High temperature symmetry nonrestoration and inverse symmetry breaking in the Cornwall-Jackiw-Tomboulis formalism

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The Cornwall-Jackiw-Tomboulis (CJT) effective action at finite temperature is applied to study the symmetry nonrestoration (SNR) and inverse symmetry breaking (ISB) at high temperature in the $Z_2 \times Z_2$ model. A renormalization prescription is developed for the CJT effective action in the double bubble approximation. It is shown that the triviality related feature of the model does not show up, and the temperature effects do not alter the conditions for SNR/ISB in a broad range of temperatures.

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I. INTRODUCTION

At present it is well known that there are several categories of physical systems:

(a) One of these includes systems in which the symmetry, broken at $T = 0$, is restored at high temperatures [1–3]. In addition, there is another alternative phenomenon, the behavior of which associates with broken symmetry as temperature is increased. This is the so-called inverse symmetry breaking (ISB). Here high temperature means that $T/M \gg 1$ for mass scale M of the system in question.

(b) Another category includes systems displaying symmetry nonrestoration (SNR) at high temperatures. There exist many such systems associated with numerous different materials [4]. High temperature SNR has been studied in the framework of quantum field theory [5–9], and recently developed in connection with cosmological applications [10–22].

Accordingly, interest in developing a formalism allowing for an adequate and reliable description of SNR and ISB at high temperature has been growing. As was pointed out in Ref. [9], the CJT effective action is a particularly well-suited tool for such a task. Indeed, the present work uses the finite temperature CJT effective action [23] to shed new light on the $Z_2 \times Z_2$ model in connection with the domain wall [10,17] and other [24,25] problems. Section II is dedicated to the calculation and normalization of the CJT effective action and Sec. III to the study of high temperature SNR and ISB. The results are discussed in Sec. IV where some conclusions are spelled out.

II. RENORMALIZED CJT EFFECTIVE ACTION AT FINITE T

We use a simple Lagrangian to describe the system

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\mu_1^2}{2}\phi^2 + \frac{\lambda_1}{4!}\phi^4 + \frac{1}{2}(\partial_\mu \psi)^2 + \frac{\mu_2^2}{2}\psi^2 + \frac{\lambda_2}{4!}\psi^4 + \frac{\lambda}{4}\phi^2\psi^2 + \Delta\mathcal{L}. \tag{2.1}$$

The counterterms are taken as

$$\Delta\mathcal{L} = \frac{\delta\mu_1^2}{2}\phi^2 + \frac{\delta\lambda_1}{4!}\phi^4 + \frac{\delta\mu_2^2}{2}\psi^2 + \frac{\delta\lambda_2}{4!}\psi^4 + \frac{\delta\lambda}{4}\phi^2\psi^2.$$

The boundedness of the potential requires

$$\lambda_1 > 0, \qquad \lambda_2 > 0 \quad \text{and} \quad \lambda_1\lambda_2 > 9\lambda^2. \tag{2.2}$$

Shifting $\{\phi, \psi\} \to \{\phi + \phi_0, \psi + \psi_0\}$ leads to the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = \frac{\lambda_1 + \delta\lambda_1}{24}\phi^4 + \frac{\lambda_1 + \delta\lambda_1}{6}\phi_0\phi^3 + \frac{\lambda_2 + \delta\lambda_2}{24}\psi^4 + \frac{\lambda_2 + \delta\lambda_2}{6}\psi_0\psi^3 + \frac{\lambda + \delta\lambda}{4}\phi^2\psi^2 + \frac{\lambda + \delta\lambda}{2}\phi_0\phi\psi^2 + \frac{\lambda + \delta\lambda}{2}\psi_0\psi\phi^2$$

and the tree-level propagators

$$D_0^{-1}(k;\phi_0,\psi_0) = k^2 + \mu_1^2 + \delta\mu_1^2 + \frac{\lambda_1 + \delta\lambda_1}{2}\phi_0^2 + \frac{\lambda + \delta\lambda}{2}\psi_0^2,$$

$$G_0^{-1}(k;\phi_0,\psi_0) = k^2 + \mu_2^2 + \delta\mu_2^2 + \frac{\lambda_2 + \delta\lambda_2}{2}\psi_0^2 + \frac{\lambda + \delta\lambda}{2}\phi_0^2.$$

Next the two-loop double-bubble approximation, which is the leading two-loop contribution within a systematic expansion in loops is used to yield the truncated expression for CJT effective potential $V_\beta^{\rm CJT}[\phi_0, \psi_0, D, G]$ at finite temperature

$$\begin{aligned}V_\beta^{\rm CJT}[\phi_0,\psi_0,D,G] &= \delta\Omega + \frac{\mu_1^2+\delta\mu_1^2}{2}\phi_0^2 + \frac{\lambda_1+\delta\lambda_1}{24}\phi_0^4 + \frac{\mu_2^2+\delta\mu_2^2}{2}\psi_0^2 + \frac{\lambda_2+\delta\lambda_2}{24}\psi_0^4 + \frac{\lambda+\delta\lambda}{4}\phi_0^2\psi_0^2 \\ &+ \frac{1}{2}\int_\beta\{\ln D^{-1}(k) + \ln G^{-1}(k) + D_0^{-1}(k;\phi_0,\psi_0)D + G_0^{-1}(k;\phi_0,\psi_0)G - 2\} + \frac{\lambda_1+\delta\lambda_1}{8}\Big[\int_\beta D(k)\Big]^2 \\ &+ \frac{\lambda_2+\delta\lambda_2}{8}\Big[\int_\beta G(k)\Big]^2 + \frac{\lambda+\delta\lambda}{4}\Big[\int_\beta D(k)\Big]\Big[\int_\beta G(k)\Big], \end{aligned} \tag{2.3}$$

where the usual notation is implied

$$\int_\beta \equiv T\sum_{-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}.$$

The validity of the double-bubble approximation and its relevant applications were discussed in [23,26].

Following [9] we introduce the temperature dependent effective masses M_1 and M_2

$$D^{-1}(p) = p^2 + M_1^2; \qquad G^{-1}(p) = p^2 + M_2^2,$$

where

$$\begin{aligned} M_1^2 &= \mu_1^2 + \delta\mu_1^2 + \frac{\lambda_1+\delta\lambda_1}{2}[\phi_0^2 + P(M_1)] + \frac{\lambda+\delta\lambda}{2}[\psi_0^2 + P(M_2)], \\ M_2^2 &= \mu_2^2 + \delta\mu_2^2 + \frac{\lambda_2+\delta\lambda_2}{2}[\psi_0^2 + P(M_2)] + \frac{\lambda+\delta\lambda}{2}[\phi_0^2 + P(M_1)]. \end{aligned} \tag{2.4}$$

in which

$$P(M) = \int_\beta \frac{1}{k^2+M^2}.$$

Inserting (2.4) into (2.3) we get

$$\begin{aligned}V_\beta^{\rm CJT}[\phi_0,\psi_0,M_1,M_2] &= \frac{\mu_1^2+\delta\mu_1^2}{2}\phi_0^2 + \frac{\lambda_1+\delta\lambda_1}{24}\phi_0^4 + \frac{\mu_2^2+\delta\mu_2^2}{2}\psi_0^2 + \frac{\lambda_2+\delta\lambda_2}{24}\psi_0^4 + \frac{\lambda+\delta\lambda}{4}\phi_0^2\psi_0^2 + Q(M_1) \\ &+ Q(M_2) + \frac{1}{2}\Big(\mu_1^2 + \delta\mu_1^2 + \frac{\lambda_1+\delta\lambda_1}{2}\phi_0^2 + \frac{\lambda+\delta\lambda}{2}\psi_0^2 - M_1^2\Big)P(M_1) \\ &+ \frac{1}{2}\Big(\mu_2^2 + \delta\mu_2^2 + \frac{\lambda_2+\delta\lambda_2}{2}\psi_0^2 + \frac{\lambda+\delta\lambda}{2}\phi_0^2 - M_2^2\Big)P(M_2) + \frac{\lambda_1+\delta\lambda_1}{8}[P(M_1)]^2 \\ &+ \frac{\lambda_2+\delta\lambda_2}{8}[P(M_2)]^2 + \frac{\lambda+\delta\lambda}{4}P(M_1)P(M_2), \end{aligned} \tag{2.5}$$

where

$$Q(M) = \frac{1}{2}\int_\beta \ln(k^2+M^2).$$

Regularizing the divergent integrals $P(M)$ and $Q(M)$, appearing in (2.5), we make use of the three-dimensional momentum cutoff scheme. Each divergent integral is written as the sum of divergent and finite parts, namely

$$Q(M) = \mathrm{Div}Q(M) + Q_f(M), \qquad \mathrm{Div}Q(M) = -\frac{M^4}{4}I_2 + \frac{M^2}{2}I_1,$$
$$Q_f(M) = \frac{M^4}{64\pi^2}\left(\ln\frac{M^2}{\mu^2} - \frac{1}{2}\right) + T\int\frac{d^3k}{(2\pi)^3}\ln(1 - e^{-(E(\vec{k})/T)}), \qquad P(M) = \mathrm{Div}P(M) + P_f(M),$$
$$\mathrm{Div}P(M) = I_1 - M^2 I_2, \qquad P_f(M) = \frac{M^2}{16\pi^2}\ln\frac{M^2}{\mu^2} - \int\frac{d^3k}{(2\pi)^3}[E(\vec{k})(1 - e^{(E(\vec{k})/T)})]^{-1},$$
$$I_1 = \frac{\Lambda^2}{8\pi^2}, \qquad I_2 = \frac{1}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}, \qquad E(\vec{k}) = (\vec{k}^2 + M^2)^{1/2}.$$

Developing the renormalization carried out in [27] requires that all divergent terms have to be absorbed into counterterms, corresponding to renormalizing masses and coupling constants. To this end, the renormalized masses and coupling constants are defined as

$$\left.\frac{\delta^2 V}{\delta\phi^2}\right|_{\phi=\phi_0;\psi=\psi_0} = \mu_{1R}^2, \qquad \left.\frac{\delta^2 V}{\delta\psi^2}\right|_{\phi=\phi_0;\psi=\psi_0} = \mu_{2R}^2, \qquad \left.\frac{\delta^4 V}{\delta\phi^4}\right|_{\phi=\phi_0;\psi=\psi_0} = \lambda_{1R},$$
$$\left.\frac{\delta^4 V}{\delta\psi^4}\right|_{\phi=\phi_0;\psi=\psi_0} = \lambda_{2R}, \qquad \left.\frac{\delta^4 V}{\delta\phi^2\delta\psi^2}\right|_{\phi=\phi_0;\psi=\psi_0} = \lambda_R,$$

which lead to a pair of equations for $\delta\lambda_1$, $\delta\lambda_2$, and $\delta\lambda$

$$\delta\lambda_1[\phi_0^2 + P(M_1)] + \delta\lambda[\psi_0^2 + P(M_2)] + \lambda_1[\phi_0^2 + P(M_1)] + \lambda[\psi_0^2 + P(M_2)] = 0,$$
$$\delta\lambda_2 P(M_2) + \delta\lambda[\phi_0^2 + P(M_1)] + \lambda_2[\psi_0^2 + P(M_2)] + \lambda[\phi_0^2 + P(M_1)] = 0.$$

We impose in addition,

$$\begin{aligned}&\delta\mu_1^2\mathrm{Div}P(M_1) + \delta\mu_2^2\mathrm{Div}P(M_2) + \frac{\delta\lambda_1}{4}[P^2(M_1) + 2\phi_0^2\mathrm{Div}P(M_1)] + \frac{\delta\lambda_2}{4}[P^2(M_2) + 2\psi_0^2\mathrm{Div}P(M_2)]\\ &\quad + \frac{\delta\lambda}{4}[P(M_1)P(M_2) + \phi_0^2\mathrm{Div}P(M_2) + \psi_0^2\mathrm{Div}P(M_1)] + \left(\mu_1^2 + \frac{\lambda_1}{2}\phi_0^2 + \frac{\lambda}{2}\psi_0^2 - M_1^2\right)\mathrm{Div}P(M_1)\\ &\quad + \left(\mu_2^2 + \frac{\lambda}{2}\phi_0^2 + \frac{\lambda_2}{2}\psi_0^2 - M_2^2\right)\mathrm{Div}P(M_2) + 2[\mathrm{Div}Q(M_1) + \mathrm{Div}Q(M_2)] = 0.\end{aligned}$$

Altogether, we have a system of three linear equations for five unknown quantities $\delta\mu_1^2$, $\delta\mu_2^2$, $\delta\lambda_1$, $\delta\lambda_2$, and $\delta\lambda$. The existence of nontrivial solutions ensures that only finite terms would be present in the renormalized effective potential

$$\begin{aligned}V_\beta^{\mathrm{CJT}}[\phi_0, \psi_0, M_{1R}^2, M_{2R}^2] &= \frac{\mu_{1R}^2}{2}\phi_0^2 + \frac{\lambda_{1R}}{24}\phi_0^4 + \frac{\mu_{2R}^2}{2}\psi_0^2 + \frac{\lambda_{2R}}{24}\psi_0^4 + \frac{\lambda_R}{4}\phi_0^2\psi_0^2 + Q_f(M_{1R}) + Q_f(M_{2R})\\ &\quad + \frac{1}{2}\left(\mu_{1R}^2 + \frac{\lambda_{1R}}{2}\phi_0^2 + \frac{\lambda_R}{2}\psi_0^2 - M_{1R}^2\right)P_f(M_{1R}) + \frac{1}{2}\left(\mu_{2R}^2 + \frac{\lambda_{2R}}{2}\psi_0^2 + \frac{\lambda_R}{2}\phi_0^2 - M_{2R}^2\right)P_f(M_{2R})\\ &\quad + \frac{\lambda_{1R}}{8}[P_f(M_{1R})]^2 + \frac{\lambda_{2R}}{8}[P_f(M_{2R})]^2 + \frac{\lambda_R}{4}P_f(M_{1R})P_f(M_{2R}).\end{aligned} \tag{2.6}$$

From (2.6) the renormalized gap equations are obtained

$$\begin{aligned}&\left[\mu_{1R}^2 + \frac{\lambda_{1R}}{6}\phi_0^2 + \frac{\lambda_R}{2}\psi_0^2 + \frac{\lambda_{1R}}{2}P_f(M_{1R}) + \frac{\lambda_R}{2}P_f(M_{2R})\right]\phi_0 = 0,\\ &\left[\mu_{2R}^2 + \frac{\lambda_{2R}}{6}\psi_0^2 + \frac{\lambda_R}{2}\phi_0^2 + \frac{\lambda_{2R}}{2}P_f(M_{2R}) + \frac{\lambda_R}{2}P_f(M_{1R})\right]\psi_0 = 0.\end{aligned} \tag{2.7}$$

and

$$\begin{aligned}M_{1R}^2 &= \mu_{1R}^2 + \frac{\lambda_{1R}}{2}[\phi_0^2 + P_f(M_{1R})] + \frac{\lambda_R}{2}[\psi_0^2 + P_f(M_{2R})],\\ M_{2R}^2 &= \mu_{2R}^2 + \frac{\lambda_{2R}}{2}[\psi_0^2 + P_f(M_{2R})] + \frac{\lambda_R}{2}[\phi_0^2 + P_f(M_{1R})].\end{aligned} \tag{2.8}$$

The equations used for SNR analysis in [5] are directly derived from (2.8) at high temperature. Substituting (2.8) into (2.6) we arrive at

$$\begin{aligned} V_\beta[\phi_0, \psi_0] = {} & \frac{\mu_{1R}^2}{2}\phi_0^2 + \frac{\lambda_{1R}}{24}\phi_0^4 + \frac{\mu_{2R}^2}{2}\psi_0^2 + \frac{\lambda_{2R}}{24}\psi_0^4 \\ & + \frac{\lambda_R}{4}\phi_0^2\psi_0^2 + Q_f(M_{1R}) + Q_f(M_{2R}) \\ & - \frac{\lambda_{1R}}{8}[P_f(M_{1R})]^2 - \frac{\lambda_{2R}}{8}[P_f(M_{2R})]^2 \\ & - \frac{\lambda_R}{4}P_f(M_{1R})P_f(M_{2R}), \end{aligned} \tag{2.9}$$

For convenience the subscript R will be omitted in what follows.

It is worth emphasizing that the present renormalization prescription leads to two important results:

(i) The expression (2.6) for the renormalized $V_\beta^{\rm CJT}$ does not contain any cutoff dependent term.

(ii) The so-called triviality-related features of the model under consideration do not show up.

These are the main improvements that have been obtained with respect to [9].

III. HIGH TEMPERATURE SNR/ISB

Considering high temperature SNR/ISB we assume that $\mu_1^2 < 0$ and $\mu_2^2 > 0$. As a consequence,

$$\phi_0 \neq 0 \quad \text{and} \quad \psi_0 = 0,$$

which implies that at $T = 0$ the symmetry of the system is spontaneously broken in the ϕ sector and unbroken in the ψ sector.

At high temperature the necessary condition for symmetry restoration (SR) in sector ϕ is that

$$M_1^2(T)|_{T=T_1} = 0$$

at some value $T = T_1$.

In the vicinity of T_1, $T^2/M_1^2 \gg 1$, and the high temperature behavior of $M_1^2(T)$ looks like

$$M_1^2(T) \simeq \mu_1^2 + \frac{T^2}{24}(\lambda_1 + \lambda), \tag{3.1}$$

which gives immediately

$$T_1^2 \simeq -\frac{24\mu_1^2}{\lambda_1 + \lambda}.$$

For $\lambda_1 + \lambda > 0$, T_1 is real and the symmetry gets restored at $T = T_1$. Inversely, the symmetry nonrestoration occurs in the ϕ sector if

$$\lambda_1 + \lambda < 0$$

or

$$\lambda < 0 \quad \text{and} \quad \lambda_1 < |\lambda|.$$

Analogously, the symmetry in the ψ sector is spontaneously broken at high temperature $T = T_{c2}$ only if

$$M_2^2(T_2) = 0.$$

In the vicinity of T_2 we obtain the high temperature expansion of $M_2^2(T)$

$$M_2^2(T) \simeq \mu_2^2 + \frac{T^2}{24}(\lambda_2 + \lambda)$$

which leads to

$$T_2^2 \simeq -\frac{24\mu_2^2}{\lambda_2 + \lambda}.$$

Therefore the condition for ISB is

$$\lambda < 0, \quad \lambda_2 < |\lambda|.$$

In summary, the parameters are constrained by

$$\begin{aligned} &\lambda_1 > 0, \quad \lambda_2 > 0, \quad \mu_1^2 < 0, \quad \mu_2^2 > 0, \\ &\lambda_1\lambda_2 > 9\lambda^2, \quad \lambda < 0, \quad |\lambda| > \lambda_1, \quad \lambda_2, \end{aligned} \tag{3.2}$$

for the present model, in which both SNR and ISB simultaneously take place at high temperature in the corresponding sector.

A question that arises is whether the constraints obeyed by the coupling constants are altered by thermal effects. The calculations carried out in [6,13,14] revealed that the possibility of SNR/ISB occurring at high temperature is not affected by thermal contributions to the coupling constants.

We reconsider this issue using the effective potential $V_\beta[\phi_0, \psi_0]$ (2.9). The T dependent coupling constants are

$$\begin{aligned} \lambda_1(T) &= \frac{\delta^4 V_\beta}{\delta\phi_0^4}\bigg|_{\phi_0=0,\psi_0=0}, \\ \lambda_2(T) &= \frac{\delta^4 V_\beta}{\delta\psi_0^4}\bigg|_{\phi_0=0,\psi_0=0}, \\ \lambda(T) &= \frac{\delta^4 V_\beta}{\delta\phi_0^2\delta\psi_0^2}\bigg|_{\phi_0=0,\psi_0=0}. \end{aligned} \tag{3.3}$$

Inserting the high temperature expansion for $V_\beta[\phi_0, \psi_0]$ into (3.3) we get the leading terms of the thermal contributions to $\lambda_1(T)$, $\lambda_2(T)$, and $\lambda(T)$,

$$\lambda_1(T) \approx \lambda_1 + \frac{3\lambda_1^2}{32\pi^2}\ln\frac{T^2}{24\mu^2} + \frac{3\lambda^2}{32\pi^4}\ln\frac{T^2}{24\mu^2} - \frac{9\lambda_1^3}{896\pi^4}\ln\frac{T^2}{24\mu^2} - \frac{9\lambda^2\lambda_2}{896\pi^4}\ln\frac{T^2}{24\mu^2} - \frac{3\lambda}{896\pi^4}\left[\lambda_1^2\frac{\lambda_2+\lambda}{\lambda_1+\lambda} + \lambda^2\frac{\lambda_1+\lambda}{\lambda_2+\lambda} + 4\lambda\lambda_1\right]\ln\frac{T^2}{24\mu^2},$$

$$\lambda_2(T) \approx \lambda_2 + \frac{3\lambda_2^2}{32\pi^2}\ln\frac{T^2}{24\mu^2} + \frac{3\lambda^2}{32\pi^2}\ln\frac{T^2}{24\mu^2} - \frac{9\lambda_2^3}{896\pi^4}\ln\frac{T^2}{24\mu^2} - \frac{9\lambda^2\lambda_1}{896\pi^4}\ln\frac{T^2}{24\mu^2} - \frac{3\lambda}{896\pi^4}\left[\lambda_2^2\frac{\lambda_1+\lambda}{\lambda_2+\lambda} + \lambda^2\frac{\lambda_2+\lambda}{\lambda_1+\lambda} + 4\lambda\lambda_2\right]\ln\frac{T^2}{24\mu^2},$$

$$\lambda(T) \approx \lambda + \frac{\lambda\lambda_1}{32\pi^2}\ln\frac{T^2}{24\mu^2} + \frac{\lambda\lambda_2}{32\pi^2}\ln\frac{T^2}{24\mu^2} - \frac{3\lambda\lambda_1^2}{896\pi^4}\ln\frac{T^2}{24\mu^2} - \frac{3\lambda\lambda_2^2}{896\pi^4}\ln\frac{T^2}{24\mu^2} - \frac{\lambda}{896\pi^4}\left[2\lambda^2 + \lambda\lambda_1\frac{\lambda_2+\lambda}{\lambda_1+\lambda} + \lambda\lambda_2\frac{\lambda_1+\lambda}{\lambda_2+\lambda} + 2\lambda_1\lambda_2\right]\ln\frac{T^2}{24\mu^2}.$$

The logarithmic dependence of the coupling constants on temperature makes it clear that the conditions (3.2) for SNR/ISB to occur are very stable, strengthening the results of [6,13,14].

Finally, we evaluate the critical exponents β and γ associated with phase transitions at $T = T_1$ in the ϕ sector. From their definitions and the fluctuation-dissipation theorem it follows that

$$\phi_0(T_1 + \Delta T) \sim |\Delta T|^\beta, \qquad M_1^2(T_1 + \Delta T) \sim |\Delta T|^\gamma.$$

The value of γ is easily derived from (3.1),

$$M_1^2(T_1 + \Delta T) \approx \frac{\lambda_1 + \lambda}{12}T_1|\Delta T|$$

yielding $\gamma = 1$.

Next, combining the expressions for ϕ_0 and M_1^2 in (2.7) and (2.8) we get

$$\frac{\lambda_1}{3}\phi_0^2 - M_1^2 = 0$$

which immediately leads to

$$\phi_0(T_1 + \Delta T) \approx \sqrt{\frac{\lambda_1 + \lambda}{4\lambda_1}T_1}|\Delta T|^{1/2}$$

giving $\beta = \frac{1}{2}$.

The preceding result indicates that in the double-bubble approximation the calculated values for β and γ are not better than those obtained in mean field theory. This is the shortcoming of our approximation.

IV. CONCLUSION AND DISCUSSION

The high temperature occurrence of symmetry nonrestoration and inverse symmetry breaking has been studied in the framework of the $Z_2 \times Z_2$ model using the CJT effective action at finite temperature. The application of the renormalization prescription to the T dependent CJT effective potential in the double-bubble approximation has been shown to solve two problems arising in the treatment of Ref. [9]: the cutoff dependence of the renormalized effective potential and the triviality related feature of the model, the latter being probably an artifact of the renormalization method that had been used.

The conditions to be obeyed by the coupling constants for SNR or ISB to occur have been shown to be

$$\lambda < 0, \qquad |\lambda| > \lambda_1 \quad \text{or} \quad \lambda < 0, \qquad |\lambda| > \lambda_2$$

respectively.

Earlier results [6,13,14] concerning the stability of these conditions have been strengthened by our finding, using the double-bubble approximation of the CJT effective potential, that they depend logarithmically on temperature. However, the expressions obtained for the critical exponents, β and γ, give the same values as does mean field theory.

Generalization to the $O(M) \times O(N)$ model is straightforward and gives similar results.

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