Consistent Kaluza-Klein reductions for general supersymmetric AdS solutions

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For the most general supersymmetric solutions of type IIB supergravity consisting of a warped product of AdS_5 with a five-dimensional manifold M_5, we construct an explicit consistent Kaluza-Klein reduction on M_5 to minimal $D = 5$ gauged supergravity. Thus, any solution of the gauged supergravity can be uplifted on M_5 to obtain an exact solution of type IIB supergravity. We also show that for general $AdS_4 \times SE_7$ solutions, where SE_7 is a seven-dimensional Sasaki-Einstein manifold, and for a general class of supersymmetric solutions that are a warped product of AdS_4 with a seven-dimensional manifold N_7, there is an analogous consistent reduction to minimal $D = 4$ gauged supergravity.

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I. INTRODUCTION

A powerful method to construct solutions of $D = 10$ or $D = 11$ supergravity is to uplift solutions of simpler theories in lower-dimensions. For this to work it is necessary that there is an appropriate *consistent* Kaluza-Klein (KK) reduction on some internal manifold M from $D = 10$ or $D = 11$ down to the lower-dimensional theory. In general, a KK expansion on M leads to a lower-dimensional theory involving an infinite tower of fields. Splitting these fields into a finite number of "light" fields and an infinite tower of "heavy" fields,[1] the KK reduction is called consistent if it is in fact consistent to set all of the heavy fields to zero in the equations of motion, leaving equations of motion for the light fields only. Clearly this is only possible if the on-shell light fields do not source the heavy fields.

KK reductions on a circle or more generally on an n-dimensional torus are always consistent. The heavy fields, which arise from modes with nontrivial dependence on the coordinates of the torus, are all charged under the $U(1)^n$ gauge symmetry, while the light fields, which in this case are actually massless fields, are independent of these coordinates and hence uncharged under the gauge symmetry. As a consequence, the heavy fields can never be sourced by the light fields alone and so the truncation to the light fields is consistent. Since this argument also extends to fermions, one concludes that a KK reduction of a higher-dimensional supergravity theory on a torus can always be consistently truncated to a lower-dimensional supergravity theory. Moreover, solutions of the lower-dimensional supergravity theory that preserve supersymmetry will uplift to supersymmetric solutions of the higher-dimensional supergravity theory.

More generally, however, consistent KK reductions are very much the exception rather than the rule. For example, it is only in very special circumstances that there is a consistent KK reduction on a sphere (for further discussion see [1]). An interesting class of examples are those associated with the maximally supersymmetric solutions of $D = 10$ and $D = 11$ supergravity that consist of products of AdS spaces and spheres. Corresponding to the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ solutions of $D = 11$ supergravity, there are consistent KK reductions on S^7 [2] and S^4 [3,4] to $D = 4$ $SO(8)$ gauged supergravity and $D = 7$ $SO(5)$ gauged supergravity, respectively. Similarly, starting with the $AdS_5 \times S^5$ solution of type IIB supergravity there is expected to be a consistent KK reduction to $SO(6)$ gauged supergravity: various additional truncations were shown to be consistent in [5–7] and an ansatz for the full metric was constructed in [8].

We would like to view these examples as special cases of the following conjecture:

For any supersymmetric solution of $D = 10$ *or* $D = 11$ *supergravity that consists of a warped product of* $d + 1$ *dimensional anti–de Sitter space with a Riemannian manifold* M, $AdS_{d+1} \times_w M$, *there is a consistent Kaluza-Klein truncation on* M *to a gauged supergravity theory* in $d + 1$-*dimensions for which the fields are dual to those in the superconformal current multiplet of the d-dimensional dual superconformal field theory (SCFT).*

Equivalently, one can characterize the fields of the gauged supergravity as those that contain the $d + 1$-dimensional graviton and fill out an irreducible representation of the superisometry algebra of the $D = 10$ or $D = 11$ supergravity solution. This conjecture is essentially a restricted version of one that appeared long ago in [9], for which general arguments supporting it were put forward in [10].

For example the $AdS_5 \times S^5$ solution of type IIB, which has superisometry algebra $SU(2, 2|4)$, is dual to $N = 4$ superYang-Mills theory in $d = 4$. The superconformal current multiplet of the latter theory includes the energy momentum tensor, $SO(6)$ R-symmetry currents, along with scalars and fermions. These are dual to the metric, $SO(6)$ gauge fields along with scalar and fermion fields, and are precisely the fields of the maximally supersymmetric $SO(6)$ gauged supergravity in five-dimensions.

[1] In general there is not a sharp separation of energy scales, and hence the quotation marks.

As we have phrased the conjecture above, it is natural to try and prove the conjecture directly from the SCFT point of view. For the case of AdS_3 solutions, an argument has been made by [11,12], but this needs to be modified for higher dimension AdS solutions. While we think that this is an interesting avenue to pursue, in this paper we will verify the conjecture for a number of cases by constructing an explicit consistent KK reduction ansatz. By this we mean an explicit ansatz for the higher-dimensional fields that is built from the fields of the lower-dimensional theory with the property that it solves the equations of motion of the higher-dimensional theory provided that the equations of the lower-dimensional theory are satisfied. This approach has the advantage that it allows one to uplift an explicit solution of the lower-dimensional gauged supergravity to obtain an explicit solution[2] of $D = 10$ or $D = 11$ supergravity.

Often, for simplicity, such explicit KK reduction Ansätze are constructed for the bosonic fields only. This is thought to provide very strong evidence that the ansatz can be extended to the fermionic fields also. In fact an argument was constructed in [1], based on [10], which shows that if a consistent KK reduction has been constructed for the bosonic fields, then the supersymmetry of the higher-dimensional theory will guarantee that the reduction can be consistently extended to the fermionic sector. In any event, a bosonic KK ansatz certainly allows one to uplift bosonic solutions which is the most interesting class of solutions. One can go further and construct an ansatz for the fermion fields and demand that the supersymmetry variation of a bosonic configuration in higher dimensions leads to the correct supersymmetry variation of the bosonic configuration in lower dimensions. This explicitly demonstrates that a supersymmetric bosonic solution of the lower-dimensional theory will uplift to a supersymmetric solution of $D = 10$ or $D = 11$ supergravity which will preserve at least the same amount of supersymmetry as in the lower-dimensional theory.

In this paper we will verify the conjecture for a general class of AdS_5 solutions which are dual to $N = 1$ SCFTs in $d = 4$ dimensions. For this case, the bosonic fields in the superconformal current multiplet are the energy momentum tensor and the Abelian R-symmetry current. Thus we seek a consistent truncation to minimal $D = 5$ gauged supergravity whose bosonic fields are the metric (dual to the energy momentum tensor of the SCFT) and an Abelian gauge field (dual to the R-symmetry current). For the special class of solutions of type IIB of the form $AdS_5 \times SE_5$, where SE_5 is a five-dimensional Sasaki-Einstein manifold, and only the self-dual five-form is nonvanishing, a consistent KK reduction was constructed in [13] (see also [14]). Here we will extend this result by showing that for the most general $AdS_5 \times_w M_5$ supersymmetric solution of type IIB supergravity with all of the fluxes active, that were analyzed in [15], the KK reduction is also consistent. We will construct a KK ansatz for the bosonic fields, and we will also verify the consistency of the supersymmetry variations. The analogous result for the most general supersymmetric solutions of $D = 11$ supergravity of the form $AdS_5 \times_w M_6$ with nonvanishing four-form flux [16] was shown in [17]. Given that any AdS_5 solution of type IIA supergravity can be considered to be a special case of one in $D = 11$, if we are to assume that there are no AdS_5 solutions in type I supergravity, the results here combined with [13,17] covers all AdS_5 solutions in $D = 10$ and $D = 11$ dimensions.

We will also prove similar results for two classes of AdS_4 solutions of $D = 11$ supergravity, both of which are dual to $N = 2$ SCFTs in $d = 3$. The first, and the simplest, is the Freund-Rubin class of solutions which take the form $AdS_4 \times SE_7$ where SE_7 is a seven-dimensional Sasaki-Einstein manifold and the four-form flux is proportional to the volume form of the AdS_4 factor. A discussion of this case appears in [18]. Furthermore, our analysis is a simple extension of the analysis in [19] which considered the seven-sphere viewed as a $U(1)$ fibration over CP^3. The second is the class of $AdS_4 \times_w N_7$ solutions, corresponding to M5-branes wrapping SLAG 3-cycles, that were classified in [20]. It is very plausible that this class of solutions are the most general class of solutions with this amount of supersymmetry and with purely magnetic four-form flux. In both cases we show that there is a consistent KK reduction on the SE_7 or the N_7 to minimal gauged supergravity in four spacetime dimensions. The bosonic fields of the latter theory again consist of a metric and a $U(1)$ gauge field which are dual to the bosonic fields in the superconformal current multiplet. For these examples, we will be content to present the KK ansatz for the bosonic fields only.

The general classes of supersymmetric solutions that we consider have been analyzed using G-structure techniques [21,22]. In particular, the G-structure can be characterized in terms of bi-linears constructed from the Killing spinors. Since the results we obtain only assume supersymmetry and AdS factors one might expect that the explicit KK reduction ansatz involves these bi-linears, and this is indeed the case. In fact it might be illuminating to recast the known consistent KK truncations on spheres in terms of this language, but we shall not investigate that here.

The plan of the rest of the paper is as follows. We begin in Secs. II and III by considering the AdS_4 solutions of $D = 11$ supergravity. In Sec. IV we consider the general class of AdS_5 solutions of type IIB supergravity. Only for the latter class we will present details of our calculations and these can be found in the appendices. In Sec. V we briefly conclude.

[2]Note that since the uplifting formulae are local, in general, even if the lower-dimensional solution is free from singularities one still needs to check that the higher-dimensional solution is also.

II. REDUCTION OF $D = 11$ SUPERGRAVITY ON SE_7

Our starting point in this section is the class of supersymmetric solutions of $D = 11$ supergravity of the form $AdS_4 \times SE_7$ where SE_7 is a Sasaki-Einstein 7-manifold:

$$ds^2_{11} = \tfrac{1}{4}ds^2(AdS_4) + ds^2(SE_7), \qquad G = \tfrac{3}{8}\mathrm{vol}(AdS_4). \tag{2.1}$$

Here $\mathrm{vol}(AdS_4)$ is the volume four-form of the unit radius AdS_4 metric $ds^2(AdS_4)$ and we have normalized the Sasaki-Einstein metric $ds^2(SE_7)$ so that $\mathrm{Ric}(SE_7) = 6g(SE_7)$ (the same as for the unit radius metric on the round seven-sphere). The Sasaki-Einstein metric has a Killing vector which is dual to the R-symmetry of the dual $N = 2$ SCFT in $d = 3$. Introducing coordinates so that this Killing vector is ∂_ψ, locally, the Sasaki-Einstein metric can be written

$$ds^2(SE_7) = (d\psi + \sigma)^2 + ds^2(M_6), \tag{2.2}$$

where $ds^2(M_6)$ is locally Kähler-Einstein with Kähler form J, normalized so that $\mathrm{Ric}(M_6) = 8g(M_6)$ and $d\sigma = 2J$.

We now construct an ansatz which leads to a consistent truncation, at the level of bosonic fields, to gauged supergravity in $D = 4$. Specifically, we consider

$$\begin{aligned} ds^2_{11} &= \tfrac{1}{4}ds^2_4 + (d\psi + \sigma + \tfrac{1}{4}A)^2 + ds^2(M_6), \\ G &= \tfrac{3}{8}\mathrm{vol}_4 - \tfrac{1}{4} *_4 F_2 \wedge J, \end{aligned} \tag{2.3}$$

where ds^2_4 is an arbitrary metric on a four-dimensional spacetime, vol_4 is its associated volume form, and A and $F_2 = dA$ are one- and two-forms on this spacetime with a normalization chosen for convenience. Substituting this into the $D = 11$ equations of motion [23] (we use the conventions of [22]),

$$\begin{aligned} R_{AB} - \tfrac{1}{12}(G_{AC_1C_2C_3}G_B{}^{C_1C_2C_3} - \tfrac{1}{12}g_{AB}G^2) &= 0, \\ d *_{11} G + \tfrac{1}{2}G \wedge G = 0, \qquad dG &= 0 \end{aligned} \tag{2.4}$$

where $G^2 = G_{C_1C_2C_3C_4}G^{C_1C_2C_3C_4}$, we find that the metric $g_{\mu\nu}$, corresponding to ds^2_4, and F_2 must satisfy

$$R_{\mu\nu} = -3g_{\mu\nu} + \tfrac{1}{2}F_{\mu\rho}F^\rho_\nu - \tfrac{1}{8}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}, \qquad d *_4 F_2 = 0. \tag{2.5}$$

These are precisely the equations of motion of minimal gauged supergravity in $D = 4$ [24,25].

Thus we have shown the consistency of the KK reduction at the level of the bosonic fields. In particular, any solution of the minimal gauged supergravity, which were systematically studied in [26], can be uplifted on an arbitrary seven-dimensional Sasaki-Einstein manifold to a solution of $D = 11$ supergravity.

III. REDUCTION OF $D = 11$ SUPERGRAVITY ON A SLAG-3 FLUX GEOMETRY

Let us now consider the general class of supersymmetric warped product solutions of the form $AdS_4 \times_w \mathcal{N}_7$ with purely magnetic four-form flux which are dual to $N = 2$ SCFTs in $d = 3$ [20]. We call these geometries SLAG-3 flux geometries, since they can be derived from a class of geometries that correspond to M5-branes wrapping special Lagrangian (SLAG) three-cycles in a $SU(3)$ holonomy manifold—for further details see [20]. It is quite possible that this class of geometries is the most general class of AdS_4 geometries with this amount of supersymmetry and with purely magnetic four-form flux, but this has not been proven.

The $D = 11$ metric of the SLAG-3 flux geometry is given by

$$ds^2_{11} = \lambda^{-1}ds^2(AdS_4) + ds^2(\mathcal{N}_7), \tag{3.1}$$

where $ds^2(AdS_4)$ has unit radius and the warp factor λ is independent of the coordinates of AdS_4. $\mathcal{N}_7$ has a local $SU(2)$ structure which is specified by three one-forms and three self-dual two-forms J^1, J^2, J^3. One of the one-forms is dual to a Killing vector that also preserves the flux: this is dual to the R-symmetry of the corresponding $N = 2$ SCFT. Introducing local coordinates so that this Killing vector is given by ∂_ϕ we have

$$\begin{aligned} ds^2(\mathcal{N}_7) = ds^2(\mathcal{M}_{SU(2)}) + w \otimes w + \frac{\lambda^2 d\rho^2}{4(1 - \lambda^3\rho^2)} \\ + \frac{\lambda^2\rho^2}{4}d\phi^2, \end{aligned} \tag{3.2}$$

$\mathcal{M}_{SU(2)}$ is a four-dimensional space where the J^a live. The three one-forms mentioned above are w, $(\lambda/2\sqrt{1 - \lambda^3\rho^2})d\rho$, and $(\lambda\rho/2)d\phi$. In addition we must have

$$\begin{aligned} d\Big[\lambda^{-1}\sqrt{1 - \lambda^3\rho^2}\,w\Big] &= \lambda^{-1/2}J^1 + \frac{\lambda^2\rho}{2\sqrt{1 - \lambda^3\rho^2}}w \wedge d\rho, \\ d\Big(\lambda^{-3/2}J^3 \wedge w - \frac{\lambda\rho}{2\sqrt{1 - \lambda^3\rho^2}}J^2 \wedge d\rho\Big) &= 0, \\ d\Big(J^2 \wedge w + \frac{1}{2\lambda^{1/2}\rho\sqrt{1 - \lambda^3\rho^2}}J^3 \wedge d\rho\Big) &= 0. \end{aligned} \tag{3.3}$$

Finally the 4-form flux is given by

$$G = d\phi \wedge d\Big(\frac{1}{2}\lambda^{-1/2}\sqrt{1 - \lambda^3\rho^2}J^3\Big). \tag{3.4}$$

An explicit example of a solution to these equations was given in [27] as discussed in [20].

We now consider the KK reduction ansatz:

$$ds^2_{11} = \lambda^{-1} ds^2_4 + ds^2(\hat{\mathcal{N}}_7), \quad G = \hat{G} + F_2 \wedge Y + *_4 F_2 \wedge X, \tag{3.5}$$

where ds^2_4 is a line element and $F_2 = dA$ is a two-form on a four-dimensional spacetime. In addition $ds^2(\hat{\mathcal{N}}_7)$ is the expected deformation of $ds^2(\mathcal{N}_7)$, given by

$$ds^2(\hat{\mathcal{N}}_7) = ds^2(\mathcal{M}_{SU(2)}) + w \otimes w + \frac{\lambda^2 d\rho^2}{4(1-\lambda^3\rho^2)} + \frac{\lambda^2\rho^2}{4}(d\phi + A)^2, \tag{3.6}$$

$\hat{G}$ is the expected deformation of the four-form flux appearing in (3.4)

$$\hat{G} = (d\phi + A) \wedge d\left(\frac{1}{2}\lambda^{-1/2}\sqrt{1-\lambda^3\rho^2} J^3\right), \tag{3.7}$$

and the two-forms X and Y are given by

$$X = -\frac{1}{2}\left(\lambda^{-1/2} J^1 + \frac{\lambda^2\rho}{2\sqrt{1-\lambda^3\rho^2}}\omega \wedge d\rho\right), \quad Y = -\frac{1}{2}\lambda^{-1/2}\sqrt{1-\lambda^3\rho^2} J^3. \tag{3.8}$$

Substituting this ansatz into the equations of motion of $D = 11$ supergravity (2.4) and using (3.3) we find that all equations are satisfied provided that the equations of motion (2.5) of minimal gauged supergravity in $D = 4$ are satisfied. This again shows the consistency of the truncation, at the level of the bosonic fields.

IV. REDUCTION OF IIB ON GENERAL M_5

We now turn to the general class of supersymmetric $AdS_5 \times_w M_5$ solutions of IIB supergravity with all fluxes active that were analyzed in [15]. Such solutions are dual to $N = 1$ SCFTs in $d = 4$ which all have a $U(1)$ R-symmetry. We will show that there is a consistent KK reduction on M_5 to minimal gauged supergravity in $D = 5$. This case is more involved than the previous two and so we have included some details of the calculation in the appendices.

A. Internal geometry and fluxes

We begin by summarizing the results of [15]. The ten-dimensional metric is a warped product of AdS_5 with a five-dimensional Riemannian manifold M_5,

$$ds^2_{10} = e^{2\Delta}[ds^2(AdS_5) + ds^2(M_5)], \tag{4.1}$$

where the warp factor Δ is a real function on M_5. All fluxes are active: in order to preserve the spatial $SO(4, 2)$ isometry, the one-forms P, Q and the complex three-form G lie entirely on the internal M_5, and the five-form is taken to be

$$F = f(\mathrm{vol}_{AdS_5} + \mathrm{vol}_{M_5}), \tag{4.2}$$

where f is a constant and vol is the volume form corresponding to each of the metrics in the right-hand side of (4.1). We use the same conventions as in [15] and some of this is recorded in Appendix A.

The manifold M_5 is equipped with two spinors ξ_1, ξ_2 of Spin(5) subject to a set of differential and algebraic constraints arising from the IIB Killing spinor equations. The spinors ξ_1, ξ_2 define a local identity structure on M_5, which can be conveniently characterized in terms of a set of forms, bi-linear in ξ_1, ξ_2, consisting of a real scalar $\sin\zeta$, a complex scalar S, a real one-form K_5, and two complex one-forms K, K_3. These satisfy the following differential conditions:

$$e^{-4\Delta} d(e^{4\Delta} S) = 3iK, \quad e^{-6\Delta} D(e^{6\Delta} K_3) = P \wedge K_3^* - 4iW - e^{-2\Delta} * G, \quad e^{-8\Delta} d(e^{8\Delta} K_5) = 4\sin\zeta V - 6U, \tag{4.3}$$

where $D(e^{6\Delta}K_3) \equiv d(e^{6\Delta}K_3) - iQ \wedge e^{6\Delta}K_3$. In (4.3), U, V are real two-forms and W is a complex two-form that can be constructed as bi-linears in ξ and moreover can be expressed in terms of the identity structure:

$$\begin{aligned}U &= \frac{1}{2(\cos^2\zeta - |S|^2)}(\mathrm{i}\sin\zeta K_3 \wedge K_3^* + \mathrm{i} K \wedge K^* - 2\,\mathrm{Im} S^* K \wedge K_5),\\ V &= \frac{1}{2\sin\zeta(\cos^2\zeta - |S|^2)}(\mathrm{i}\sin\zeta K_3 \wedge K_3^* + \mathrm{i}[\sin^2\zeta + |S|^2] K \wedge K^* - 2\,\mathrm{Im} S^* K \wedge K_5),\\ W &= \frac{1}{\sin\zeta(\cos^2\zeta - |S|^2)}(\cos^2\zeta K_5 + \mathrm{Re} S^* K + \mathrm{i}\sin\zeta\,\mathrm{Im} S^* K) \wedge K_3.\end{aligned} \tag{4.4}$$

In addition, one also has the algebraic constraint

$$\mathrm{i}_{K_3^*} P = 2\mathrm{i}_{K_3} d\Delta, \tag{4.5}$$

the five-form flux is given by (4.2) with

$$f = 4e^{4\Delta}\sin\zeta, \tag{4.6}$$

the three-form flux is given by

$$(\cos^2\zeta - |S|^2)e^{-2\Delta} * G = 2P \wedge K_3^* - (4d\Delta + 4iK_4 - 4i\sin\zeta K_5) \wedge K_3 + 2 * (P \wedge K_3^* \wedge K_5 - 2d\Delta \wedge K_3 \wedge K_5), \tag{4.7}$$

where $\sin\zeta K_4 = K_5 + \mathrm{Re}(S^* K)$, and the metric can be written

$$ds^2(M_5) = \frac{(K_5)^2}{\sin^2\zeta + |S|^2} + \frac{K_3 \otimes K_3^*}{\cos^2\zeta - |S|^2} + \frac{|S|^2}{\cos^2\zeta - |S|^2}(\mathrm{Im}S^{-1}K)^2 + \frac{|S|^2}{\sin^2\zeta}\frac{\sin^2\zeta + |S|^2}{\cos^2\zeta - |S|^2}\left(\mathrm{Re}S^{-1}K + \frac{1}{\sin^2\zeta + |S|^2}K_5\right)^2. \tag{4.8}$$

Finally, the vector dual to K_5 is a Killing vector of the metric (4.8) that also generates a symmetry of the full solution: $\mathcal{L}_{K_5}\Delta = i_{K_5}P = \mathcal{L}_{K_5}G = 0$. The above constraints arising from supersymmetry ensure that all equations of motion and Bianchi identities are satisfied.

B. KK reduction

We now construct the ansatz for a KK reduction from type IIB on the general M_5 that we discussed in the last subsection. We shall show that there is a consistent reduction to minimal $D = 5$ gauged supergravity.

On M_5 the vector field dual to the one-form K_5 is Killing and corresponds to the R-symmetry in the $d = 4$ dual SCFT. If one introduces coordinates such that this dual vector field is $3\partial_\psi$, we would like to shift $d\psi$ by the gauge field A: noting that $\|K_5\|^2 = (\sin^2\zeta + |S|^2)$ this means that we should make the shift

$$K_5 \to \hat{K}_5 = K_5 + (\sin^2\zeta + |S|^2)\frac{A}{3}. \tag{4.9}$$

In particular, given (4.1), our ansatz for the $D = 10$ type IIB metric is then

$$ds^2_{10} = e^{2\Delta}[ds^2_5 + ds^2(\hat{M}_5)], \tag{4.10}$$

where ds^2_5 is an arbitrary metric on five-dimensional spacetime, and $ds^2(\hat{M}_5)$ is the metric $ds^2(M_5)$ in (4.8) after the shift (4.9).

The KK ansatz for the five-form and the complex three-form of type IIB reads:

$$\begin{aligned} F_5 &= \hat{F}_5 + F_2 \wedge \tfrac{1}{3}e^{4\Delta}\hat{*}_5 V + *_5F_2 \wedge \tfrac{1}{3}e^{4\Delta}V, \\ G &= \hat{G} + F_2 \wedge \tfrac{1}{3}e^{2\Delta}K_3, \end{aligned} \tag{4.11}$$

where $F_2 = dA$, $\hat{F}_5$ and $\hat{G}$ are the five-form and three-form flux of the undeformed solution on M_5 after we make the shift (4.9), V, K_3 are the bi-linears on M_5 introduced in the previous subsection,[3] and $\hat{*}_5$ and $*_5$ are, respectively, the Hodge duals with respect to the metrics $ds^2(\hat{M}_5)$ and ds^2_5 in (4.10). Notice that since the one-forms P and Q of the undeformed solution on M_5 are independent of K_5, they remain the same as they were.

In appendix B we provide some details of how we constructed this particular ansatz. In particular, a long calculation shows that the ansatz (4.10) and (4.11) with P, Q unchanged satisfies all of the IIB equations of motion and Bianchi identities, provided that ds^2_5 and F_2 satisfy

$$R_{\mu\nu} = -4g_{\mu\nu} + \tfrac{1}{6}F_{\mu\lambda}F_\nu^{\lambda} - \tfrac{1}{36}g_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho}, \tag{4.12}$$

$$d *_5 F_2 - \tfrac{1}{3}F_2 \wedge F_2 = 0. \tag{4.13}$$

These are precisely the equations of motion of minimal $D = 5$ gauged supergravity [28]. This shows the consistency of the truncation of the bosonic sector.

The truncation is, moreover, consistent at the level of the variations of the IIB fermion fields (see Appendix C for the details). On the one hand we find that the supersymmetry variations of the dilatino λ and of the internal components of the gravitino Ψ_M identically vanish. On the other hand, the external components of the IIB gravitino variation reduce to

$$\delta\psi_\alpha = D_\alpha\varepsilon - \frac{1}{2}\rho_\alpha\varepsilon + \frac{i}{2}A_\alpha\varepsilon + \frac{i}{24}F_{\beta\gamma}(\rho_\alpha^{\beta\gamma} - 4\delta_\alpha^\beta\rho^\gamma)\varepsilon, \tag{4.14}$$

where ψ_α is the $D = 5$ gravitino and ε a $D = 5$ spinor. This is the gravitino variation corresponding to minimal $D = 5$ gauged supergravity.

To summarize, we have shown that any bosonic solution of $D = 5$ supergravity can be uplifted to $D = 10$ using a general supersymmetric solution by means of the KK ansatz (4.10) and (4.11). Moreover, if the five-dimensional bosonic solution is supersymmetric[4] then so will be the uplifted ten-dimensional solution.

V. CONCLUSION

In this paper we have constructed explicit consistent KK reduction Ansätze for general classes of AdS_5 solutions in type IIB supergravity and AdS_4 solutions in $D = 11$ supergravity. Our results can be extended to other classes of supersymmetric solutions that have been classified. It would be nice to show for the $AdS_5 \times_w M_6$ solutions of $D = 11$ supergravity, classified in [30], which are dual to $N = 2$ SCFTs in $d = 4$, that there is a consistent KK reduction to the $SU(2) \times U(1)$ gauged supergravity of [31]. A similar result in type IIB requires an analogous classification of $AdS_5 \times_w M_5$ solutions that are dual to $N = 2$ SCFTs in $d = 4$, which has not yet been carried out.

There are several classes of AdS_4 solutions of $D = 11$ supergravity that can be considered. For example, one can consider $AdS_4 \times N_7$ solutions of $D = 11$ where N_7 has weak G_2 holonomy [32,33] or the $AdS_4 \times_w N_7$ solutions that arise from $M5$-branes wrapping associative 3-cycles that were analyzed in [20]. These solutions are dual to $N = 1$ SCFTs in $d = 3$, which have no R-symmetry, and so one expects a consistent KK reduction on N_7 to a $N = 1$ supergravity whose field content is just the metric and

[3] The bi-linear V is not affected by the shift (4.9): choosing the convenient frame of Appendix B of [20], one can check that all K_5 dependence of V in Eq. (4.4) drops out.

[4] Such solutions were classified in [29].

fermions. In fact it is easy to show that there is a consistent reduction to the $N = 1$ supergravity of [34]. Similarly, the $AdS_4 \times N_7$ solutions of $D = 11$ where N_7 is tri-Sasaki [32,33], are dual to $N = 3$ SCFTs in $d = 3$ and there should be a consistent KK reduction to a $SO(3)$ gauged supergravity in $D = 4$. Additional AdS_3 and AdS_2 solutions of $D = 11$ supergravity studied in [20,35,36] can also be considered.

The consistency of the KK truncation makes it manifest from the gravity side that SCFTs with a type IIB or $D = 11$ dual share common sectors. For example, if we consider such SCFTs in $d = 4$, the black hole solutions of minimal gauged supergravity constructed in [37] should be relevant for any of the SCFTs. It would be interesting to pursue this further.

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APPENDIX A: IIB SUPERGRAVITY CONVENTIONS

We quote here our conventions for IIB supergravity [38,39], that follow those of [15]. The bosonic ten-dimensional fields consist of a metric and the following set of form field stregths: a complex one-form P, a complex three-form G, and a real five-form F_5, subject to the following equations of motion:

$$\begin{aligned} R_{MN} &= P_M P_N^* + P_N P_M^* + \tfrac{1}{96} F_{MP_1P_2P_3P_4} F_N{}^{P_1P_2P_3P_4} \\ &\quad + \tfrac{1}{8}(G_M{}^{P_1P_2} G^*_{NP_1P_2} + G_N{}^{P_1P_2} G^*_{MP_1P_2} \\ &\quad - \tfrac{1}{6} g_{MN} G^{P_1P_2P_3} G^*_{P_1P_2P_3}), \end{aligned} \tag{A1}$$

$$* F_5 = F_5, \tag{A2}$$

$$D * G - P \wedge *G^* + iG \wedge F_5 = 0, \tag{A3}$$

$$D * P + \tfrac{1}{4} G \wedge *G = 0. \tag{A4}$$

We are working in the formalism where $SU(1,1)$ is realized linearly. In particular there is a local $U(1)$ invariance and Q_M acts as the corresponding gauge field. Note that Q_M is a composite gauge field with field strength given by $dQ = -\mathrm{i} P \wedge P^*$. Since G has charge 1 and P has charge 2 under this $U(1)$ we have the covariant derivatives: $D * G \equiv d * G - iQ \wedge *G$ and $D * P \equiv d * P - 2iQ \wedge *P$. We also need to impose the Bianchi identities

$$dF_5 - \frac{i}{2} G \wedge G^* = 0, \qquad DG + P \wedge G^* = 0, \qquad DP = 0. \tag{A5}$$

The IIB fermionic fields consist of a gravitino Ψ_M and a dilatino λ. For supersymmetric bosonic solutions, the variations under supersymmetry of the fermion fields,

$$\delta\lambda = i\Gamma^M P_M \epsilon^c + \frac{i}{24} \Gamma^{P_1P_2P_3} G_{P_1P_2P_3} \epsilon, \tag{A6}$$

$$\begin{aligned} \delta\Psi_M &= D_M \epsilon - \frac{1}{96} (\Gamma_M{}^{P_1P_2P_3} G_{P_1P_2P_3} - 9\Gamma^{P_1P_2} G_{MP_1P_2}) \epsilon^c \\ &\quad + \frac{i}{192} \Gamma^{P_1P_2P_3P_4} F_{MP_1P_2P_3P_4} \epsilon, \end{aligned} \tag{A7}$$

must vanish. The spinor ϵ has composite $U(1)$ charge $+1/2$ so that $D_M \epsilon = (\nabla_M - \frac{\mathrm{i}}{2} Q_M)\epsilon$.

APPENDIX B: IIB REDUCTION: BOSONIC SECTOR

We now derive the KK reduction ansatz (4.11) for the type IIB bosonic fields. Recall that the vector field dual to the bi-linear K_5 is Killing and that $\|K_5\|^2 = (\sin^2\zeta + |S|^2)$. We therefore need to make the shift

$$K_5 \rightarrow \hat{K}_5 = K_5 + (\sin^2\zeta + |S|^2)\frac{A}{3} \tag{B1}$$

in the metric of the undeformed solution to obtain

$$ds^2_{10} = e^{2\Delta}[ds^2_5 + ds^2(\hat{M}_5)]. \tag{B2}$$

In fact for any p-form β_p on M_5 we can define a $\hat{\beta}_p$ in $\hat{M}_5$ via

$$\hat{\beta}_p = \beta_p + \tfrac{1}{3} A \wedge \mathrm{i}_{K_5} \beta_p, \tag{B3}$$

where i_{K_5} is the interior product with respect to the vector dual to the one-form K_5. If we restrict to forms β_p whose Lie-derivative with respect to the Killing vector dual to K_5 vanish, it is useful in the calculations below to note that

$$\begin{aligned} d\hat{\beta}_p &= d\beta_p - \tfrac{1}{3} A \wedge d\mathrm{i}_{K_5}\beta_p + \tfrac{1}{3} F_2 \wedge \mathrm{i}_{K_5}\beta_p \\ &= d\beta_p + \tfrac{1}{3} A \wedge \mathrm{i}_{K_5} d\beta_p + \tfrac{1}{3} F_2 \wedge \mathrm{i}_{K_5}\beta_p \\ &\equiv \widehat{d\beta_p} + \tfrac{1}{3} F_2 \wedge \mathrm{i}_{K_5}\beta_p. \end{aligned} \tag{B4}$$

We now propose the following KK ansatz for the five-form and complex three-form field strengths. We first take the fluxes of the undeformed $AdS_5 \times_w M_5$ solution, and make the shift (B3) to obtain $\hat{F}_5$ and $\hat{G}$. We then introduce a set of forms β_3, β_2, α_1, α_0 on M_5, which we take to be invariant under the action of the Killing vector,[5] and write

[5]This is a natural condition to impose. If we introduce coordinates so that the Killing vector field dual to K_5 is $3\partial_\psi$, then the condition says that the components of the forms must be independent of ψ.

$$F_5 = \hat{F}_5 + F_2 \wedge \hat{\beta}_3 + *_5 F_2 \wedge \hat{\beta}_2, \qquad G = \hat{G} + F_2 \wedge \hat{\alpha}_1 + *_5 F_2 \hat{\alpha}_0. \tag{B5}$$

The IIB forms P and Q in the KK ansatz are taken to be the same as those in the undeformed solution.

For the KK reduction ansatz (B2) and (B5) to be consistent, it must satisfy the IIB field Eqs. (A1)–(A5) when the $D = 5$ Eqs. (4.12) and (4.13) for ds_5^2, F_2 are satisfied. To carry out these calculations it is useful to use the orthonormal frame e^a, $a = 1, \ldots, 5$, on M_5 that was introduced in Appendix B of [15] which, in particular, contains

$$e^1 = \frac{3}{h} K_5, \qquad h = \frac{1}{3}\sqrt{\sin^2\zeta + |S|^2}. \tag{B6}$$

For $\hat{M}_5$ we use the corresponding frame obtained by the prescription (B1).

The requirement that the fields obey the field Eqs. (A2)–(A5) translates into a set of differential and algebraic equations relating the undeformed forms β_3, β_2, α_1, α_0 to the undeformed fluxes G, F_5, P, Q and metric on M_5:

$$\begin{gathered} d\beta_2 = \frac{i}{2}(\alpha_0^* G - \alpha_0 G^*), \qquad \frac{1}{3}\mathrm{i}_{K_5}\beta_3 = -\frac{1}{3}\beta_2 + \frac{i}{2}\alpha_1 \wedge \alpha_1^*, \qquad d\beta_3 = \frac{i}{2}(G\wedge\alpha_1^* - G^*\wedge\alpha_1) - \frac{1}{3}\mathrm{i}_{K_5}F_5, \\ \frac{1}{3}\mathrm{i}_{K_5}\beta_2 = \frac{i}{2}(\alpha_0^*\alpha_1 - \alpha_0\alpha_1^*), \qquad D\alpha_1 + P\wedge\alpha_1^* + \frac{1}{3}\mathrm{i}_{K_5}G = 0, \qquad D\alpha_0 + P\alpha_0^* = 0, \qquad \mathrm{i}_{K_5}\alpha_1 = -\alpha_0, \\ \beta_3 = *_5\beta_2, \qquad \frac{1}{3}e^{4\Delta}\mathrm{i}_{K_5} *_5 \alpha_1 = -i\alpha_1\wedge\beta_2 + i\alpha_0\beta_3, \qquad -\frac{1}{3}e^{4\Delta} *_5 \alpha_1 = \frac{1}{3}e^{4\Delta}\alpha_0 \mathrm{i}_{K_5}\mathrm{vol}_{M_5} + i\alpha_1\wedge\beta_3, \\ D(e^{4\Delta} *_5 \alpha_1) - P\wedge e^{4\Delta} *_5 \alpha_1^* + iG\wedge\beta_2 - i\alpha_0 f \mathrm{vol}_{M_5} = 0, \qquad \alpha_1\wedge *_5\alpha_1 = \alpha_0^2\mathrm{vol}_{M_5}, \end{gathered} \tag{B7}$$

where α_0, α_1 both carry charge 1 under the composite $U(1)$ gauge field so that e.g. $D\alpha_1 \equiv d\alpha_1 - iQ\wedge\alpha_1$.

We must also demand that the KK ansatz satisfies the Einstein equations. After substitution of (B5), and imposing for simplicity $\beta_3 = *_5\beta_2$ [one of the conditions in (B7)] we find that the external, $\mu\nu$, components of the Einstein Eq. (A1) read

$$R_{\mu\nu} = -4g_{\mu\nu} - k_1 F_{\mu\lambda}F_\nu^{\lambda} - k_2 g_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho}, \tag{B8}$$

where k_1, k_2 are functions on M_5 given by

$$k_1 = \tfrac{1}{4}[e^{-8\Delta}\beta_{2ab}\beta_2^{ab} + 2e^{-4\Delta}\alpha_0\alpha_0^* + 2e^{-4\Delta}\alpha_1^a\alpha_{1a}^* + 2h^2], \tag{B9}$$

$$k_2 = \tfrac{1}{16}[e^{-8\Delta}\beta_{2ab}\beta_2^{ab} + 3e^{-4\Delta}\alpha_0\alpha_0^* + e^{-4\Delta}\alpha_1^a\alpha_{1a}^*]. \tag{B10}$$

Comparing with (4.12) we see that we require[6] that $k_1 = 1/6$ and $k_2 = 1/36$.

The mixed, μa, components of the Einstein Eqs. (A1) give

$$\nabla^\rho F_{\rho\mu} + \frac{k_3}{4}\epsilon_{\mu\nu\lambda\rho\sigma}F^{\nu\lambda}F^{\rho\sigma} = 0 \tag{B11}$$

with

$$k_3 = \frac{1}{8h}\delta^{1a}[e^{-8\Delta}\epsilon_{abcde}\beta_2^{bc}\beta_2^{de} + 4e^{-4\Delta}(\alpha_0\alpha_{1a}^* + \alpha_0^*\alpha_{1a})]. \tag{B12}$$

Comparing with (4.13) we see that we demand $k_3 = -1/3$.

[6]The possibility that k_1, k_2, and k_3, below, cannot be chosen to be constant is a potential source of inconsistency of the KK reduction; a similar issue has been discussed for other reductions in [18,40].

Finally, the internal, ab, components of the Einstein Eqs. (A1) give one more relation among the unknown coefficients in the KK ansatz:

$$\begin{aligned} &4e^{-8\Delta}\beta_{2ac}\beta_{2b}^c + 2e^{-4\Delta}(\alpha_{1a}\alpha_{1b}^* + \alpha_{1a}^*\alpha_{1b}) \\ &\quad + \delta_{ab}(e^{-8\Delta}\beta_{2cd}\beta_2^{cd} + e^{-4\Delta}(\alpha_0\alpha_0^* - \alpha_1^c\alpha_{1c}^*)) \\ &\quad = 4h^2\delta_{a1}\delta_{b1}. \end{aligned} \tag{B13}$$

After considering the spinor bi-linears that characterize the identity structure on M_5 [15], we find that all of the above conditions are satisfied if we choose

$$\alpha_0 = 0, \qquad \alpha_1 = \tfrac{1}{3}e^{2\Delta}K_3, \qquad \beta_2 = \tfrac{1}{3}e^{4\Delta}V, \qquad \beta_3 = \tfrac{1}{3}e^{4\Delta} *_5 V. \tag{B14}$$

The most convenient way to prove this is to again use the specific frame on M_5 introduced in Appendix B of [15].

APPENDIX C: IIB REDUCTION: FERMIONS

Now we show that the KK ansatz (4.10) and (4.11) is also consistent at the level of the supersymmetry variations of the fermions. For this we follow the spinor conventions of Appendix A of [15] which we refer the reader to for more details (we will correct a typo in [15] below).

The undeformed $AdS_5 \times_w M_5$ solution admits Killing spinors of the form

$$\epsilon = \psi \otimes e^{\Delta/2}\xi_1 \otimes \theta + \psi^c \otimes e^{\Delta/2}\xi_2^c \otimes \theta, \tag{C1}$$

where ψ is a Killing spinor on AdS_5, θ is a constant two-component spinor and, most importantly, ξ_1, ξ_2 are spin(5) spinors on M_5 that satisfy two differential conditions

$$D_m\xi_1 + \frac{i}{4}(e^{-4\Delta}f - 2)\gamma_m\xi_1 + \frac{1}{8}e^{-2\Delta}G_{mnp}\gamma^{np}\xi_2 = 0,$$
$$\bar{D}_m\xi_2 - \frac{i}{4}(e^{-4\Delta}f + 2)\gamma_m\xi_2 + \frac{1}{8}e^{-2\Delta}G^*_{mnp}\gamma^{np}\xi_1 = 0, \tag{C2}$$

and four algebraic conditions

$$\gamma^m\partial_m\Delta\xi_1 - \frac{1}{48}e^{-2\Delta}\gamma^{mnp}G_{mnp}\xi_2 - \frac{i}{4}(e^{-4\Delta}f - 4)\xi_1 = 0,$$
$$\gamma^m\partial_m\Delta\xi_2 - \frac{1}{48}e^{-2\Delta}\gamma^{mnp}G^*_{mnp}\xi_1 + \frac{i}{4}(e^{-4\Delta}f + 4)\xi_2 = 0,$$
$$\gamma^m P_m\xi_2 + \frac{1}{24}e^{-2\Delta}\gamma^{mnp}G_{mnp}\xi_1 = 0,$$
$$\gamma^m P^*_m\xi_1 + \frac{1}{24}e^{-2\Delta}\gamma^{mnp}G^*_{mnp}\xi_2 = 0, \tag{C3}$$

where γ^m generate Cliff(5) with $\gamma_{12345} = +1$. Note that $\psi^c = C_{1,4}\psi^*$, $\xi_i^c = C_5\xi_i^*$, $i = 1, 2$, where $C_{1,4}$, C_5 are charge conjugation matrices.

The KK ansatz for the $D = 10$ Killing spinor is then simply

$$\epsilon = \varepsilon \otimes e^{\Delta/2}\xi_1 \otimes \theta + \varepsilon^c \otimes e^{\Delta/2}\xi_2^c \otimes \theta. \tag{C4}$$

Here ε is an arbitrary $D = 5$ spacetime spinor and the rest is as in the undeformed case. For the gravitino, we shall only need a KK reduction ansatz for the external components, namely, (in tangent space):

$$\Psi_\alpha = \psi_\alpha \otimes e^{-\Delta/2}\xi_1 \otimes \theta + \psi^c_\alpha \otimes e^{-\Delta/2}\xi_2^c \otimes \theta, \tag{C5}$$

where ψ_α is the $D = 5$ gravitino.

We now demand that the conditions for the KK ansatz to preserve supersymmetry, namely, that the supersymmetry variations of λ and Ψ_M vanish, is the same as the conditions for preservation of supersymmetry in the $D = 5$ gauged supergravity. We will use (B5) but with $\alpha_0 = 0$ and $\beta_3 = *_5\beta_2$.

First consider the variations of the dilatino and of the internal components Ψ_a of the gravitino. After substituting (B5) into (A6) and (A7) and using (C2) and (C3), we find that these variations vanish providing that

$$\alpha_{1a}\gamma^a\xi_1 = 0, \qquad \alpha^*_{1a}\gamma^a\xi_2 = 0,$$
$$-4h\delta_{a1}\xi_1 + 2ie^{-4\Delta}\beta_{2ab}\gamma^b\xi_1 - ie^{-4\Delta}\gamma_{abc}\beta_2^{bc}\xi_1 - e^{-2\Delta}\alpha_1^b\gamma_{ab}\xi_2 + 3e^{-2\Delta}\alpha_{1a}\xi_2 = 0,$$
$$-4h\delta_{a1}\xi_2 - 2ie^{-4\Delta}\beta_{2ab}\gamma^b\xi_2 + ie^{-4\Delta}\gamma_{abc}\beta_2^{bc}\xi_2 - e^{-2\Delta}\alpha_1^{*b}\gamma_{ab}\xi_1 + 3e^{-2\Delta}\alpha^*_{1a}\xi_1 = 0. \tag{C6}$$

One can check that these relations are indeed satisfied[7] given our expressions (B14) for α_1 and β_2.

Next consider the variation of the external components of the gravitino. After substituting (B5) into (A7), one finds

$$\begin{aligned}\delta\Psi_\alpha ={}& \frac{1}{2}e^{-\Delta/2}\rho_\alpha\varepsilon \otimes \left(-\frac{1}{4}(e^{-4\Delta}f - 4)\xi_1 - i\gamma_a\partial^a\Delta\xi_1 + \frac{i}{48}e^{-2\Delta}\gamma^{abc}G_{abc}\xi_2\right)\otimes\theta \\ &+ \frac{1}{2}e^{-\Delta/2}\rho_\alpha\varepsilon^c \otimes \left(-\frac{1}{4}(e^{-4\Delta}f + 4)\xi_2^c - i\gamma_a\partial^a\Delta\xi_2^c + \frac{i}{48}e^{-2\Delta}\gamma^{abc}G_{abc}\xi_1^c\right)\otimes\theta + e^{-\Delta/2}\Big[D_\alpha\varepsilon \otimes \xi_1 - \frac{1}{2}\rho_\alpha\varepsilon\otimes\xi_1 \\ &- A_\alpha\varepsilon\otimes\partial_\psi\xi_1 + \frac{1}{16}F_{\alpha\beta}\rho^\beta\varepsilon\otimes(-4ih\gamma_1\xi_1 - e^{-4\Delta}\beta_{2ab}\gamma^{ab}\xi_1 - 3ie^{-2\Delta}\alpha_{1a}\gamma^a\xi_2) \\ &+ \frac{1}{32}\rho_{\alpha\beta\gamma}F^{\beta\gamma}\varepsilon\otimes(ie^{-2\Delta}\alpha_{1a}\gamma^a\xi_2 + e^{-4\Delta}\beta_{2bc}\gamma^{bc}\xi_1)\Big]\otimes\theta + e^{-\Delta/2}\Big[D_\alpha\varepsilon^c\otimes\xi_2^c + \frac{1}{2}\rho_\alpha\varepsilon^c\otimes\xi_2^c - A_\alpha\varepsilon^c\otimes\partial_\psi\xi_2^c \\ &+ \frac{1}{16}F_{\alpha\beta}\rho^\beta\varepsilon^c\otimes(-4ih\gamma_1\xi_2^c - e^{-4\Delta}\beta_{2ab}\gamma^{ab}\xi_2^c - 3ie^{-2\Delta}\alpha_{1a}\gamma^a\xi_1^c) \\ &+ \frac{1}{32}\rho_{\alpha\beta\gamma}F^{\beta\gamma}\varepsilon^c\otimes(ie^{-2\Delta}\alpha_{1a}\gamma^a\xi_1^c + e^{-4\Delta}\beta_{2bc}\gamma^{bc}\xi_2^c)\Big]\otimes\theta,\end{aligned} \tag{C7}$$

where we are using the coordinate ψ so that the Killing vector dual to K_5 is $3\partial_\psi$. In this expression the ρ^α generate Cliff(4,1) and satisfy $\rho_{01234} = -i$ (this corrects a sign in [15]). We also have $\epsilon_{01234} = +1$.

We now observe that for the choice of forms given in (B14) one has

$$-4ih\gamma_1\xi_1 - e^{-4\Delta}\beta_{2ab}\gamma^{ab}\xi_1 - 3ie^{-2\Delta}\alpha_{1a}\gamma^a\xi_2 = -\frac{8i}{3}\xi_1,$$
$$ie^{-2\Delta}\alpha_{1a}\gamma^a\xi_2 + e^{-4\Delta}\beta_{2bc}\gamma^{bc}\xi_1 = \frac{4i}{3}\xi_1 \tag{C8}$$

and similar expressions for the last two terms of (C7). Using these results, the fact that $\partial_\psi\xi_1 = -\frac{i}{2}\xi_1$ and Eqs. (C3), after introducing the KK ansatz (C5) for the gravitino we deduce that

$$\delta\psi_\alpha \otimes e^{-\Delta/2}\xi_1 \otimes \theta + \delta\psi^c_\alpha \otimes e^{-\Delta/2}\xi^c_2 \otimes \theta = \left(D_\alpha\varepsilon - \frac{1}{2}\rho_\alpha\varepsilon + \frac{i}{2}A_\alpha\varepsilon + \frac{i}{24}F_{\beta\gamma}(\rho_\alpha^{\beta\gamma} - 4\delta^\beta_\alpha\rho^\gamma)\varepsilon\right) \otimes e^{-\Delta/2}\xi_1 \otimes \theta + \left(D_\alpha\varepsilon^c + \frac{1}{2}\rho_\alpha\varepsilon^c - \frac{i}{2}A_\alpha\varepsilon^c + \frac{i}{24}F_{\beta\gamma}(\rho_\alpha^{\beta\gamma} - 4\delta^\beta_\alpha\rho^\gamma)\varepsilon^c\right) \otimes e^{-\Delta/2}\xi^c_2 \otimes \theta, \tag{C9}$$

which implies

$$\delta\psi_\alpha = D_\alpha\varepsilon - \frac{1}{2}\rho_\alpha\varepsilon + \frac{i}{2}A_\alpha\varepsilon + \frac{i}{24}F_{\beta\gamma}(\rho_\alpha^{\beta\gamma} - 4\delta^\beta_\alpha\rho^\gamma)\varepsilon, \tag{C10}$$

as claimed in the text.

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