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Viability criterion for modified gravity with an extra force

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A recently proposed theory of modified gravity with an explicit anomalous coupling of the Ricci curvature to matter is discussed, and an inequality is derived which expresses a necessary and sufficient condition to avoid the notorious Dolgov-Kawasaki instability.

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I. INTRODUCTION

Modifications of gravity have received attention [1–3] in order to explain the cosmic acceleration [4]. The alternative is a mysterious form of dark energy [5] with negative pressure $P < 0$ and energy density ρ satisfying $P \approx -\rho$ and perhaps even $P < -\rho$ (phantom energy [6]). The latter easily leads to a big rip singularity [7]. Modified gravity allows one to avoid such exotica. The Einstein-Hilbert action [8] is modified to [1,2]

$$S = \int d^4x\sqrt{-g}\left[\frac{f(R)}{2\kappa} + \mathcal{L}_m\right], \tag{1}$$

where $f(R)$ is an (*a priori*) arbitrary function of R, and the modifications are designed to affect cosmological scales and stay tiny at smaller scales in order not to violate the solar system constraints [9]. The prototype $f(R) = R - \mu^4/R$ (with $\mu \sim H_0 \sim 10^{-33}$ eV) is now regarded as a toy model at best because of a violent instability [10] and its violation of the experimental constraints [11]. Modified gravities must be free of instabilities and ghosts [10,12–14] and have a well-posed Cauchy problem [15] and the correct cosmological dynamics (there are often problems with the exit from the radiation era [16]).

Modified $f(R)$ gravity comes in three versions: metric formalism, in which the action (1) is varied with respect to the (inverse) metric tensor g^{ab}; Palatini $f(R)$ gravity, in which variation is with respect to both g^{ab} and an independent, nonmetric, connection Γ^a_{bc} but the matter part of the action does not depend on Γ^a_{bc} [17]; and metric-affine gravity, in which also $\mathcal{L}_m$ depends on the nonmetric connection [18]. Here I focus on the metric approach.

Recently, Bertolami, Böhmer, Harko, and Lobo (hereafter BBHL) [19] put a new twist on $f(R)$ gravity by considering the action

$$S = \int d^4x\sqrt{-g}\left\{\frac{f_1(R)}{2} + [1 + \lambda f_2(R)]\mathcal{L}_m\right\}, \tag{2}$$

where $f_{1,2}(R)$ are arbitrary functions of the Ricci curvature and λ is a small parameter (from now on I follow [19] and set $\kappa \equiv 8\pi G = 1$). The novelty consists of the coupling function $f_2(R)$ which adds extra freedom and new features.

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The field equations are

$$\begin{aligned} f_1'(R)R_{ab} - \frac{f_1(R)}{2}g_{ab} = {}& \nabla_a\nabla_b f_1'(R) - g_{ab}\Box f_1'(R) \\ & - 2\lambda f_2'(R)\mathcal{L}_m R_{ab} \\ & + 2\lambda(\nabla_a\nabla_b - g_{ab}\Box)(\mathcal{L}_m f_2'(R)) \\ & + [1 + \lambda f_2(R)]T^{(m)}_{ab}, \end{aligned} \tag{3}$$

where a prime denotes differentiation with respect to R, $\Box \equiv g^{cd}\nabla_c\nabla_d$, and $T^{(m)}_{ab} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{ab}}$. Because of the extra explicit coupling to matter $\lambda f_2(R)$, $T^{(m)}_{ab}$ is not covariantly conserved and energy is exchanged between ordinary matter ($T^{(m)}_{ab}$) and the "effective matter" represented by terms in $f_2'(R)$ in Eq. (3). $T^{(m)}_{ab}$ obeys [19]

$$\nabla^b T^{(m)}_{ab} = \frac{\lambda f_2'(R)}{1 + \lambda f_2(R)}[g_{ab}\mathcal{L}_m - T^{(m)}_{ab}]\nabla^b R. \tag{4}$$

The BBHL theory contains intriguing phenomenology: all massive particles are subject to an extra force, similar to the one arising in scalar-tensor (ST) gravity in the Einstein frame [20,21]. In the latter, the extra force is due to an "anomalous" coupling of the matter Lagrangian to the Brans-Dicke-like scalar ϕ,

$$S_{\rm EF} = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\nabla^c\phi\nabla_c\phi - e^{-\alpha\phi}\mathcal{L}_m\right). \tag{5}$$

There is, however, an important difference between Einstein frame ST gravity and BBHL theory: while in the former the units of time, length, and mass are not constant but scale with appropriate powers of the conformal factor of the conformal transformation defining the Einstein frame [20,22], in the latter there is no such scaling of units. For this reason, the BBHL theory [19] cannot be reduced to "ordinary" ST or string gravity (in this respect, string theory has the same phenomenology of ST gravity).

The extra force on massive particles generated by the $\lambda f_2(R)$ coupling is always present and causes a deviation from geodesic paths; therefore, massive test particles simply do not exist. Because of this extra force, the acceleration law in the weak-field limit of BBHL theory assumes a form similar to the one of modified Newtonian dynamics (MOND) [23]. MOND has recently received a relativistic formulation in the rather complicated tensor-vector-scalar (TeVeS) theory of [24]. The BBHL proposal exhibits MOND-like phenomenology but has a simpler formal structure than TeVeS: as shown below, it amounts to a ST theory with two coupling functions, one of which is the coupling of the scalar degree of freedom $\phi = R$ to matter (unorthodox in ST gravity [25]). Of course, in order to be viable, the BBHL theory must pass the tests mentioned above for $f(R)$ gravity, and it is not clear yet whether this is possible. In this paper I study one of these criteria, namely, the stability of the theory with respect to local perturbations. In pure $f(R)$ gravity, a fatal instability develops on time scales $\sim 10^{-26}$ s [10] when $f''(R) < 0$. This instability, the "Dolgov-Kawasaki phenomenon," was discovered in the prototype model $f(R) = R - \mu^4/R$ [10] which is ruled out (and only cured by adding extra terms to $f(R)$ [12,13,26]), and then generalized to arbitrary $f(R)$ models [27].

II. EQUIVALENCE WITH AN ANOMALOUS ST THEORY

It is well-known that pure $f(R)$ gravity (1) is equivalent to a ST theory [28]; here I revisit this equivalence and generalize it to BBHL theory.

By introducing a new field ϕ, the action (2) is written as

$$S = \int d^4x\sqrt{-g}\left\{\frac{f_1(\phi)}{2} + \frac{1}{2}\frac{df_1}{d\phi}(R-\phi) + [1 + \lambda f_2(\phi)]\mathcal{L}_m\right\} \quad (6)$$

and, further introducing the field $\psi(\phi) \equiv f_1'(\phi)$ (where now a prime denotes differentiation with respect to ϕ [29]), one can write

$$S = \int d^4x\sqrt{-g}\left[\frac{\psi R}{2} - V(\psi) + U(\psi)\mathcal{L}_m\right], \quad (7)$$

where

$$V(\psi) = \frac{\phi(\psi) f_1'[\phi(\psi)] - f_1[\phi(\psi)]}{2}, \quad (8)$$

$$U(\psi) = 1 + \lambda f_2[\phi(\psi)], \quad (9)$$

with $\phi(\psi)$ given by inverting $\psi(\phi) \equiv f_1'(\phi)$. The actions (2) and (7) are equivalent when $f_1''(R) \neq 0$: in fact, by setting $\phi = R$, Eq. (7) reduces trivially to Eq. (2). Viceversa, variation of (6) with respect to ϕ yields

$$(R - \phi)f_1''(\phi) + 2\lambda f_2'(\phi)\mathcal{L}_m = 0. \quad (10)$$

In vacuo ($\mathcal{L}_m = 0$), this equation yields $\phi = R$ whenever $f_1'' \neq 0$ [28]. In the presence of matter there seem to be other possibilities which are, however, excluded as follows. When $\mathcal{L}_m \neq 0$, the actions (2) and (6) are equivalent if $(R-\phi)f_1''(\phi) + 2\lambda f_2''(\phi)\mathcal{L}_m \neq 0$. When Eq. (10) is satisfied, there is a pathological case which, upon integration of this equation, corresponds to

$$\lambda f_2(\phi)\mathcal{L}_m = \frac{f_1'(\phi)}{2}(\phi - R) - \frac{f_1(\phi)}{2}. \quad (11)$$

But if Eq. (11) holds, then the action (6) reduces to pure matter without the gravity sector, and I dismiss this case. Then, the actions (2) and (7) are equivalent when $f_1''(R) \neq 0$, as in pure $f(R)$ gravity [28]. The action (7) corresponds to a Brans-Dicke theory [30] with a single scalar field, vanishing Brans-Dicke parameter ω, and an unorthodox coupling $U(\psi)$ to matter. Actions of this kind have been contemplated before [31–33], but little is known about them.

III. BBHL THEORY AND INSTABILITIES

The trace of the field equations is, in terms of R,

$$\begin{aligned}3[f_1''(R) &+ 2\lambda\mathcal{L}_m f_2''(R)]\Box R \\ &+ 3[f_1'''(R) + 2\lambda\mathcal{L}_m f_2''(R)]\nabla^c R\nabla_c R \\ &+ 12\lambda f_2''(R)\nabla^c\mathcal{L}_m\nabla_c R + f_1'(R)R - 2f_1(R) \\ &+ 2\lambda\mathcal{L}_m f_2'(R)R = [1 + \lambda f_2(R)]T^{(m)} - 6\lambda f_2'(R)\Box\mathcal{L}_m,\end{aligned} \quad (12)$$

where $T^{(m)} \equiv T^{(m)a}{}_a$. As customary in $f(R)$ gravity, I parametrize the function $f_1(R)$ as $f_1(R) = R + \epsilon\varphi(R)$, where ϵ and λ must necessarily be small to respect the solar system constraints [34]. Following [10], I expand the spacetime quantities of interest as the sum of a background with constant curvature and a small perturbation: $R = R_0 + R_1$, $T = T_0 + T_1$, $\mathcal{L}_m = \mathcal{L}_0 + \mathcal{L}_1$, and the spacetime metric can *locally* be approximated by $g_{ab} = \eta_{ab} + h_{ab}$, where η_{ab} is the Minkowski metric. Accordingly, $f_1(R) = R_0 + R_1 + \epsilon\varphi(R_0) + \epsilon\varphi'(R_0)R_1 + \ldots$, $f_1'(R) = 1 + \epsilon\varphi'(R_0) + \epsilon\varphi''(R_0)R_1 + \ldots$ and the linearized version of the trace Eq. (12) in the perturbations becomes

$$\begin{aligned}3[\epsilon\varphi''(R_0) &+ 2\lambda f_2''(R_0)]\Box R_1 + [\epsilon\varphi''(R_0)R_0 - 1 - \epsilon\varphi'(R_0) \\ &+ 2\lambda f_2'(R_0)\mathcal{L}_0 + 2\lambda\mathcal{L}_0 f_2''(R_0)R_0 - \lambda f_2'(R_0)T_0 \\ &+ 6\lambda f_2''(R_0)(\Box\mathcal{L}_0)]R_1 \\ &= -2\lambda f_2'(R_0)R_0\mathcal{L}_1 + [1 + \lambda f_2(R_0)]T_1 - 6\lambda f_2'(R_0)\Box\mathcal{L}_1,\end{aligned} \quad (13)$$

where the zero order equation

$$\begin{aligned}f_1'(R_0)R_0 &- 2f_1(R_0) + 2\lambda\mathcal{L}_0 f_2'(R_0)R_0 \\ &= [1 + \lambda f_2(R_0)]T_0\end{aligned} \quad (14)$$

has been used. Equation (13) is further rewritten as

$$\ddot{R}_1 - \nabla^2 R_1 + m_{\rm eff}^2 R_1 = \{3[\epsilon\varphi''(R_0) + 2\lambda f_2''(R_0)]\}^{-1} \times \{2\lambda f_2'(R_0)\mathcal{L}_1 - [1 + \lambda f_2(R_0)]T_1 + 6\lambda f_2'(R_0)\Box\mathcal{L}_1\}, \quad (15)$$

where the effective mass $m_{\rm eff}$ of the dynamical degree of freedom R_1 is given by

$$m_{\rm eff}^2 = \{3[\epsilon\varphi''(R_0) + 2\lambda f_2''(R_0)]\}^{-1}\{1 + \epsilon\varphi'(R_0) + \epsilon\varphi''(R_0)R_0 - 2\lambda\mathcal{L}_0[f_2'(R_0) + f_2''(R_0)R_0] + \lambda f_2'(R_0)T_0\}.$$

The dominant term on the right-hand side is $\{3[\epsilon\varphi''(R_0) + 2\lambda f_2''(R_0)]\}^{-1}$ and the effective mass squared must be non-negative for stability. Therefore, $\epsilon\varphi''(R) + 2\lambda f_2''(R) \geq 0$ is the stability criterion for the BBHL theory against Dolgov-Kawasaki instabilities.

IV. OUTLOOKS

The inequality $\epsilon\varphi''(R) + 2\lambda f_2''(R) \geq 0$ generalizes the stability condition $f''(R) = \epsilon\varphi''(R) \geq 0$ found in pure $f(R)$ gravity [27,35]. The survival of BBHL theory [19] is subject to satisfying the other (physically independent) viability criteria mentioned above, which require a separate analysis and will be analyzed in future publications.

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