 8\xi^2 + 5\xi^4 + 4g(-4 + 3\xi^2) + 2g^2(-4 + 5\xi^2))] \\ & + 2x^2[24 + 48\eta - \eta^2(-64 + 48g + g^4 + 8\xi^2 + \xi^4 + 2g^2(-20 + \xi^2)) - \eta^3(-80 - 16g^3 + g^4 + 8\xi^2 + \xi^4 \\ & + 2g^2(-20 + \xi^2) - 16g(-5 + \xi^2)) + \eta^4(48 - 28g^3 + 19g^4 + 40\xi^2 + 19\xi^4 - 4g(12 + 7\xi^2) \\ & + g^2(40 + 38\xi^2))] - 2x\eta[48 + 16\eta(7 + g^3 + g(-2 + \xi^2)) + \eta^3(80 - 76g^3 + 25g^4 + 88\xi^2 + 25\xi^4 \\ & + 10g^2(12 + 5\xi^2) - 4g(28 + 19\xi^2)) + \eta^2(16g^3 + 3g^4 + 16g(-5 + \xi^2) + g^2(8 + 6\xi^2) \\ & + 3(48 + 8\xi^2 + \xi^4))]\}, \end{aligned} \tag{A4}$$

where $r_0 = \frac{\alpha}{m}$ stands for the classical radius, $\eta = \omega/m$, $x = \cos\theta$, and we used

$$\omega' = \frac{\omega}{(m + \omega(1 - x))}, \tag{A5}$$

for the energy of the final photon ω'.

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