PHYSICAL REVIEW D **76,** 116004 (2007)

Excited $(70, L^+)$ baryons in a relativistic quark model

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(Received 23 June 2007; revised manuscript received 11 September 2007; published 17 December 2007)

The masses of positive parity $(70, 0^+)$ and $(70, 2^+)$ nonstrange and strange baryons are calculated in the relativistic quark model. The relativistic three-quark equations of the $(70, L^+)$ multiplets are found in the framework of the dispersion relation technique. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. The calculated mass values of the $(70, L^+)$ multiplets are in good agreement with the experimental ones.

DOI: 10.1103/PhysRevD.76.116004 PACS numbers: 11.55.Fv, 12.39.Ki, 12.40.Yx, 14.20.−c

I. INTRODUCTION

Hadron spectroscopy has always played an important role in the revealing mechanisms underlying the dynamic of strong interactions.

Recently the $(70, L^+)$ baryon multiplet has been analyzed in the $1/N_c$ expansion. At low energies, typical for baryon spectroscopy, QCD does not admit a perturbative expansion in the strong coupling constant. In 1974 't Hooft [1] suggested a perturbative expansion of QCD in terms of the parameter $1/N_c$ where N_c is the number of colors. This suggestion together with the power counting rules of Witten [2] has lead to the $1/N_c$ expansion method which allows us to systematically analyze baryon properties. The success of the method stems from the discovery that the ground state baryons have an exact contracted $SU(2N_f)$ symmetry when $N_c \to \infty$ [3,4], N_f being the number of flavors. For $N_c \to \infty$ the baryon masses are degenerated. For large N_c the mass splitting starts at order $1/N_c$. Operator reduction rules simplify the $1/N_c$ expansion [5,6].

A considerable number of works have been devoted to the ground state baryons, described by the symmetric representation 56 of $SU(6)$ [7–11]. The excited baryons belonging to $(56, L)$ multiplets can be studied by analogy with the ground state. In this case both the orbital and the spin-flavor parts of the wave functions are symmetric. Explicit forms for such wave functions were given, for example, in Ref [12]. Together with the color part, they generate antisymmetric wave functions.

The states belonging to $(70, L)$ multiplets are apparently more difficult. In this case the general practice was to split the baryon into excited quark and a symmetric core, the latter being either in the ground state for the $N = 1$ or in an excited state for the $N \geq 2$ bands. Recently Matagne and Stancu have suggested the new approach [13] for the excited $(70, 1^-)$ multiplet. They solved the problem by removing the splitting of generators and using orbital-flavor-spin wave functions. The excited baryons are considered as bound states.

Details about the application of the $1/N_c$ expansion method to mixed symmetric states can be found in Refs. [14–16].

In the series of papers [17–21] a practical treatment of relativistic three-hadron systems has been developed. The physics of the three-hadron system is usefully described in terms of the pairwise interactions among the three particles. The theory is based on the two principles of unitarity and analyticity, as applied to the two-body subenergy channels. The linear integral equations in a single variable are obtained for the isobar amplitudes. Instead of the quadrature methods of obtaining solution the set of suitable functions is identified and used as a basis set for the expansion of the desired solutions. By this means the couple integral equations are solved in terms of simple algebra.

In our papers [22,23] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting particles. The mass spectrum of S-wave baryons including u, d, s quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such an approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions of the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In our paper [24] the construction of the orbital-flavor-spin wave functions for the $(70, 1^-)$ multiplet are given. We deal with a three-quark system having one unit of orbital excitation. The orbital part of wave function must have a mixed symmetry. The spin-flavor part of the wave function must have the same symmetry in order to obtain a totally symmetric state in the orbital-flavor-spin space. The integral equations using the orbital-flavor-spin wave func-

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tions was constructed. It allows us to calculate the mass spectra for all baryons of $(70, 1^-)$ multiplet. We take into account the u, d, s quarks. We have represented the 30 nonstrange and strange resonances belonging to the $(70, 1^-)$ multiplet. The 15 resonances are in good agreement with experimental data [25]. We have predicted 15 masses of baryons. In our model the four parameters are used: gluon coupling constants g_+ and g_- for the various parity, cutoff energy parameters λ, λ_s for the nonstrange and strange diquarks.

The paper is organized as follows. After this introduction, we discuss the construction of the orbital-flavor-spin wave functions for the $(70, 0^+)$ and $(70, 2^+)$ multiplets.

In Sec. III the relativistic three-quark equations are obtained in the form of the dispersion relation over the two-body subenergy.

In Sec. IV the systems of equations for the reduced amplitudes are derived.

Section V is devoted to the calculation results for the mass spectrum of the $(70, 0^+)$ and $(70, 2^+)$ multiplets (Tables I, II, III, IV, V, VI, VII, VIII, IX, X, XI, and XII).

In the conclusion, the status of the considered model is discussed.

In Appendix A the wave functions of $(70, 0^+)$ and $(70, 2^+)$ baryon resonances are given.

In Appendix B the reduced equations for the $(70, 0^+)$ and $(70, 2^+)$ multiplets are obtained.

II. THE WAVE FUNCTION OF $(70, 0^+)$ AND $(70, 2^+)$ EXCITED STATES

The multiplet $(70, 2^+)$ consists of the excited baryon resonances with the orbital angular momentum $L = 2$ and positive parity. According to the nonrelativistic approach [15], the $(70, 2^+)$ multiplet states include the two quarks on the $1s$ levels and one quark on the $1d$ level (002) or the two quarks on the $1p$ levels and one quark on the $1s$ level (110). Then the baryons of multiplet $(70, 2^+)$ consist of the superposition of the 002 and 110 states. The transition of these states with the projection of orbital angular momentum $L_z = 2$ is considered.

The multiplet $(70, 2^+)$ of $SU(6)$ includes the decuplet $(10, 2)$ with the spin $S = \frac{1}{2}$, octet $(8, 2)$ with the spin $S = \frac{1}{2}$, octet $(8, 4)$ with the spin $S = \frac{3}{2}$ and singlet $(1, 2)$ with the spin $S = \frac{1}{2}$. Taking into account the orbital angular momentum and spin $\vec{J} = \vec{L} + \vec{S}$, we obtain total angular momentum for the $S = \frac{1}{2}$ $J = \frac{3}{2}, \frac{5}{2}$ and for the $S = \frac{3}{2}$ $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$. We can represent the total multiplet $(70, 2^+)$:

$$(10, 2)\colon \frac{3^+}{2}, \frac{5^+}{2} \qquad (8, 2)\colon \frac{3^+}{2}, \frac{5^+}{2}$$
$$(8, 4)\colon \frac{1^+}{2}, \frac{3^+}{2}, \frac{5^+}{2}, \frac{7^+}{2} \qquad (1, 2)\colon \frac{3^+}{2}, \frac{5^+}{2}.$$

The $(70, 2^+)$ multiplet includes the 34 baryons with different masses.

The $(70, 0^+)$ multiplet includes the excited baryon resonances with the orbital angular momentum $L = 0$ and the positive parity. The states of this multiplet consist of the two quarks on the $1s$ levels and one radial excited quark on the level $2s$, or the two quarks on the $1p$ levels with the projection of orbital angular momentum $L_z = 0$ and one quark on the $1s$ level. We consider the spin $S = \frac{1}{2}$ and $J = \frac{1}{2}$, and $S = \frac{3}{2}$, $J = \frac{3}{2}$. We can represent the total multiplet $(70, 0^+)$:

$$(10, 2)\colon \frac{1^+}{2} \qquad (8, 2)\colon \frac{1^+}{2} \qquad (8, 4)\colon \frac{3^+}{2} \qquad (1, 2)\colon \frac{1^+}{2}.$$

The $(70, 0^+)$ multiplet includes the 13 baryons with different masses.

The three-quark wave function of the excited baryon possesses the symmetry $SU(6) \times O(3) \times SU(3)_c$, where the $SU(3)_c$ group determines the color symmetry, therefore the total wave function is antisymmetric. The part of wave function $SU(6) \times O(3)$ must be total symmetric.

The $O(3)$ wave functions with the mixed symmetry allow us to construct two states with the mixed symmetry and the positive parity. Then we use these states and two mixed multiplets 70 and 70′ of group $SU(6)$. We can construct the total symmetric state of multiplet $(70, 2^+)$.

III. THE THREE-QUARK INTEGRAL EQUATIONS FOR THE $(70, 0^+)$ AND $(70, 2^+)$ MULTIPLETS

We calculate the masses of the baryon resonances belonging to $(70, L^+)$ multiplet in a relativistic approach using the dispersion relation technique. The relativistic three-quark integral equations are constructed in the form of the dispersion relations over the two-body subenergy.

By consideration of $(70, 0^+)$ and $(70, 2^+)$ baryons integral equations we need to use the projectors for the different diquark states. The projectors to the symmetric and antisymmetric states can be obtained as

$$\tfrac{1}{2}(q_1 q_2 + q_2 q_1), \qquad \tfrac{1}{2}(q_1 q_2 - q_2 q_1). \tag{1}$$

The spin projectors are the following:

$$\tfrac{1}{2}(\uparrow\downarrow + \downarrow\uparrow), \qquad \tfrac{1}{2}(\uparrow\downarrow - \downarrow\uparrow). \tag{2}$$

The orbital angular momentum projectors take into account the transition of diquarks $20 \leftrightarrow 11$ with the projection of orbital angular momentum $L_z = 2$.

$L_z = 2$:

$$200\colon A^{s0+}, A^{d2+}, \qquad 011\colon A^{p2+}, A^{p1-}. \tag{3}$$

$L_z = 1$:

$$1^*00\colon A^{s0+}, A^{d1+}, \qquad 00^*1\colon A^{p1+}, \tfrac{1}{2}(A^{p0-} + A^{p1-}). \tag{4}$$

$L_z = 0$:

$$0^*00\colon A^{s0+}, A^{d0+},$$
$$00^*0^*, 01(-1)^*\colon A^{p0+}, \tfrac{1}{4}(2 \cdot A^{p0-} + A^{p1-} + A^{p(-1)-}). \tag{5}$$

$L_z = -1$:

$$(-1)^*00\colon A^{s0+}, A^{d(-1)+}, \qquad 00^*(-1)^*\colon A^{p(-1)+}, \tfrac{1}{2}(A^{p0-} + A^{p(-1)-}). \tag{6}$$

The upper index corresponds the s, p, d-wave, orbital angular momentum and p parity. The excited quark is determined by (*). The A is the three-quark amplitude. The orbital angular momentum projectors can be considered as follows:

$$11\colon \frac{A^{\rm sym}}{4}\left(2\cdot 11 + \frac{1}{\sqrt{2}}\cdot(20+02)\right), \tag{7}$$

$$10\colon \frac{A^{\rm sym}}{2}(10+01) + \frac{A^{\rm asym}}{2}(10-01), \tag{8}$$

$$00\colon A^{\rm sym}\cdot 00, \tag{9}$$

$$20\colon \frac{A^{\rm sym}}{4}(20+02+2\cdot\sqrt{2}\cdot 11) + \frac{A^{\rm asym}}{2}(20-02), \tag{10}$$

here

$$A^{\rm sym} = \frac{A^{s0+} + A^{p2+}}{2}, \qquad A^{\rm asym} = \frac{A^{d2+} + A^{p1-}}{2}. \tag{11}$$

For example, the projector to the diquark $u^2\uparrow s\downarrow$ is the following:

$$\begin{aligned}&\frac{A_1^{\rm syms}}{16}(us+su)(\uparrow\downarrow+\downarrow\uparrow)(20+02+2\cdot\sqrt{2}\cdot 11)\\ &\quad+\frac{A_0^{\rm syms}}{16}(us-su)(\uparrow\downarrow-\downarrow\uparrow)(20+02+2\cdot\sqrt{2}\cdot 11)\\ &\quad+\frac{A_1^{\rm asyms}}{8}(us-su)(\uparrow\downarrow+\downarrow\uparrow)(20-02)\\ &\quad+\frac{A_0^{\rm asyms}}{8}(us+su)(\uparrow\downarrow-\downarrow\uparrow)(20-02).\end{aligned} \tag{12}$$

here the lower index of amplitude corresponds to the diquark spin (1 or 0), and the index s points out the strangeness of diquark.

For the sake of simplicity we derive the relativistic Faddeev equations using the Σ hyperon with $J^p = \frac{5}{2}^+$ of the (10, 2) multiplets. We use the graphic equations for the functions $A_J(s, s_{ik})$ [22,23]. In order to represent the amplitude $A_J(s, s_{ik})$ in the form of dispersion relations, it is necessary to define the amplitudes of quark-quark interaction $a_J(s_{ik})$. The pair quarks amplitudes $qq \to qq$ are calculated in the framework of the dispersion N/D method with the input four-fermion interaction with quantum numbers of the gluon [26]. We use results of our relativistic quark model [27] and write down the pair quark amplitudes in the following form:

$$a_J(s_{ik}) = \frac{G_J^2(s_{ik})}{1 - B_J(s_{ik})}, \tag{13}$$

$$B_J(s_{ik}) = \int_{(m_i+m_k)^2}^{\Lambda_J(i,k)} \frac{ds'_{ik}}{\pi}\frac{\rho_J(s'_{ik})G_J^2(s'_{ik})}{s'_{ik} - s_{ik}}, \tag{14}$$

$$\rho_J(s_{ik}) = \frac{(m_i+m_k)^2}{4\pi}\left(\alpha_J\frac{s_{ik}}{(m_i+m_k)^2} + \beta_J + \frac{\delta_J}{s_{ik}}\right) \times \frac{\sqrt{(s_{ik}-(m_i+m_k)^2)(s_{ik}-(m_i-m_k)^2)}}{s_{ik}}. \tag{15}$$

Here G_J is the diquark vertex function; $B_J(s_{ik})$, $\rho_J(s_{ik})$ are the Chew-Mandelstam function [28] and the phase space consequently. s_{ik} is the two-particle subenergy squared (i, $k = 1, 2, 3$), s is the systems total energy squared. $\Lambda_J(i, k)$ is the pair energy cutoff. The coefficient of Chew-Mandelstam function are given in Table XIII.

In the case in question the interacting quarks do not produce bound state, then the integration in dispersion integrals is carried out from $(m_i + m_k)^2$ to $\Lambda_J(i, k)$. All diagrams are classified over the last quark pair (Fig. 1).

We use the diquark projectors. Then we consider the particle $\Sigma\ \frac{5}{2}^+$ of the (10, 2) $(70, 2^+)$ multiplet again. This wave function contains the contribution to $u^2\downarrow u\uparrow s\uparrow$, which includes three diquarks: $u^2\downarrow u\uparrow$, $u^2\downarrow s\uparrow$, and $u\uparrow s\uparrow$. The diquark projectors allow us to obtain Eqs. (16)–(18) [with the definition $\rho_J(s_{ij}) \equiv k_{ij}$].

$$\begin{aligned}k_{12}\Bigg(&\frac{A_1^{s0+} + A_1^{p2+} + 2\cdot A_0^{d2+} + 2\cdot A_0^{p1-}}{16}\\ &\times(u^2\downarrow u\uparrow s\uparrow + u\uparrow u^2\downarrow s\uparrow)\\ &+\frac{A_1^{s0+} + A_1^{p2+} - 2\cdot A_0^{d2+} - 2\cdot A_0^{p1-}}{16}\\ &\times(u^2\uparrow u\downarrow s\uparrow + u\downarrow u^2\uparrow s\uparrow)\\ &+\frac{A_1^{s0+} + A_1^{p2+}}{8}\cdot\sqrt{2}\cdot(u^1\downarrow u^1\uparrow s\uparrow + u^1\uparrow u^1\downarrow s\uparrow)\Bigg),\end{aligned} \tag{16}$$

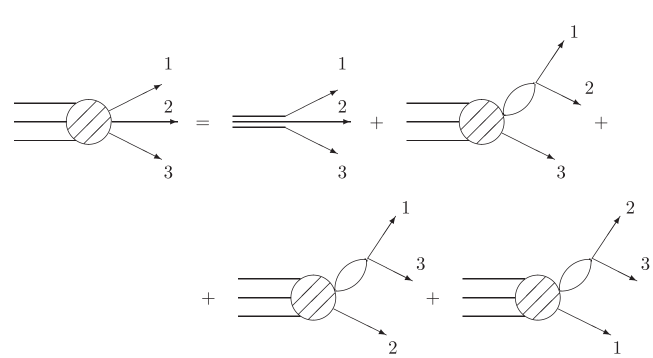

FIG. 1. The contribution of diagrams at the last pair of the interacting particles.

$$k_{13}\Big(\frac{A_1^{s0s+}+A_1^{p2s+}+2\cdot A_0^{d2+}+2\cdot A_0^{p1-}+A_0^{s0s+}+A_0^{p2s+}+2\cdot A_1^{d2+}+2\cdot A_1^{p1-}}{32}(u^2\downarrow u\uparrow s\uparrow +s\uparrow u\uparrow u^2\downarrow)$$
$$+\frac{A_1^{s0s+}+A_1^{p2s+}-2\cdot A_0^{d2+}-2\cdot A_0^{p1-}-A_0^{s0s+}-A_0^{p2s+}+2\cdot A_1^{d2+}+2\cdot A_1^{p1-}}{32}(u^2\uparrow u\uparrow s\downarrow +s\downarrow u\uparrow u^2\uparrow)$$
$$+\frac{A_1^{s0s+}+A_1^{p2s+}+2\cdot A_0^{d2+}+2\cdot A_0^{p1-}-A_0^{s0s+}-A_0^{p2s+}-2\cdot A_1^{d2+}-2\cdot A_1^{p1-}}{32}(s^2\downarrow u\uparrow u\uparrow +u\uparrow u\uparrow s^2\downarrow)$$
$$+\frac{A_1^{s0s+}+A_1^{p2s+}-2\cdot A_0^{d2+}-2\cdot A_0^{p1-}+A_0^{s0s+}+A_0^{p2s+}-2\cdot A_1^{d2+}-2\cdot A_1^{p1-}}{32}(s^2\uparrow u\uparrow u\downarrow +u\downarrow u\uparrow s^2\uparrow)$$
$$+\frac{A_1^{s0s+}+A_1^{p2s+}+A_0^{s0s+}+A_0^{p2s+}}{16}\cdot\sqrt{2}\cdot(u^1\downarrow u\uparrow s^1\uparrow +s^1\uparrow u\uparrow u^1\downarrow)$$
$$+\frac{A_1^{s0s+}+A_1^{p2s+}-A_0^{s0s+}-A_0^{p2s+}}{16}\cdot\sqrt{2}\cdot(u^1\uparrow u\uparrow s^1\downarrow +s^1\downarrow u\uparrow u^1\uparrow)\Big), \tag{17}$$

$$k_{23}\Big(\frac{A_1^{s0s+}+A_1^{p2s+}}{4}(u^2\downarrow u\uparrow s\uparrow +u^2\downarrow s\uparrow u\uparrow)\Big). \tag{18}$$

All members of wave function can be considered. After the grouping of these members we can obtain

$$u^2\downarrow u\uparrow s\uparrow\Big\{k_{12}\frac{A_1^{s0+}+A_1^{p2+}+3A_0^{d2+}+3A_0^{p1-}}{8}+k_{13}\frac{A_1^{s0s+}+A_1^{p2s+}+3A_0^{d2s+}+3A_0^{p1s-}}{8}+k_{23}\frac{A_1^{sos+}+A_1^{p2s+}}{2}\Big\}. \tag{19}$$

The left side of the diagram (Fig. 2) corresponds to the quark interactions. The right side of Fig. 2 determines the zero approximation (first diagram) and the subsequent pair interactions (second diagram). The contribution to $u^2\downarrow u\uparrow s\uparrow$ is shown in Fig. 3. If we group the same members, we obtain the system integral equations for the Σ state with the $J^p=\frac{5}{2}^+$ of the (10, 2) $(70, 2^+)$ multiplet:

$$\begin{aligned}
A_1^{s0+}(s,s_{12}) &= \lambda b_{1^{s+}}(s_{12})L_{1^{s+}}(s_{12})+K_{1^{s+}}(s_{12})[\tfrac18 A_1^{s0s+}(s,s_{13})+\tfrac18 A_1^{p2s+}(s,s_{13})+\tfrac38 A_0^{d2s+}(s,s_{13})+\tfrac38 A_0^{p1s-}(s,s_{13})\\
&\quad+\tfrac18 A_1^{s0s+}(s,s_{23})+\tfrac18 A_1^{p2s+}(s,s_{23})+\tfrac38 A_0^{d2s+}(s,s_{23})+\tfrac38 A_0^{p1s-}(s,s_{23})],\\
A_1^{p2+}(s,s_{12}) &= \lambda b_{3^{d+}}(s_{12})L_{3^{d+}}(s_{12})+K_{3^{d+}}(s_{12})[\tfrac18 A_1^{s0s+}(s,s_{13})+\tfrac18 A_1^{p2s+}(s,s_{13})+\tfrac38 A_0^{d2s+}(s,s_{13})+\tfrac38 A_0^{p1s+}(s,s_{13})\\
&\quad+\tfrac18 A_1^{s0s+}(s,s_{23})+\tfrac18 A_1^{p2s+}(s,s_{23})+\tfrac38 A_0^{d2s+}(s,s_{23})+\tfrac38 A_0^{p1s-}(s,s_{23})],\\
A_1^{s0s+}(s,s_{12}) &= \lambda b_{1_s^{s+}}(s_{12})L_{1_s^{s+}}(s_{12})+K_{1_s^{s+}}(s_{12})[\tfrac14 A_1^{s0+}(s,s_{13})+\tfrac14 A_1^{p2+}(s,s_{13})-\tfrac18 A_1^{s0s+}(s,s_{13})-\tfrac18 A_1^{p2s+}(s,s_{13})\\
&\quad+\tfrac38 A_0^{d2s+}(s,s_{13})+\tfrac38 A_0^{p1s-}(s,s_{13})+\tfrac14 A_1^{s0+}(s,s_{23})+\tfrac14 A_1^{p2+}(s,s_{23})-\tfrac18 A_1^{s0s+}(s,s_{23})-\tfrac18 A_1^{p2s+}(s,s_{23})\\
&\quad+\tfrac38 A_0^{d2s+}(s,s_{23})+\tfrac38 A_0^{p1s-}(s,s_{23})\Big]\\
A_1^{p2s+}(s,s_{12}) &= \lambda b_{3_s^{d+}}(s_{12})L_{3_s^{d+}}(s_{12})+K_{3_s^{d+}}(s_{12})[\tfrac14 A_1^{s0+}(s,s_{13})+\tfrac14 A_1^{p2+}(s,s_{13})-\tfrac18 A_1^{s0s+}(s,s_{13})-\tfrac18 A_1^{p2s+}(s,s_{13})\\
&\quad+\tfrac38 A_0^{d2s+}(s,s_{13})+\tfrac38 A_0^{p1s-}(s,s_{13})+\tfrac14 A_1^{s0+}(s,s_{23})+\tfrac14 A_1^{p2+}(s,s_{23})-\tfrac18 A_1^{s0s+}(s,s_{23})-\tfrac18 A_1^{p2s+}(s,s_{23})\\
&\quad+\tfrac38 A_0^{d2s+}(s,s_{23})+\tfrac38 A_0^{p1s-}(s,s_{23})],\\
A_0^{d2s+}(s,s_{12}) &= \lambda b_{2_s^{d+}}(s_{12})L_{2_s^{d+}}(s_{12})+K_{2_s^{d+}}(s_{12})[\tfrac14 A_1^{s0+}(s,s_{13})+\tfrac14 A_1^{p2+}(s,s_{13})+\tfrac18 A_1^{s0s+}(s,s_{13})+\tfrac18 A_1^{p2s+}(s,s_{13})\\
&\quad+\tfrac18 A_0^{d2s+}(s,s_{13})+\tfrac18 A_0^{p1s-}(s,s_{13})+\tfrac14 A_1^{s0+}(s,s_{23})+\tfrac14 A_1^{p2+}(s,s_{23})+\tfrac18 A_1^{s0s+}(s,s_{23})+\tfrac18 A_1^{p2s+}(s,s_{23})\\
&\quad+\tfrac18 A_0^{d2s+}(s,s_{23})+\tfrac18 A_0^{p1s-}(s,s_{23})],\\
A_0^{p1s-}(s,s_{12}) &= \lambda b_{1_s^{p-}}(s_{12})L_{1_s^{p-}}(s_{12})+K_{1_s^{p-}}(s_{12})[\tfrac14 A_1^{s0+}(s,s_{13})+\tfrac14 A_1^{p2+}(s,s_{13})+\tfrac18 A_1^{s0s+}(s,s_{13})+\tfrac18 A_1^{p2s+}(s,s_{13})\\
&\quad+\tfrac18 A_0^{d2s+}(s,s_{13})+\tfrac18 A_0^{p1s-}(s,s_{13})+\tfrac14 A_1^{s0+}(s,s_{23})+\tfrac14 A_1^{p2+}(s,s_{23})+\tfrac18 A_1^{s0s+}(s,s_{23})+\tfrac18 A_1^{p2s+}(s,s_{23})\\
&\quad+\tfrac18 A_0^{d2s+}(s,s_{23})+\tfrac18 A_0^{p1s-}(s,s_{23})].
\end{aligned} \tag{20}$$

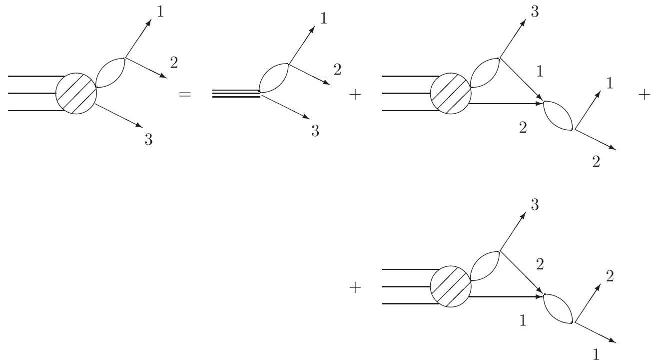

FIG. 2. Graphic representation of the equations for the amplitude $A_1(s, s_{ik})$.

Here the function $L_J(s_{ik})$ has the form

$$L_J(s_{ik}) = \frac{G_J(s_{ik})}{1 - B_J(s_{ik})}. \tag{21}$$

The integral operator $K_J(s_{ik})$ is

$$K_J(s_{ik}) = L_J(s_{ik}) \int_{(m_i+m_k)^2}^{\infty} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik}) G_J(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^{1} \frac{dz}{2}, \tag{22}$$

$$b_J(s_{ik}) = \int_{(m_i+m_k)^2}^{\Lambda_J(i,k)} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik}) G_J^2(s'_{ik})}{s'_{ik} - s_{ik}}. \tag{23}$$

The function $b_J(s_{ik})$ is the truncated function of Chew-Mandelstam. z is the cosine of the angle between the relative momentum of particles i and k in the intermediate state and the momentum of particle j in the final state, taken in the c.m. of the particles i and k. Let some current produce three quarks (first diagram Fig. 1) with vertex constant λ. This constant do not affect to the spectra mass of excited baryons. By analogy with the $\Sigma\ \frac{5}{2}^+$ (10, 2) $(70, 2^+)$ state we obtain the rescattering amplitudes of the three various quarks for all $(70, 2^+)$ and $(70, 0^+)$ states which satisfy the system of integral equations (Appendix A).

IV. THE REDUCED EQUATIONS FOR THE $(70, 0^+)$ AND $(70, 2^+)$ MULTIPLETS

Let us extract two-particle singularities in $A_J(s, s_{ik})$:

$$A_J(s, s_{ik}) = \frac{\alpha_J(s, s_{ik}) b_J(s_{ik}) G_J(s_{ik})}{1 - B_J(s_{ik})}, \tag{24}$$

$\alpha_J(s, s_{ik})$ is the reduced amplitude. Accordingly to this, all integral equations can be rewritten using the reduced amplitudes. The function $\alpha_J(s, s_{ik})$ is a smooth function of s_{ik} as compared with the singular part of the amplitude. We do not extract the three-body singularities, because they are weaker than the two-particle singularities. The poles of the reduced amplitudes α_i, $i = 1, 2, 3, \ldots$ correspond to the bound states and determine the masses of excited baryons of $(70, 0^+)$ and $(70, 2^+)$ multiplets. For instance, one considers the first equation of system for the $\Sigma\ J^p = \frac{5}{2}^+$ of the (10, 2) $(70, 2^+)$ multiplet:

$$\begin{aligned} \alpha_1^{s0+}(s, s_{12}) = \lambda + \frac{1}{b_{1^{s+}}(s_{12})} \int_{(m_1+m_2)^2}^{\Lambda_{1^+}(1,2)} \frac{ds'_{12}}{\pi} \\ \times \frac{\rho_{1^{s+}}(s'_{12}) G_{1^{s+}}(s'_{12})}{s'_{12} - s_{12}} \int_{-1}^{1} \frac{dz}{2} \\ \times \Bigg(\frac{G_{1_s^{s+}}(s'_{13}) b_{1_s^{s+}}(s'_{13})}{1 - B_{1_s^{s+}}(s'_{13})} \frac{1}{4} \alpha_1^{s0s+}(s, s'_{13}) \\ + \frac{G_{3_s^{d+}}(s'_{13}) b_{3_s^{d+}}(s'_{13})}{1 - B_{3_s^{d+}}(s'_{13})} \frac{1}{4} \alpha_1^{p2s+}(s, s'_{13}) \\ + \frac{G_{2_s^{d+}}(s'_{13}) b_{2_s^{d+}}(s'_{13})}{1 - B_{2_s^{d+}}(s'_{13})} \frac{3}{4} \alpha_0^{d2s+}(s, s'_{13}) \\ + \frac{G_{1_s^{p-}}(s'_{13}) b_{1_s^{p-}}(s'_{13})}{1 - B_{1_s^{p-}}(s'_{13})} \frac{3}{4} \alpha_0^{p1s-}(s, s'_{13}) \Bigg). \end{aligned} \tag{25}$$

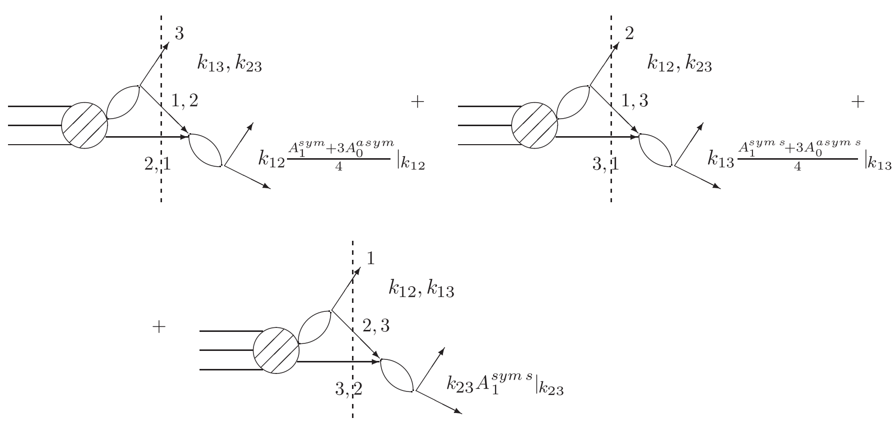

FIG. 3. The contribution of the diagrams with the rescattering.

The connection between s'_{12} and s'_{13} is

$$s'_{13} = m_1^2 + m_3^2 - \frac{(s'_{12} + m_3^2 - s)(s'_{12} + m_1^2 - m_2^2)}{2s'_{12}} \pm \frac{z}{2s'_{12}} \times \sqrt{(s'_{12} - (m_1 + m_2)^2)(s'_{12} - (m_1 - m_2)^2)}\sqrt{(s'_{12} - (\sqrt{s} + m_3)^2)(s'_{12} - (\sqrt{s} - m_3)^2)}. \quad (26)$$

The formula for s'_{23} is similar to (26) with z replaced by $-z$. Thus $A_1^{s0s+}(s, s'_{13}) + A_1^{s0s+}(s, s'_{23})$ must be replaced by $2A_1^{s0s+}(s, s'_{13})$. $\Lambda_J(i, k)$ is the cutoff at the large value of s_{ik}, which determines the contribution from small distances.

The construction of the approximate solution of system of equations is based on the extraction of the leading singularities which are close to the region $s_{ik} = (m_i + m_k)^2$ [29]. Amplitudes with different number of rescattering have the following structure of singularities. The main singularities in s_{ik} are from pair rescattering of the particles i and k. First of all there are threshold square root singularities. Pole singularities are also possible which correspond to the bound states. The diagrams in Fig. 2 apart from two-particle singularities have their own specific triangle singularities. Such classification allows us to search the approximate solution of Eq. (24) by taking into account some definite number of leading singularities and neglecting all the weaker ones.

We consider the approximation, which corresponds to the single interaction of all three particles (two-particle and triangle singularities) and neglecting all the weaker ones.

The functions $\alpha_J(s, s_{ik})$ are the smooth functions of s_{ik} as compared with the singular part of the amplitude, hence it can be expanded in a series in the singularity point and only the first term of these series should be employed further. As s_0 it is convenient to take the middle point of physical region of the Dalitz plot in which $z = 0$. In this case we get from (26) $s_{ik} = s_0 = \frac{s + m_1^2 + m_2^2 + m_3^2}{m_{12}^2 + m_{13}^2 + m_{23}^2}$, where $m_{ik} = \frac{m_i + m_k}{2}$. We define the functions $\alpha_J(s, s_{ik})$ and $b_J(s_{ik})$ at the point s_0. Such a choice of point s_0 allows us to replace integral Eqs. (20) by the algebraic equations for the state Σ with $J^P = \frac{5}{2}^+$ of (10, 2) $(70, 2^+)$:

$$\begin{cases}
\alpha_1^{s0+} = \lambda + \frac{1}{4}\alpha_1^{s0s+} M_{1^{s+}1_s^{s+}} + \frac{1}{4}\alpha_1^{p2s+} M_{1^{s+}3_s^{d+}} + \frac{3}{4}\alpha_0^{d2s+} M_{1^{s+}2_s^{d+}} + \frac{3}{4}\alpha_0^{p1s-} M_{1^{s+}1_s^{p-}} & 1^{s+} \\
\alpha_1^{p2+} = \lambda + \frac{1}{4}\alpha_1^{s0s+} M_{3^{d+}1_s^{s+}} + \frac{1}{4}\alpha_1^{p2s+} M_{3^{d+}3_s^{d+}} + \frac{3}{4}\alpha_0^{d2s+} M_{3^{d+}2_s^{d+}} + \frac{3}{4}\alpha_0^{p1s-} M_{3^{d+}1_s^{p-}} & 3^{d+} \\
\alpha_1^{s0s+} = \lambda + \frac{1}{2}\alpha_1^{s0+} M_{1_s^{s+}1^{s+}} + \frac{1}{2}\alpha_1^{p2+} M_{1_s^{s+}3^{d+}} - \frac{1}{4}\alpha_1^{s0s+} M_{1_s^{s+}1_s^{s+}} - \frac{1}{4}\alpha_1^{p2s+} M_{1_s^{s+}3_s^{d+}1_s^{s+}} + \frac{3}{4}\alpha_0^{d2s+} M_{1_s^{s+}2_s^{d+}} + \frac{3}{4}\alpha_0^{p1s-} M_{1_s^{s+}1_s^{p-}} & 1_s^{s+} \\
\alpha_1^{p2s+} = \lambda + \frac{1}{2}\alpha_1^{s0+} M_{3_s^{d+}1^{s+}} + \frac{1}{2}\alpha_1^{p2+} M_{3_s^{d+}3^{d+}} - \frac{1}{4}\alpha_1^{s0s+} M_{3_s^{d+}1_s^{s+}} - \frac{1}{4}\alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}3_s^{d+}} + \frac{3}{4}\alpha_0^{d2s+} M_{3_s^{d+}2_s^{d+}} + \frac{3}{4}\alpha_0^{p1s-} M_{3_s^{d+}1_s^{p-}} & 3_s^{d+} \\
\alpha_0^{d2s+} = \lambda + \frac{1}{2}\alpha_1^{s0+} M_{2_s^{d+}1^{s+}} + \frac{1}{2}\alpha_1^{p2+} M_{2_s^{d+}3^{d+}} + \frac{1}{4}\alpha_1^{s0s+} M_{2_s^{d+}1_s^{s+}} + \frac{1}{4}\alpha_1^{p2s+} M_{2_s^{d+}3_s^{d+}2_s^{d+}} + \frac{1}{4}\alpha_0^{d2s+} M_{2_s^{d+}2_s^{d+}} + \frac{1}{4}\alpha_0^{p1s-} M_{2_s^{d+}1_s^{p-}} & 2_s^{d+} \\
\alpha_0^{p1s-} = \lambda + \frac{1}{2}\alpha_1^{s0+} M_{1_s^{p-}1^{s+}} + \frac{1}{2}\alpha_1^{p2+} M_{1_s^{p-}3^{d+}} + \frac{1}{4}\alpha_1^{s0s+} M_{1_s^{p-}1_s^{s+}} + \frac{1}{4}\alpha_1^{p2s+} M_{1_s^{p-}3_s^{d+}1_s^{p-}} + \frac{1}{4}\alpha_0^{d2s+} M_{1_s^{p-}2_s^{d+}} + \frac{1}{4}\alpha_0^{p1s-} M_{1_s^{p-}1_s^{p-}}, & 1_s^{p-}
\end{cases} \quad (27)$$

Here the reduced amplitudes for the diquarks 1^{s+}, 3^{d+}, 1_s^{s+}, 3_s^{d+}, 2_s^{d+}, 1_s^{p-} are given.

We used the following form:

$$M_{X_m^{ip} Y_n^{jq}} \equiv M_{X_m^{ip} Y_n^{jq}}(s, s_0) = I_{X_m^{ip} Y_n^{jq}}(s, s_0) \frac{b_{Y_n^{jq}}(s_0)}{b_{X_m^{ip}}(s_0)}, \quad (28)$$

here X_m^{ip} corresponds to the diquark with total momentum X $(X = 0, 1, 2, 3)$; $i = s, p, d$ for the s-, p-, d-wave consequently; $p = +, -$ is the p parity of diquark; $m = s$ for the strange diquark and this index is absent in other case.

The reduced amplitude $\alpha_s^{\mathrm{clmp}} \equiv \alpha_s^{\mathrm{clmp}}(s, s_0)$ for the $p = +, -$ of parity of diquark; $c = s$ if the diquark is determined as $1s1s$, $c = p$ if we consider $1s1p$ or $1p1p$ states, $c = d$ if we have $1s1d$; $s = 1, 0$ corresponds to the diquark spin $(\uparrow\uparrow, \uparrow\downarrow)$, $l = 2, 1, 0, -1$ are the values of projection orbital angular momentum at definite axes, $m = s$ for the strange diquark.

The function $I_{J_1 J_2}(s, s_0)$ takes into account the singularity which corresponds to the simultaneous vanishing of all propagators in the triangle diagrams.

$$I_{J_1 J_2}(s, s_0) = \int_{(m_i + m_k)^2}^{\Lambda_{J_1}} \frac{ds'_{ik}}{\pi} \frac{\rho_{J_1}(s'_{ik}) G_{J_1}^2(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^{1} \frac{dz}{2} \times \frac{1}{1 - B_{J_2}(s_{ij})}. \quad (29)$$

The $G_J(s_{ik})$ functions have the smooth dependence from energy s_{ik} [27] therefore we suggest them as constants. The parameters of model: g_J vertex constants, λ_J cutoff parameters are chosen dimensionless.

$$g_J = \frac{m_i + m_k}{2\pi} G_J, \qquad \lambda_J = \frac{4\Lambda_J}{(m_i + m_k)^2}. \quad (30)$$

Here m_i and m_k are quark masses in the intermediate state of the quark loop. Dimensionless parameters g_J and λ_J are supposed to be the constants independent of the quark interaction type. We calculate the system of equations and can determine the mass values of the Σ $J^p = \frac{5}{2}^+$ (10, 2) $(70, 2^+)$. We calculate a pole in s which corresponds to the bound state of the three quarks.

By analogy with the Σ-hyperon we obtain the system of equations for the reduced amplitudes for all particles $(70, 0^+)$ and $(70, 2^+)$ multiplets (Appendix B).

The solutions of the system of equations are considered as

$$\alpha_J = \frac{F_J(s, \lambda_J)}{D(s)}, \tag{31}$$

where the zeros of the $D(s)$ determines the value of masses of bound states of baryons. $F_J(s, \lambda_J)$ are the functions of s and λ_J. The functions $F_J(s, \lambda_J)$ determine the contributions of subamplitudes to the excited baryon amplitude.

The parameters of model (Tables I, II, III, IV, V, VI, VII, VIII, IX, X, XI, and XII): gluon coupling constants $g_s^+ = g_p^- = 0.6$, $g_d^+ = 0.5$, cutoff energy parameters $\lambda = 13.0$, $\lambda_{ss} = 11.1$.

V. CALCULATION RESULTS

The quark masses ($m_u = m_d = m$ and m_s) are not fixed. In order to fix m and m_s, in any way we assume $m = \frac{1}{3} m_\Delta(1.232)$ and $m = \frac{1}{3} m_\Omega(1.672)$ i.e. the quark masses are $m = 0.410$ GeV and $m_s = 0.557$ GeV.

The S-wave baryon mass spectra are obtained in good agreement with the experimental data. When we research the excited states the confinement potential cannot be neglected. In our case the confinement potential is imitated by the simple increasing of constituent quark masses [30]. The shift of quark mass (parameter $\Delta = 340$ MeV) effectively takes into account the changing of the confinement potential. We have shown that inclusion of only gluon

TABLE I. The Δ-isobar masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{5}{2}^+$ (10, 2)	F_{35}	1964	1976 ± 237	$F_{35}(2000)^{**}$
$\frac{3}{2}^+$ (10, 2)	P_{33}	2108	...	...

TABLE II. The nucleon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{5}{2}^+$ (8, 2)	F_{15}	1973	...	...
$\frac{3}{2}^+$ (8, 2)	P_{13}	2049	...	...
$\frac{7}{2}^+$ (8, 4)	F_{17}	2086	2016 ± 104	$F_{17}(1990)^{**}$
$\frac{5}{2}^+$ (8, 4)	F_{15}	1981	1981 ± 200	$F_{15}(2000)^{**}$
$\frac{3}{2}^+$ (8, 4)	P_{13}	2028	...	...
$\frac{1}{2}^+$ (8, 4)	P_{11}	1703	1986 ± 26	$P_{11}(2100)^{*}$

TABLE III. The Σ-hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{5}{2}^+$ (10, 2)	F_{15}	2075	...	...
$\frac{3}{2}^+$ (10, 2)	P_{13}	2237	...	...
$\frac{5}{2}^+$ (8, 2)	F_{15}	2084	...	...
$\frac{3}{2}^+$ (8, 2)	P_{13}	2171	...	...
$\frac{7}{2}^+$ (8, 4)	F_{17}	2113	...	...
$\frac{5}{2}^+$ (8, 4)	F_{15}	2083	...	...
$\frac{3}{2}^+$ (8, 4)	P_{13}	2149	...	...
$\frac{1}{2}^+$ (8, 4)	P_{11}	1786	...	...

TABLE IV. The Ξ-hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{5}{2}^+$ (10, 2)	F_{15}	2193	...	...
$\frac{3}{2}^+$ (10, 2)	P_{13}	2367	...	...
$\frac{5}{2}^+$ (8, 2)	F_{15}	2203	...	...
$\frac{3}{2}^+$ (8, 2)	P_{13}	2296	...	...
$\frac{7}{2}^+$ (8, 4)	F_{17}	2341	...	...
$\frac{5}{2}^+$ (8, 4)	F_{15}	2200	...	...
$\frac{3}{2}^+$ (8, 4)	P_{13}	2271	...	...
$\frac{1}{2}^+$ (8, 4)	P_{11}	1876	...	...

TABLE V. The Λ-hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{5}{2}^+$ (8, 2)	F_{05}	2169	...	...
$\frac{3}{2}^+$ (8, 2)	P_{03}	2082	...	...
$\frac{7}{2}^+$ (8, 4)	F_{07}	2213	2094 ± 78	$F_{07}(2020)^{*}$
$\frac{5}{2}^+$ (8, 4)	F_{05}	2084	2112 ± 40	$F_{05}(2110)^{***}$
$\frac{3}{2}^+$ (8, 4)	P_{03}	2146	...	...
$\frac{1}{2}^+$ (8, 4)	P_{01}	1786	...	...
$\frac{5}{2}^+$ (1, 2)	F_{05}	2097	...	...
$\frac{3}{2}^+$ (1, 2)	P_{03}	2094	...	...

TABLE VI. The Ω-hyperon masses of multiplet $(70, 2^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{5}{2}^+$ (10, 2)	F_{05}	2311	...	...
$\frac{3}{2}^+$ (10, 2)	P_{03}	2498	...	...

TABLE VII. The Δ-isobar masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{1}{2}^+$ (10, 2)	P_{31}	1803	1744 ± 36	$P_{31}(1750)^{*}$

TABLE VIII. The nucleon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{1}{2}^+$ (8, 2)	P_{11}	1710	1710 ± 30	$P_{11}(1710)^{***}$
$\frac{3}{2}^+$ (8, 4)	P_{13}	1879	1879 ± 17	$P_{13}(1900)^{**}$

TABLE IX. The Σ-hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{1}{2}^+$ (10, 2)	P_{11}	1896	1896 ± 95	$P_{11}(1880)^{**}$
$\frac{1}{2}^+$ (8, 2)	P_{11}	1826	1760 ± 27	$P_{11}(1770)^{*}$
$\frac{3}{2}^+$ (8, 4)	P_{13}	1962	...	...

TABLE X. The Ξ-hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{1}{2}^+$ (10, 2)	P_{11}	2000	...	...
$\frac{1}{2}^+$ (8, 2)	P_{11}	1923	...	...
$\frac{3}{2}^+$ (8, 4)	P_{13}	2070	...	...

TABLE XI. The Λ-hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{1}{2}^+$ (8, 2)	P_{01}	1878	...	...
$\frac{3}{2}^+$ (8, 4)	P_{03}	1964	...	...
$\frac{1}{2}^+$ (1, 2)	P_{01}	1753	1791 ± 64	$P_{01}(1810)^{***}$

TABLE XII. The Ω-hyperon masses of multiplet $(70, 0^+)$.

Multiplet	Baryon	Mass (MeV)	Mass (MeV) (exp.)	Name, status
$\frac{1}{2}^+$ (10, 2)	P_{01}	2105	...	...

TABLE XIII. Coefficient of Ghew-Mandelstam function for the different diquarks.

	α_J	β_J	δ_J
3^+	$\frac{5}{14}$	$\frac{2}{14} - \frac{5}{14}\frac{(m_i-m_k)^2}{(m_i+m_k)^2}$	$-\frac{2}{14}(m_i - m_k)^2$
2^+	$\frac{1}{2}$	$-\frac{1}{2}\frac{(m_i-m_k)^2}{(m_i+m_k)^2}$	0
1^{d+}	$\frac{1}{7}$	$\frac{3}{14}(1 + \frac{8m_im_k}{3(m_i+m_k)^2})$	$-\frac{5}{14}(m_i - m_k)^2$
1^{s+}	$\frac{1}{3}$	$\frac{4m_im_k}{3(m_i+m_k)^2} - \frac{1}{6}$	$-\frac{1}{6}(m_i - m_k)^2$
0^+	$\frac{1}{2}$	$-\frac{1}{2}\frac{(m_i-m_k)^2}{(m_i+m_k)^2}$	0
0^-	0	$\frac{1}{2}$	$-\frac{1}{2}(m_i - m_k)^2$
1^-	$\frac{1}{2}$	$-\frac{1}{2}\frac{(m_i-m_k)^2}{(m_i+m_k)^2}$	0
2^-	$\frac{3}{10}$	$\frac{1}{5}(1 - \frac{3}{2}\frac{(m_i-m_k)^2}{(m_i+m_k)^2})$	$-\frac{1}{5}(m_i - m_k)^2$

exchange does not lead to the appearance of bound states corresponding to the excited baryons in the $(70, 0^+)$ and $(70, 2^+)$ multiplets. The mass shift Δ allows us to obtain the mass spectra of these states. The similar result for the P-wave baryons was obtained [24].

In the case considered the same parameters Δ for the u, d, s quarks are chosen. Then the quark masses $m_u = m_d = 0.750$ GeV and $m_s = 0.897$ GeV are given.

In our model the four parameters are used: gluon coupling constants $g_s^+ \equiv g_p^-$ for the s- and p-wave diquarks, g_d^+ for d-wave diquarks, cutoff parameters λ, λ_{ss} for the nonstrange and strange diquarks. The parameter of λ_s was calculated using λ and λ_{ss} parameters.

The parameters $g_s^+ \equiv g_p^- = 0.6$, $g_d+ = 0.5$, $\lambda = 13.0$, $\lambda_{ss} = 11.1$ have been determined by the baryon masses: $M_{N(5/2)^+(8,4)(70,2^+)} = 1981$ MeV, $M_{N(1/2)^+(8,2)(70,0^+)} = 1710$ MeV, $M_{N(3/2)^+(8,4)(70,0^+)} = 1879$ MeV, and $M_{\Sigma(1/2)^+(10,2)(70,0^+)} = 1896$ MeV.

In the Tables I, II, III, IV, V, VI, VII, VIII, IX, X, XI, and XII we represent the masses of the nonstrange and strange resonances belonging to the $(70, 0^+)$ and $(70, 2^+)$ multiplets obtained using the fit of the experimental values [15,25].

The $(70, 0^+)$ and $(70, 2^+)$ multiplets include 414 particles, only 47 baryons have different masses. The 12 resonances are in good agreement with experimental data [15,25]. We have predicted 35 masses of baryons.

In the framework of the proposed approximate method of solving the relativistic three-particle problem, we have obtained a satisfactory spectrum of $N = 2$ level baryons. The important problem is the mixing of the states of baryons and the five quark systems (cryptoexotic baryons [31] or hybrid baryons [32]).

VI. CONCLUSION

In strongly bound systems of light quarks, such as the baryons considered, where $p/m \sim 1$, the approximation by nonrelativistic kinematics and dynamics is not justified.

In the papers [22,23] the relativistic generalization of Faddeev equations in the framework of dispersion relation are constructed. We calculated the S-wave baryon masses using the method based on the extraction of leading singularities of the amplitude. The behavior of electromagnetic form factor of the nucleon and hyperon in the region of low and intermediate momentum transfers is determined by [33]. In the framework of the dispersion relation approach the charge radii of S-wave baryon multiplets with $J^P = \frac{1}{2}^+$ are calculated.

In our paper the relativistic description of three particles amplitudes of P-wave baryons are considered. We take into account the u, d, s quarks. The mass spectrum of nonstrange and strange states of multiplet $(\mathbf{70}, \mathbf{1}^-)$ are calculated. We use only four parameters for the calculation of 30 baryon masses. We take into account the mass shift of u, d, s quarks which allows us to obtain the P-wave baryon bound states.

In the present paper the relativistic consideration of three particles amplitudes of $(70, 0^+)$ and $(70, 2^+)$ excited baryons are given. We take into account the u, d, s quarks. We have calculated the 47 masses of resonances $(70, 0^+)$, $(70, 2^+)$ with only four parameters. We take into account the mass shift (similar to [24]) for u, d, s quarks which allows us to obtain the $N = 2$ level excited baryon bound states.

The baryon resonances $(70, 2^+)$ multiplet heavier than ones of the $(70, 0^+)$ multiplet that is similar to the results of papers [15,16].

ACKNOWLEDGMENTS

The authors would like to thank T. Barnes, S. Capstick, S. Chekanov, and Fl. Stancu for useful discussions. The work was carried with the support of the Russian Ministry of Education (Grant No. 2.1.1.68.26).

APPENDIX A: THE WAVE FUNCTIONS

We consider, for instance, the wave functions of the upper submultiplets of decuplet (10, 2) $J^p = \frac{5}{2}^+$, octets (8, 2) $J^p = \frac{5}{2}^+$, (8, 4) $J^p = \frac{7}{2}^+$ and singlet (1, 2) $J^p = \frac{5}{2}^+$, which are corresponded to the projection orbital angular momentum $L_z = 2$. For the lower multiplets one must use the corresponding wave functions. $O(3)$ wave functions possess mixed symmetry and can be written as

$$\varphi_{\rm MA}^{O(3)} = \frac{1}{\sqrt{6}}(020 - 200 + \sqrt{2}(011 - 101)),$$
$$\varphi_{\rm MS}^{O(3)} = \frac{1}{\sqrt{18}}(020 + 200 - 2 \cdot 002 - \sqrt{2}(011 + 101 - 2 \cdot 110)). \quad \text{(A1)}$$

here 0, 1, 2 are the values of the projections of quark orbital angular momentum. MA and MS correspond to the mixed antisymmetric and symmetric part of wave function.

The $SU(2)$ wave functions have the following form:

$$\varphi_{\rm MA}^{SU(2)} = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$
$$\varphi_{\rm MS}^{SU(2)} = \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow), \quad \text{(A2)}$$

$\uparrow$ and $\downarrow$ determine the spin directions.

The $SU(3)_f$ wave functions are different for each particles.

1. Multiplet (10, 2)

The totally symmetric $SU(6) \times O(3)$ wave function for each decuplet particle is constructed as

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_{\rm MA}^{SU(6)}\varphi_{\rm MA}^{O(3)} + \varphi_{\rm MS}^{SU(6)}\varphi_{\rm MS}^{O(3)}) = \frac{1}{\sqrt{2}}\varphi_S^{SU(3)}(\varphi_{\rm MA}^{SU(2)}\varphi_{\rm MA}^{O(3)} + \varphi_{\rm MS}^{SU(2)}\varphi_{\rm MS}^{O(3)}), \quad \text{(A3)}$$

here by analogy of the paper [24]:

$$\varphi_{\rm MA}^{SU(6)} = \varphi_S^{SU(3)}\varphi_{\rm MA}^{SU(2)}, \qquad \varphi_{\rm MS}^{SU(6)} = \varphi_S^{SU(3)}\varphi_{\rm MS}^{SU(2)}. \quad \text{(A4)}$$

For the Σ^+ hyperon belonging to the decuplet $SU(3)$ the wave function is

$$\varphi_S^{SU(3)} = \frac{1}{\sqrt{3}}(usu + suu + uus). \quad \text{(A5)}$$

The totally symmetric $SU(6) \times O(3)$ function of Σ^+ from multiplet (10, 2) is given:

$$\varphi_{\Sigma^+(10,2)} = \frac{\sqrt{2}}{18}(2\{u^2\downarrow u\uparrow s\uparrow\} + \{s^2\downarrow u\uparrow u\uparrow\} - \{u^2\uparrow u\downarrow s\uparrow\} - \{u^2\uparrow u\uparrow s\downarrow\} - \{s^2\uparrow u\uparrow u\downarrow\} - \sqrt{2}(2\{u\downarrow u^1\uparrow s^1\uparrow\} + \{s\downarrow u^1\uparrow u^1\uparrow\} - \{u\uparrow u^1\downarrow s^1\uparrow\} - \{u\uparrow u^1\uparrow s^1\downarrow\} - \{s\uparrow u^1\uparrow u^1\downarrow\})). \quad \text{(A6)}$$

Here the parentheses determine the symmetrical function:

$$\{abc\} \equiv abc + acb + bac + cab + bca + cba. \quad \text{(A7)}$$

For the Δ^{++} of multiplet (10, 2) the $SU(6) \times O(3)$ wave function can be obtained by the replacement $u \leftrightarrow s$:

$$\varphi_{\Delta^{++}(10,2)} = \frac{\sqrt{6}}{18}(\{u^2\downarrow u\uparrow u\uparrow\} - \{u^2\uparrow u\uparrow u\downarrow\} - \sqrt{2}(\{u\downarrow u^1\uparrow u^1\uparrow\} - \{u\uparrow u^1\uparrow u^1\downarrow\})). \quad \text{(A8)}$$

For the Ξ of decuplet (10, 2) the results are similar to Σ by the replacement $u \leftrightarrow s$ or $d \leftrightarrow s$. The Ω^- wave function of (10, 2) coincides with Δ by the replacement $u \longrightarrow s$.

2. Multiplet (8, 2)

The wave functions of octet (8, 2) can be constructed to the following method:

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_{\rm MA}^{SU(6)}\varphi_{\rm MA}^{O(3)} + \varphi_{\rm MS}^{SU(6)}\varphi_{\rm MS}^{O(3)}), \quad \text{(A9)}$$

where

$$\varphi_{\rm MA}^{SU(6)} = \frac{1}{\sqrt{2}}(\varphi_{\rm MS}^{SU(3)}\varphi_{\rm MA}^{SU(2)} + \varphi_{\rm MA}^{SU(3)}\varphi_{\rm MS}^{SU(2)}), \quad \text{(A10)}$$

$$\varphi_{\rm MS}^{SU(6)} = \frac{1}{\sqrt{2}}(-\varphi_{\rm MS}^{SU(3)}\varphi_{\rm MS}^{SU(2)} + \varphi_{\rm MA}^{SU(3)}\varphi_{\rm MA}^{SU(2)}). \quad \text{(A11)}$$

In the case of Σ^+ octet the wave functions $\varphi_{\rm MS}^{SU(3)}$ and $\varphi_{\rm MA}^{SU(3)}$ are

$$\varphi_{\rm MS}^{SU(3)} = \frac{1}{\sqrt{6}}(usu + suu - 2uus),$$
$$\varphi_{\rm MA}^{SU(3)} = \frac{1}{\sqrt{2}}(usu - suu). \quad \text{(A12)}$$

Then the symmetric wave function for Σ^+ of (8, 2) have the following form:

$$\varphi_{\Sigma^+(8,2)} = \frac{\sqrt{2}}{18}(2\{u^2\uparrow u\downarrow s\Uparrow\} + \{s^2\downarrow u\uparrow u\Uparrow\} - \{u^2\uparrow u\uparrow s\Downarrow\} - \{u^2\downarrow u\uparrow s\Uparrow\} - \{s^2\uparrow u\uparrow u\Downarrow\} - \sqrt{2}(2\{u\uparrow u^1\downarrow s^1\Uparrow\} + \{s\downarrow u^1\uparrow u^1\Uparrow\} - \{u\uparrow u^1\uparrow s^1\Downarrow\} - \{u\downarrow u^1\uparrow s^1\Uparrow\} - \{s\uparrow u^1\uparrow u^1\Downarrow\})). \quad \text{(A13)}$$

The nucleon functions of (8, 2) can be constructed as similar to Σ^+ by the replacement $s \leftrightarrow d$, and the functions of Ξ^0 by the replacement $u \leftrightarrow s$.

In the case of the Λ^0 the $SU(3)$ wave functions $\varphi_{\mathrm{MS}}^{SU(3)}$ and $\varphi_{\mathrm{MA}}^{SU(3)}$ are

$$\varphi_{\mathrm{MS}}^{SU(3)} = \frac{1}{2}(dsu - usd + sdu - sud), \quad \text{(A14)}$$

$$\varphi_{\mathrm{MA}}^{SU(3)} = \frac{\sqrt{3}}{6}(sdu - sud + usd - dsu - 2dus + 2uds). \quad \text{(A15)}$$

As result, we have obtained the symmetric $SU(6) \times O(3)$ wave function for Λ^0 of (8, 2):

$$\varphi_{\Lambda^0(8,2)} = \frac{\sqrt{3}}{18}(\{u^2\uparrow d\uparrow s\Downarrow\} - \{u^2\downarrow d\uparrow s\Uparrow\} - \{d^2\uparrow u\uparrow s\Downarrow\} + \{d^2\downarrow u\uparrow s\Uparrow\} - \{s^2\uparrow u\uparrow d\Downarrow\} + \{s^2\uparrow u\downarrow d\Uparrow\} - \sqrt{2}(\{u\uparrow d^1\uparrow s^1\Downarrow\} - \{u\downarrow d^1\uparrow s^1\Uparrow\} - \{d\uparrow u^1\uparrow s^1\Downarrow\} + \{d\downarrow u^1\uparrow s^1\Uparrow\} - \{s\uparrow u^1\uparrow d^1\Downarrow\} + \{s\uparrow u^1\downarrow d^1\Uparrow\})). \quad \text{(A16)}$$

3. Multiplet (8, 4)

The wave functions of octet (8, 4) are constructed as similar to the cases of (10, 2) and (8, 2) multiplets:

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_{\mathrm{MA}}^{SU(6)}\varphi_{\mathrm{MA}}^{O(3)} + \varphi_{\mathrm{MS}}^{SU(6)}\varphi_{\mathrm{MS}}^{O(3)}), \quad \text{(A17)}$$

here

$$\varphi_{\mathrm{MA}}^{SU(6)} = \varphi_{\mathrm{MA}}^{SU(3)}\varphi_S^{SU(2)}, \qquad \varphi_{\mathrm{MS}}^{SU(6)} = \varphi_{\mathrm{MS}}^{SU(3)}\varphi_S^{SU(2)}. \quad \text{(A18)}$$

The $SU(2)$ function is totally symmetric:

$$\varphi_S^{SU(2)} = \Uparrow\Uparrow\Uparrow, \quad \text{(A19)}$$

and $\varphi_{\mathrm{MS}}^{SU(3)}$, $\varphi_{\mathrm{MA}}^{SU(3)}$ similar to (8, 2).

For the Σ^+ of (8, 4) we can calculate:

$$\varphi_{\Sigma^+(8,4)} = \frac{\sqrt{6}}{18}(\{s^2\uparrow u\uparrow u\Uparrow\} - \{u^2\uparrow u\uparrow s\Uparrow\} - \sqrt{2}(\{s\uparrow u^1\uparrow u^1\Uparrow\} - \{u\uparrow u^1\uparrow s^1\Uparrow\})). \quad \text{(A20)}$$

For the nucleon N of (8, 4) the results are similar to Σ^+ of (8, 4) by replacement $s \to d$; and for Ξ^0 by replacement $u \leftrightarrow s$.

For Λ^0 of (8, 4):

$$\varphi_{\Lambda^0(8,4)} = \tfrac{1}{6}(-\{u^2\uparrow d\uparrow s\Uparrow\} + \{d^2\uparrow u\uparrow s\Uparrow\} - \sqrt{2}(-\{u\uparrow d^1\uparrow s^1\Uparrow\} + \{d\uparrow u^1\uparrow s^1\Uparrow\})). \quad \text{(A21)}$$

4. Multiplet (1, 2)

In the case of Λ_1^0 singlet of (1, 2) the totally symmetric $SU(6) \times O(3)$ function must be constructed in the form:

$$\varphi = \varphi_A^{SU(3)}\varphi_A^{SU(2)\times O(3)}, \quad \text{(A22)}$$

$$\varphi_A^{SU(3)} = \frac{1}{\sqrt{6}}(sdu - sud + usd - dsu + dus - uds), \quad \text{(A23)}$$

$$\varphi_A^{SU(2)\times O(3)} = \frac{1}{\sqrt{2}}(\varphi_{\mathrm{MS}}^{SU(2)}\varphi_{\mathrm{MA}}^{O(3)} - \varphi_{\mathrm{MA}}^{SU(2)}\varphi_{\mathrm{MS}}^{O(3)}). \quad \text{(A24)}$$

Then, we have calculated the $\varphi_{\Lambda_1^0(1,2)}$:

$$\varphi_{\Lambda_1^0(1,2)} = \frac{\sqrt{2}}{6}(-\{u^2\uparrow d\uparrow s\Downarrow\} + \{u^2\uparrow d\downarrow s\Uparrow\} + \{d^2\uparrow u\uparrow s\Downarrow\} - \{d^2\uparrow u\downarrow s\Uparrow\} - \{s^2\uparrow u\uparrow d\Downarrow\} + \{s^2\uparrow u\downarrow d\Uparrow\} - \sqrt{2}(-\{u\uparrow d^1\uparrow s^1\Downarrow\} + \{u\uparrow d^1\downarrow s^1\Uparrow\} + \{d\uparrow u^1\uparrow s^1\Downarrow\} - \{d\uparrow u^1\downarrow s^1\Uparrow\} - \{s\uparrow u^1\uparrow d^1\Downarrow\} + \{s\uparrow u^1\downarrow d^1\Uparrow\})). \quad \text{(A25)}$$

APPENDIX B: THE SYSTEM EQUATIONS OF REDUCED AMPLITUDES OF THE MULTIPLETS ($70, 0^+$) AND ($70, 2^+$)

1. Multiplet (10, 2)

$\Delta\,\frac{5}{2}^+$ (10, 2) ($70, 2^+$):

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0+}M_{1^{s+}1^{s+}} + \tfrac{1}{4}\alpha_1^{p2+}M_{1^{s+}3^{d+}} + \tfrac{3}{4}\alpha_0^{d2+}M_{1^{s+}2^{d+}} + \tfrac{3}{4}\alpha_0^{p1-}M_{1^{s+}1^{p-}} \qquad 1^{s+}\\
\alpha_1^{p2+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0+}M_{3^{d+}1^{s+}} + \tfrac{1}{4}\alpha_1^{p2+}M_{3^{d+}3^{d+}} + \tfrac{3}{4}\alpha_0^{d2+}M_{3^{d+}2^{d+}} + \tfrac{3}{4}\alpha_0^{p1-}M_{3^{d+}1^{p-}} \qquad 3^{d+}\\
\alpha_0^{d2+} &= \lambda + \tfrac{3}{4}\alpha_1^{s0+}M_{2^{d+}1^{s+}} + \tfrac{3}{4}\alpha_1^{p2+}M_{2^{d+}3^{d+}} + \tfrac{1}{4}\alpha_0^{d2+}M_{2^{d+}2^{d+}} + \tfrac{1}{4}\alpha_0^{p1-}M_{2^{d+}1^{p-}} \qquad 2^{d+}\\
\alpha_0^{p1-} &= \lambda + \tfrac{3}{4}\alpha_1^{s0+}M_{1^{p-}1^{s+}} + \tfrac{3}{4}\alpha_1^{p2+}M_{1^{p-}3^{d+}} + \tfrac{1}{4}\alpha_0^{d2+}M_{1^{p-}2^{d+}} + \tfrac{1}{4}\alpha_0^{p1-}M_{1^{p-}1^{p-}}. \qquad 1^{p-}
\end{aligned} \tag{A26}$$

$\Sigma\ \frac{5}{2}^+$ (10, 2) (70, 2^+):

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0s+}M_{1^{s+}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+}M_{1^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_0^{d2s+}M_{1^{s+}2_s^{d+}} + \tfrac{3}{4}\alpha_0^{p1s-}M_{1^{s+}1_s^{p-}} \qquad 1^{s+}\\
\alpha_1^{p2+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0s+}M_{3^{d+}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+}M_{3^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_0^{d2s+}M_{3^{d+}2_s^{d+}} + \tfrac{3}{4}\alpha_0^{p1s-}M_{3^{d+}1_s^{p-}} \qquad 3^{d+}\\
\alpha_1^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{1_s^{s+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{1_s^{s+}3^{d+}} - \tfrac{1}{4}\alpha_1^{s0s+}M_{1_s^{s+}1_s^{s+}} - \tfrac{1}{4}\alpha_1^{p2s+}M_{1_s^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_0^{d2s+}M_{1_s^{s+}2_s^{d+}} + \tfrac{3}{4}\alpha_0^{p1s-}M_{1_s^{s+}1_s^{p-}} \qquad 1_s^{s+}\\
\alpha_1^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{3_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{3_s^{d+}3^{d+}} - \tfrac{1}{4}\alpha_1^{s0s+}M_{3_s^{d+}1_s^{s+}} - \tfrac{1}{4}\alpha_1^{p2s+}M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_0^{d2s+}M_{3_s^{d+}2_s^{d+}} + \tfrac{3}{4}\alpha_0^{p1s-}M_{3_s^{d+}1_s^{p-}} \qquad 3_s^{d+}\\
\alpha_1^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{2_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{2_s^{d+}3^{d+}} + \tfrac{1}{4}\alpha_1^{s0s+}M_{2_s^{d+}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+}M_{2_s^{d+}3_s^{d+}} + \tfrac{1}{4}\alpha_0^{d2s+}M_{2_s^{d+}2_s^{d+}} + \tfrac{1}{4}\alpha_0^{p1s-}M_{2_s^{d+}1_s^{p-}} \qquad 2_s^{d+}\\
\alpha_0^{p1s-} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{1_s^{p-}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{1_s^{p-}3^{d+}} + \tfrac{1}{4}\alpha_1^{s0s+}M_{1_s^{p-}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+}M_{1_s^{p-}3_s^{d+}} + \tfrac{1}{4}\alpha_0^{d2s+}M_{1_s^{p-}2_s^{d+}} + \tfrac{1}{4}\alpha_0^{p1s-}M_{1_s^{p-}1_s^{p-}}. \qquad 1_s^{p-}
\end{aligned} \tag{A27}$$

The $\Xi\ \frac{5}{2}^+$ (10, 2) (70, 2^+) reduced equations are similar to the $\Sigma\ \frac{5}{2}^+$ (10, 2) (70, 2^+) with the replacement $u \leftrightarrow s$. The $\Omega\ \frac{5}{2}^+$ (10, 2) (70, 2^+) reduced equations are constructed by the replacement $s \leftrightarrow u$ for the $\Delta\ \frac{5}{2}^+$ (10, 2) (70, 2^+).

The analogous results are obtained if we have considered the spin $J^p = \frac{3}{2}^+((70, 2^+))$, $J^p = \frac{1}{2}^+((70, 0^+))$.

2. Multiplet (8, 2)

$N\ \frac{5}{2}^+$ (8, 2) (70, 2^+):

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda - \tfrac{1}{8}\alpha_1^{s0+}M_{1^{s+}1^{s+}} - \tfrac{1}{8}\alpha_1^{p2+}M_{1^{s+}3^{d+}} + \tfrac{3}{8}\alpha_0^{s0+}M_{1^{s+}0^{s+}} + \tfrac{3}{8}\alpha_0^{p2+}M_{1^{s+}2^{d+}} + \tfrac{3}{8}\alpha_1^{d2+}M_{1^{s}3^{d+}} + \tfrac{3}{8}\alpha_1^{p1-}M_{1^{s+}2^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2+}M_{1^{s+}2^{d+}} + \tfrac{3}{8}\alpha_0^{p1-}M_{1^{s+}1^{p-}} \qquad 1^{s+}\\
\alpha_1^{p2+} &= \lambda - \tfrac{1}{8}\alpha_1^{s0+}M_{3^{d+}1^{s+}} - \tfrac{1}{8}\alpha_1^{p2+}M_{3^{d+}3^{d+}} + \tfrac{3}{8}\alpha_0^{s0+}M_{3^{d+}0^{s+}} + \tfrac{3}{8}\alpha_0^{p2+}M_{3^{d+}2^{d+}} + \tfrac{3}{8}\alpha_1^{d2+}M_{3^{d+}3^{d+}} + \tfrac{3}{8}\alpha_1^{p1-}M_{3^{d+}2^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2+}M_{3^{d+}2^{d+}} + \tfrac{3}{8}\alpha_0^{p1-}M_{3^{d+}1^{p-}} \qquad 3^{d+}\\
\alpha_0^{s0+} &= \lambda + \tfrac{3}{8}\alpha_1^{s0+}M_{0^{s+}1^{s+}} + \tfrac{3}{8}\alpha_1^{p2+}M_{0^{s+}3^{d+}} - \tfrac{1}{8}\alpha_0^{s0+}M_{0^{s+}0^{s+}} - \tfrac{1}{8}\alpha_0^{p2+}M_{0^{s+}2^{d+}} + \tfrac{3}{8}\alpha_1^{d2+}M_{0^{s+}3^{d+}} + \tfrac{3}{8}\alpha_1^{p1-}M_{0^{s+}2^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2+}M_{0^{s+}2^{d+}} + \tfrac{3}{8}\alpha_0^{p1-}M_{0^{s+}1^{p-}} \qquad 0^{s+}\\
\alpha_0^{p2+} &= \lambda + \tfrac{3}{8}\alpha_1^{s0+}M_{2^{d+}1^{s+}} + \tfrac{3}{8}\alpha_1^{p2+}M_{2^{d+}3^{d+}} - \tfrac{1}{8}\alpha_0^{s0+}M_{2^{d+}0^{s+}} - \tfrac{1}{8}\alpha_0^{p2+}M_{2^{d+}2^{d+}} + \tfrac{3}{8}\alpha_1^{d2+}M_{2^{d+}3^{d+}} + \tfrac{3}{8}\alpha_1^{p1-}M_{2^{d+}2^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2+}M_{2^{d+}2^{d+}} + \tfrac{3}{8}\alpha_0^{p1-}M_{2^{d+}1^{p-}} \qquad 2^{d+}\\
\alpha_1^{d2+} &= \lambda + \tfrac{3}{8}\alpha_1^{s0+}M_{3^{d+}1^{s+}} + \tfrac{3}{8}\alpha_1^{p2+}M_{3^{d+}3^{d+}} + \tfrac{3}{8}\alpha_0^{s0+}M_{3^{d+}0^{s+}} + \tfrac{3}{8}\alpha_0^{p2+}M_{3^{d+}2^{d+}} - \tfrac{1}{8}\alpha_1^{d2+}M_{3^{d+}3^{d+}} - \tfrac{1}{8}\alpha_1^{p1-}M_{3^{d+}2^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2+}M_{3^{d+}2^{d+}} + \tfrac{3}{8}\alpha_0^{p1-}M_{3^{d+}1^{p-}} \qquad 3^{d+}\\
\alpha_1^{p1-} &= \lambda + \tfrac{3}{8}\alpha_1^{s0+}M_{2^{p-}1^{s+}} + \tfrac{3}{8}\alpha_1^{p2+}M_{2^{p-}3^{d+}} + \tfrac{3}{8}\alpha_0^{s0+}M_{2^{p-}0^{s+}} + \tfrac{3}{8}\alpha_0^{p2+}M_{2^{p-}2^{d+}} - \tfrac{1}{8}\alpha_1^{d2+}M_{2^{p-}3^{d+}} - \tfrac{1}{8}\alpha_1^{p1-}M_{2^{p-}2^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2+}M_{2^{p-}2^{d+}} + \tfrac{3}{8}\alpha_0^{p1-}M_{2^{p-}1^{p-}} \qquad 2^{p-}\\
\alpha_0^{d2+} &= \lambda + \tfrac{3}{8}\alpha_1^{s0+}M_{2^{d+}1^{s+}} + \tfrac{3}{8}\alpha_1^{p2+}M_{2^{d+}3^{d+}} + \tfrac{3}{8}\alpha_0^{s0+}M_{2^{d+}0^{s+}} + \tfrac{3}{8}\alpha_0^{p2+}M_{2^{d+}2^{d+}} + \tfrac{3}{8}\alpha_1^{d2+}M_{2^{d+}3^{d+}} + \tfrac{3}{8}\alpha_1^{p1-}M_{2^{d+}2^{p-}}\\
&\quad - \tfrac{1}{8}\alpha_0^{d2+}M_{2^{d+}2^{d+}} - \tfrac{1}{8}\alpha_0^{p1-}M_{2^{d+}1^{p-}} \qquad 2^{d+}\\
\alpha_0^{p1-} &= \lambda + \tfrac{3}{8}\alpha_1^{s0+}M_{1^{p-}1^{s+}} + \tfrac{3}{8}\alpha_1^{p2+}M_{1^{p-}3^{d+}} + \tfrac{3}{8}\alpha_0^{s0+}M_{1^{p-}0^{s+}} + \tfrac{3}{8}\alpha_0^{p2+}M_{1^{p-}2^{d+}} + \tfrac{3}{8}\alpha_1^{d2+}M_{1^{p-}3^{d+}} + \tfrac{3}{8}\alpha_1^{p1-}M_{1^{p-}2^{p-}}\\
&\quad - \tfrac{1}{8}\alpha_0^{d2+}M_{1^{p-}2^{d+}} - \tfrac{1}{8}\alpha_0^{p1-}M_{1^{p-}1^{p-}}. \qquad 1^{p-}
\end{aligned} \tag{A28}$$

$\Sigma\ \frac{5}{2}^+$ (8, 2) (70, 2^+):

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda - \tfrac{1}{8}\alpha_1^{s0s+}M_{1^{s+}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{1^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{1^{s+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{1^{s+}2_s^{d+}} + \tfrac{3}{8}\alpha_1^{d2s+}M_{1^{s}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{1^{s+}2_s^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2s+}M_{1^{s+}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{1^{s+}1_s^{p-}} \qquad 1^{s+}\\
\alpha_1^{p2+} &= \lambda - \tfrac{1}{8}\alpha_1^{s0s+}M_{3^{d+}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{3^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{3^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{3^{d+}2_s^{d+}} + \tfrac{3}{8}\alpha_1^{d2s+}M_{3^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{3^{d+}2_s^{p-}}\\
&\quad + \tfrac{3}{8}\alpha_0^{d2s+}M_{3^{d+}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{3^{d+}1_s^{p-}} \qquad 3^{d+}
\end{aligned}$$

$$\begin{aligned}\alpha_1^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{1_s^{s+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{1_s^{s+}3^{d+}} - \tfrac{5}{8}\alpha_1^{s0s+}M_{1_s^{s+}1_s^{s+}} - \tfrac{5}{8}\alpha_0^{p2s+}M_{1_s^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{1_s^{s+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{1_s^{s+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{1_s^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{1_s^{s+}2_s^{p-}} + \tfrac{3}{8}\alpha_0^{d2s+}M_{1_s^{s+}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{1_s^{s+}1_s^{p-}} \qquad 1_s^{s+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{3_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{3_s^{d+}3^{d+}} - \tfrac{5}{8}\alpha_1^{s0s+}M_{3_s^{d+}1_s^{s+}} - \tfrac{5}{8}\alpha_0^{p2s+}M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{3_s^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{3_s^{d+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{3_s^{d+}2_s^{p-}} + \tfrac{3}{8}\alpha_0^{d2s+}M_{3_s^{d+}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{3_s^{d+}1_s^{p-}} \qquad 3_s^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_0^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{0_s^{s+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{0_s^{s+}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{0_s^{s+}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{0_s^{s+}3_s^{d+}} - \tfrac{1}{8}\alpha_0^{s0s+}M_{0_s^{s+}0_s^{s+}} - \tfrac{1}{8}\alpha_0^{p2s+}M_{0_s^{s+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{0_s^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{0_s^{s+}2_s^{p-}} + \tfrac{3}{8}\alpha_0^{d2s+}M_{0_s^{s+}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{0_s^{s+}1_s^{p-}} \qquad 0_s^{s+}\end{aligned}$$

$$\begin{aligned}\alpha_0^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{2_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{2_s^{d+}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{2_s^{d+}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{2_s^{d+}3_s^{d+}} - \tfrac{1}{8}\alpha_0^{s0s+}M_{2_s^{d+}0_s^{s+}} - \tfrac{1}{8}\alpha_0^{p2s+}M_{2_s^{d+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{2_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{2_s^{d+}2_s^{p-}} + \tfrac{3}{8}\alpha_0^{d2s+}M_{2_s^{d+}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{2_s^{d+}1_s^{p-}} \qquad 2_s^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{3_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{3_s^{d+}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{3_s^{d+}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{3_s^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{3_s^{d+}2_s^{d+}} \\ &\quad - \tfrac{1}{8}\alpha_1^{d2s+}M_{3_s^{d+}3_s^{d+}} - \tfrac{1}{8}\alpha_1^{p1s-}M_{3_s^{d+}2_s^{p-}} + \tfrac{3}{8}\alpha_0^{d2s+}M_{3_s^{d+}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{3_s^{d+}1_s^{p-}} \qquad 3_s^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{p1s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{2_s^{p-}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{2_s^{p-}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{2_s^{p-}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{2_s^{p-}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{2_s^{p-}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{2_s^{p-}2_s^{d+}} \\ &\quad - \tfrac{1}{8}\alpha_1^{d2s+}M_{2_s^{p-}3_s^{d+}} - \tfrac{1}{8}\alpha_1^{p1s-}M_{2_s^{p-}2_s^{p-}} + \tfrac{3}{8}\alpha_0^{d2s+}M_{2_s^{p-}2_s^{d+}} + \tfrac{3}{8}\alpha_0^{p1s-}M_{2_s^{p-}1_s^{p-}} \qquad 2_s^{p-}\end{aligned}$$

$$\begin{aligned}\alpha_0^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{2_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{2_s^{d+}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{2_s^{d+}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{2_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{2_s^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{2_s^{d+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{2_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{2_s^{d+}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{2_s^{d+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{2_s^{d+}1_s^{p-}} \qquad 2_s^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_0^{p1s-} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{1_s^{p-}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{1_s^{p-}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{1_s^{p-}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{1_s^{p-}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{1_s^{p-}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{1_s^{p-}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{1_s^{p-}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{1_s^{p-}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{1_s^{p-}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{1_s^{p-}1_s^{p-}} \qquad 1_s^{p-}.\end{aligned} \tag{A29}$$

$\Lambda\,\frac{5}{2}^{+}$ (8, 2) (70, 2^+):

$$\begin{aligned}\alpha_1^{s0+} &= \lambda + \tfrac{3}{8}\alpha_1^{s0s+}M_{1^{s+}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+}M_{1^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{1^{s+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{1^{s+}2_s^{d+}} + \tfrac{3}{8}\alpha_1^{d2s+}M_{1^{s}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{1^{s+}2_s^{p-}} \\ &\quad - \tfrac{1}{8}\alpha_0^{d2s+}M_{1^{s+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{1^{s+}1_s^{p-}} \qquad 1^{s+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{p2+} &= \lambda + \tfrac{3}{8}\alpha_1^{s0s+}M_{3^{d+}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+}M_{3^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{3^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{3^{d+}2_s^{d+}} + \tfrac{3}{8}\alpha_1^{d2s+}M_{3^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{3^{d+}2_s^{p-}} \\ &\quad - \tfrac{1}{8}\alpha_0^{d2s+}M_{3^{d+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{3^{d+}1_s^{p-}} \qquad 3^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{1_s^{s+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{1_s^{s+}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{1_s^{s+}1_s^{s+}} - \tfrac{1}{8}\alpha_1^{p2s+}M_{1_s^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{1_s^{s+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{1_s^{s+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{1_s^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{1_s^{s+}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{1_s^{s+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{1_s^{s+}1_s^{p-}} \qquad 1_s^{s+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{3_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{3_s^{d+}3^{d+}} - \tfrac{1}{8}\alpha_1^{s0s+}M_{3_s^{d+}1_s^{s+}} - \tfrac{1}{8}\alpha_0^{p2s+}M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{3_s^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{3_s^{d+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{3_s^{d+}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{3_s^{d+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{3_s^{d+}1_s^{p-}} \qquad 3_s^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_0^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{0_s^{s+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{0_s^{s+}3^{d+}} + \tfrac{3}{8}\alpha_1^{s0s+}M_{0_s^{s+}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+}M_{0_s^{s+}3_s^{d+}} - \tfrac{1}{8}\alpha_0^{s0s+}M_{0_s^{s+}0_s^{s+}} - \tfrac{1}{8}\alpha_0^{p2s+}M_{0_s^{s+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{0_s^{s+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{0_s^{s+}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{0_s^{s+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{0_s^{s+}1_s^{p-}} \qquad 0_s^{s+}\end{aligned}$$

$$\begin{aligned}\alpha_0^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{2_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{2_s^{d+}3^{d+}} + \tfrac{3}{8}\alpha_1^{s0s+}M_{2_s^{d+}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+}M_{2_s^{d+}3_s^{d+}} - \tfrac{1}{8}\alpha_0^{s0s+}M_{2_s^{d+}0_s^{s+}} - \tfrac{1}{8}\alpha_0^{p2s+}M_{2_s^{d+}2_s^{d+}} \\ &\quad + \tfrac{3}{8}\alpha_1^{d2s+}M_{2_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-}M_{2_s^{d+}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{2_s^{d+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{2_s^{d+}1_s^{p-}} \qquad 2_s^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{3_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{3_s^{d+}3^{d+}} + \tfrac{3}{8}\alpha_1^{s0s+}M_{3_s^{d+}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+}M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{3_s^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{3_s^{d+}2_s^{d+}} \\ &\quad - \tfrac{1}{8}\alpha_1^{d2s+}M_{3_s^{d+}3_s^{d+}} - \tfrac{1}{8}\alpha_1^{p1s-}M_{3_s^{d+}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{3_s^{d+}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{3_s^{d+}1_s^{p-}} \qquad 3_s^{d+}\end{aligned}$$

$$\begin{aligned}\alpha_1^{p1s-} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+}M_{2_s^{p-}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+}M_{2_s^{p-}3^{d+}} + \tfrac{3}{8}\alpha_1^{s0s+}M_{2_s^{p-}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+}M_{2_s^{p-}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+}M_{2_s^{p-}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+}M_{2_s^{p-}2_s^{d+}} \\ &\quad - \tfrac{1}{8}\alpha_1^{d2s+}M_{2_s^{p-}3_s^{d+}} - \tfrac{1}{8}\alpha_1^{p1s-}M_{2_s^{p-}2_s^{p-}} - \tfrac{1}{8}\alpha_0^{d2s+}M_{2_s^{p-}2_s^{d+}} - \tfrac{1}{8}\alpha_0^{p1s-}M_{2_s^{p-}1_s^{p-}} \qquad 2_s^{p-}\end{aligned}$$

$$\begin{aligned}
\alpha_0^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+} M_{2_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+} M_{2_s^{d+}3^{d+}} + \tfrac{3}{8}\alpha_1^{s0s+} M_{2_s^{d+}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+} M_{2_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+} M_{2_s^{d+}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+} M_{2_s^{d+}2_s^{d+}} \\
&\quad + \tfrac{3}{8}\alpha_1^{d2s+} M_{2_s^{d+}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-} M_{2_s^{d+}2_s^{p-}} - \tfrac{5}{8}\alpha_0^{d2s+} M_{2_s^{d+}2_s^{d+}} - \tfrac{5}{8}\alpha_0^{p1s-} M_{2_s^{d+}1_s^{p-}} \qquad 2_s^{d+} \\
\alpha_0^{p1s-} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+} M_{1_s^{p-}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+} M_{1_s^{p-}3^{d+}} + \tfrac{3}{8}\alpha_1^{s0s+} M_{1_s^{p-}1_s^{s+}} + \tfrac{3}{8}\alpha_1^{p2s+} M_{1_s^{p-}3_s^{d+}} + \tfrac{3}{8}\alpha_0^{s0s+} M_{1_s^{p-}0_s^{s+}} + \tfrac{3}{8}\alpha_0^{p2s+} M_{1_s^{p-}2_s^{d+}} \\
&\quad + \tfrac{3}{8}\alpha_1^{d2s+} M_{1_s^{p-}3_s^{d+}} + \tfrac{3}{8}\alpha_1^{p1s-} M_{1_s^{p-}2_s^{p-}} - \tfrac{5}{8}\alpha_0^{d2s+} M_{1_s^{p-}2_s^{d+}} - \tfrac{5}{8}\alpha_0^{p1s-} M_{1_s^{p-}1_s^{p-}} \qquad 1_s^{p-}.
\end{aligned} \tag{A30}$$

3. Multiplet (8, 4)

$N\,\frac{7}{2}^+$ (8, 4) (70, 2^+):

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0+} M_{1^{s+}1^{s+}} + \tfrac{1}{4}\alpha_1^{p2+} M_{1^{s+}3^{d+}} + \tfrac{3}{4}\alpha_1^{d2+} M_{1^{s+}3^{d+}} + \tfrac{3}{4}\alpha_1^{p1-} M_{1^{s+}2^{p-}} \qquad 1^{s+} \\
\alpha_1^{p2+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0+} M_{3^{d+}1^{s+}} + \tfrac{1}{4}\alpha_1^{p2+} M_{3^{d+}3^{d+}} + \tfrac{3}{4}\alpha_1^{d2+} M_{3^{d+}3^{d+}} + \tfrac{3}{4}\alpha_1^{p1-} M_{3^{d+}2^{p-}} \qquad 3^{d+} \\
\alpha_1^{d2+} &= \lambda + \tfrac{3}{4}\alpha_1^{s0+} M_{3^{d+}1^{s+}} + \tfrac{3}{4}\alpha_1^{p2+} M_{3^{d+}3^{d+}} + \tfrac{1}{4}\alpha_1^{d2+} M_{3^{d+}3^{d+}} + \tfrac{1}{4}\alpha_1^{p1-} M_{3^{d+}2^{p-}} \qquad 3^{d+} \\
\alpha_1^{p1-} &= \lambda + \tfrac{3}{4}\alpha_1^{s0+} M_{2^{p-}1^{s+}} + \tfrac{3}{4}\alpha_1^{p2+} M_{2^{p-}3^{d+}} + \tfrac{1}{4}\alpha_1^{d2+} M_{2^{p-}3^{d+}} + \tfrac{1}{4}\alpha_1^{p1-} M_{2^{p-}2^{p-}}. \qquad 2^{p-}
\end{aligned} \tag{A31}$$

$\Sigma\,\frac{7}{2}^+$ (8, 4) (70, 2^+):

$$\begin{aligned}
\alpha_1^{s0+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0s+} M_{1^{s+}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+} M_{1^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{1^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{1^{s+}2_s^{p-}} \qquad 1^{s+} \\
\alpha_1^{p2+} &= \lambda + \tfrac{1}{4}\alpha_1^{s0s+} M_{3^{d+}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+} M_{3^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{3^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{3^{d+}2_s^{p-}} \qquad 3^{d+} \\
\alpha_1^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+} M_{1_s^{s+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+} M_{1_s^{s+}3^{d+}} - \tfrac{1}{4}\alpha_1^{s0s+} M_{1_s^{s+}1_s^{s+}} - \tfrac{1}{4}\alpha_1^{p2s+} M_{1_s^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{1_s^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{1_s^{s+}2_s^{p-}} \qquad 1_s^{s+} \\
\alpha_1^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+} M_{3_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+} M_{3_s^{d+}3^{d+}} - \tfrac{1}{4}\alpha_1^{s0s+} M_{3_s^{d+}1_s^{s+}} - \tfrac{1}{4}\alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \qquad 3_s^{d+} \\
\alpha_1^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+} M_{3_s^{d+}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+} M_{3_s^{d+}3^{d+}} + \tfrac{1}{4}\alpha_1^{s0s+} M_{3_s^{d+}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \qquad 3_s^{d+} \\
\alpha_1^{p1s-} &= \lambda + \tfrac{1}{2}\alpha_1^{s0+} M_{2_s^{p-}1^{s+}} + \tfrac{1}{2}\alpha_1^{p2+} M_{2_s^{p-}3^{d+}} + \tfrac{1}{4}\alpha_1^{s0s+} M_{2_s^{p-}1_s^{s+}} + \tfrac{1}{4}\alpha_1^{p2s+} M_{2_s^{p-}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{d2s+} M_{2_s^{p-}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{p1s-} M_{2_s^{p-}2_s^{p-}}. \qquad 2_s^{p-}
\end{aligned} \tag{A32}$$

$\Lambda\,\frac{7}{2}^+$ (8, 4) (70, 2^+):

$$\begin{aligned}
\alpha_1^{d2+} &= \lambda + \tfrac{1}{4}\alpha_1^{d2s+} M_{3^{d+}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{p1s-} M_{3^{d+}2_s^{p-}} + \tfrac{3}{4}\alpha_1^{s0s+} M_{3^{d+}1_s^{s+}} + \tfrac{3}{4}\alpha_1^{p2s+} M_{3^{d+}3_s^{d+}} \qquad 3^{d+} \\
\alpha_1^{p1-} &= \lambda + \tfrac{1}{4}\alpha_1^{d2s+} M_{2^{p-}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{p1s-} M_{2^{p-}2_s^{p-}} + \tfrac{3}{4}\alpha_1^{s0s+} M_{2^{p-}1_s^{s+}} + \tfrac{3}{4}\alpha_1^{p2s+} M_{2^{p-}3_s^{d+}} \qquad 2^{p-} \\
\alpha_1^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{d2+} M_{3_s^{d+}3^{d+}} + \tfrac{1}{2}\alpha_1^{p1-} M_{3_s^{d+}2^{p-}} + \tfrac{1}{2}\alpha_1^{s0s+} M_{3_s^{d+}1_s^{s+}} + \tfrac{1}{2}\alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}} \qquad 3_s^{d+} \\
\alpha_1^{p1s-} &= \lambda + \tfrac{1}{2}\alpha_1^{d2+} M_{2_s^{p-}3^{d+}} + \tfrac{1}{2}\alpha_1^{p1-} M_{2_s^{p-}2^{p-}} + \tfrac{1}{2}\alpha_1^{s0s+} M_{2_s^{p-}1_s^{s+}} + \tfrac{1}{2}\alpha_1^{p2s+} M_{2_s^{p-}3_s^{d+}} \qquad 2_s^{p-} \\
\alpha_1^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_1^{d2+} M_{1_s^{s+}3^{d+}} + \tfrac{1}{2}\alpha_1^{p1-} M_{1_s^{s+}2^{p-}} + \tfrac{1}{6}\alpha_1^{d2s+} M_{1_s^{s+}3_s^{d+}} + \tfrac{1}{6}\alpha_1^{p1s-} M_{1_s^{s+}2_s^{p-}} + \tfrac{1}{3}\alpha_1^{s0s+} M_{1_s^{s+}1_s^{s+}} + \tfrac{1}{3}\alpha_1^{p2s+} M_{1_s^{s+}3_s^{d+}} \qquad 1_s^{s+} \\
\alpha_1^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_1^{d2+} M_{3_s^{d+}3^{d+}} + \tfrac{1}{2}\alpha_1^{p1-} M_{3_s^{d+}2^{p-}} + \tfrac{1}{6}\alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \tfrac{1}{6}\alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} + \tfrac{1}{3}\alpha_1^{s0s+} M_{3_s^{d+}1_s^{s+}} + \tfrac{1}{3}\alpha_1^{p2s+} M_{3_s^{d+}3_s^{d+}}. \qquad 3_s^{d+}
\end{aligned} \tag{A33}$$

4. Multiplet (1, 2)

$\Lambda\,\frac{5}{2}^+$ (1, 2) (70, 2^+):

$$\begin{aligned}
\alpha_0^{s0+} &= \lambda + \tfrac{1}{4}\alpha_0^{s0s+} M_{0^{s+}0_s^{s+}} + \tfrac{1}{4}\alpha_0^{p2s+} M_{0^{s+}2_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{0^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{0^{s+}2_s^{p-}} \qquad 0^{s+} \\
\alpha_0^{p2+} &= \lambda + \tfrac{1}{4}\alpha_0^{s0s+} M_{2^{d+}0_s^{s+}} + \tfrac{1}{4}\alpha_0^{p2s+} M_{2^{d+}2_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{2^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{2^{d+}2_s^{p-}} \qquad 2^{d+} \\
\alpha_0^{s0s+} &= \lambda + \tfrac{1}{2}\alpha_0^{s0+} M_{0_s^{s+}0^{s+}} + \tfrac{1}{2}\alpha_0^{p2+} M_{0_s^{s+}2^{d+}} - \tfrac{1}{4}\alpha_0^{s0s+} M_{0_s^{s+}0_s^{s+}} - \tfrac{1}{4}\alpha_0^{p2s+} M_{0_s^{s+}2_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{0_s^{s+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{0_s^{s+}2_s^{p-}} \qquad 0_s^{s+} \\
\alpha_0^{p2s+} &= \lambda + \tfrac{1}{2}\alpha_0^{s0+} M_{2_s^{d+}0^{s+}} + \tfrac{1}{2}\alpha_0^{p2+} M_{2_s^{d+}2^{d+}} - \tfrac{1}{4}\alpha_0^{s0s+} M_{2_s^{d+}0_s^{s+}} - \tfrac{1}{4}\alpha_0^{p2s+} M_{2_s^{d+}2_s^{d+}} + \tfrac{3}{4}\alpha_1^{d2s+} M_{2_s^{d+}3_s^{d+}} + \tfrac{3}{4}\alpha_1^{p1s-} M_{2_s^{d+}2_s^{p-}} \qquad 2_s^{d+} \\
\alpha_1^{d2s+} &= \lambda + \tfrac{1}{2}\alpha_0^{s0+} M_{3_s^{d+}0^{s+}} + \tfrac{1}{2}\alpha_0^{p2+} M_{3_s^{d+}2^{d+}} + \tfrac{1}{4}\alpha_0^{s0s+} M_{3_s^{d+}0_s^{s+}} + \tfrac{1}{4}\alpha_0^{p2s+} M_{3_s^{d+}2_s^{d+}} + \tfrac{1}{4}\alpha_1^{d2s+} M_{3_s^{d+}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{p1s-} M_{3_s^{d+}2_s^{p-}} \qquad 3_s^{d+} \\
\alpha_1^{p1s-} &= \lambda + \tfrac{1}{2}\alpha_0^{s0+} M_{2_s^{p-}0^{s+}} + \tfrac{1}{2}\alpha_0^{p2+} M_{2_s^{p-}2^{d+}} + \tfrac{1}{4}\alpha_0^{s0s+} M_{2_s^{p-}0_s^{s+}} + \tfrac{1}{4}\alpha_0^{p2s+} M_{2_s^{p-}2_s^{d+}} + \tfrac{1}{4}\alpha_1^{d2s+} M_{2_s^{p-}3_s^{d+}} + \tfrac{1}{4}\alpha_1^{p1s-} M_{2_s^{p-}2_s^{p-}}. \qquad 2_s^{p-}
\end{aligned} \tag{A34}$$

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