$\mu - \tau$ symmetry, sterile right-handed neutrinos, and leptogenesis

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Leptogenesis is studied in a seesaw model with $\mu - \tau$ symmetry for $SU_L(2)$-singlet right-handed neutrinos. It is shown that lepton asymmetry is not zero and is given by the square of the solar neutrino mass difference and can be of the right order of magnitude. Further it involves the same Majorana phase which appears in the neutrinoless double β-decay. In this framework one of the right-handed seesaw partners of light neutrinos can be made massless. This can be identified with a sterile neutrino, once it acquires a tiny mass ($\approx$ 1 eV) when $\mu - \tau$ symmetry is broken in the right-handed neutrino sector. The above mentioned sterile neutrino together with another one can be identified to explain the MiniBooNE and LSND results. The light 5×5 neutrino mass matrix is completely fixed if CP is conserved and so is the effective mass for neutrinoless double β-decay.

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There is a compelling evidence [1] that neutrinos change flavor, have nonzero masses, and the neutrino mass eigenstates are different from weak eigenstates. As such they undergo oscillations.

All neutrino data [1] with the exception of the LSND anomaly [2] is explained by three active neutrino flavor oscillations with the mass squared differences and mixing angles having the following values [3]:

$$\begin{aligned}
&\Delta m^2_{\rm scalar} = \Delta m^2_{12} = (8.1 \times 1.0) \times 10^{-5}\ {\rm eV}^2,\\
&\sin^2\theta_{12} = 0.30 \pm 0.08,\\
&\Delta m^2_{\rm atm} = |\Delta m_{13}|^2 \simeq |\Delta m^2_{23}| = (2.2 \pm 1.1) \times 10^{-3}\ {\rm eV}^2,\\
&\sin^2\theta_{23} = 0.50 \pm 0.18,\\
&\sin^2\theta_{13} \leq 0.047. \qquad (1)
\end{aligned}$$

The MiniBooNE (MB) [4] and LSND experiments would require that there are two sterile neutrinos [5] that must mix with the active ones. The sterile neutrinos mass is in the range of an eV. There exist many proposals in literature to explain the light mass of the sterile neutrinos [6].

As is well-known, the seesaw mechanism, which can explain tiny masses of three light active neutrinos, introduces three right-handed neutrinos to the standard model. Since the right-handed neutrinos are sterile with respect to known weak interactions, there have been speculations that one of them can be made ultralight so as to play the role of the sterile neutrino without affecting the conventional seesaw mechanism for the active neutrinos. There exist models in literature by which this can be achieved [6,7].

The purpose of this paper is to consider a simple extension of the standard model, namely, $SU_L(2) \times U_e(1) \times U_\mu(1) \times U_\tau(1)$, which has been used [8,9] to get light neutrino masses within the framework of the seesaw mechanism.

Now the experimental observation of near maximal atomospheric mixing ($\sin\theta_{23} = \frac{1}{2}$) and small upper limit on θ_{13} indicate that there may be an approximate $\mu - \tau$ interchange symmetry in the neutrino sector [10]. This symmetry also has interesting implications for leptogenesis [11–13]. I assume this symmetry only for the right-handed neutrinos which are $SU_L(2)$ singlets and in that case I have to consider the gauge group $SU_L(2) \times U_e(1) \times U_{\mu-\tau}(1)$. I show that this leads to a 3 + 1 scenario for the LSND anomaly if there is a single Yukawa coupling for right-handed neutrinos with relevant Higgs in its ($\mu - \tau$) subsector (see below). But the MB results rule this out. However, I can overcome this by postulating a purely singlet right-handed neutrino in the above gauge group to provide the second sterile neutrino needed. I also discuss the implications of this model in neutrinoless double beta decay and leptogenesis.

In addition to the usual fermions and Higgs, I consider $SU_L(2)$-singlet right-handed neutrinos $N^i_R (i = e, \mu, \tau)$ and the Higgs with quantum numbers given below:

$$\begin{aligned}
&L_e\colon (2, -1, 0), \quad \phi^{(1)}\colon (2, -1, 0), \quad N^e_R\colon (1, -1, 1),\\
&e_R\colon (1, -2, 0), \quad L_{\mu-\tau}\colon (2, 0, -1), \quad \phi^{(2)}\colon (2, 0, -1),\\
&N^{\mu,\tau}_R\colon (1, 1, -1), \quad \mu_R, \tau_R\colon (1, 0, -2),\\
&\Sigma\colon (1, 0, 0), \quad \Sigma'\colon (1, 2, -2). \qquad (2)
\end{aligned}$$

The Yukawa couplings of neutrinos with Higgs, using $\mu - \tau$ symmetry for right-handed neutrinos only, is given by (suppressing subscripts L and R)

$$\begin{aligned}
\mathcal{L}_Y = {}& h_{11}\bar{L}_e N_e \phi^{(2)} + [h_{22}\bar{L}_\mu (N_\mu + N_\tau)\\
&+ h_{32}\bar{L}_\tau (N_\mu + N_\tau)]\phi^{(1)} + {\rm H.c.} + f_{11} N^T_e C N_e \Sigma'\\
&+ f_{12} N^T_e C (N_\mu + N_\tau)\Sigma + {\rm H.c.}\\
&+ f_{22}[(N^T_\mu C N_\mu + N^T_\tau C N_\tau)\\
&+ f_{23}(N^\mu_\tau C N_\tau + N^T_\tau C N_\mu)]\bar{\Sigma}'. \qquad (3)
\end{aligned}$$

Writing

$$< \phi^1 > = v_1, \qquad < \phi^2 > = v_2,$$
$$< \Sigma > = \Lambda, \qquad < \Sigma' > = \Lambda'$$

the Dirac and Majorana mass terms are

$$H_D = \{h_{11} v_2 \bar{\nu}_e N_e + v_1(h_{22}\bar{\nu}_\mu + h_{32}\bar{\nu}_\tau)(N_\mu + N_\tau)\} + \text{H.c.}, \tag{4}$$

$$\begin{aligned} H_M = {} & f_{12}\Lambda(N_e^T C(N_\mu + N_\tau)) + \text{H.c.} \\ & + \Lambda'\{f_{11}(N_e^T C N_e) + f_{22}(N_\mu^T C N_\mu + N_\tau^T C N_\tau) \\ & + f_{23}(N_\mu^T C N_\tau + N_\tau^T C N_\mu)\}. \end{aligned} \tag{5}$$

Before I proceed further I wish to remark that by imposing $\mu - \tau$ symmetry only on the $SU_L(2)$-singlet right-handed neutrinos, I have avoided the well-known problem [14,15] associated with simultaneous imposition of $\mu - \tau$ symmetry on charged lepton and neutrino mass matrices. Later on I will show that by requiring only maximal atmospheric mixing and zero U_{e3}, when diagonalizing light neutrino mass matrix obtained in the seesaw mechanism, I obtain $h_{22} = h_{32}$. The same result is obtained if $\nu_\mu \leftrightarrow \nu_\tau$ symmetry is imposed in Eq. (4) independent of $\mu - \tau$ symmetry for the right-handed neutrino sector, as also noticed previously [14]. Then $\nu_- = \frac{\nu_\mu - \nu_\tau}{\sqrt{2}}$ will be massless.

The Eqs. (4) and (5) give the Dirac and Majorana mass matrices as

$$m_D = \begin{pmatrix} h_{11}v_2 & 0 & 0 \\ 0 & h_{22}v_1 & h_{22}v_1 \\ 0 & h_{32}v_1 & h_{32}v_1 \end{pmatrix}, \tag{6}$$

$$M_R = \begin{pmatrix} f_{11}\Lambda' & f_{12}\Lambda & f_{12}\Lambda \\ f_{12}\Lambda & f_{22}\Lambda' & f_{23}\Lambda' \\ f_{12}\Lambda & f_{23}\Lambda' & f_{22}\Lambda' \end{pmatrix}. \tag{7}$$

I diagonalize M_R in Eq. (7) by the unitary matrix V, thereby defining the mass eigenstate N_1, N_2, N_3:

$$\begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} = V \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} \tag{8}$$

where the most general 3×3 unitary matrix, consistent with $\mu - \tau$ symmetry, is

$$V = \begin{pmatrix} c' & s' & 0 \\ -\frac{s'}{\sqrt{2}} & \frac{c'}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{s'}{\sqrt{2}} & \frac{c'}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} P(\gamma) \tag{9}$$

where $P(\gamma)$ is a diagonal phase matrix (consisting of three nontrivial Majorana phases γ_1, γ_2, γ_3). Then

$$V^T M_R V = \hat{M}_R = \text{diag}(M_1, M_2, M_3) \tag{10}$$

where

$$M_3 = e^{2i\gamma_3}[f_{22} - f_{23}]\Lambda'. \tag{11}$$

Using Eqs. (6) and (8), the Dirac matrix in

$$(\bar{N}_1 \quad \bar{N}_2 \quad \bar{N}_3) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

basis is

$$\hat{m}_D^\dagger = V^T m_D^\dagger,$$

or

$$\hat{m}_D = m_D V^*. \tag{12}$$

One finds that due to the structure of m_D in Eq. (6), the third column of matrix $\hat{m}_D$ has zero values. Taking out this column, $\hat{m}_D$ is a 3×2 matrix

$$\hat{m}_D = \begin{pmatrix} c' e^{-i\gamma_1}(h_{11}v_2) & s' e^{-i\gamma_2}(h_{11}v_2) \\ -\frac{s'}{\sqrt{2}}(2h_{22}v_1) & \frac{c'}{\sqrt{2}} e^{-i\gamma_1}(2h_{22}v_1) \\ -\frac{s'}{\sqrt{2}}(2h_{32}v_1) & \frac{c'}{\sqrt{2}} e^{-i\gamma_1}(2h_{32}v_1) \end{pmatrix}$$

so that [12]

$$R = \hat{m}_D^\dagger \hat{m}_D \tag{13}$$

$$= \begin{pmatrix} c'^2|h_{11}|^2 v_2^2 + \frac{s'^2}{2}[|2h_{22}|^2 v_1^2 + |2h_{32}|^2 v_1^2] & c's'[|h_{11}|^2 v_2^2 - \frac{1}{2}(|2h_{22}|^2 v_1^2 + |2h_{32}|^2 v_1^2) e^{i(\gamma_1 - \gamma_2)}] \\ c's'(|h_{11}|^2 v_2^2 - \frac{1}{2}(|2h_{22}|^2 v_1^2 + |2h_{32}|^2 v_1^2) e^{-i(\gamma_1 - \gamma_2)} & s'^2|h_{11}|^2 v_2^2 + \frac{c'^2}{2}[|2h_{22}|^2 v_1^2 + |2h_{32}|^2 v_1^2] \end{pmatrix}. \tag{14}$$

Then the effective Majorana mass matrix for light neutrinos is

$$M_\nu = \hat{m}_D \hat{M}_R^{-1} \hat{m}_D^T = \hat{A} \tag{15}$$

where $\hat{A}$ is a 3×3 matrix with matrix elements

$$\begin{aligned} & a_{11} = h_{11}^2 v_2^2 A, \qquad \sqrt{2}a_{12} = h_{11}(2h_{22})v_1 v_2 B, \\ & \sqrt{2}a_{13} = h_{11}(2h_{32})v_1 v_2 B, \qquad a_{22} = \frac{1}{2}(4h_{22}^2 v_1^2)C, \\ & a_{23} = \frac{1}{2}(2h_{22})(2h_{32})v_1^2 C, \qquad a_{33} = \frac{1}{2}(4h_{32}^2)v_1^2 C. \end{aligned} \tag{16}$$

Here

$$
\begin{aligned}
A &= e^{-2i\gamma_1}\left[\frac{c'^2}{M_1} + \frac{s'^2}{M_2}e^{2i(\gamma_1-\gamma_2)}\right],\\
B &= -e^{-2i\gamma_1}c's'\left[\frac{1}{M_1} - \frac{1}{M_2}e^{2i(\gamma_1-\gamma_2)}\right],\\
C &= e^{-2i\gamma_1}\left[\frac{s'^2}{M_1} + \frac{c'^2}{M_2}e^{2i(\gamma_1-\gamma_2)}\right].
\end{aligned}
\tag{17}
$$

In order to diagonalize M_ν as given in Eq. (15) I now make the assumption of maximal atmospheric mixing and zero U_{e3}. This requires the 3×3 unitary matrix for diagonalization to be

$$
U = \begin{pmatrix} c & s & 0 \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\mathrm{diag}(e^{i\beta_1}, e^{i\beta_3}, e^{i\beta_3}) \tag{18}
$$

so that

$$
U^T M_\nu U = \mathrm{diag}(m_1 m_2 m_3). \tag{19}
$$

This gives

$$
\begin{gathered}
a_{12} = a_{13} = b, \qquad a_{22} = a_{33} = d,\\
m_3 = 2e^{i\beta_3}(a_{22} - a_{23}).
\end{gathered}
$$

For my case these relations in turn imply [cf. Eq. (16)] $h_{22} = h_{32}$ so that $a_{22} = a_{23} = d$.

Finally then I have

$$
M_\nu = \begin{pmatrix} a & b & b \\ b & d & d \\ b & d & d \end{pmatrix} \tag{20}
$$

where

$$
\begin{aligned}
a \equiv a_{11} &= e^{-2i\beta_1}[c^2 m_1 + s^2 m_2 e^{i\Delta}],\\
2d &= e^{-2i\beta_1}[s^2 m_1 + c^2 m_2 e^{i\Delta}],\\
\sqrt{2}b &= -cse^{-2i\beta_1}[m_1 - m_2 e^{i\Delta}],\\
m_3 &= 0,\\
\Delta &= 2(\beta_1 - \beta_2).
\end{aligned}
\tag{21}
$$

I wish to emphasize that form (20) for M_ν is a consequence of $\mu - \tau$ symmetry for right-handed $SU_L(2)$-singlet neutrinos, and maximal atmospheric mixing and vanishing of U_{e3}.

Taking out the state corresponding to eigenvalue zero, i.e., in (ν_{eL}, ν_{+L}) basis

$$
M_\nu = \begin{pmatrix} a & \sqrt{2}b \\ \sqrt{2}b & 2d \end{pmatrix}. \tag{22}
$$

I now list some useful relations, which I shall be using. Calculating $\mathrm{Im}[(\sqrt{2}b)^2 a^* 2d^*]$ from Eqs. (16) and (22) and equating them, I obtain

$$
\begin{aligned}
c'^2 s'^2 \sin[2(\gamma_1 - \gamma_2)] = &-\frac{1}{|h_{11}v_2|^4 |2h_{22}v_1|^4}\\
&\times\left[c^2 s^2 \frac{m_1 m_2 (m_2^2 - m_1^2)}{(M_2^2 - M_1^2)}\right]\\
&\times M_2^3 M_1^3 \sin\Delta.
\end{aligned}
\tag{23}
$$

Further from Eqs. (21) and (23)

$$
|\det M_\nu| = |a(2d) - (\sqrt{2}b)^2| = m_1 m_2. \tag{24}
$$

Calculating $|a(2d) - (\sqrt{2}b)^2|$ from Eqs. (16) and (17), I obtain

$$
|h_{11}v_2|^2 |2h_{22}v_1|^2 \frac{1}{M_1 M_2} \tag{25}
$$

giving

$$
|h_{11}v_2|^2 |2h_{22}v_1|^2 = M_1 M_2 m_1 m_2 \tag{26}
$$

so that from Eq. (23)

$$
c'^2 s'^2 \sin[2(\gamma_1 - \gamma_2)] = -c^2 s^2 \frac{m_2^2 - m_1^2}{(M_2^2 - M_1^2)} \frac{M_1 M_2}{m_1 m_2}. \tag{27}
$$

Another useful relation comes from calculating $|\sqrt{2}b|^2$ from Eqs. (16), (17), and (21) and equating them

$$
\begin{aligned}
&c^2 s^2\left[(m_2 - m_1)^2 + 4m_1 m_2 \sin^2\frac{\Delta}{2}\right]\\
&\quad = c'^2 s'^2 \frac{m_1 m_2}{M_1 M_2}[(M_2 - M_1)^2 + 4M_1 M_2 \sin^2(\gamma_1 - \gamma_2)].
\end{aligned}
\tag{28}
$$

Finally I have to express $|h_{11}v_2|^2$ and $|2h_{22}v_1|^2$ in terms of observables $\Delta m^2_{\mathrm{solar}}$ and $\Delta m^2_{\mathrm{atm}}$ and $\sin^2\theta_{12}$ (I have already employed $\theta_{13} = 0$ and $\sin^2\theta_{23} = \frac{1}{2}$).

Equating the expressions for $|a|$ in Eqs. (16), (17), and (21) and using the relation (28), I obtain

$$
\begin{aligned}
|h_{11}v_2|^2 = &\left[(c^2 m_1 + s^2 m_2)^2 - 4c^2 s^2 m_1 m_2 \sin^2\frac{\Delta}{2}\right]^{1/2}\\
&\times\left\{\frac{c'^2}{M_1^2} + \frac{s'^2}{M_2^2} - \frac{c^2 s^2}{M_1 M_2 m_1 m_2}\right.\\
&\left.\times\left[(m_2 - m_1)^2 + 4m_1 m_2 \sin^2\frac{\Delta}{2}\right]\right\}^{-1/2}.
\end{aligned}
\tag{29}
$$

$|2h_{22}v_1|^2$ can then be obtained from Eq. (26):

$$
\begin{aligned}
|2h_{22}v_1|^2 = m_1 m_2&\left[(c^2 m_1 + s^2 m_2)^2\right.\\
&\left.- 4c^2 s^2 m_1 m_2 \sin^2\frac{\Delta}{2}\right]^{-1/2}\\
&\times\left\{c'^2 M_2^2 + s'^2 M_1^2 - c^2 s^2 \frac{M_1 M_2}{m_1 m_2}\right.\\
&\left.\times\left[(m_2 - m_1)^2 + 4m_1 m_2 \sin^2\frac{\Delta}{2}\right]\right\}^{1/2}.
\end{aligned}
\tag{30}
$$

I now discuss leptogenesis in my scenario. For the decay of a heavy Majorana neutrino N_i, the CP asymmetry is generated through the interference between tree-level and one-loop N_i decay diagrams and is given by [11–13,16]

$$\epsilon_i = \frac{1}{8\pi}\sum_{k\neq i}\frac{1}{v_i^2 R_{ii}}\,\mathrm{Im}\Big[(R_{ik})^2 f\Big(\frac{M_k^2}{M_i^2}\Big)\Big] \tag{31}$$

where M_i denotes the heavy Majorana neutrino masses, R_{ij} are defined in Eq. (15), and the loop function containing vertex and self-energy correction is

$$f(x) = \sqrt{x}\Big(\frac{2-x}{1-x} - (1+x)\ln\frac{1+x}{x}\Big) \to -\frac{3}{2}x^{-1/2}, \quad x \gg 1. \tag{32}$$

I consider the case that $M_1 \ll M_2$ (M_3 plays no role). I have $|v_1|^2 + |v_2|^2 = (174\ \mathrm{GeV})^2 = |v|^2$. I take $|v_1|^2 = |v_2|^2 = \frac{1}{2}v^2$, so that

$$\epsilon_1 = -\frac{3}{16\pi}\frac{M_1}{M_2}\frac{1}{v_1^2 R_{11}}\,\mathrm{Im}[(R_{12})^2] \tag{33}$$

where from Eq. (15) (with $h_{22} = h_{32}$),

$$R_{11} = c'^2|h_{11}v_1|^2 - s'^2|2h_{22}v_1|^2, \tag{34}$$

$$\begin{aligned}\mathrm{Im}[(R_{12})]^2 &= c'^2 s'^2[|h_{11}v_1|^2 - |2h_{22}v_1|^2]^2 \sin 2(\gamma_1 - \gamma_2)\\ &= -[|h_{11}v_1|^2 - |2h_{22}v_1|^2]^2 c^2 s^2\\ &\quad\times\frac{(m_2-m_1)^2 M_2 M_1}{(M_2-M_1)^2 m_1 m_2}\sin\Delta\end{aligned} \tag{35}$$

where I have used Eq. (27).

There is another constraint from out of equilibrium decay of the lightest right-handed neutrino, which I take N_1. This is given by [11,12]

$$\Gamma 1 = \frac{R_{11}M_1}{8\pi v_1^2} \leq H \tag{36}$$

where H is the Hubble constant at temperature $T = M_1$:

$$H = 1.66 g^{*1/2}\frac{T^2}{M_{pl}} \simeq 17\frac{M_1^2}{M_{pl}} \tag{37}$$

in a radiation dominated Universe. Here I have used g^* (the effective number of relativistic degrees of freedom) $\simeq 100$. The constraint (37) gives, for $M_1 \simeq 10^{10}$ GeV,

$$R_{11} < 4.3\times 10^{-7} v_1^2. \tag{38}$$

Hence from Eqs. (33) and (35), the lower limit on ϵ_1 is

$$\begin{aligned}\epsilon_1 &= \frac{3}{16\pi}\frac{M_1}{M_2}\frac{2.3\times 10^6}{v_1^4}c^2 s^2[|h_{11}v_1|^2 - |2h_{22}v_1|^2]^2\\ &\quad\times\frac{(m_2^2-m_1^2)M_2 M_1}{(M_2^2-M_1^2)m_1 m_2}\sin\Delta.\end{aligned} \tag{39}$$

With $M_1 \ll M_2$ and writing $m = \frac{m_1+m_2}{2}$, $\Delta m = \frac{m_2-m_1}{2}$, $(m_2^2 - m_1^2) = 4m\Delta m = \Delta m^2_{\mathrm{solar}}$, I obtain from Eqs. (29) and (30) and neglecting $(\frac{\Delta m}{m})^2$

$$|h_{11}v_1|^2 \simeq mM_1\Big[1 - \cos 2\theta\frac{\Delta m}{m} - \sin^2 2\theta \sin^2\frac{\Delta}{2}\Big]^{1/2}, \tag{40}$$

$$|2h_{22}v_1|^2 \simeq mM_2\Big[1 - \cos 2\theta\frac{\Delta m}{m} - \sin^2 2\theta \sin^2\frac{\Delta}{2}\Big]^{-1/2}, \tag{41}$$

where θ is the solar mixing angle. Since Δm^2 and $\sin\Delta$ already appear in Eq. (40), I have, with $s^2 = \frac{1}{3}$, $c^2 = \frac{2}{3}$ in leading order,

$$\epsilon_1 \simeq 3\times 10^4\frac{M_1^2}{v_1^4}\Delta m^2_{\mathrm{solar}}\sin\Delta. \tag{42}$$

The Majorana phase Δ is unknown, but is the same as would appear in double beta decay [cf. first part of Eq. (21)]. With $\sin\Delta \simeq 0.14$ [11], and $\Delta m^2_{\mathrm{solar}}$ given in Eq. (1), I can write

$$\epsilon_1 \simeq 3.4\times 10^{-8}\Big(\frac{M_1}{10^{10}\ \mathrm{GeV}}\Big)^2\frac{\Delta m^2_{\mathrm{solar}}}{8\times 10^{-5}\ \mathrm{eV}^2}\Big(\frac{174\ \mathrm{GeV}}{v_1}\Big)^4\times\frac{\sin\Delta}{0.14}. \tag{43}$$

With $v_1^2 = \frac{1}{2}v^2 = \frac{1}{2}(174\ \mathrm{GeV})^2$, I get

$$\epsilon_1 \simeq 10^{-7} \tag{44}$$

which is of the right order of magnitude [11,12] to explain baryogenesis through leptogenesis. It is important to note that CP violation responsible for the generation of the baryogenesis parameter η_B comes entirely from Majorana phase Δ.

The effective electron neutrinos mass in neutrinoless double beta decay is given by

$$\begin{aligned}m_{ee} &\simeq |a| = |c^2 m_1^2 + s^2 e^{i\Delta}m_2| \simeq m\Big[1 - 4c^2 s^2\sin^2\frac{\Delta}{2}\Big]^{1/2}\\ &= m\Big[1 - \frac{8}{9}\sin^2\frac{\Delta}{2}\Big]^{1/2} \simeq \sqrt{\Delta m^2_{\mathrm{atm}}} \simeq 4.5\times 10^{-3}\ \mathrm{eV}.\end{aligned} \tag{45}$$

Finally using the neutrino oscillation data given in Eq. (1), I have ($s^2 \simeq \frac{1}{3}$)

$$m \simeq (\Delta m^2_{\mathrm{atm}})^{1/2} = 4.7\times 10^{-2}\ \mathrm{eV}, \tag{46}$$

$$\Delta m = \frac{\Delta m^2_{\mathrm{solar}}}{4(\Delta m^2_{\mathrm{atm}})^{1/2}} = 4.3\times 10^{-4}\ \mathrm{eV}. \tag{47}$$

Then the light active neutrino mass matrix in ν_e, ν_μ, ν_τ basis given in Eq. (22) is completely determined [cf. Eqs. (21)] except for Majorana phase Δ. Putting $\Delta = 0$

$$a = 4.7 \times 10^{-2}\ \text{eV}, \qquad b = \frac{2}{3} = 2.9 \times 10^{-4}\ \text{eV}, \qquad d = 2.35 \times 10^{-2}\ \text{eV}, \tag{48}$$

in agreement with [17].

To proceed with the identification of one of the right-handed neutrinos, namely, N_- with a sterile neutrino of mass ≈ 0, I see from Eq. (11) that to make its mass M_3 zero I need an additional assumption $f_{23} = f_{22}$, which implies $(M_R)_{23} = (M_R)_{22} = (M_R)_{33}$. The $\mu - \tau$ symmetry alone does not ensure the equality. In other words there is a single Yukawa coupling for right-handed neutrinos with the Higgs $\sum'$ in the $(\mu - \tau)$ subsector of L_Y. The assumed equality at the tree level does not receive infinite radiative corrections and as such is safe.

In order to give mass to $N_- \equiv \nu_{s2}$, I now break $(\mu - \tau)$ symmetry in M_{12} and M_{13} elements in Eq. (7) since these matrix elements involve Λ, which I take to be $\ll \Lambda'$ as I want the mass of N_- to be of order 1 eV (see below). Thus Eq. (7) is replaced by

$$M_R = \begin{pmatrix} f_{11}\Lambda' & f_{12}\Lambda & f_{13}\Lambda \\ f_{12}\Lambda & f_{22}\Lambda' & f_{22}\Lambda' \\ f_{13}\Lambda & f_{22}\Lambda' & f_{22}\Lambda' \end{pmatrix}. \tag{49}$$

In the (N_e, N_+, N_-) basis it takes the form

$$\tilde{M}_R = \begin{pmatrix} f_{11}\Lambda' & f_{12+}\Lambda & f_{12-}\Lambda \\ f_{12}\Lambda & 2f_{22}\Lambda' & 0 \\ f_{12-}\Lambda & 0 & 0 \end{pmatrix} \tag{50}$$

where $f_{12\pm} = \frac{f_{12} \pm f_{13}}{\sqrt{2}}$ so that $f_{12+} \simeq \sqrt{2} f_{12}$. I diagonalize it by (putting Majorana phases equal to zero)

$$V = \begin{pmatrix} c' & s' & \delta_1' \\ -s' & c' & \delta_2' \\ -c'\delta_1' + s'\delta_2' & -s'\delta_1' - c'\delta_2' & 1 \end{pmatrix} \tag{51}$$

[cf. Eq. (9) where in the N_e, N_+ basis

$$V = \begin{pmatrix} c' & s' \\ -s' & c' \end{pmatrix}$$

]. Then [cf. Eq. (50)]

$$V^T \tilde{M}_R V = \hat{M}_R = \text{diag}(M_1, M_2, M_3)$$

where M_3 is the mass of $N_- \equiv \nu_{s_2}$. This gives in the leading order

$$c's' = \frac{(c'^2 - s'^2)\sqrt{2} f_{12}\Lambda}{2f_{22}\Lambda' - f_{11}\Lambda'}, \tag{52}$$

$$M_1 \simeq f_{11}\Lambda', \qquad M_2 \simeq 2f_{22}\Lambda',$$
$$M_3 = M_{N-} \simeq -\frac{f_{12-}^2}{f_{11}\Lambda'}\Lambda^2$$

so that I can take

$$m_{\nu_{s2}} = \frac{f_{12-}^2 \Lambda^2}{M_1}. \tag{53}$$

Taking

$$\Lambda \simeq v = 174\ \text{GeV}, \qquad M_1 \simeq 10^{10}\ \text{GeV}, \qquad m_{\nu_{s2}} = 1\ \text{eV} \tag{54}$$

if $f_{12-} \sim 1.7 \times 10^{-2}$ and as such no fine-tuning is needed. Then Eq. (12) is replaced by

$$\hat{M}_D = M_D V^*$$

so that the effective Majorana mass matrix for light neutrinos in (ν_e, ν_+, ν_s) basis is

$$M_\nu = \hat{M}_D \hat{M}_R^{-1} \hat{M}_D^T = \begin{pmatrix} \tilde{a} & \sqrt{2}\tilde{b} & \sigma \\ \sqrt{2}\tilde{b} & 2\tilde{d} & \epsilon \\ \sigma & \epsilon & f \end{pmatrix} \tag{55}$$

where $f \simeq M_3 \simeq m_{\nu_{s2}}$ and

$$\sigma \simeq (h_{11} v_2)\delta_1', \qquad \epsilon \simeq (2h_{22} v_1)\delta_2' \tag{56}$$

and

$$\tilde{a} = a + \Delta a, \qquad \tilde{b} = b + \Delta b, \qquad 2\tilde{d} = 2d + \Delta(2d),$$
$$\Delta a = \frac{\sigma^2}{f}, \qquad \Delta b = \frac{1}{\sqrt{2}} \frac{\sigma\epsilon}{f}, \qquad \Delta(2d) \simeq \frac{\epsilon^2}{f}. \tag{57}$$

Finally the diagonalization of M_ν given in Eq. (55) [cf. Eq. (18) where in the (ν_e, ν_+) basis

$$U = \begin{pmatrix} c & s \\ -s & c \end{pmatrix},$$

putting Majorana phases = 0] by

$$U = \begin{pmatrix} c & s & \delta_1 \\ -s & c & \delta_2 \\ -c\delta_1 + s\delta_2 & -c\delta_2 - s\delta_1 & 1 \end{pmatrix} \tag{58}$$

gives

$$\delta_1 \simeq \frac{\sigma}{f}, \qquad \delta_2 \simeq \frac{\epsilon}{f}, \tag{59}$$

$$\frac{cs}{c^2 - s^2} = \frac{\sqrt{2}\tilde{b} - \frac{\sigma\epsilon}{f}}{2\tilde{d} - \tilde{a} + \frac{(\sigma^2 - \epsilon^2)}{f}}. \tag{60}$$

This is identically satisfied if I use the relations (57) and $\frac{cs}{c^2 - s^2} = \frac{\sqrt{2}b}{2d - a}$ [cf. Eqs. (21) with $\beta_1 = 0 = \beta_2$]. Further $\tilde{m}_1 = m_1$, $\tilde{m}_2 = m_2$.

Using [5] $\Delta m_{s_2}^2 = m_4^2 - m_3^2 = m_4^2 \simeq f^2 = 0.90\ \text{eV}^2$,

$$\delta_1 \equiv |U_{e4}| = 0.11 \quad \text{and} \quad \delta_2 \equiv |U_{\nu_+ 4}|, \qquad \delta_2 \simeq \sqrt{2}|U_{\mu 4}| = \sqrt{2}(0.12), \tag{61}$$

I obtain, on using Eqs. (57) and (59)

$$\tilde{a} = 0.058 \text{ eV}, \qquad \tilde{b} = 0.017, \qquad \tilde{d} = 0.048 \text{ eV},$$
$$\sigma = 0.10 \text{ eV}, \qquad \frac{\epsilon}{\sqrt{2}} = 0.15 \text{ eV}. \tag{62}$$

Thus in the $(\nu_e, \nu_\mu, v_+, \nu_s)$ basis,

$$M_\nu = \begin{pmatrix} 0.058 & 0.017 & 0.017 & 0.10 \\ 0.017 & 0.048 & 0.048 & 0.15 \\ 0.017 & 0.048 & 0.048 & 0.15 \\ 0.10 & 0.15 & 0.15 & 0.95 \end{pmatrix} \text{eV} \tag{63}$$

which may be compared with that given in [17].

To fit both LSND and MB data I need another sterile neutrino. This I can easily include in my formalism. I postulate a right-handed neutrino which is singlet, i.e., has quantum numbers (1, 0, 0) with respect to the gauge group $SU_L(2) \times U_e(1) \times U_{\mu-\tau}(1)$.

Then the additional terms in Eqs. (4) (with $h_{22} = h_{32}$) and (5) are

$$H'_D = h_{10}v_1\bar{\nu}_e N_0 + \sqrt{2}h_{20}v_2(\bar{\nu}_+ N_0), \tag{64}$$

$$H'_M = f_{00}N_0^T C N_0 \Lambda. \tag{65}$$

Neglecting any mixing of N_0 with N_-, the additional contributions to the parameters $\tilde{a}$, $\tilde{b}$, $\tilde{d}$ are

$$\begin{aligned} \Delta a' &= \frac{(h_{10}v_1)^2}{m_{s_1}} \equiv |U_{e5}|^2 m_{s_1}, \\ \sqrt{2}\Delta b' &= \frac{(h_{10}v_1)(\sqrt{2}h_{20}v_2)}{m_{s_1}} = |U_{e5}||U_{\nu_+5}|m_{s_1} \\ &= \sqrt{2}|U_{e5}||U_{\mu5}|m_{s_1}, \end{aligned} \tag{66}$$

$$2\Delta d' = \frac{(\sqrt{2}h_{20}v_2)^2}{m_{s_1}} = 2|U_{\mu5}|^2 m_{s_1}$$

where $m_{s_1} = m_5 = f_{00}\Lambda$, $\Delta m^2_{s_1} = m_5^2 - m_3^2 \simeq m_5^2$. Taking [5] $m_{s_1} = \sqrt{6.49} \simeq 2.55$ eV, $|U_{e5}| = |U_{\mu5}| = 0.12$, I obtain

$$\begin{aligned} \Delta a' &= (0.12)^2 m_{s_1} = 3.6 \times 10^{-2} \text{ eV} = \Delta b' = \Delta d', \\ \sigma' &= (0.12)m_{s_1} = 0.30 \text{ eV} = \frac{\epsilon'}{\sqrt{2}}. \end{aligned} \tag{67}$$

Thus Eq. (63) is replaced by

$$M_\nu = \begin{pmatrix} 0.094 & 0.053 & 0.053 & 0.1 & 0.30 \\ 0.053 & 0.084 & 0.084 & 0.15 & 0.30 \\ 0.053 & 0.084 & 0.084 & 0.15 & 0.30 \\ 0.10 & 0.15 & 0.15 & 0.95 & 0.00 \\ 0.30 & 0.30 & 0.30 & 0.00 & 2.55 \end{pmatrix} \text{eV}.$$

This corresponds to mass orderings: $m_5 > m_4 > m_2 > m_1 > m_3$ [6].

I may note that in the absence of CP violation, the effective mass for neutrinoless double beta decay is

$$m_{ee} = 94 \times 10^{-3} \text{ eV}, \tag{68}$$

which is enhanced compared to (44) due to the presence of sterile neutrinos s_1 and s_2.

One final comment is that although I have used the model $SU_L(2) \times U_e(1) \times U_{\mu-\tau}(1)$ as a guide, the results are independent of the details of this model.

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