# ${\bm C}{\bm P}$  violation at one loop in the polarization-independent chargino production in  $e^+e^-$  collisions

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Recently Osland and Vereshagin noticed, based on sample calculations of some box diagrams, that in unpolarized  $e^+e^-$  collisions *CP*-odd effects in the nondiagonal chargino-pair production process are generated at one loop. Here we perform a full one-loop analysis of these effects and point out that in some cases the neglected vertex and self-energy contributions may play a dominant role. We also show that *CP* asymmetries in chargino production are sensitive not only to the phase of  $\mu$  parameter in the chargino sector but also to the phase of stop trilinear coupling *At*.

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# **I. INTRODUCTION**

The electroweak sector of the standard model (SM) contains only one *CP*-violating phase which arises in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Adding right-handed neutrinos to account for nonzero neutrino masses and their mixing opens up a possibility of new *CP*-violating phases in the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix. While the observed amount of *CP* violation in the *K* and *B* can be accommodated within the SM, another (indirect) piece of evidence of *CP* violation, the baryon asymmetry in the universe, requires a new source of *CP* violation [[1\]](#page-6-0). Thus new *CP*-violating phases must exist in nature.

Supersymmetric extensions of the SM introduce a plethora of *CP* phases in soft supersymmetry breaking terms. This poses a SUSY *CP* problem, since if the phases are large  $\mathcal{O}(1)$ , SUSY contributions to the lepton and neutron electric dipole moments (EDMs) can be too large to satisfy current experimental constraints [\[2\]](#page-6-1). Many models have been proposed [\[3](#page-6-2)] to overcome this problem: fine tune phases to be small, push sparticle spectra (especially squarks and sleptons) above a TeV scale to suppress effects of large phases on the EDM, arrange for internal cancellations, etc.

In the absence of any reliable theory that forces in a natural way the phases to be vanishing or small, it is mandatory to consider scenarios with some of the phases large and arranged consistent with experimental EDM data. In such *CP*-violating scenarios charginos and neutralinos (denoted generically by  $\tilde{\chi}$ ) might be light enough to be produced at  $e^+e^-$  colliders, and many phenomena will be affected by nonvanishing phases: sparticle masses, their decay rates and production cross sections, SUSY contributions to SM processes, etc. However, the most unambiguous way to study the presence of *CP*-violating phases would be in some *CP*-odd observables measurable at future accelerators.

To build a  $CP$ -odd observable in a two-fermion  $\rightarrow$ two-fermion process, e.g.  $e^+e^- \rightarrow \tilde{\chi}_i \tilde{\chi}_j$ , typically one uses spin information of one of the particles involved. For example, a measurement of the fermion polarization *s* transverse to the production plane [[4\]](#page-6-3) allows one to build a *CP*-odd observable  $s \cdot (p_e \times p_{\tilde{\chi}})$ . This requires either transverse beam polarization and/or spin-analyzer of produced  $\tilde{\chi}$ 's via angular distributions of their decay products [\[5\]](#page-6-4). Another possibility is to look into triple products involving momenta of the decay products of charginos in case of longitudinal polarization of the beams [[6\]](#page-7-0).

However, *CP*-odd effects can also be detected in simple event-counting experiments if several processes are measured. One example is provided by nondiagonal neutralinopair production in  $e^+e^-$  annihilation with unpolarized beams: observing the  $\tilde{\chi}^0_i \tilde{\chi}^0_j$ ,  $\tilde{\chi}^0_i \tilde{\chi}^0_k$  and  $\tilde{\chi}^0_j \tilde{\chi}^0_k$  pairs to be excited *all* in S-wave near respective thresholds signals *CP* violation in the neutralino sector at tree level [[7\]](#page-7-1). Alternatively, unambiguous evidence for *CP* violation in the neutralino system is provided by the observation of simultaneous sharp S-wave excitations of both the production of any nondiagonal neutralino pair  $\tilde{\chi}^0_i \tilde{\chi}^0_j$  near threshold and the  $f\bar{f}$  invariant mass distribution of the decay  $\tilde{\chi}^0_j \rightarrow \tilde{\chi}^0_i f \bar{f}$  near the end point [[8\]](#page-7-2).

Recently Osland and Vereshagin pointed out that in the nondiagonal chargino-pair production process  $e^+e^- \rightarrow$  $\tilde{\chi}_1^{\pm} \tilde{\chi}_2^{\mp}$  a *CP*-odd observable can be constructed from unpolarized cross sections at one loop [[9\]](#page-7-3). Their simplified numerical analysis based on only some of the box diagrams shows that, indeed, the *CP* violation induced by the complex Higgsino mass parameter  $\mu$  may in principle be observed in this reaction without any spin detection and with unpolarized initial beams.

In this note we perform a full one-loop analysis of the nondiagonal chargino-pair production. First we recapitulate the minimal supersymmetric standard model (MSSM) chargino sector at tree level and show explicitly that such a *CP* asymmetry vanishes. In Sec. III we discuss the *CP* asymmetry at one loop. We note that a nonzero asymmetry requires not only complex couplings but also absorptive parts of Feynman diagrams. In Sec. IV we present numerical results for the *CP* asymmetry and discuss relative weights of various contributions. We consider effects of

both the complex Higgsino mass parameter and the complex trilinear scalar coupling in the top squark sector. Section V summarizes and concludes our analysis.

### **II. MSSM CHARGINO SECTOR AT TREE LEVEL**

In the MSSM, the tree-level mass matrix of the spin-1/2 partners of the charged gauge and Higgs bosons,  $\tilde{W}^-$  and  $\tilde{H}^-$ , takes the form

$$
\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \cos\beta \\ \sqrt{2}m_W \sin\beta & \mu \end{pmatrix}, \quad (1)
$$

where  $M_2$  is the SU(2) gaugino mass,  $\mu$  is the Higgsino mass parameter, and  $\tan\beta$  is the ratio  $v_2/v_1$  of the vacuum expectation values of the two neutral Higgs fields. By reparametrization of the fields,  $M_2$  can be taken real and positive, while  $\mu$  can be complex  $\mu = |\mu|e^{i\Phi_{\mu}}$ . Since the chargino mass matrix  $\mathcal{M}_C$  is not symmetric, two different unitary matrices acting on the left- and right-chiral  $(W, H)_{L,R}$  two-component states

$$
U_{L,R}\left(\begin{array}{c}\tilde{W}^-\\\tilde{H}^-\end{array}\right)_{L,R} = \left(\begin{array}{c}\tilde{\chi}_1^-\\\tilde{\chi}_2^-\end{array}\right)_{L,R} \tag{2}
$$

are needed to diagonalize it. The unitary matrices  $U_L$  and  $U_R$  can be parametrized in the following way [\[10\]](#page-7-4) ( $s_\alpha$  =  $\sin\alpha$ ,  $c_{\alpha} = \cos\alpha$ ):

$$
U_L = \begin{pmatrix} c_{\phi_L} & e^{-i\beta_L} s_{\phi_L} \\ -e^{i\beta_L} s_{\phi_L} & c_{\phi_L} \end{pmatrix},
$$
  
\n
$$
U_R = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix} \begin{pmatrix} c_{\phi_R} & e^{-i\beta_R} s_{\phi_R} \\ -e^{i\beta_R} s_{\phi_R} & c_{\phi_R} \end{pmatrix}.
$$
\n(3)

As far as  $\Phi_{\mu}$  dependence is concerned, the mass eigenvalues and rotation angles  $\cos 2\phi_{L,R}$ ,  $\sin 2\phi_{L,R}$ , being  $CP$  even, are functions of  $cos\Phi_{\mu}$  only. On the other hand, the four phases  $\beta_{L,R}$  and  $\gamma_{1,2}$  are *CP* odd since their tangents depend linearly on  $\sin\Phi_\mu$ ; all four phases vanish in the *CP*-invariant case for which  $\Phi_{\mu} = 0$  or  $\pi$ .

Charginos can copiously be produced at prospective  $e^+e^-$  linear colliders [\[11\]](#page-7-5). At tree-level they are produced via the *s*-channel  $\gamma$ , *Z* exchange and *t*-channel electron sneutrino exchange. Photon exchange contributes only to the production of diagonal pairs  $\tilde{\chi}_1^{\pm} \tilde{\chi}_1^{-}$  and  $\tilde{\chi}_2^{\pm} \tilde{\chi}_2^{-}$ . The production amplitude, after a Fierz transformation of the *t*-channel contribution,

$$
\mathcal{A}[e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+] = \frac{e^2}{s} Q_{\alpha\beta}^{ij} [\bar{v}(e^+) \gamma_\mu P_\alpha u(e^-)]
$$
  
 
$$
\times [\bar{u}(\tilde{\chi}_i^-) \gamma^\mu P_\beta v(\tilde{\chi}_j^+)], \tag{4}
$$

is expressed in terms of four bilinear charges  $Q_{\alpha\beta}^{ij}$ , defined by the chiralities  $\alpha$ ,  $\beta = L$ , R of the lepton and chargino currents. The charges take the form

$$
Q_{RL}^{ij} = \delta_{ij} D_R + C_{ij}^L F_R,
$$
  
\n
$$
Q_{LL}^{ij} = \delta_{ij} D_L + C_{ij}^L F_L,
$$
  
\n
$$
Q_{RR}^{ij} = \delta_{ij} D_R + C_{ij}^R F_R,
$$
  
\n
$$
Q_{LR}^{ij} = \delta_{ij} D_L + C_{ij}^R F_L + \frac{D_{\tilde{\nu}}}{4s_W^2} (\delta_{ij} - C_{ij}^R),
$$
\n(5)

with *s*-, *t*-channel propagators  $D_L = 1 + \frac{D_Z}{s_W^2 c_W^2} (s_W^2 - \frac{1}{2}) \times$  $(s_W^2 - \frac{3}{4}), \quad F_L = \frac{D_Z}{4s_W^2 c_W^2} (s_W^2 - \frac{1}{2}), \quad D_R = 1 + \frac{D_Z}{c_W^2} (s_W^2 - \frac{3}{4}),$  $F_R = \frac{D_Z}{4c_w^2}$ ,  $D_Z = s/(s - m_Z^2)$ ,  $D_{\tilde{\nu}} = s/(t - m_{\tilde{\nu}}^2)$ ; the  $\tilde{\nu}_e$ exchange contributes only to the *LR* amplitude. The coefficients  $C_{ij}^L$  are functions of  $U_L$  as follows

<span id="page-1-0"></span>
$$
C_{11}^{L} = -\cos 2\phi_{L}, \qquad C_{22}^{L} = \cos 2\phi_{L},
$$
  
\n
$$
C_{12}^{L} = e^{-i\beta_{L}} \sin 2\phi_{L}, \qquad C_{21}^{L} = e^{i\beta_{L}} \sin 2\phi_{L},
$$
\n(6)

for  $C^R$  replace  $\phi_L \rightarrow \phi_R$  and  $\beta_L \rightarrow \beta_R - \gamma_1 + \gamma_2$ .

Note that the phases  $\beta_L$ ,  $\beta_R$ ,  $\gamma_1$ ,  $\gamma_2$  enter only nondiagonal  $\{12\}$  and  $\{21\}$  amplitudes. However, after summing over chargino helicities, the dependence on these phases disappears in the polarized differential cross section for the  $e^+e^- \rightarrow \tilde{\chi}_i^- \tilde{\chi}_j^+$ . Defining the polar angle  $\theta$  and the azimuthal angle  $\phi$  of  $\tilde{\chi}^-_i$  with respect to the  $e^-$  momentum direction and the  $e^{-}$  transverse polarization vector, respectively, the polarized differential cross section is given by [\[10\]](#page-7-4)

$$
d \sigma^{ij} = \frac{d\sigma^{\{ij\}}}{d\cos\theta d\phi}
$$
  
= 
$$
\frac{\alpha^2}{16s} \lambda^{1/2} [(1 - P_L \bar{P}_L) \Sigma_{\text{unp}} + (P_L - \bar{P}_L) \Sigma_L
$$
  
+ 
$$
P_T \bar{P}_T \cos(2\phi - \eta) \Sigma_T ] \tag{7}
$$

where  $P = (P_T, 0, P_L) [\bar{P} = (\bar{P}_T \cos \eta, \bar{P}_T \sin \eta, -\bar{P}_L)]$  is the electron [positron] polarization vector;  $\lambda =$  $[1 - (\mu_i + \mu_j)^2][1 - (\mu_i - \mu_j)^2]$  with  $\mu_i = m_i/\sqrt{s}$ . The distributions  $\Sigma_{\text{unp}}$ ,  $\Sigma_L$  and  $\Sigma_T$  depend only on the polar angle  $\theta$  and can be expressed as (the superscripts  $\{ij\}$ labeling the produced chargino pair are understood)

$$
\Sigma_{\text{unp}} = 4\{[1 - (\mu_i^2 - \mu_j^2)^2 + \lambda \cos^2 \theta]Q_1 + 4\mu_i \mu_j Q_2 + 2\lambda^{1/2} Q_3 \cos \theta\},
$$
  
\n
$$
\Sigma_{LL} = 4\{[1 - (\mu_i^2 - \mu_j^2)^2 + \lambda \cos^2 \theta]Q_1' + 4\mu_i \mu_j Q_2' + 2\lambda^{1/2} Q_3' \cos \theta\},
$$
  
\n
$$
\Sigma_{TT} = -4\lambda \sin^2 \theta Q_5.
$$
 (8)

The eight quartic charges for each of the production processes of the diagonal and mixed chargino pairs, expressed in terms of bilinear charges, are collected in Table [I,](#page-2-0) including the transformation properties under *P* and *CP*.

The charges  $Q_1$  to  $Q_5$  are manifestly parity even,  $Q'_1$  to  $Q_3'$  are parity odd. The charges  $Q_1$  to  $Q_3$ ,  $Q_5$ , and  $Q_1'$  to  $Q_3'$ 

<span id="page-2-0"></span>*CP* VIOLATION AT ONE LOOP IN THE ... PHYSICAL REVIEW D **76,** 115006 (2007)

TABLE I. The quartic charges of the chargino system.

$\boldsymbol{P}$	$\overline{CP}$	Quartic charges				
Even	Even	$Q_1 = \frac{1}{4} [  Q_{RR} ^2 +  Q_{LL} ^2 +  Q_{RL} ^2 +  Q_{LR} ^2 ]$ $Q_2 = \frac{1}{2} \text{Re} [Q_{RR} Q_{RL}^* + Q_{LL} Q_{LR}^*]$				
		$Q_3 = \frac{1}{4} [  Q_{RR} ^2 +  Q_{LL} ^2 -  Q_{RL} ^2 -  Q_{LR} ^2 ]$ $Q_5 = \frac{1}{2} \text{Re} [Q_{LR} Q_{RR}^* + Q_{LL} Q_{RL}^*]$				
	<b>Odd</b>	$Q_4 = \frac{1}{2} \text{Im} [Q_{RR} Q_{RI}^* + Q_{LL} Q_{LR}^*]$				
Odd	Even	$Q_1' = \frac{1}{4} [  Q_{RR} ^2 +  Q_{RL} ^2 -  Q_{LR} ^2 -  Q_{LL} ^2 ]$ $Q_2' = \frac{1}{2} \text{Re} [Q_{RR} Q_{RL}^* - Q_{LL} Q_{LR}^*]$ $Q_3' = \frac{1}{4} [  Q_{RR} ^2 +  Q_{LR} ^2 -  Q_{RL} ^2 -  Q_{LL} ^2 ]$				

are *CP* invariant. Only *Q*<sup>4</sup> changes sign under *CP* transformations.

<span id="page-2-1"></span>From the above expressions it is evident that even for transverse beam polarization the differential cross section  $\sim \Sigma_{TT}$  is *CP* even. Also the differential distributions for

nondiagonal chargino pairs  $\tilde{\chi}_1^- \tilde{\chi}_2^+$  and  $\tilde{\chi}_2^- \tilde{\chi}_1^+$  are equal, so the asymmetry

$$
A_{12} = \frac{\int_{-1}^{1} (\mathrm{d}\sigma^{\{12\}} - \mathrm{d}\sigma^{\{21\}}) \mathrm{d}\cos\theta}{\int_{-1}^{1} (\mathrm{d}\sigma^{\{12\}} + \mathrm{d}\sigma^{\{21\}}) \mathrm{d}\cos\theta} \tag{9}
$$

at tree level vanishes in *CP*-noninvariant theories. The *CP*-odd quartic charge  $Q_4$  can only be probed by observables sensitive to the chargino polarization component normal to the production plane in mixed  $e^+e^- \rightarrow$  $\tilde{\chi}_1^{\pm} \tilde{\chi}_2^{\mp}$  processes [\[10\]](#page-7-4). Thus, at tree level one cannot build a *CP*-odd observable from chargino polarized cross sections alone.

Because of Poincaré invariance the unpolarized differential cross section  $\sim \Sigma_{\text{unp}}$  may depend only on masses  $m_i$ ,  $m_j$  and on two independent scalar variables *s* and *t*. As a result, the unpolarized differential cross sections for equal-mass fermions  $m_i = m_j$  in the final state are always *CP* even. However, if the chargino species are different,



FIG. 1. Generic triangle graphs contributing to chargino pair  $\tilde{\chi}_2^-\tilde{\chi}_1^+$  production in  $e^+e^-$  collisions.

beyond tree level the *CP*-violating terms can arise even in the unpolarized cross section [\[9](#page-7-3)].

# **III.** *CP***-ODD ASYMMETRY AT ONE-LOOP**

Radiative corrections to the chargino pair production include the following generic one-loop Feynman diagrams: the virtual vertex corrections Fig. [1](#page-2-1), the self-energy corrections to the  $\tilde{\nu}$ , *Z* and  $\gamma$  propagators, and the box diagrams contributions Fig. [2.](#page-3-0) We also have to include corrections on external chargino legs. Generation and calculation of one-loop graphs is performed using FEYNARTS3.2 and FORMCALC5.2 packages [[12](#page-7-6)]. For numerical evaluation of loop integrals we use LOOPTOOLS2.2 [[13\]](#page-7-7).

In the Ref. [\[9\]](#page-7-3) sample calculations of box diagrams with only photon, *Z* and *W* boson exchanges (c.f. diagrams 5 and 10 in Fig. [2](#page-3-0)) and neglecting all sfermion contributions have been performed to demonstrate nonzero asymmetry  $A_{12}$  at one loop. Here we present the full calculation, including all possible contributions at the one-loop level taking into account *CP*-violating phases. Full calculation of radiative corrections to chargino-pair production without *CP*-violating phases can be found in [[14](#page-7-8)].

One-loop corrected matrix element squared is given by

<span id="page-3-0"></span>
$$
|\mathcal{M}_{\text{loop}}|^2 = |\mathcal{M}_{\text{tree}}|^2 + 2 \operatorname{Re}(\mathcal{M}_{\text{tree}}^* \mathcal{M}_{\text{loop}}). \tag{10}
$$

<span id="page-3-1"></span>Accordingly, the one-loop *CP* asymmetry for the nondiagonal chargino pair is defined as

$$
A_{12} = \frac{\int_{-1}^{1} (\mathrm{d}\sigma_{\text{loop}}^{\{12\}} - \mathrm{d}\sigma_{\text{loop}}^{\{21\}}) \mathrm{d}\cos\theta}{\int_{-1}^{1} (\mathrm{d}\sigma_{\text{tree}}^{\{12\}} + \mathrm{d}\sigma_{\text{tree}}^{\{21\}}) \mathrm{d}\cos\theta}.
$$
 (11)

Since, as mentioned in the previous section, the *CP*-odd contribution vanishes at tree level, it has to be UV finite. In fact one can note that the structure of counterterms is the same as the tree-level graphs, so using the same arguments as in Sec. II it can be shown that renormalization procedure will not give rise to the asymmetry. Nevertheless selfenergy and vertex corrections are UV divergent, and proper treatment of divergences is needed. We choose to work in the dimensional reduction scheme [\[15\]](#page-7-9), which preserves supersymmetry.

Loop diagrams with internal photon line also introduce infrared singularities. They can be removed by adding emission of soft photons from external charged particles. The sum of both contributions is then IR finite, however it depends on the soft photon cut. On the other hand soft photon emission part has the form of tree-level amplitude multiplied by soft photon factor  $[16]$  $[16]$  $[16]$ . Therefore, as explained in Sec. II, the terms arising due to soft photon bremsstrahlung do not affect the asymmetry  $A_{12}$ . Similar



FIG. 2. Generic box graphs contributing to chargino pair  $\tilde{\chi}_2^-\tilde{\chi}_1^+$  production in  $e^+e^-$  collisions.

arguments apply for hard photon emission from external fermions.

At this point we want to make some remarks about the origin of the *CP*-odd asymmetry  $A_{12}$ . In order to obtain a nonzero asymmetry in the chargino production it is not enough to have *CP*-violating phases in the MSSM Lagrangian. In addition it requires a nontrivial imaginary part from Feynman diagrams—the absorptive part. Such contributions appear when some of the intermediate state particles in loop diagrams go on-shell. *CP*-odd asymmetry is generated due to the interference between imaginary part of loop integrals and imaginary parts of the couplings [[17\]](#page-7-11). As one can see from Eq.  $(6)$ , the contributions for the production of nondiagonal chargino pairs  $\{12\}$  and  $\{21\}$ differ by the opposite sign of the imaginary part. Since the absorptive parts of loop integrals are the same for both processes, we clearly see that the final *real* contribution to the matrix element squared will be different in each of these final states.

# **IV. NUMERICAL ANALYSIS**

To illustrate relative weights of various contributions to the *CP* asymmetry, we consider two scenarios: (A) which is close to the SPS1a' point that has been studied particularly widely [[18](#page-7-12)]; (B) for comparison with Ref. [[9\]](#page-7-3). In both scenarios the value of the ratio of the vacuum expectation values for Higgs fields is taken to be  $tan \beta = 10$  and the parameters defined below are low-scale parameters.

In scenario A we take the following values for the gaugino and Higgsino mass parameters:

$$
|M_1| = 100 \text{ GeV}, \qquad M_2 = 200 \text{ GeV},
$$
  
 $|\mu| = 400 \text{ GeV}.$ 

For the sfermion mass parameters we assume

$$
m_{\tilde{q}} \equiv M_{\tilde{Q}_{1,2}} = M_{\tilde{U}_{1,2}} = M_{\tilde{D}_{1,2}} = 450 \text{ GeV},
$$
  
\n
$$
M_{\tilde{Q}} \equiv M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = 300 \text{ GeV},
$$
  
\n
$$
m_{\tilde{l}} \equiv M_{\tilde{L}_{1,2,3}} = M_{\tilde{E}_{1,2,3}} = 150 \text{ GeV},
$$

and for the trilinear coupling

$$
|A_t| = -A_b = -A_\tau = A = 400 \text{ GeV}.
$$

Moreover, we allow nonzero phases  $\Phi_{\mu}$  of the  $\mu$  parameter,  $\Phi_1$  of the bino mass parameter  $M_1$  and  $\Phi_t$  of the trilinear coupling in the stop sector  $A_t$ .

In scenario B we take, as in Ref. [[9\]](#page-7-3), the gaugino/ Higgsino masses

$$
|M_1| = 250
$$
 GeV,  $M_2 = 200$  GeV,  
 $|\mu| = 300$  GeV.

For comparison with Ref. [\[9](#page-7-3)] we set the universal scalar mass

$$
M_{S} = M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}} = M_{\tilde{L}} = M_{\tilde{E}} = 10 \text{ TeV},
$$

so the contributions from diagrams with exchanges of supersymmetric scalars are negligibly small. We also investigate the  $M<sub>S</sub>$  dependence of the asymmetry.

In addition, the following values of the SM parameters are used:

$$
m_W = 80.45 \text{ GeV}, \qquad m_Z = 91.1875 \text{ GeV},
$$
  
\n
$$
\cos \theta_W = m_W/m_Z, \qquad m_t = 171 \text{ GeV}, \qquad (12)
$$
  
\n
$$
\alpha = 1/127.9.
$$

The masses of charginos and neutralinos are given in Table [II](#page-4-0), and the masses of stop squarks in scenario A are  $m_{\tilde{t}_1} = 204.9 \text{ GeV}$  and  $m_{\tilde{t}_2} = 438.6 \text{ GeV}$ . The threshold for nondiagonal chargino-pair production is 608*:*5 GeV in scenario A and 510*:*1 GeV in scenario B. Therefore for all plots in the present analysis we take the center of for an piots in the present and<br>mass energy  $\sqrt{s} = 700$  GeV.

First we consider scenario A. The dependence of the *CP* asymmetry on the phase  $\Phi_{\mu}$  of the Higgsino mass parameter  $\mu$  is shown in the left panel of Fig. [3.](#page-5-0) Contributions due to box corrections, vertex corrections and self-energy corrections have been plotted in addition to the full result. The asymmetry can reach values as large as 1%. Box and selfenergy diagrams can give the asymmetry of the order 1.5%–2%, but since they are of opposite signs the total asymmetry tends to be smaller. Moreover, in this scenario the constraints from EDMs restrict the phase  $\Phi_{\mu}$  to be close to  $n\pi$ . For such values the predicted asymmetry is very small and probably unmeasurable even at high luminosity  $e^+e^-$  linear colliders.

For the case of  $CP$  asymmetry induced by the phase  $\Phi_t$ of the trilinear coupling in the top squark sector  $A_t$ , the situation is quite different, as illustrated in the right panel of Fig. [3.](#page-5-0) The box diagrams do not give rise to the *CP* asymmetry in this case, since there are no box diagrams with stop exchanges. Diagrams with top squark exchanges appear only in vertex and self-energy corrections. As for vertex corrections, only diagrams of class 1 ( $FFS = bb\tilde{t}_i$ ) and class 2 ( $SSF = \tilde{t}_i \tilde{t}_j b$ ) from Fig. [1](#page-2-1) contribute. The contributions from vertex and self-energies are of the

TABLE II. Masses of charginos and neutralinos.

<span id="page-4-0"></span>

<b>Masses</b>	$m_{\tilde{\nu}^{\pm}}$	$m_{\tilde{\nu}^{\pm}}$	$m_{z0}$	$m_{z0}$	$m_{z,0}$	$m_{z}$
Scenario A		186.7 GeV 421.8 GeV 97.5 GeV 187.0 GeV 405.8 GeV 421.2 GeV				
Scenario B		175.6 GeV 334.5 GeV 172.8 GeV 242.4 GeV 306.5 GeV 341.4 GeV				

<span id="page-5-0"></span>

FIG. 3. CP asymmetry in chargino production in scenario A as a function of  $\Phi_{\mu}$  (left) and  $\Phi_t$  (right): full asymmetry (full line) and contributions from box (dashed), vertex (dotted) and self-energy (dash-dotted) diagrams.

same sign and add coherently to give the full asymmetry of the order 6%—large enough to be measurable in future experiments.

Note that the triangle graphs induce a coupling of the photon field to fermions of different mass. To this coupling the diagrams of class [1](#page-2-1), 2, 3 and 6 with  $V = \gamma$  from Fig. 1 contribute. These diagrams give rise to the *CP* asymmetry of the order 0.1% both for  $\Phi_{\mu}$  and  $\Phi_{t}$ .

In the left panel of Fig. [4](#page-5-1) the dependence of the differential asymmetry  $a_{12}$  [defined as in Eq. ([11](#page-3-1)) but with integrals removed] as a function of the production angle  $\cos\theta$ . For comparison we show the plots for two choices of phases:  $\Phi_{\mu} = 3\pi/2$  (full line) and and  $\Phi_{t} = \pi/2$  (dotted line). Apart from the difference in magnitude, these asymmetries have different dependence on the  $\cos\theta$ :  $\Phi_{\mu}$  asymmetry decreases with  $cos\theta$ , whereas  $\Phi_t$  asymmetry increases from 5.7% to 7%.

It is also interesting to investigate the tan $\beta$  dependence of the various contributions to the asymmetry. It is shown in the right panel of Fig. [4](#page-5-1) for the range tan  $\beta \in [1, 20]$  with  $\Phi_{\mu} = \pi/2$ ,  $\Phi_1 = \Phi_t = 0$ . As tan $\beta$  increases from 1, the absolute values of box and vertex contributions to the *CP* asymmetry increase reaching maxima around  $\tan \beta = 2$ and then drop down. This behavior follows mainly from the structure of the chargino mass matrix and consequently the tan $\beta$  dependence of the chargino mixing angles and phases—imaginary parts of coefficients  $C_{12}^{R,L}$  in Eq. [\(6\)](#page-1-0) rapidly go down for small and large values of  $tan \beta$ . For  $tan \beta = 1$  the full asymmetry is close to 0, although again it is a result of cancellations between various contributions.

We now turn to scenario B as discussed in Ref. [\[9](#page-7-3)]. In the left panel of Fig. [5](#page-6-5) we show the full asymmetry and contributions from box, vertex and self-energy diagrams. The asymmetry at its maximum reaches almost 0.5%, and is significantly smaller than in scenario A. Because sfermions are very heavy at this parameter point, the main contribution to the asymmetry is due to box diagrams with exchanges of vector bosons  $\gamma$ , *Z*, *W*, namely, diagrams of class 5 and 10 with  $FFVV = e\tilde{\chi}_i \gamma Z$ ,  $e\tilde{\chi}_i Z \gamma$ ,  $e\tilde{\chi}_i ZZ$ , and diagram of class 5 with  $FFVV = \nu \tilde{\chi}_i^0 WW$  of Fig. [2.](#page-3-0) Contributions from vertex and self-energy diagrams are significantly smaller and opposite in sign and almost cancel each other. This is the reason why our results are consistent with results obtained by [\[9](#page-7-3)]. In addition, in the

<span id="page-5-1"></span>

FIG. 4. Left panel: CP asymmetry [defined as in Eq.  $(11)$  $(11)$  $(11)$  but without integration] as a function of the polar angle  $\theta$  in scenario A with  $\Phi_{\mu} = 3\pi/2$  (full line), and  $\Phi_{t} = \pi/2$  (dotted line). Right panel: *CP* asymmetry Eq. ([11](#page-3-1)) as a function of tan $\beta$  in chargino production with other parameters as in scenario A: full asymmetry (full line) and contributions from box (dashed), vertex (dotted), selfenergy (dash-dotted) diagrams.

<span id="page-6-5"></span>

FIG. 5. Left panel: CP asymmetry in chargino production in scenario B as a function of  $\Phi_{\mu}$  phase: full asymmetry (full line) and contributions from box (dashed), vertex (dotted), self-energy (dash-dotted) diagrams. Right panel: *CP* asymmetry Eq. [\(11\)](#page-3-1) as a function of  $M_S = M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}} = M_{\tilde{E}} = M_{\tilde{E}}$ . A full line is for the full result and a dotted line is for box contributions only, as in [\[9\]](#page-7-3). Other parameters are taken as in scenario B with  $\Phi_{\mu} = \pi/2$ .

right panel of Fig. [5](#page-6-5) we show the dependence of the full *CP* asymmetry on the universal soft SUSY-breaking scalar mass  $M_S = M_{\tilde{O}} = M_{\tilde{U}} = M_{\tilde{D}} = M_{\tilde{L}} = M_{\tilde{E}}$  and compare it to the approximate result obtained by Osland and Vereshagin, i.e. when only box contributions without sfermion exchanges are included. With increasing  $M<sub>S</sub>$  the full result approaches a constant value, which is slightly lower than the approximate result. This small difference (which  $\alpha$  on  $\sqrt{s}$  is due to vertex and self-energy corrections.

We have shown the influence of the phases  $\Phi_{\mu}$  and  $\Phi_{t}$ on the *CP* asymmetry Eq. [\(11\)](#page-3-1). However one can also introduce *CP* violating phases for the trilinear couplings for other sfermions, e.g. for bottom squarks  $A<sub>b</sub>$  and for tau sleptons  $A_{\tau}$ , as well as for the bino mass parameter  $M_1$  in the neutralino sector. Indeed these can give rise to the *CP* asymmetry. However calculations show that *CP* asymmetries due to these phases are typically very small, e.g. for  $\Phi_1$  of the order 0.1%, so we do not include them here.

# **V. CONCLUSIONS**

In this note we have investigated the nondiagonal chargino-pair production  $e^+e^- \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_2^{\mp}$  at one loop and

calculated the loop-generated *CP* asymmetry. The *CP*-odd observable can be constructed from unpolarized production cross sections alone without the need of measuring chargino polarizations in the final state. Our numerical analyses show that not only the box diagrams but also the vertex and self-energy diagrams can contribute to the *CP* violation if it is induced by the complex Higgsino mass parameter. For the case of *CP* violation in the top squark sector the box diagrams do not contribute at one loop and the asymmetry comes entirely from vertex and self-energy diagrams. The asymmetries can be of the order of a few percent and in principle measurable allowing to discover the *CP*-violating phases via simple event-counting experiments.

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