Exploring the holographic dark energy model with the Sandage-Loeb test

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Taking into account that the Sandage-Loeb test is unique in its coverage of the redshift desert and available in the near future, we explore the cosmic time evolution behavior of the source redshift for the holographic dark energy model, an important competing cosmological model. As a result, we find that the Sandage-Loeb test can provide an extremely strong bound on Ω_m^0, while its constraint on another dimensionless parameter λ is weak. In addition, it is proposed here for the first time that we can also constrain various cosmological models by measuring the value of $z_{\max}$ at which the peak of redshift velocity occurs. Combining this new proposed method with the traditional Sandage-Loeb test, we should be able to provide a better constraint on λ, at least from the theoretical perspective.

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I. INTRODUCTION

The recent observations of type Ia Supernovae (SN Ia) indicate that our universe is currently accelerating [1,2]. Besides the cosmological constant, there have been other various dark energy models proposed to explain this exotic phenomenon [3,4]. On the other hand, a renewed interest has also been stimulated towards classic cosmological tests, including the spacial geometry of our universe and the kinematics of the expansion. For example, the position of acoustic peaks in the cosmic microwave background (CMB) angular power spectrum shows the spacial curvature of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric is nearly flat [5]. A similar test is also carried out by the detection of baryon acoustic oscillations in the power spectrum of matter calculated from galaxy samples. In addition, the luminosity distance of SN Ia and other standard candles allows one to provide a constraint on the value of the expansion rate at near redshifts $z < 2$ [6].

Until now, however, there are still a number of theoretical models surviving such observational tests. Thus, to check the internal consistency of the underlying cosmological model and discriminate various competing candidates, some new cosmological tools have been proposed and performed, such as the lookback time to galaxy clusters, the age of the universe, and the relative ages of passively evolving galaxies [7–10]. In particular, recently Corasaniti *et al.* employed the Sandage-Loeb test to constrain dark energy models with high significance within the redshift desert $2 < z < 5$, where other dark energy probes are unable to provide useful information about the expansion history of our universe [11]. Later, Balbi and Quercellini extended this analysis to more general dark energy models [12]. But they all neglected to investigate an important and popular competing candidate, i.e., the holographic dark energy model with the Sandage-Loeb test. Thus, as a further step along this line, the purpose of this paper is to explore the potential constraint on the holographic dark energy model with the Sandage-Loeb test.

In the subsequent section, we shall provide a brief review of the holographic dark energy model, including the latest observational constraints on it. After the Sandage-Loeb test is reviewed, we shall extensively investigate its potential power in constraint on the holographic dark energy model, where we furthermore go beyond the traditional Sandage-Loeb test within the redshift desert to propose a new cosmological probe at low redshifts to constrain the model better. Concluding remarks are presented in the last section. In addition, as motivated by inflation, the flat FLRW universe is assumed in the following discussions.

II. THE HOLOGRAPHIC DARK ENERGY MODEL WITH ITS AVAILABLE OBSERVATIONAL CONSTRAINTS

Some time ago, taking into account the insightful viewpoint that the UV cutoff in effective quantum field theory is connected with the IR cutoff against the formation of black holes, Cohen *et al.* argued that, if ρ_H is the zero-point energy density induced by the UV cutoff, the total energy in a region of size L should not exceed the mass of a black hole of the same size, i.e., $\rho_H L^3 \leq L M_p^2$ with the Planck mass $M_p = \frac{1}{\sqrt{8\pi G}}$ [13]. If we apply this to our universe, then the zero-point energy can serve as dark energy, referred to as holographic dark energy. As suggested by Li [14], the corresponding energy density can further be expressed as

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$$\rho_H = \frac{3\lambda^2 M_p^2}{L^2}. \tag{1}$$

Here λ is a dimensionless parameter, to be determined by the future complete quantum gravity theory, and L takes the size of the future event horizon of our universe, i.e.,

$$L[a(t)] = a(t)\int_t^\infty \frac{dt'}{a(t')} = a(t)\int_{a(t)}^\infty \frac{da'}{H'a'^2}, \tag{2}$$

where a is the scale factor of our universe, t is the cosmic time, and H is the Hubble parameter.

Next let us consider a flat FLRW universe with a matter component ρ_m (including both baryon matter and cold dark matter) and a holographic dark energy ρ_H. Then the Friedmann equation reads

$$3M_p^2 H^2 = \rho_m + \rho_H, \tag{3}$$

which can also be expressed equivalently as

$$\frac{H^2}{H_0^2} = \frac{\Omega_m^0}{a^3} + \Omega_H \frac{H^2}{H_0^2}. \tag{4}$$

Here $\Omega_m = \frac{\rho_m}{3M_p^2 H^2}$ and $\Omega_H = \frac{\rho_H}{3M_p^2 H^2}$. In addition, the superscript (subscript) 0 denotes the value for the corresponding physical quantity at the present time. Especially, for convenience but without loss of generality, we have set the present scale factor $a_0 = 1$ here. Later combining Eq. (1) with Eq. (2), we have

$$\int_a^\infty \frac{d\ln a'}{H'a'} = \frac{\lambda}{Ha\sqrt{\Omega_H}}. \tag{5}$$

On the other hand, Eq. (4) gives

$$\frac{1}{Ha} = \frac{\sqrt{a(1-\Omega_H)}}{H_0\sqrt{\Omega_m^0}}. \tag{6}$$

Substituting Eq. (6) into Eq. (5) and taking the derivative with respect to $\ln a$ on both sides, we obtain

$$\frac{d\Omega_H}{d\ln a} = \Omega_H(1-\Omega_H)\left(1 + \frac{2}{\lambda}\sqrt{\Omega_H}\right), \tag{7}$$

which describes the dynamic evolution of holographic dark energy, and can be formulated in terms of the redshift of our universe $z = \frac{1}{a} - 1$ as

$$\frac{d\Omega_H}{\Omega_H(1-\Omega_H)(1+\frac{2}{\lambda}\sqrt{\Omega_H})} = -\frac{dz}{1+z}. \tag{8}$$

It can further be integrated analytically as follows:

$$\begin{aligned} F_\lambda(\Omega_H) \equiv{}& \ln\Omega_H - \frac{\lambda}{2+\lambda}\ln(1-\sqrt{\Omega_H}) \\ &+ \frac{\lambda}{2-\lambda}\ln(1+\sqrt{\Omega_H}) \\ &- \frac{8}{4-\lambda^2}\ln(\lambda + 2\sqrt{\Omega_H}) \\ ={}& -\ln(1+z) + F_\lambda(1-\Omega_m^0). \end{aligned} \tag{9}$$

If we write the equation of state as $w_H = \frac{p_H}{\rho_H}$ for holographic dark energy, then the conservation of energy momentum gives

$$\dot{\rho}_H + 3H(1+w_H)\rho_H = 0, \tag{10}$$

which implies

$$w_H = -1 - \frac{1}{3}\frac{d\ln\rho_H}{d\ln a} = -\frac{1}{3}\left(1 + \frac{2}{\lambda}\sqrt{\Omega_H}\right), \tag{11}$$

where we have employed $\rho_H = \frac{\Omega_H \rho_m^0}{(1-\Omega_H)a^3}$ and Eq. (7) in the second step.

Note that there are only three free parameters in this holographic dark energy model: One is the kinematic parameter H_0, whose value is taken as $72\frac{\mathrm{km}}{\mathrm{s\,Mpc}}$ in the following calculations. The others are the dynamic parameters Ω_m^0 and λ, which determine the large scale evolution of our universe, including the final fate of our universe. Thus, constraint of Ω_m^0 and λ from observational data plays a fundamental role in the diagnosis of this model [15].

Such a dynamical dark energy model has been constrained by various astronomical observation data, such as the luminosity distance of SN Ia [16], the x-ray gas mass fraction of galaxy clusters [17], and the relative ages of galaxies [18]. In addition, a joint constraint from SN Ia, CMB, and large scale structure (LSS) observational data has also been performed [19,20]. In particular, combining the latest observational data from SN Ia, and CMB plus LSS, Zhang and Wu obtained $\Omega_m^0 = 0.29 \pm 0.03$ and the dimensionless parameter $\lambda = 0.91^{+0.26}_{-0.18}$ at the 1σ confidence level [20]. In the next section, we shall focus ourselves on constraining the holographic dark energy model with the Sandage-Loeb test.

III. THE SANDAGE-LOEB TEST AND ITS POTENTIAL CONSTRAINT ON THE HOLOGRAPHIC DARK ENERGY MODEL

It is useful to first derive the redshift variation underlying the Sandage-Loeb test. Consider a given source without peculiar velocity, which emitted its light at a time t_s, then the observed redshift at time t_0 is

$$z(t_0) = \frac{a(t_0)}{a(t_s)} - 1, \tag{12}$$

which becomes after a time interval δt_0

$$z(t_0 + \delta t_0) = \frac{a(t_0 + \delta t_0)}{a(t_s + \delta t_s)} - 1. \tag{13}$$

Then in the linear approximation, the observed redshift variation of the fixed source gives

$$\begin{aligned}\delta z(t_0) &\equiv z(t_0 + \delta t_0) - z(t_0) = \frac{a(t_0 + \delta t_0)}{a(t_s + \delta t_s)} - \frac{a(t_0)}{a(t_s)} \\ &\approx \frac{a(t_0)[1 + H(t_0)\delta t_0]}{a(t_s)[1 + H(t_s)\delta t_s]} - \frac{a(t_0)}{a(t_s)} \\ &\approx \frac{a(t_0)}{a(t_s)}[1 + H(t_0)\delta t_0 - H(t_s)\delta t_s] - \frac{a(t_0)}{a(t_s)} \\ &\approx H(t_0)\delta t_0\left[1 + z(t_0) - \frac{H(t_s)}{H(t_0)}\right],\end{aligned} \tag{14}$$

where $\delta t_1 \approx [1 + z(t_1)]\delta t_s$ has been used in the last step. This redshift variation can also be expressed in terms of a spectroscopic velocity shift, i.e.,

$$\delta v = \frac{\delta z}{1 + z} = H_0 \delta t_0\left[1 - \frac{E(z)}{1 + z}\right], \tag{15}$$

where $E(z) = \frac{H(z)}{H_0}$. Note that the redshift variation is directly related to the expansion rate of our universe, which is the most essential part of any model. For example, in the holographic dark energy model considered here, by Eq. (6) the expansion rate reads

$$E(z) = \sqrt{\frac{\Omega_m^0(1 + z)^3}{1 - \Omega_H}}, \tag{16}$$

which is obviously different from the ΛCDM model with

$$E(z) = \sqrt{\Omega_m^0(1 + z)^3 + 1 - \Omega_m^0}. \tag{17}$$

Therefore different from those classical cosmological probes, which are almost exclusively sensitive to the cosmological parameters through a time integral of Hubble parameter, the measurement of velocity shift plays a critical role in investigating the physical mechanism responsible for the acceleration and discriminating various dark energy models.

This kind of astronomical observation as a possible cosmological tool, referred to as the Sandage-Loeb test, was first put forward by Sandage [21], and revisited by Loeb more recently [22]. With the foreseen development of very large telescopes, and the availability of spectrographs of unprecedented resolution, the quasar absorption lines typical of the Lyman-α forest provide a powerful tool to measure the velocity shift within the redshift desert [11,22]. Especially, invoking Monte Carlo simulations, Pasquini *et al.* estimated the statistical error on δv as measured by the cosmic dynamics experiment (CODEX) spectrograph over a period of 10 years as

$$\sigma_{\delta v} = 1.4\left(\frac{2350}{s/n}\right)\sqrt{\frac{30}{N}}\left(\frac{5}{1 + z}\right)^{1.8}\frac{\text{cm}}{s}, \tag{18}$$

where s/n denotes the signal to noise ratio per 0.0125A pixel, and N is the number of Lyman-α quasars [23]. In what follows, we assume that the future experimental configuration and uncertainties are similar to those expected from CODEX. Namely, the error bars are estimated from Eq. (18), with the assumption that a total of 240 quasars can be observed uniformly distributed in 6 equally spaced redshift bins within the redshift desert, with $s/n = 3000$ [11,12].

Now let us start to explore the behavior of redshift velocity in the holographic dark energy model. To proceed, we first plug Eq. (16) into Eq. (15), and then perform a numerical calculation for different values of Ω_m^0 and λ. The results obtained are plotted in Figs. 1 and 2. We also plot the redshift velocity curve for the ΛCDM model with $\Omega_m^0 = 0.27$ with the error bars from Eq. (18) in the figures, which is convenient for us to compare the holographic dark energy model with the fiducial concordance cosmological model.

As shown in Fig. 1, for fixed λ, the differences of redshift velocity among different values of Ω_m^0 become bigger and bigger with the increase of the source redshift, which is also explicitly supported by Fig. 3. On the contrary, the error bars from Eq. (18) are a decreasing function of the redshift. Thus, it is advantageous to employ the Sandage-Loeb test to distinguish the holographic dark energy models among different values of Ω_m^0 within the redshift desert. On the other hand, Fig. 2 demonstrates that the differences of redshift velocity decrease with the source redshift for fixed Ω_m^0 but different values of λ; furthermore the magnitude of differences is also comparable to that of error bars. It means that the Sandage-Loeb test is not very sensitive to the dimensionless parameter λ. That is, it is difficult to discriminate holographic dark energy models with different λs by the Sandage-Loeb test within the redshift desert.

In addition, if we assume the prediction of the fiducial ΛCDM model with the error bars from Eq. (18) represents the future practical measurement result of the redshift velocity within the redshift desert, Figs. 1 and 2 show that the Sandage-Loeb test seems to favor small Ω_m^0s, such as $(\Omega_m^0 = 0.25, \lambda = 0.9, 1.2, 1.5)$ and $(\Omega_m^0 = 0.27, \lambda = 0.3, 0.6, 0.9)$. In order to check this naive observation, next we would like to perform χ^2 statistics for the model parameters (Ω_m^0, λ). With the assumption considered above, we have

$$\chi^2_{SL} = \sum_{i=1}^{240}\frac{[\delta v_H(z_i) - \delta v_L(z_i)]^2}{\sigma^2_{\delta v}(z_i)}. \tag{19}$$

Here $\delta v_H(z_i)$ and $\delta v_L(z_i)$ represent the prediction value from the holographic dark energy model and the fiducial concordance model, respectively. In addition, $\sigma_{\delta v}(z_i)$ is

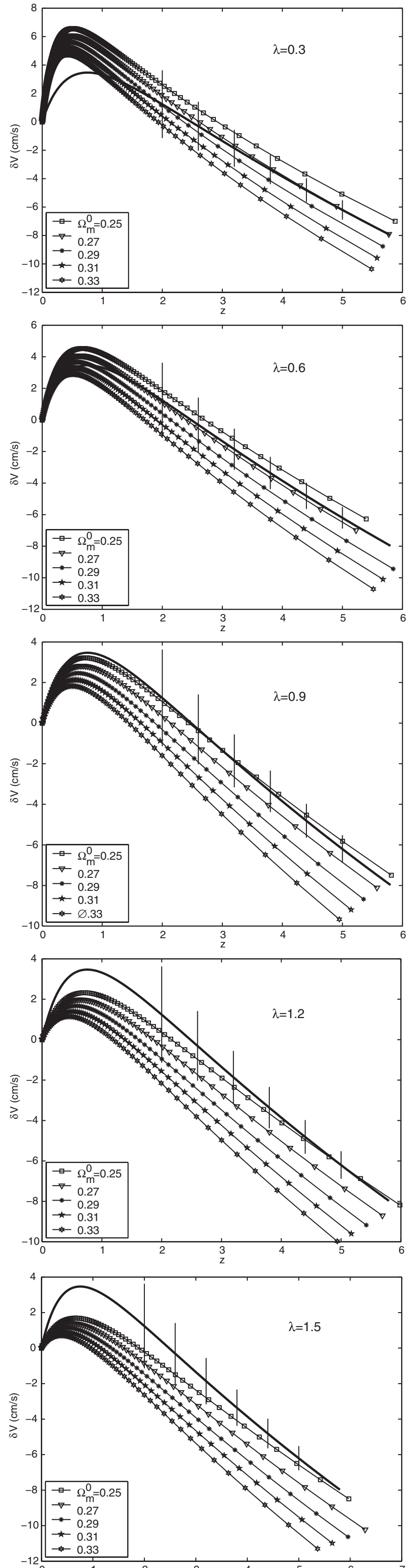

FIG. 1. The redshift velocity as a function of the source redshift for fixed λ and different values of Ω_m^0.

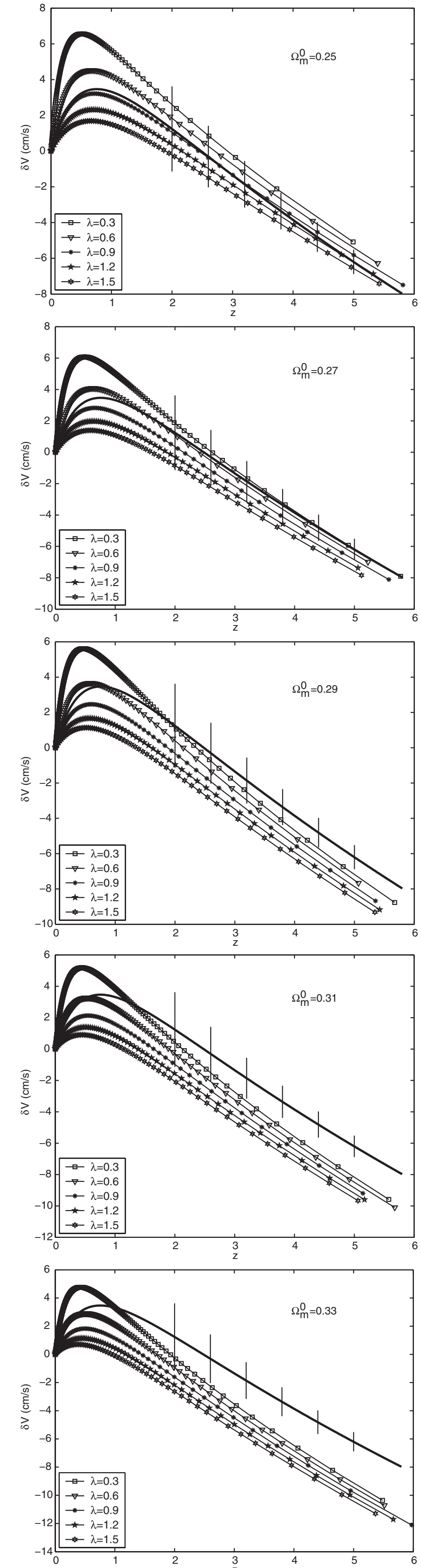

FIG. 2. The redshift velocity as a function of the source redshift for fixed Ω_m^0 and different values of λ.

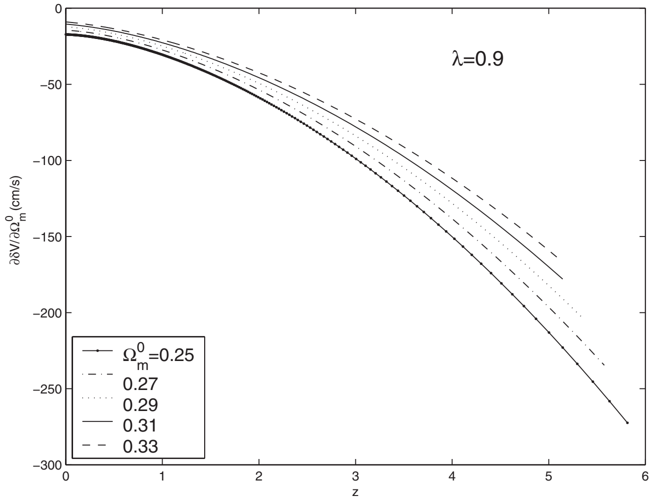

FIG. 3. The derivative of redshift velocity with respect to Ω_m^0 as a function of the source redshift for fixed $\lambda = 0.9$ and different values of Ω_m^0.

estimated from Eq. (18). Accordingly, numerical computation gives the contour diagrams as Fig. 4. Especially, the 1σ fit value for the model parameters: $\Omega_m^0 = 0.264^{+0.007}_{-0.006}$, and $\lambda = 0.611^{+0.215}_{-0.233}$ with $\chi^2_{\rm min} = 0.086$, which also confirms the aforementioned observation that the Sandage-Loeb test is very sensitive to Ω_m^0, while the constraint on λ is weaker.

Last but definitely not least, if we do not restrict ourselves within the redshift desert, it is noteworthy that there is something interesting appearing in Figs. 1 and 2: In either the ΛCDM model or the holographic dark energy model, there always exists some low redshift $z_{\rm max}$ at which the redshift velocity reaches its maximum [24]. Obviously $z_{\rm max}$ can be obtained by requiring the usual conditions satisfied, i.e., $\frac{d\delta v}{dz}|_{z_{\rm max}} = 0$. We have then from Eq. (15)

$$\frac{dE}{dz}\bigg|_{z_{\rm max}} = \frac{E}{1+z}\bigg|_{z_{\rm max}}. \tag{20}$$

Hereby we find

$$z_{\rm max} = \sqrt[3]{\frac{2(1-\Omega_m^0)}{\Omega_m^0}} - 1 \tag{21}$$

for the ΛCDM model. Similarly, an implicit but analytic formula can be obtained for $z_{\rm max}$ in the holographic dark energy model, i.e.,

$$\Omega_H\left(1 + \frac{2}{\lambda}\sqrt{\Omega_H}\right)\bigg|_{z_{\rm max}} = 1. \tag{22}$$

Note that $z_{\rm max}$ is related to the dynamic cosmological parameter(s) in such a direct way, but independent of H_0. For example, if the fiducial concordance model is really correct, we should find observationally $z_{\rm max} = 0.755$ by taking $\Omega_m^0 = 0.27$ in Eq. (21). It is thus suggested that the measurement of $z_{\rm max}$ may provide another strong potential test of various cosmological models, at least from the theoretical perspective. Different from the traditional Sandage-Loeb test, where precise measurement of amplitude of redshift velocity is needed within the redshift desert, this new possible test need only to determine $z_{\rm max}$ by discerning a narrow low redshift region of $z \leq 1$ where the peak of redshift velocity occurs, regardless of the specific value of amplitude of redshift velocity, including the magnitude of peak. For the holographic dark energy model considered here, if the value of $z_{\rm max}$ can be measured precisely, it is obvious that we can employ Eq. (22) to provide a stronger constraint on the dimensionless parameter λ, in combination with the traditional Sandage-Loeb test.

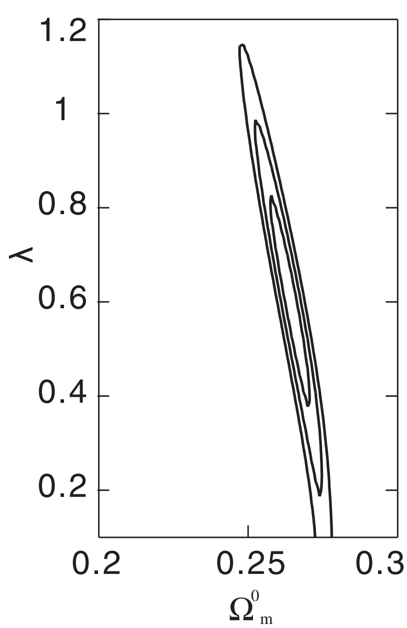

FIG. 4. The Sandage-Loeb test contours for 1σ, 2σ, and 3σ, respectively.

IV. CONCLUDING REMARKS

We have explored the holographic dark energy model with the Sandage-Loeb test. The obtained result shows that the Sandage-Loeb test from the redshift desert can impose a strong bound on Ω_m^0, while its constraint on λ is weaker. Especially, if we fit the holographic dark energy model to the fiducial ΛCDM model, which is assumed to provide a prediction of future measurement value with the error estimated from Monte Carlo stimulations, we find $\Omega_m^0 = 0.264^{+0.007}_{-0.006}$, and $\lambda = 0.611^{+0.215}_{-0.233}$ with $\chi^2_{\rm min} = 0.086$ at the 1σ accuracy level. In addition, we notice an interesting and significant behavior for the redshift velocity function, i.e., the peak of redshift velocity seems to always occur at some low redshift, which may be employed to provide another strong potential test of various cosmological models. A more detailed analysis of this new suggested cosmological tool, such as its prospects of observational availability and feasible constraints on various cosmological models, is worthy of further investigation but beyond the scope of this paper. We expect to report it elsewhere.

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