

## Supersymmetric QCD corrections to dark matter annihilation in the Higgs funnel region

Björn Herrmann and Michael Klasen\*

*Laboratoire de Physique Subatomique et de Cosmologie, Université Joseph Fourier/CNRS-IN2P3/INPG,  
53 Avenue des Martyrs, F-38026 Grenoble, France*

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We compute the full  $\mathcal{O}(\alpha_s)$  SUSY-QCD corrections to dark matter annihilation in the Higgs-funnel, resumming potentially large  $\mu \tan\beta$  and  $A_b$  contributions and keeping all finite  $\mathcal{O}(m_b, s, 1/\tan^2\beta)$  terms. We demonstrate numerically that these corrections strongly influence the extraction of SUSY mass parameters from cosmological data and must therefore be included in common analysis tools such as DARKSUSY or MICROMEAS.

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## I. INTRODUCTION

Thanks to the recent mission of the Wilkinson microwave anisotropy probe (WMAP) satellite and other cosmological observations, the matter and energy decomposition of our Universe is known today with unprecedented precision [1]. Direct evidence for the existence of cold dark matter (CDM) is accumulating [2,3], and its relic density  $\Omega_{\text{CDM}}$  can now be constrained to the rather narrow range [4]

$$0.094 < \Omega_{\text{CDM}} h^2 < 0.136 \quad (1)$$

at 95% ( $2\sigma$ ) confidence level. Here,  $h$  denotes the present Hubble expansion rate  $H_0$  in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

Although the nature of cold dark matter still remains unknown, it is likely to be composed of weakly interacting massive particles (WIMPs), as proposed by various extensions of the standard model (SM) of particle physics. In supersymmetry (SUSY), a natural candidate is the lightest supersymmetric particle (LSP), which is stable, if  $R$ -parity is conserved. It is usually the lightest of the four neutralinos, denoted  $\tilde{\chi}_1^0$  or shortly  $\chi$ .

The minimal supersymmetric standard model (MSSM) depends *a priori* on 124 soft SUSY-breaking parameters, which are often restricted to five universal parameters that are imposed at the unification scale and can be constrained using data from high-energy colliders. As the lightest neutralino relic density depends also on these parameters, its computation is another powerful tool to put constraints on the parameter space and provide complementary information, in particular, at high energies or masses that would otherwise not be accessible at colliders.

To evaluate the number density  $n$  of the relic particle with velocity  $v$ , one has to solve the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2) \quad (2)$$

with the Hubble rate  $H$  and the thermal equilibrium density  $n_{\text{eq}}$ . The present number density  $n_0$  is directly related to the relic density  $\Omega_{\text{CDM}} h^2 = m_\chi n_0 / \rho_c \propto \langle \sigma_{\text{eff}} v \rangle^{-1}$ , where  $m_\chi$

is the LSP mass,  $\rho_c = 3H_0^2 / (8\pi G_N)$  is the critical density of our Universe, and  $G_N$  is the gravitational constant [5]. The effective cross section  $\sigma_{\text{eff}}$  involves all annihilation and coannihilation processes of the relic particle  $\chi$  into SM particles, and  $\langle \sigma_{\text{eff}} v \rangle$  in Eq. (2) signifies the thermal average of its nonrelativistic expansion ( $v \ll c$ ).

The most important processes contributing to  $\sigma_{\text{eff}}$  are those that have two-particle final states and that occur at the tree level [6]. Possible final states include those with fermion-antifermion pairs as well as with combinations of gauge ( $W^\pm, Z^0$ ) and Higgs ( $h^0, H^0, A^0, H^\pm$ ) bosons, depending on the region of the parameter space. Processes producing fermions or antifermions may be detectable either directly or through their annihilation into photons. In addition, these channels are always open (for  $b$ -quarks if  $m_\chi \geq 4.5 \text{ GeV}$ ) in contrast to the other channels, which may be suppressed or even closed [5,6].

Several public codes perform a calculation of the dark matter relic density within supersymmetric models. The most developed and most popular ones are DARKSUSY [7] and MICROMEAS [8]. All relevant processes are implemented in these codes, but for most of them no (or at least not the full) higher order corrections are included. However, due to the large magnitude of the strong coupling constant, QCD and SUSY-QCD corrections are bound to affect the annihilation cross section in a significant way. They may even be enhanced logarithmically by kinematics or in certain regions of the parameter space.

In this Brief Report, we compute these corrections for neutralino-pair annihilation into a bottom quark-antiquark pair through the  $s$ -channel exchange of a pseudoscalar Higgs-boson  $A^0$ . This process dominates in the so-called  $A$ -funnel region of minimal supergravity (mSUGRA) parameter space at large  $\tan\beta$ , which is theoretically favored by the unification of Yukawa couplings in grand unified theories (GUTs) [9]. Supposing a WIMP mass of 50–70 GeV, this process has also been claimed to be compatible with the gamma-ray excess observed in all sky directions by the EGRET satellite [10]. However, the corresponding scenarios may lead to antiproton overproduction, so that they would not be compatible with the observed antiproton flux [11].

\*klasen@lpsc.in2p3.fr

## II. ANALYTICAL RESULTS

Denoting the neutralino and  $b$ -quark velocities by

$$\beta_\chi = \frac{v}{2} = \sqrt{1 - \frac{4m_\chi^2}{s}} \quad \text{and} \quad \beta_b = \sqrt{1 - \frac{4m_b^2}{s}} \quad (3)$$

and the squared total center-of-mass energy by  $s$ , the properly antisymmetrized neutralino annihilation cross section can be written at leading order (LO) of perturbation theory as

$$\sigma_{\text{LO}} v = \frac{1}{2} \frac{\beta_b}{8\pi s} \frac{N_C g^2 T_{A11}^2 h_{Abb}^2 s^2}{|s - m_A^2 + im_A \Gamma_A|^2}. \quad (4)$$

It is proportional to the inverse of the flux factor  $sv$ , the integrated two-particle phase space  $s\beta_b/(8\pi s)$ , the number of quark colors  $N_C = 3$  and the squares of the weak coupling constant  $g$ , a neutralino mixing factor

$$T_{Aij} = \frac{1}{2}(N_{2j} - \tan\theta_W N_{1j})(N_{4i} \cos\beta - N_{3i} \sin\beta) + (i \leftrightarrow j), \quad (5)$$

the bottom-quark mass  $m_b$  through the Yukawa coupling  $h_{Abb} = -gm_b \tan\beta/(2m_W)$  and the Higgs-boson propagator. Expanding in powers of  $v^2$ , we obtain  $\sigma_{\text{LO}} v \doteq a_{\text{LO}} + b_{\text{LO}} v^2 + \mathcal{O}(v^4)$  with

$$a_{\text{LO}} = 2b_{\text{LO}} = \frac{N_C g^2 T_{A11}^2 h_{Abb}^2}{4\pi m_\chi^2 |4 - \frac{m_A^2}{m_\chi^2} + \frac{im_A \Gamma_A}{m_\chi^2}|^2} \sqrt{1 - \frac{m_b^2}{m_\chi^2}} \quad (6)$$

in agreement with Ref. [6].

Using standard methods for the virtual one-loop and real emission contributions shown on the left-hand side of Fig. 1, we compute the  $\mathcal{O}(\alpha_s)$  QCD correction in the *on-shell scheme*

$$\begin{aligned} \Delta_{\text{QCD}}^{(1)} = & \left(\frac{\alpha_s(s)}{\pi}\right) C_F \left[ \frac{1 + \beta_b^2}{\beta_b} \left( 4\text{Li}_2 \frac{1 - \beta_b}{1 + \beta_b} + 2\text{Li}_2 \frac{\beta_b - 1}{1 + \beta_b} \right. \right. \\ & - 3 \log \frac{2}{1 + \beta_b} \log \frac{1 + \beta_b}{1 - \beta_b} - 2 \log \beta_b \log \frac{1 + \beta_b}{1 - \beta_b} \left. \right) \\ & - 3 \log \frac{4}{1 - \beta_b^2} - 4 \log \beta_b + \frac{3}{8} (7 - \beta_b^2) \\ & \left. + \frac{1}{16\beta_b} (19 + 2\beta_b^2 + 3\beta_b^4) \log \frac{1 + \beta_b}{1 - \beta_b} \right], \quad (7) \end{aligned}$$

which agrees with the known result for pseudoscalar Higgs-boson decays [12] and contributes to the total correction

$$\sigma = \sigma_{\text{LO}} [1 + \Delta_{\text{QCD}} + \Delta_{\text{top}} + \Delta_{\text{SUSY}}] \quad (8)$$

and equivalently for  $\Gamma_A$ . Here,  $\Delta_{\text{QCD}} = (\frac{\alpha_s}{\pi}) \Delta_{\text{QCD}}^{(1)} + (\frac{\alpha_s}{\pi})^2 \Delta_{\text{QCD}}^{(2)} + (\frac{\alpha_s}{\pi})^3 \Delta_{\text{QCD}}^{(3)} + \dots$

In the limit  $m_b^2 \ll s$  ( $\beta_b \rightarrow 1$ ), the correction in Eq. (7) develops a logarithmic mass singularity

$$\Delta_{\text{QCD}}^{(1)} \simeq \left(\frac{\alpha_s(s)}{\pi}\right) C_F \left[ -\frac{3}{2} \log \frac{s}{m_b^2} + \frac{9}{4} \right], \quad (9)$$

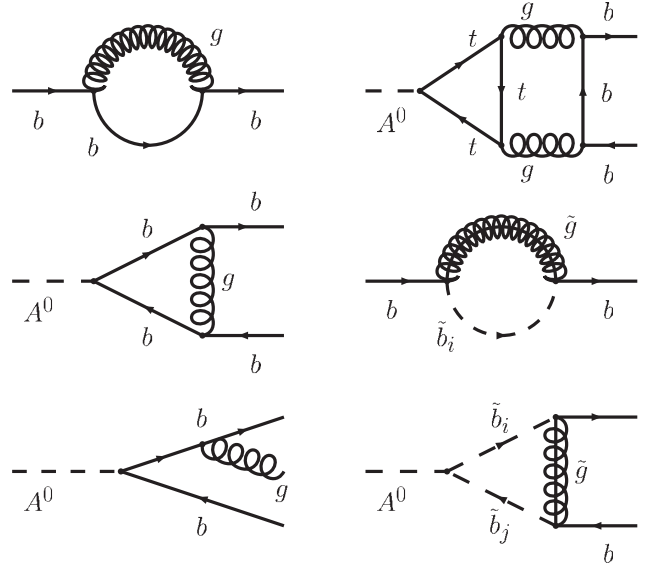


FIG. 1. Left: Diagrams for self-energy (top), vertex (center), and real (bottom) QCD corrections. Right: Diagrams for two-loop top-quark (top) and one-loop self-energy (center) and vertex (bottom) SUSY-QCD corrections to the process  $\chi\chi \rightarrow A^0 \rightarrow b\bar{b}$ .

which can be resummed to all orders using the renormalization group, i.e. by replacing  $m_b$  with the running mass  $\bar{m}_b(s)$  in the Yukawa coupling  $h_{Abb}$  [13]. The remaining finite QCD-corrections in the  $\overline{MS}$ -scheme are known up to  $\mathcal{O}(\alpha_s^3)$  [14]

$$\begin{aligned} \Delta_{\text{QCD}} = & \left(\frac{\alpha_s(s)}{\pi}\right) C_F \frac{17}{4} + \left(\frac{\alpha_s(s)}{\pi}\right)^2 (35.94 - 1.36n_f) \\ & + \left(\frac{\alpha_s(s)}{\pi}\right)^3 (164.14 - 25.76n_f + 0.259n_f^2). \quad (10) \end{aligned}$$

A separately gauge-independent  $\mathcal{O}(\alpha_s^2)$  correction is induced by the top-quark loop diagram shown on the upper right-hand side of Fig. 1. Its contribution [15]

$$\Delta_{\text{top}} = \frac{1}{\tan^2\beta} \left(\frac{\alpha_s(s)}{\pi}\right)^2 \left[ \frac{23}{6} - \log \frac{s}{m_t^2} + \frac{1}{6} \log^2 \frac{\bar{m}_b^2(s)}{s} \right] \quad (11)$$

can be important at small values of  $\tan\beta$ , but is largely suppressed in the Higgs-funnel region considered here.

In SUSY, additional  $\mathcal{O}(\alpha_s)$  corrections arise through the sbottom-gluino exchanges shown in the central and lower right-hand side diagrams of Fig. 1. The self-energy diagram leads to the mass renormalization [9]

$$\Delta m_b = \left(\frac{\alpha_s(s)}{\pi}\right) C_F \frac{m_{\tilde{g}}}{2} (A_b - \mu \tan\beta) I(m_{b_1}^2, m_{b_2}^2, m_{\tilde{g}}^2), \quad (12)$$

which is for  $m_b \ll m_{\text{SUSY}}$  proportional to

$$\sin 2\theta_b = \frac{2m_b(A_b - \mu \tan\beta)}{m_{b_1}^2 - m_{b_2}^2}, \quad (13)$$

i.e. the off-diagonal component of the sbottom mass matrix, and the 3-point function at zero external momentum

$$I(a, b, c) = \frac{ab \log \frac{a}{b} + bc \log \frac{b}{c} + ca \log \frac{c}{a}}{(a-b)(b-c)(c-a)}. \quad (14)$$

In this low-energy (LE) limit, and neglecting  $A_b$  with respect to the  $\tan\beta$ -enhanced  $\mu$ , the vertex correction equals the mass renormalization [16] up to a factor  $1/\tan^2\beta$ , so that the total SUSY correction becomes

$$\Delta_{\text{SUSY}}^{(\text{LE})} = \left(\frac{\alpha_s(s)}{\pi}\right) C_F \left(1 + \frac{1}{\tan^2\beta}\right) m_{\bar{g}} \mu \tan\beta \times I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2). \quad (15)$$

It has long been known that for large  $\tan\beta$ ,  $\Delta m_b$  can be significant and must be resummed by replacing  $m_b \rightarrow m_b/(1 + \lim_{A_b \rightarrow 0} \Delta m_b)$  in the Yukawa coupling  $h_{Abb}$  [16]. More recently it has been observed that  $A_b$  may be of similar size as  $\mu \tan\beta$ , e.g. in no-mixing scenarios, so that its contribution must also be resummed by replacing  $\lim_{A_b \rightarrow 0} \Delta m_b \rightarrow (\lim_{A_b \rightarrow 0} \Delta m_b)/(1 + \lim_{\mu \tan\beta \rightarrow 0} \Delta m_b)$  [17]. Our result for the full SUSY-QCD correction  $\Delta_{\text{SUSY}}$  agrees with those in [18,19], and we implement the finite  $\mathcal{O}(m_b, s, 1/\tan^2\beta)$  remainder as described in [17].

### III. NUMERICAL EVALUATION

For our numerical study of the impact of QCD, top-quark loop, and SUSY-QCD corrections on dark matter annihilation in the Higgs-funnel, we place ourselves in a minimal supergravity (mSUGRA) scenario with  $A_0 = 0$  and large  $\tan\beta = 44.5$  (54) for  $\mu < 0$  ( $\mu > 0$ ), which still allows for electroweak symmetry breaking (EWSB) in a large region of the scanned  $m_{1/2} - m_0$  plane. The weak-scale MSSM parameters are then determined with SPHENO [20], which includes resummed  $\lim_{A_b \rightarrow 0} \Delta m_b$  corrections, and the physical Higgs and SUSY masses with FEYNHIGGS [21] after imposing the current SM masses (in particular  $\bar{m}_b(m_b) = 4.2$  GeV and  $m_t = 174.2$  GeV), gauge couplings ( $\alpha$  and  $\sin^2\theta_W$  in the improved Born approximation,  $\alpha_s(M_Z) = 0.1176$ ), and direct and indirect SUSY mass limits [22]. For the impact of SUSY spectrum calculations on dark matter annihilation see [23].

Comparing the observed CDM relic density in Eq. (1) to the one calculated with DARKSUSY [7], which includes the QCD corrections up to  $\mathcal{O}(\alpha_s^2)$  and where we have added the  $\mathcal{O}(\alpha_s^3)$  QCD and  $\mathcal{O}(\alpha_s)$  SUSY-QCD corrections described above, we determine the allowed regions in the  $m_{1/2} - m_0$  plane shown in Fig. 2.

The Higgs-funnel contribution to  $\sigma_{\text{eff}}$  rises from 40% for low values of  $m_{1/2}$  (or  $m_0$  for  $\mu < 0$ ) to more than 95%, when  $m_{1/2}$  (and  $m_0$ ) is (are) large. For  $m_0 = 1200$  GeV (dashed line), the condition  $(m_A - 2m_\chi)/\Gamma_A = 0$  is, e.g., satisfied when  $m_{1/2} = 840$  (970) GeV for  $\mu < 0$  ( $\mu > 0$ ), where  $m_\chi \simeq 360$  (420) GeV. It is obvious from Fig. 2 that the LO allowed regions (light) are dramatically changed by the  $\mathcal{O}(\alpha_s^2)$  (medium) and  $\mathcal{O}(\alpha_s^3)$  QCD and  $\mathcal{O}(\alpha_s)$  SUSY-QCD corrections (dark), which reduce  $\sigma_{\text{eff}}$  by more than a

factor of 2. The increase in  $\Omega_{\text{CDM}}$  must therefore be compensated by smaller masses. However,  $\Gamma_A$  is reduced by approximately the same amount, so that on the Higgs pole, where  $\Gamma_A$  is of particular importance, the effect is reversed. As expected, the effect is negligible in the focus point (very low  $m_{1/2}$ ) and coannihilation (very low  $m_0$ , see also [24]) regions.

In Fig. 3 we plot the relic density for  $m_0 = 1200$  GeV as a function of  $m_{1/2}$ . Since  $\Gamma_A$  increases with  $m_{1/2}$ , in particular for  $\mu > 0$ ,  $\sigma_{\text{eff}}$  ( $\Omega_{\text{CDM}}$ ) reaches its maximum (minimum) at some distance from the pole (vertical dashed line). Further away, the (SUSY-)QCD corrections decrease (increase)  $\sigma_{\text{eff}}$  ( $\Omega_{\text{CDM}}$ ) as expected. Closer to the pole, the reduced width becomes important, so that the maximum (minimum) approaches the pole. The effect of the  $\mathcal{O}(\alpha_s^2)$  QCD corrections already included in DARKSUSY and MICROMEAS [25,26] is considerably enhanced by our

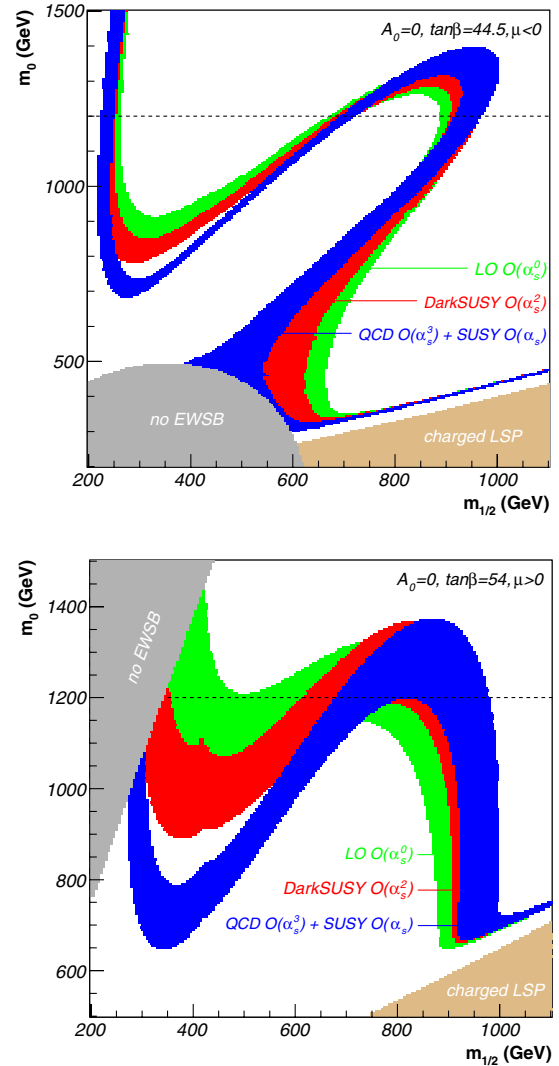


FIG. 2 (color online). Regions in the mSUGRA  $m_{1/2} - m_0$  plane forbidden by a charged LSP/no EWSB and favored by the observed  $\Omega_{\text{CDM}} h^2$  for large  $\tan\beta = 44.5(54)$  and  $\mu < 0$  ( $\mu > 0$ ).

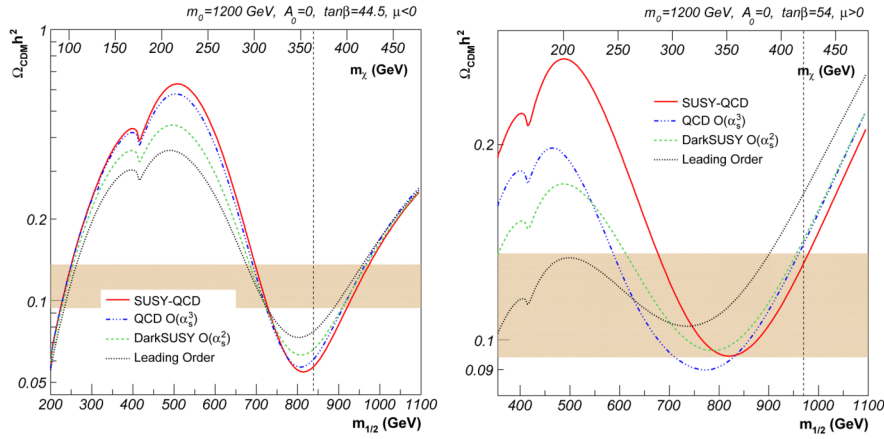


FIG. 3 (color online). LO (dotted line),  $\mathcal{O}(\alpha_s^2)$  (dashed line), and  $\mathcal{O}(\alpha_s^3)$  (dot-dashed line) QCD and  $\mathcal{O}(\alpha_s)$  SUSY-QCD (full) corrected predictions for  $\Omega_{\text{CDM}} h^2$  compared to the experimentally allowed range as a function of  $m_{1/2}$ . The position of the Higgs pole is indicated by a vertical dashed line.

$\mathcal{O}(\alpha_s^3)$  QCD and SUSY-QCD corrections, while the top-quark loop contributes less than 0.01% for the large values of  $\tan\beta$  under consideration here. Note that the dip at  $m_{1/2} = 420$  GeV occurs when  $m_\chi = m_t$ , where the  $t\bar{t}$  annihilation channel is opened [27].

#### IV. CONCLUSION

In summary, we have computed the full  $\mathcal{O}(\alpha_s)$  SUSY-QCD corrections to dark matter annihilation in the Higgs-funnel, resumming potentially large  $\mu \tan\beta$  and  $A_b$  con-

tributions and keeping all finite  $\mathcal{O}(m_b, s, 1/\tan^2\beta)$  terms. We have demonstrated numerically that these corrections strongly influence the extraction of SUSY mass parameters from cosmological data and must therefore be included in common analysis tools like DARKSUSY or MICROMEGAS.

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