Investigation of the bottomonium ground state η_b via its inclusive charm decays

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(Received 19 October 2007; published 17 December 2007)

Based on the nonrelativistic QCD factorization formalism, we calculate the bottomonium ground state η_b, inclusive charm decays at the leading order in the strong coupling constant α_s, and quarkonium internal relative velocity v. The inclusive charm pair production in η_b decay is mainly realized through the $\eta_b \to c\bar{c}g$ process, where the charm and anticharm quarks then dominantly hadronize into charmed hadrons. The momentum distributions of the final states are presented. In this work, we also calculate the J/ψ inclusive production rate in the η_b decays, where the color-octet contribution is found to be very important. We expect this study may shed some light on finding η_b or knowing more about its nature.

DOI: 10.1103/PhysRevD.76.125013 PACS numbers: 12.38.Bx, 12.39.Hg, 13.20.Gd

I. INTRODUCTION

In high energy research, heavy-quarkonium physics is one of the most interesting fields and it plays an important role in understanding the hadron configuration and the microcosmos. Theoretically, due to the nonrelativistic nature of heavy quarkonia, it is convenient to research their properties in the framework of the nonrelativistic potential model and nonrelativistic QCD [1], or other theories which work well in the nonrelativistic limit. Experimentally, heavy quarkonia have a relatively high production rate in both electronic and hadronic collisions and the vector members can be easily seen through their bilepton decays. Recently, some new resonances have been observed in the charmonium energy region [2], which enrich the heavy-quarkonium spectroscopy and make heavy-quarkonium physics more interesting.

After the spin-triplet bottomonium Υ was discovered three decades ago, its pseudoscalar partner η_b had been looked for in various experiments. Unfortunately, there is still no conclusive evidence that this elusive particle has been found. As a solid prediction from the quark model, the existence of η_b is indubitable, but its mass and decay channels remain undetermined experimentally at the present time. To search η_b, several experiments have been conducted both in e^+e^- collisions at the CLEO and the LEP, and hadronic collisions at the Fermilab Tevatron. The advantage of e^+e^- collisions is its clear background, which is impaired by the fact that production rates for spin-singlet states are generally small. Based on the 2.4 fb^{-1} data taken at the $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances, CLEO has searched distinctive single photons from hindered $M1$ transitions of $\Upsilon(2S)$ and $\Upsilon(3S)$ to $\eta_b\gamma$, and also from the cascade decay $\Upsilon(3S) \to h_b\pi^0, h_b\pi^+\pi^-$ followed by $E1$ transition $h_b \to \eta_b\gamma$, but no signals have been found [3]. In the experiments at LEP II, the $\eta_b(1S)$ is expected to be produced in a two-photon process. The ALPHA Collaborations analyzed the $\gamma\gamma$ interaction data, but found no evident signal in the four- and six-charged-particle final states [4]. The results of the L3 Collaboration and the DELPHI Collaboration were also negative and the upper limits of the products $\Gamma_{\gamma\gamma} \times \text{Br}(\eta_b)$ were set [5–7]. In comparison to e^+e^- experiments, the hadronic collision at the Fermilab Tevatron gives a large η_b production rate, but due to the complicated hadronic interaction background, the search for η_b there is also hard. Using the full 1992–1996 (run I) data, the CDF collaboration searched the η_b through its exclusive decay to double J/ψ and found some oblique evidences, but far from conclusive [8]. Right now, further efforts are being pursued in the run II data there.

Considering the situation of η_b experiments, theoretical research on its properties is still necessary, such as its mass, the production cross sections at different colliders, and its various decay channels. Among all properties, the η_b mass is believed to be predicted by potential model, effective theory, and lattice calculation without much ambiguity, which is very important for experimental observation. Recent theoretical work fixes the $\Upsilon - \eta_b$ mass splitting in the range of 40–60 MeV [9–12]. In Ref. [13], Braaten, Fleming, and Leibovich calculated the η_b production cross section at the Fermilab Tevatron in the framework of nonrelativistic QCD (NRQCD) and evaluated the branching ratio of the decay $\eta_b \to J/\psi J/\psi$. They suggested that η_b should be observable through this decay because of its large branching ratio of $7 \times 10^{-4\pm1}$. In Ref. [14], Maltoni and Polosa also evaluated the observation potential for η_b at the Tevatron, but they found and suggested that the decay $\eta_b \to D^*D^{(*)}$ might be the most optimistic channel to observe the η_b signal. In Ref. [15], the relativistic correction to the $\eta_b \to J/\psi J/\psi$ decay process was calculated and a much smaller branching ratio in comparison to Ref. [13] was obtained. The author also discussed the $\eta_b \to D^*D^{(*)}$ process and got a smaller rate than that in

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Ref. [14]. However, Santorelli finds that the final state interaction may enhance $\eta_b \to J/\psi J/\psi$ decay width by about 2 orders of magnitude [16]. Instead of η_b decays into hadronic final states, Hao *et al.* calculated the branching ratio of the η_b radiative decay process, i.e. $\eta_b \to \gamma J/\psi$ [17]. They claimed that this channel is also a hopeful one in the η_b hunting.

Comparing to the exclusive process, the inclusive process has a large branching ratio, nevertheless normally also has large uncertainties in pinning down the parent particle. Fortunately, the final state distributions of experimental observables are helpful to the inclusive process for the aim. For instance, recently, the inclusive charm production in the χ_b and Υ decays have been studied in [18,19], respectively.

At the leading order in α_s and nonrelativistic expansion, the η_b inclusive open charm decay happens through $b\bar{b} \to gg^*$ followed by $g^* \to c\bar{c}$, where the initial $b\bar{b}$ is configured in color singlet. Based on the result of $b\bar{b} \to gc\bar{c}$, we can roughly estimate the branching fraction of $b\bar{b} \to X+$ charmed hadrons. We calculate this process in the framework of NRQCD factorization formalism, in which the uncalculable nonperturbative effects are represented by the matrix elements of NRQCD operators. According to NRQCD, the color-octet configurations appear as higher order Fock states in η_b decays, which are suppressed by orders of the small magnitude in relative velocity v^2. Hence, for a leading order calculation, we can safely treat the $b\bar{b}$ pair inside the η_b to be in color singlet. However, for the J/ψ production in η_b decays, although the higher order Fock state, the $(c\bar{c})(^3S_1^{[8]})$, is suppressed by v^4 relative to the leading color-singlet configuration, it may be compensated by a factor of α_s in regarding to color-singlet process.

This paper is organized as follows: the inclusive charm and charmed hadron production in η_b decay is evaluated at leading order in Sec. II. In Sec. III, calculation of the process $\eta_b \to J/\psi + X$ will be presented. The last section is remained for summary.

II. THE CHARM QUARK PRODUCTION IN η_b DECAYS

In this section we calculate the charm quark production in the inclusive η_b decays. At the leading order in v, according to the NRQCD factorization formulism, its open charm decay width takes the form

$$\Gamma[\eta_b \to c\bar{c} + X] = C_1^c \frac{\langle \mathcal{O}_1(^1S_0)\rangle_{\eta_b}}{m_b^2}, \tag{2.1}$$

where $\langle \mathcal{O}_1^{\eta_b}(^1S_0)\rangle$ is a NRQCD matrix element, which represents the long-distance effect and gives the probability for finding the heavy quark and antiquark in specific configuration within the meson, and can be evaluated by a nonperturbative method such as lattice simulation. The dimensionless short-distance coefficient C_1^c can be calculated in perturbative QCD (pQCD). The dominant source of C_1^c comes from the decay of a color-singlet $b\bar{b}(^1S_0)$ pair into gg^*, followed by $g^* \to c\bar{c}$.

We can calculate the process in the nonrelativistic limit, in which the $b\bar{b}(^1S_0)$ pair can be taken as no relative momentum within η_b, i.e., $p_b = p_{\bar{b}} = Q/2$, where Q is the momentum of the η_b. In this situation, for the $b\bar{b}$ pair to form η_b, when it is in a color-singlet state, one can replace the product of the Dirac spinors for b and $\bar{b}$ in the initial state with the projector:

$$u(p_b)\bar{v}(p_{\bar{b}}) \to \frac{1}{2\sqrt{4\pi}}(\not{Q} + M_{\eta_b})i\gamma_5\left(\frac{1}{\sqrt{M_{\eta_b}}}R_{\eta_b}(0)\right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}}\right), \tag{2.2}$$

where $N_c = 3$, and 1_c stands for the unit color matrix. M_{η_b} is the mass of η_b. At leading order in nonrelativistic expansion, it could be understood that $M_{\eta_b} \approx 2m_b$. The nonperturbative parameters, $R(0)_{\eta_b}$ are color-singlet radial wave functions at the origin for η_b, which can be either reached from phenomenological potential models or directly extracted from experiments. The relation between the $R(0)_{\eta_b}$ and $\langle \mathcal{O}_1^{\eta_b}(^3S_1)\rangle$ reads $\langle \mathcal{O}_1^{\eta_b}(^3S_1)\rangle = (N_c/2\pi)|R(0)_{\eta_b}|^2(1 + O(v^4))$.

One can then get the partial decay width straightforwardly for the process $b\bar{b}(^1S_0) \to c(p_1) + \bar{c}(p_2) + g(k)$. That is,

$$d\Gamma[b\bar{b}(^1S_0) \to c\bar{c}g] = \frac{1}{2M_{\eta_b}}\sum_{ccg}|M_{\rm str}|^2 d\Phi_3, \tag{2.3}$$

where $M_{\rm str}$ is the amplitude for the process and the Φ_3 represents the three-body phase space, which shows

$$d\Phi_3 = \frac{1}{(2\pi)^9}\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3k}{2k_0}(2\pi)^4\delta^4(Q - p_1 - p_2 - k). \tag{2.4}$$

In the initial state rest frame, after integrating over the variables which are independent of the amplitude, the phase space integration can be further simplified as

$$d\Phi_3' = \frac{1}{(2\pi)^3}\frac{1}{4M_{\eta_b}}dE_1 dS_{13}. \tag{2.5}$$

Here, $S_{13} = p_1 \cdot k$.

In the numerical calculation, we take $m_b = 4.65 \pm 0.15$ GeV, $m_c = 1.50 \pm 0.05$ GeV, and $\alpha_s(m_b) = 0.22$. The magnitude of the radial wave function at the origin $R(0)$ of η_b equals approximately that of its spin-triplet partner Υ, which can be determined from the experimental data on the decay width of $\Gamma(\Upsilon \to e^+e^-) = (1.340 \pm 0.018) \times 10^{-6}$ GeV [20]. That is: $|R(0)_{\eta_b}|^2 = |R(0)_\Upsilon|^2 = 4.89 \pm 0.07$ GeV^3. With these inputs and by varying the strong coupling scale from $m_b/2$ to $2m_b$, for the aim of error estimation, we have

$$\Gamma[b\bar{b}(^1S_0) \to c\bar{c}g] = 190.8^{+190.0}_{-86.3}\ \text{KeV}. \tag{2.6}$$

Since at leading order the total decay width of η_b is

$$\Gamma_{\text{tot}}(\eta_b) \approx \Gamma(\eta_b \to gg) = \frac{8}{3}\frac{\alpha_s^2}{M_{\eta_b}^2}|R_{\eta_b}(0)|^2, \tag{2.7}$$

the branching ratio hence is readily obtained to be

$$\text{Br}[b\bar{b}(^1S_0) \to c\bar{c}g] = 2.6^{+0.9}_{-0.6} \times 10^{-2}. \tag{2.8}$$

For the inclusive decay process, giving out the experiment observable differential distribution will be useful. For this purpose, We define two fractions $x_1 = E_1/E_b$ and $r_c = m_c^2/m_b^2$, where E_1 and E_b stand for the energy of charm quark and bottom quark, and in nonrelativistic approximation the $E_b = m_b = M_{\eta_b}/2$. The region of variable x_1 is $\sqrt{r_c} < x_1 < 1$. In some cases, instead of x_1 it is convenient to use another variable y_1, which is the momentum of the charm quark divided by its kinematically allowed maximum value in η_b decays [18,19]. The relation between these two variables reads

$$x_1 = \sqrt{(1 - r_c)y_1^2 + r_c}, \tag{2.9}$$

$$y_1 = \sqrt{\frac{x_1^2 - r_c}{1 - r_c}}. \tag{2.10}$$

The range of y_1 is $0 < y_1 < 1$. Figure 1 exhibits the decay rate distribution over the momentum fraction y_1.

Because almost all the charm quarks may eventually hadronize into charmed hadrons, like D^0, $D^{\pm}$, D_s, and Λ_c, etc., we schematically show the D^+ meson differential distribution in η_b decays in the fragmentation approximation, similar as done in Refs. [18,19]. It is well known that the fragmentation function $D_{c\to h}(z)$ represents the probability of a charm quark fragmenting into the charmed hadron h. Here, the z is a Lorentz boost invariant variable, defined as $z = \frac{E_h + p_h}{E_1 + p_1}$. In practice calculation, we will simply neglect the difference between fragmenting charm quark mass and the charmed hadron mass. The z can be reexpressed as

$$z = \frac{z_h}{z_1} \tag{2.11}$$

with

$$z_1 = \frac{\sqrt{(1 - r_c)y_1^2 + r_c} + y_1\sqrt{1 - r_c}}{1 + \sqrt{1 - r_c}}, \tag{2.12}$$

$$z_h = \frac{\sqrt{(1 - r_c)y_h^2 + r_c} + y_h\sqrt{1 - r_c}}{1 + \sqrt{1 - r_c}}. \tag{2.13}$$

Then the momentum distribution of the charmed hadron can be expressed as [18,19]

$$\frac{d\Gamma}{dy_h} = \frac{dz_h}{dy_h}\int_{z_h}^{1}\frac{dz_1}{z_1}D(z_h/z_1)\frac{dy_1}{dz_1}\frac{d\Gamma}{dy_1}. \tag{2.14}$$

According to the Kartvelishvili-Likhoded-Petrov (KLP) [21] fitting for fragmentation function

$$D_{c\to h}(z) = N_h z^{\alpha_c}(1 - z). \tag{2.15}$$

Here, by using the optimal value of $\alpha_c = 4$ for D^+ fitted by the Belle collaboration [22], one gets the normalization coefficient $N_h = 8.04$ [18].

We show in Fig. 2 the D^+ meson differential production distribution in η_b decays in the fragmentation approximation. For other charmed hadrons, the corresponding distributions can be obtained similarly.

It should be mentioned that since the NRQCD factorization breaks down in the $y_1 \to 1$ limit, the velocity and

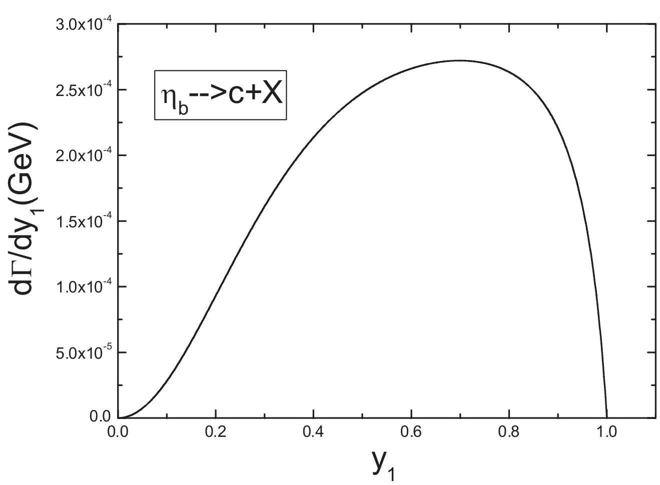

FIG. 1. The decay rate variation over momentum fraction y_1 in the inclusive process $\eta_b \to c + X$, by taking the central values of inputs.

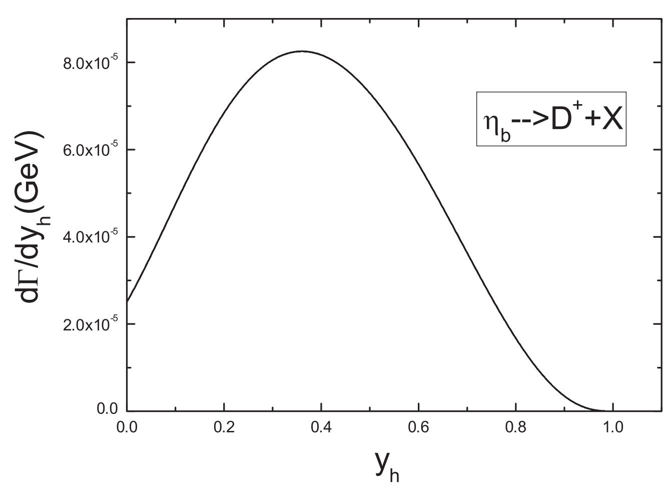

FIG. 2. The decay rate variation over momentum fraction y_h in the inclusive process $\eta_b \to D^+ + X$ with central values of inputs.

coupling expansions are no more valid in the vicinity of the end point. A proper treatment for this end point illness is to resum the large logarithms of $\log(1-y_1)$ [23–28] and invoke the shape function [29]. For these kinds of content, readers should refer to the related references in the literature; and in this situation, the final state distribution results at the end point in this work should not be taken seriously. Roughly speaking, when $y_1 < 0.7$, the end point effects become weak and our predictions turn to be robust [30]. Fortunately, for total decay widths, the end point influence is minor and the predictions are quite reliable. For the charmed hadron production, apart from the upper point problem, the lower limit also poses a problem for the fragmentation approximation, although (2.15) only comes from the phenomenological fitting. Therefore, the lower end point prediction should not be taken seriously as well. In Ref. [18], there are detailed discussions about the validity of the fragmentation approximation.

III. η_b INCLUSIVE DECAY TO $J/\psi + X$

In this section, we present the calculation of J/ψ inclusive production and its momentum distribution in the η_b decays, as shown in Fig. 3. As mentioned above, at the leading order in v only the color-singlet $b\bar{b}(^1S_0)$ contribution in the initial state is the necessary ingredient to be considered, since the higher Fock state contribution cannot get big enhancement from kinematic or dynamic reasons. However, for the final state J/ψ, both the color-singlet and color-octet effects should be included, because the latter one can get one α_s and kinematic compensation in comparison with the color-singlet process. Since it is the $b\bar{b}$ pair annihilation in the initial state and the creation of the $c\bar{c}$ pair, as well as the emission of gluon or quarks, takes place at the hard scales set by the heavy quark masses, it is legitimate to tackle this semi-inclusive process in the pQCD framework.

The NRQCD formalism allows the systematic calculation of inclusive cross sections for quarkonium decays in perturbation QCD to any order in α_s and v^2, where v is the typical relative velocity of the heavy quark inside the quarkonium. Using the NRQCD velocity scaling rules [1], we know the color-singlet process, Fig. 3(1), is at order of $\alpha_s^4 v^3$, since the matrix element $\langle O_1^V(^3S_1)\rangle$ is of order v^3, whereas, the color-octet process, Fig. 3(2), is at order $\alpha_s^3 v^7$. Here, v denotes the heavy quark relative velocity in the charmonium system. Potential model calculation indicates that the average value of v^2 is about 0.3, and the QCD coupling constant $\alpha_s(m_b) \approx 0.22$. In addition to the α_s compensation, the color-octet mechanism may also be enhanced by the single gluon propagator. Our following calculation really shows that the color-octet process should be included in this calculation.

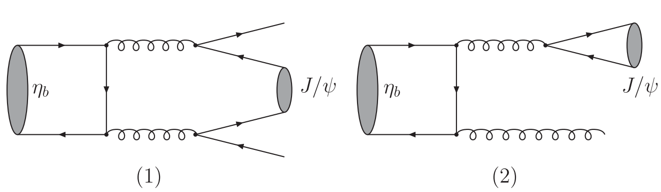

FIG. 3. Lowest-order diagrams that contribute to the inclusive process: $\eta_b \to J/\psi X$. The J/Ψ in diagram (1) is a color-singlet state, while the one in (2) is a color-octet state.

The decay width can be formulated as

$$\Gamma[\eta_b \to J/\psi + X] = A_1\langle O_1^{J/\psi}(^3S_1)\rangle + A_2\langle O_8^{J/\psi}(^3S_1)\rangle, \tag{3.1}$$

where the A_1 and A_2 are perturbative calculable short-distance coefficients. We first calculate the color-singlet coefficient. It is customary to start with the parton process; here it is $b(p_b)\bar{b}(p_{\bar{b}}) \to c(p_c)\bar{c}(p_{\bar{c}}) + c(k_1) + \bar{c}(k_2)$. Then project the matrix element onto corresponding color-singlet configurations. For the $b\bar{b}$ in initial state, it is the same as in the last section. For the J/ψ production, the color-singlet projector is

$$v(p_{\bar{c}})\bar{u}(p_c) \to \frac{1}{2\sqrt{4\pi}}\not{\epsilon}^*_{J/\psi}(\not{P} + M_{J/\psi}) \times \left(\frac{1}{\sqrt{M_{J/\psi}}}R_{J/\psi}(0)\right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}}\right), \tag{3.2}$$

where $\epsilon^\mu_{J/\psi}$ is the J/ψ polarization vector satisfying $\epsilon_{J/\psi}(\lambda)\cdot\epsilon^*_{J/\psi}(\lambda') = -\delta^{\lambda\lambda'}$ and $P\cdot\epsilon_{J/\psi} = 0$. $R_{J/\psi}(0)$ is the radial wave function at the origin, which is also valued through the J/ψ leptonic decay width. Combining all these together, one can easily get the η_b to $J/\psi + c + \bar{c}$ decay amplitude for the color-singlet case, i.e.

$$M^1_{\rm str} = C_1 g_s^4 \frac{R_{\eta_b}(0)R_{J/\psi}(0)}{4\pi\sqrt{M_{J/\psi}M_{\eta_b}}}\,{\rm Tr}[(\not{Q} + M_{\eta_b})\gamma_5\gamma_\mu((k_2-k_1)\cdot\gamma + M_{\eta_b})\gamma_\nu]\frac{1}{(k_2-k_1)^2 - M^2_{\eta_b}}\frac{1}{(k_1+P/2)^2} \times \frac{1}{(k_2+P/2)^2}\bar{u}(k_1)\gamma^\mu\not{\epsilon}_{J/\psi}(\not{P} + M_{J/\psi})\gamma^\nu v(k_2), \tag{3.3}$$

where C_1 is the corresponding color factor, k_1 and k_2 are the momenta carried by the external charm quark and antiquark, respectively.

Next, we present the calculation for the color-octet process. At the parton level it is the $b(p_b)\bar{b}(p_{\bar{b}}) \to c(p_c)\bar{c}(p_{\bar{c}}) + g(k)$ process followed by projecting the $c\ \bar{c}$ spinors onto the color-octet configuration, $^3S_1^{[8]}$, while keeping on configuring the initial $b\ \bar{b}$ in color singlet. The color-octet projector is

$$v(p_{\bar{c}})\bar{u}(p_c) \to \frac{1}{2\sqrt{4\pi}} \not\epsilon^*_{J/\psi}(\not P + M_{J/\psi}) \times \left(\frac{1}{\sqrt{M_{J/\psi}}} R^8_{J/\psi}(0)\right) \otimes \sqrt{2} T^a_{ij}. \qquad (3.4)$$

Here, T^a_{ij} denotes the SU(3) generator. We introduce another phenomenological parameter $R^8_{J/\psi}(0)$, which stands for the color-octet nonperturbative effect. The relation between $R^8_{J/\psi}(0)$ and the NRQCD matrix element $\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle$ is defined as

$$\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle = \frac{3N_c}{2\pi}|R^8_{J/\psi}(0)|^2. \qquad (3.5)$$

From the fitted value of $\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle \approx 1.5 \times 10^{-2}$ GeV^3 [31], we have $|R^8_{J/\psi}(0)| = 0.102$ GeV$^{3/2}$. Then the decay amplitude for the color-octet case is

$$M^{8(a)}_{\rm str}(\lambda_1, \lambda_2) = C_8 g_s^3 \frac{R_{\eta_b}(0) R^8_{J/\psi}(0)}{4\pi\sqrt{M_{J/\psi} M_{\eta_b}}} \mathrm{Tr}[(\not Q + M_{\eta_b}) \times \gamma_5 \not\epsilon^a_g(\lambda_2)((Q/2 - k)\cdot\gamma + M_{\eta_b}/2)\gamma^\nu] \times \frac{1}{M^2_{J/\psi}} \frac{1}{M^2_{\eta_b} - M^2_{J/\psi}} \times \mathrm{Tr}[\not\epsilon_{J/\psi}(\lambda_1)(\not P + M_{J/\psi})\gamma_\nu], \qquad (3.6)$$

where C_8 is the color factor, k is the momentum carried by the external gluon, and ϵ^μ_g is the gluon polarization satisfying $k \cdot \epsilon_g = 0$.

With the matrix elements $M^1_{\rm str}$ and $M^8_{\rm str}$, we can immediately get the $\eta_b \to J/\psi + X$ decay width. The analytical result for it is a bit lengthy, and will not be presented here. For our numerical estimation, the nonrelativistic limit is also enforced for the charmonium. That is, we take the $M_{J/\psi} \approx 2m_c$ approximation. From $\Gamma(J/\psi \to e^+e^-) = (5.55 \pm 0.14) \times 10^{-6}$ GeV [20], we get $|R_{J/\psi}(0)|^2 = 0.527 \pm 0.013$ GeV^3. Then, the decay width for the concerned process reads

$$\Gamma(\eta_b \to J/\psi_{\text{color-singlet}} + X) = 0.13^{+0.26}_{-0.08}\ \text{KeV}, \qquad (3.7)$$

$$\Gamma(\eta_b \to J/\psi_{\text{color-octet}} + X) \cong 2.16\ \text{KeV}, \qquad (3.8)$$

$$\Gamma_{\rm total}(\eta_b \to J/\psi + X) \cong 2.29\ \text{KeV}. \qquad (3.9)$$

That means that the $\eta_b \to J/\psi + X$ process has a branching ratio of 3.23×10^{-4} or so in the η_b decays. Here, the uncertainty estimate of the color-singlet process is performed in the same way as in the preceding section. Whereas, considering of the large uncertainties remaining in the color-octet matrix element fitting, we carry the numerical calculation for the color-octet process by only taking the central values of the inputs.

Like in Sec. II, to give out the differential decay width we define three new variables, the x_2, $r_{J/\psi}$, and y_2, as

$$x_2 = E_{J/\psi}/E_b, \qquad (3.10)$$

$$r_{J/\psi} = M^2_{J/\psi}/m_b^2, \qquad (3.11)$$

$$y_2 = \sqrt{\frac{x_2^2 - r_{J/\psi}}{1 - r_{J/\psi}}}. \qquad (3.12)$$

Then we can express the partial decay width of $\eta_b \to J/\psi_{\text{color-singlet}} + X$ as

$$d\Gamma[\eta_b \to J/\psi_{\text{color-single}} + X] = \frac{1}{2M_{\eta_b}} \sum_{J/\psi c\bar{c}} |M^1_{\rm str}|^2 d\Phi_3. \qquad (3.13)$$

In analogy to what is performed in the last section, we get the momentum distribution $d\Gamma(\eta_b \to J/\psi_{\text{color-single}} + X)/dy_2$, as shown in Fig. 4. For the process $\eta_b \to J/\psi_{\text{color-octet}} + X$,

$$d\Gamma[\eta_b \to J/\psi_{\text{color-octet}} + X] = \frac{1}{2M_{\eta_b}} \sum_{\lambda_1,\lambda_2} |M^8_{\rm str}(\lambda_1, \lambda_2)|^2 d\Phi_2. \qquad (3.14)$$

Since this is a two-body decay process, the J/ψ momentum distribution $d\Gamma(\eta_b \to J/\psi_{\text{color-octet}} + X)/dy_2$ is only a delta function peaked at $y_2 = \sqrt{\frac{(M^2_{\eta_b} - M^2_{J/\psi})^2}{4M^2_{\eta_b}(M^2_{\eta_b} - 4M^2_{J/\psi})}} = 0.6$. Again, for the η_b to J/ψ inclusive decay distribution, one should pay attention to the end point problem [32]. In particular, for the color-octet contribution, the η_b two-body decay at leading order results in a delta function distribution, which is smeared out by the nonperturbative effects and resulting in a shape function [33].

The numerical result shows that the branching ratio for the color-octet process is larger than the one for the color-singlet process by about an order, which offers an opportunity to check the existence of the color-octet mechanism

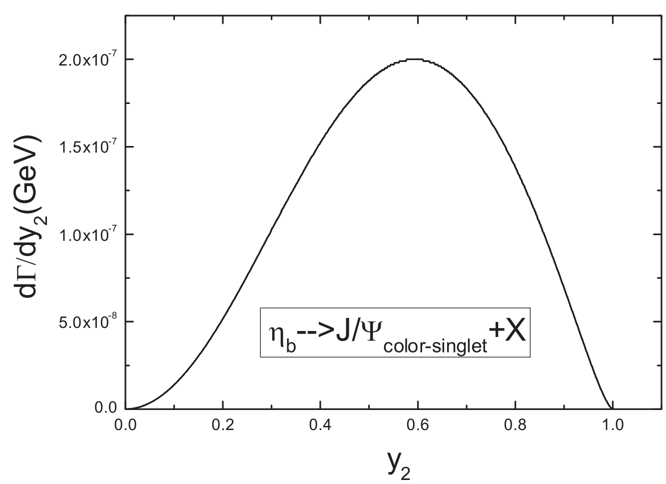

FIG. 4. The J/ψ momentum distribution in the inclusive process $\eta_b \to J/\psi_{\text{color-singlet}} + X$.

experimentally. Considering the uncertainties existing in the magnitude of the color-octet matrix element, the numerical difference between these two processes might shrink down; nevertheless, they give a distinctively different momentum distribution, which may also help experimenters to distinguish them in the future experiment.

IV. SUMMARY

We have studied in the framework of NRQCD the inclusive charm production in the decay of the pseudoscalar bottomonium state η_b. We find that it gives a quite large branching fraction. Since the produced charm quarks will dominantly evolve into charmed hadrons, by employing the KLP fragmentation function, we give out the momentum distribution of D^+, as an example.

We have also calculated the decay width and the momentum distribution of the inclusive J/ψ production in the η_b decay. We find that in this case the color-octet process should be taken into consideration. However, these two different J/ψ production schemes have obviously different momentum distributions. This is a distinct character of this process, which will be helpful for future experiment to investigate the η_b and to study the J/ψ production.

In all, to investigate the elusive η_b is still an interesting task for both theory and experiment. Our explicit calculation shows that η_b inclusive decays to charm pair (in experiment the charmed hadron pair) and J/ψ have quite large branching fractions. These processes can be helpful for people to hunt for the η_b at the Fermilab Tevatron, or LHC, where copious η_b are expected.

ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China, by the Scientific Research Fund of GUCAS (No. 055101BM03), and by a fund provided by the U.S. Department of Energy (DOE) under Cooperative Research Agreement No. DEFG02-05ER41360 under Particle.

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