Unparticles: Scales and high energy probes

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Unparticles from hidden conformal sectors provide qualitatively new possibilities for physics beyond the standard model. In the theoretical framework of minimal models, we clarify the relation between energy scales entering various phenomenological analyses. We show that these relations always counteract the effective field theory intuition that higher dimension operators are more highly suppressed, and that the requirement of a significant conformal window places strong constraints on possible unparticle signals. With these considerations in mind, we examine some of the most robust and sensitive probes and explore novel effects of unparticles on gauge coupling evolution and fermion production at high energy colliders. These constraints are presented both as bounds on four-fermion interaction scales and as constraints on the fundamental parameter space of minimal models.

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I. INTRODUCTION

Among the many candidates for new physics are hidden sectors coupled to the standard model through nonrenormalizable interactions

$$\frac{O_{\mathrm{UV}} O_{\mathrm{SM}}}{M^{m+n-4}}, \qquad (1)$$

where O_{UV} and O_{SM} are hidden sector and standard model operators with mass dimensions m and n, respectively. M is the energy scale characterizing the new physics, which may range from the weak scale to the Planck scale. Hidden sectors that become either weakly coupled or strongly coupled at low energies have several interesting motivations and have been well studied from various viewpoints.

Recently a novel possibility was introduced in Refs. [1,2], which suggested that the hidden sector could be conformal at an energy scale $\Lambda_{\mathcal{U}}$. Conformal hidden sectors have bizarre implications, including, for example, kinematic distributions in the production and decay of standard model particles that have no conventional particle interpretation. This possibility is therefore qualitatively different from other candidates for new physics, and "unparticles," the degrees of freedom of the conformal sector, have recently been the subject of several phenomenological studies [3–23].

Depending on the properties of the unparticle operators, the general interactions of Eq. (1) may generate many specific nonrenormalizable interactions when O_{UV} flows at low energies to an operator O with mass dimension d. As argued in [1,2], these interactions, such as

$$\frac{OH^2}{\Lambda^{d-2}}, \qquad \frac{O^\mu \bar{f}\gamma_\mu f}{\Lambda^{d-1}}, \qquad \frac{OH\bar{f}f}{\Lambda^{d}}, \qquad \frac{OF_{\mu\nu}F^{\mu\nu}}{\Lambda^{d}}, \qquad (2)$$

have implications for a plethora of experiments, and lower bounds on the scales Λ have already been derived by considering a wide range of topics, from anomalous magnetic moments to CP violation to production rates at high energy colliders.

In this paper, we begin by investigating the theoretical framework of unparticles from a general point of view. We note that the scales Λ appearing in Eq. (2) are not identical or even generically comparable. In fact, when expressed in terms of the fundamental energy scales M and $\Lambda_{\mathcal{U}}$, the scales Λ of Eq. (2) are typically hierarchically separated, and this hierarchy always counteracts the standard intuition from effective field theory that operators suppressed by more powers of Λ are less important. Following the work of Refs. [1,2], we explore minimal unparticle models to provide simple frameworks for phenomenological studies. This approach clarifies certain issues. For example, some phenomenological observables become sensitive to arbitrarily high scales Λ; we show that this sensitivity is artificial, and there is no singularity when bounds are expressed in terms of the fundamental parameters M and $\Lambda_{\mathcal{U}}$.

Another essential point is that the conformal symmetry does not generically hold to arbitrarily low energies [13]. In fact, the first operator of Eq. (2) breaks conformal invariance at a scale $\Lambda_{\not{\mathcal{U}}}$ [13]. Experimental probes of the conformal hidden sector must probe energies in the conformal window $\Lambda_{\not{\mathcal{U}}} < E < \Lambda_{\mathcal{U}}$. As we will see, this criterion is very restrictive. In natural models, which we define more precisely below, it implies that only experiments at energies near the weak scale $v \simeq 246$ GeV are viable probes of conformal hidden sectors. Furthermore, requiring a reasonably wide conformal window sets $\Lambda_{\mathcal{U}} \ll M$, which implies that all the couplings of the unparticle sector to the standard model are extremely suppressed and there are no accessible experimental signatures. Experimental signatures in a significant conformal window are possible only if the first coupling of Eq. (2) is absent, for reasons we discuss, or is fine-tuned to unnaturally small values.

Last, we examine in detail several leading constraints on unparticle physics, considering both scalar unparticles and vector unparticles in turn. Given the considerations noted above, we focus on high energy probes, which are most likely to be in the conformal window. In particular, we consider bounds from e^+e^- colliders in the energy range 30 to 200 GeV, and derive bounds on the scales Λ in Eq. (1) and also their implications for the fundamental parameters M and $\Lambda_{\mathcal{U}}$. We also find a novel signature of scalar unparticles. These operators lead to a coupling between the Higgs field and gauge bosons, which in turn leads to a modification of the gauge couplings that currently provides a severe constraint and is in the future potentially observable.

We close with a summary of our main results and outline directions for further work.

II. THEORETICAL FRAMEWORK

A. Scales

Following Refs. [1,2], we assume that in the ultraviolet theory, a hidden sector operator O_{UV} with dimension d_{UV} couples to standard model operators O_n^i with dimension n through the coupling

$$c_n^i \frac{O_{\mathrm{UV}} O_n^i}{M^{d_{\mathrm{UV}}+n-4}}. \tag{3}$$

The hidden sector becomes conformal at energy $\Lambda_{\mathcal{U}}$, and the operator O_{UV} flows to an operator O with dimension d. At low energies, then, the couplings of Eq. (3) flow to

$$c_n^i \frac{O O_n^i \Lambda_{\mathcal{U}}^{d_{\mathrm{UV}}-d}}{M^{d_{\mathrm{UV}}+n-4}} \equiv c_n^i \frac{O O_n^i}{\Lambda_n^{d+n-4}}. \tag{4}$$

The scales Λ_n determine the strengths of the couplings between the unparticle operator and standard model operators of dimension n.

We emphasize that operators of different dimensions couple with different strengths. For example, standard model fermions couple to vector unparticles through interactions

$$c_3^{f_i f_j} \frac{O^\mu \bar{f}_i \gamma_\mu f_j}{\Lambda_3^{d-1}} \tag{5}$$

suppressed by Λ_3. (Note that these operators become almost renormalizable for d near 1.) On the other hand, standard model gauge bosons couple to scalar unparticles through interactions

$$c_4^{F^i} \frac{O F_{\mu\nu}^i F^{i\mu\nu}}{\Lambda_4^d} \tag{6}$$

suppressed by Λ_4, where i labels the gauge group. Standard model fermions may also couple to scalar unparticles through an interaction derived from a 4-point coupling when the Higgs boson gets a vacuum expectation value (VEV) $\langle H \rangle = v \simeq 246$ GeV after electroweak symmetry breaking:

$$c_4^{f_i f_j} \frac{O H \bar{f}_i f_j}{\Lambda_4^d} \to c_4^{f_i f_j} \frac{O v \bar{f}_i f_j}{\Lambda_4^d} \equiv c_4^{f_i f_j} \frac{O \bar{f}_i f_j}{\Lambda_3^{\prime d-1}}, \tag{7}$$

where the last form defines another scale Λ_3', which is sometimes constrained in phenomenological studies. Finally, the Higgs couples through the operator

$$c_2 \Lambda_2^{2-d} O H^2. \tag{8}$$

This coupling is special in many contexts, as it is the unique super-renormalizable coupling to gauge singlet new physics [24,25]. In the present context, it also plays an essential role, because once the Higgs develops a VEV, this operator breaks conformal symmetry in the hidden sector [13]. This breaking occurs at the scale

$$\Lambda_{\not{\mathcal{U}}} = (c_2 \Lambda_2^{2-d} v^2)^{1/(4-d)}. \tag{9}$$

B. Minimal models

To unify the many scales and couplings discussed above, we may assume a single unparticle operator O_{UV} coupling to the standard model. This assumption of a single unparticle operator defines minimal unparticle models, which are fully specified by the fundamental parameters

$$\{S, M, \Lambda_{\mathcal{U}}, d_{\mathrm{UV}}, d, c_2[, c_{n\geq 3}^i]\}, \tag{10}$$

where S is the spin of the hidden sector operator O, and all other parameters are defined in Sec. II A. The many parameters $c_{n\geq 3}^i$ enter into interaction terms. However, as we will see below, very few enter any given process, and for the purposes of setting bounds consistent with conventions in the literature [26,27], we may simply set $|c_{n\geq 3}^i| = \sqrt{2\pi}/e$.

Neglecting the $c_{n\geq 3}^i$, the minimal model is defined by the discrete parameter S, two fundamental scales M and $\Lambda_{\mathcal{U}}$, and three continuous parameters, d_{UV}, d, and c_2. These fundamental parameters completely determine the couplings of unparticles to the standard model, and provide a complete, yet simple, framework for studying the phenomenology and cosmology of unparticles and hidden conformal sectors.

It is convenient to define dimensionless ratios

$$r \equiv \frac{\Lambda_{\mathcal{U}}}{M} \leq 1 \qquad s \equiv \frac{\Lambda_{\mathcal{U}}}{v}. \tag{11}$$

All remaining mass scales of the minimal model are, then, related to the mass scale $\Lambda_{\mathcal{U}}$ through

$$\Lambda_2 = r^{(d_{\mathrm{UV}}-2)/(2-d)} \Lambda_{\mathcal{U}} \tag{12}$$

$$\Lambda_3 = \left(\frac{1}{r}\right)^{(d_{\mathrm{UV}}-1)/(d-1)} \Lambda_{\mathcal{U}} \tag{13}$$

$$\Lambda_4 = \left(\frac{1}{r}\right)^{d_{\mathrm{UV}}/d} \Lambda_{\mathcal{U}} \tag{14}$$

$$\Lambda_3' = \left(\frac{1}{r}\right)^{d_{\rm UV}/(d-1)} s^{1/(d-1)} \Lambda_{\mathcal{U}} \tag{15}$$

$$\Lambda_{\not{\mathcal{U}}} = \left(\frac{c_2 r^{d_{\rm UV}-2}}{s^2}\right)^{1/(4-d)} \Lambda_{\mathcal{U}}, \tag{16}$$

where we have written all expressions with positive exponents, assuming $1 < d < 2 < d_{\rm UV}$.

From these expressions, we see that $\Lambda_2 < M < \Lambda_4 < \Lambda_3$. This hierarchy (partially) offsets the standard intuition of effective field theories that operators suppressed by more powers of the characteristic energy scale are less promising to probe. For example, $\Lambda_4 < \Lambda_3$ implies that gauge couplings probed by, for example, gluon-gluon collisions at hadron colliders, may yield more promising signals than couplings to fermions.

Note that we have assumed $r \leq 1$, that is, that the theory enters the conformal regime below the scale where it is coupled to the standard model. This is required for theoretical consistency. If $\Lambda_{\mathcal{U}} > M$, then the theory is already conformal at the scale M, and so the effective couplings Λ_n should be replaced simply by M. This is equivalent to taking $\Lambda_{\mathcal{U}} = M$ in all the formulas, and so we may take $\Lambda_{\mathcal{U}} \leq M$ without loss of generality.

C. Naturalness

As argued in Ref. [13], the breaking of conformal invariance at energy scale $\Lambda_{\not{\mathcal{U}}}$ means that unparticle physics is relevant only for experiments that probe energies $\Lambda_{\not{\mathcal{U}}} < E < \Lambda_{\mathcal{U}}$. From Eq. (16), we see that creating a conformal window spanning, say, 1 order of magnitude requires

$$\frac{c_2 r^{d_{\rm UV}-2}}{s^2} < \left(\frac{1}{10}\right)^{4-d}. \tag{17}$$

This is a significant constraint, given $1 < d < 2$.

One way to satisfy Eq. (17) is to take large s, that is, $\Lambda_{\mathcal{U}} \gg v$. This raises the energy scale of all unparticle interactions and rapidly decouples unparticles from accessible experiments. For unparticles to be accessible through weak-scale experiments, one must take $\Lambda_{\mathcal{U}} \sim v$. (Note that precision experiments, for example, those probing flavor, *CP*, and baryon number violation, probe scales far above the weak scale; however, these are typically conducted at very low energies outside the conformal window.)

A second option is small r, that is, $M \gg \Lambda_{\mathcal{U}}$. As evident from Eqs. (13)–(15), this raises Λ_3, Λ_4, and Λ_3' rapidly, again making unparticle physics inaccessible. This is illustrated in Fig. 1, where we have plotted all relevant energy scales as functions of d for $\Lambda_{\mathcal{U}} = v$, $d_{\rm UV} = 3$, and $M = 10\Lambda_{\mathcal{U}}$. Even for this relatively small hierarchy between M and $\Lambda_{\mathcal{U}}$, which creates only a slight conformal window, we find Λ_4, Λ_3, $\Lambda_3' \gtrsim 10$ TeV, which is likely beyond the reach of foreseeable experiments.

The third and final logical possibility is $c_2 \lll 1$. Naturalness suggests $c_2 \sim 1$. Furthermore, similar to quadratic divergences in the Higgs boson mass, quantum corrections to c_2 have a power law divergence, which scales as $\Lambda_{\mathcal{U}}^{2-d}$. Thus even if one sets $c_2 = 0$ at tree level, it becomes of order $\lambda_t c_t/(16\pi^2) \sim 0.01$ at loop level. Of course, if there are no scalar unparticle operators O with dimension $d < 2$, then $c_2 = 0$. Consideration of scalar operators with $d > 2$ requires extending the standard range $1 < d < 2$ through the singularities at $d = 2$. Alternatively, one may simply consider vector unparticles without scalar

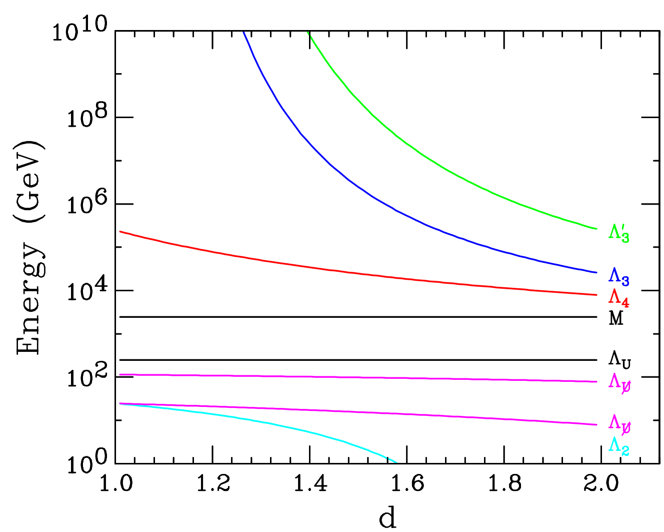

FIG. 1 (color online). Energy scales in the minimal unparticle model as functions of d, assuming $\Lambda_{\mathcal{U}} = v \simeq 246$ GeV, $M = 10v$, and $d_{\rm UV} = 3$. The two lines for $\Lambda_{\not{\mathcal{U}}}$ are for $c_2 = 1$ (upper) and $c_2 = 0.01$ (lower).

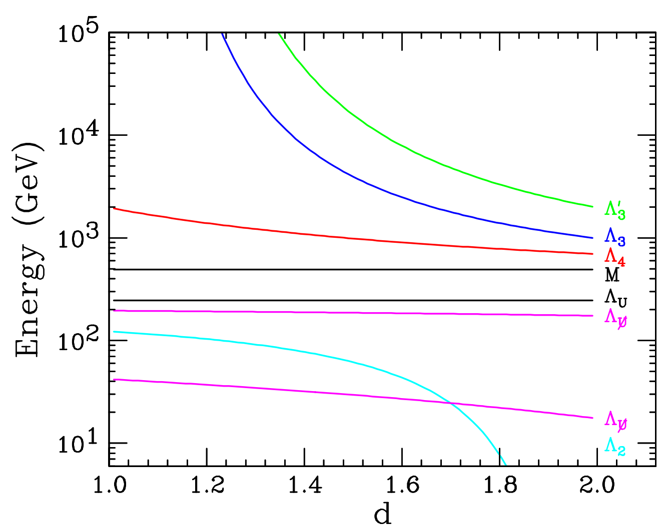

FIG. 2 (color online). Energy scales in the minimal unparticle model as functions of d, assuming $\Lambda_{\mathcal{U}} = v \simeq 246$ GeV, $M = 2v$, and $d_{\rm UV} = 3$. The two lines for $\Lambda_{\not{\mathcal{U}}}$ are for $c_2 = 1$ (upper) and $c_2 = 0.01$ (lower).

unparticles. These cases provide natural mechanisms to suppress c_2, and would imply that the conformal window extends down to very low energies.

For whatever reason, either assuming fine-tuning or one of the more natural possibilities noted above, one may consider $c_2 \ll 1$. We can then take $\Lambda_U \sim M$, implying $\Lambda_4 \sim \Lambda_3 \sim \Lambda_3' \sim \Lambda_U$. This possibility is illustrated in Fig. 2, where we have taken $M = 2\Lambda_{\mathcal{U}} = 2v$. For $c_2 \sim 0.01$, the conformal window may extend down to $\sim$10 GeV. (See also Fig. 1.) At the same time, Λ_4, Λ_3, $\Lambda_3' \sim$ TeV, a scale at which colliders and other high energy experiments can potentially probe fermion and gauge boson couplings to unparticles.

III. SCALAR UNPARTICLES

In this section, we assume that the conformal hidden sector couples through a single scalar operator ($S = 0$). We consider $r \sim 1$ (and therefore also $\Lambda_2 \sim \Lambda_3 \sim \Lambda_4 \sim \Lambda_{\mathcal{U}}$), and explore two classes of signatures in the conformal window at energies near the weak scale v. We show that scalar unparticles can modify gauge couplings, possibly leading to an exotic signal. Scalar unparticles can also be seen in modifications to cross sections at high energy colliders.

A. Contributions to gauge coupling evolution

Consider an unparticle operator O that couples both to Higgs bosons (as in [13]) and to gauge fields

$$c_2 \Lambda_2^{2-d} O H^2 + c_4^F \frac{OF^2}{\Lambda_4^d}. \tag{18}$$

Below the scale of electroweak symmetry breaking, the first of these interactions turns into a tadpole for O, which leads to the breaking of the conformal invariance at the scale $\Lambda_{\mathcal{U}}$ [13]. Generically, O obtains a VEV of the same order of magnitude, modifying the gauge kinetic term to

$$\left[\frac{1}{4} + \mathcal{O}(1)\left(\frac{\Lambda_{\mathcal{U}}}{\Lambda_4}\right)^d\right] F^2, \tag{19}$$

where we have assumed that $c_4^F \sim 1$ and $\Lambda_{\mathcal{U}}$ depend on c_2. Alternatively, one could obtain the same result by integrating out O at the threshold $\mu = \Lambda_{\mathcal{U}}$. This can be interpreted as a threshold correction to the gauge coupling

$$\Delta(\alpha^{-1})|_{\mu_2}^{\mu_1} \sim 4\alpha^{-1}\left(\frac{\Lambda_{\mathcal{U}}}{\Lambda_4}\right)^d, \qquad \mu_1 < \Lambda_{\mathcal{U}} < \mu_2. \tag{20}$$

Thus it is possible to probe scales of the unparticle physics by comparing values of the gauge coupling above and below $\Lambda_{\mathcal{U}}$.

From the phenomenological perspective, one is most interested in the case where the conformal invariance of the unparticle sector is only broken below the electroweak scale, $\Lambda_{\mathcal{U}} < M_Z$. Existing measurements of the fine structure constant at zero energy and at the Z pole are consistent with the standard model renormalization group evolution within experimental and theoretical uncertainties. In this comparison, the largest uncertainty arises from the value of the coupling at the Z pole [27]

$$\alpha^{-1}(M_Z) = 127.918 \pm 0.018. \tag{21}$$

Comparing Eqs. (20) and (21) we find that the scales of unparticle physics must be quite large. For example, taking $d = 1.5$ we find

$$\left(\frac{\Lambda_{\mathcal{U}}}{\Lambda_4}\right) \lesssim 10^{-3}. \tag{22}$$

Written in this form, constraints on the fundamental scales of the unparticle sector implicitly depend on several parameters: c_2, r, s and $d_{\rm UV}$. Thus, it is useful to rephrase this result in terms of the required fine-tuning of c_2. Choosing values of the remaining parameters as in Fig. 1 allows for c_2 of the order of a one-loop contribution, but as a result, $\Lambda_3' > \Lambda_3 > \Lambda_4 \sim 25$ TeV. If we choose the same parameters as in Fig. 2, we find $c_2 < 7 \times 10^{-7}$, which is significantly smaller that its natural one-loop value. As a final example, consider the situation where all scales in the unparticle sector are comparable. In this case, $c_2 < 1.3 \times 10^{-5}(\Lambda_{\mathcal{U}}/5\ \text{TeV})^2$ is required, again a significant fine-tuning.

B. Enhancements of collider cross sections

Scalar unparticles may also be probed more directly by looking at their effects on high energy processes. To analyze these effects, we conservatively consider only gauge-invariant operators that are also B, L, and flavor conserving. In general, one could also consider operators that violate one or more of these global symmetries—these will be much more stringently constrained.

For scalar unparticles, at leading order in Λ there are two types of interactions with standard model fermions:

$$\frac{ec_4^f}{\Lambda_4^d} OH\overline{f_L} f_R, \qquad \frac{ec_4^{\prime f_{L,R}}}{\Lambda_4^d} \partial^\mu O \overline{f_{L,R}} \gamma_\mu f_{L,R}, \tag{23}$$

where we have inserted the electromagnetic coupling e for future convenience. When electroweak symmetry is broken, the first term generates $O\overline{f_L} f_R$ couplings proportional to v. For the second term, integrating by parts, the vector contribution vanishes and the axial vector contribution is proportional to m_f. Since $m_f \ll v \simeq 246$ GeV for all but the top quark, we expect the first class of operators to dominate and focus on those operators here.

The scalar unparticle Feynman rules are

$$O\bar{f}_L f_R \text{ vertex: } ie\frac{c_4^f v}{\Lambda_4^d} P_R, \tag{24}$$

$$O \text{ propagator: } \frac{i}{(q^2)^{2-d}} B_d, \tag{25}$$

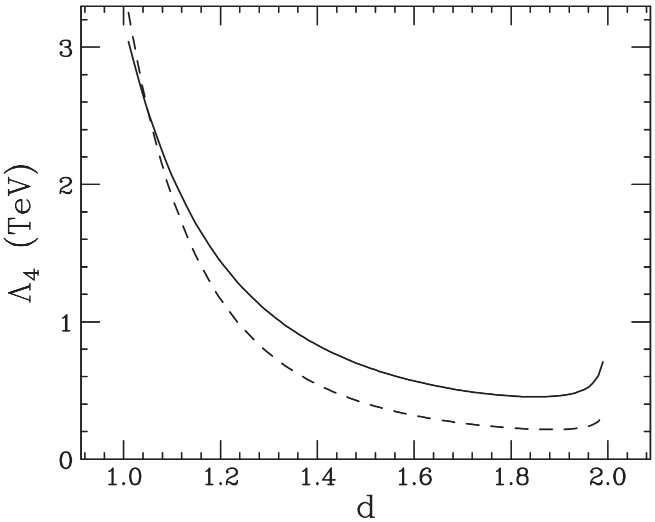

FIG. 3. Lower bounds from LEP/SLC [28] (solid) and JADE [29] (dashed) on the scalar O interaction scale Λ_4 from processes $e^+e^- \to \mu^+\mu^-$ as a function of the dimension d of the O operator.

where [2,3]

$$B_d \equiv A_d \frac{(e^{-i\pi})^{d-2}}{2\sin d\pi}, \qquad A_d \equiv \frac{16\pi^{5/2}\Gamma(d+\frac{1}{2})}{(2\pi)^{2d}\Gamma(d-1)\Gamma(2d)}. \tag{26}$$

The O interactions contribute to fermion pair production at colliders through $f_1\bar{f}_1 \to O \to f_2\bar{f}_2$. In contrast to the case of vector unparticles discussed below, scalar unparticles do not interfere with standard model γ and Z diagrams. Specializing to the case of massless initial state fermions, we find

$$\frac{d\sigma^O}{dx}\bigg|_{\rm CM} = N\frac{\pi\alpha^2 s}{2}v_f\frac{1+v_f^2}{2}\frac{|B_d c_4^1 c_4^2|^2 v^4}{s^{4-2d}\Lambda_4^{4d}}, \tag{27}$$

where N is the numerical spin/color factor from averaging over initial states and summing over final states, $\sqrt{s}$ is the (parton-level) center-of-mass energy, $v_f = (1-4m_{f_2}^2/s)^{1/2}$ is the final state particles' velocity in the center-of-mass frame, and $x = \cos\theta$, where θ is the angle between the incoming f_1 and outgoing f_2. Scalar O unparticles simply produce an isotropic increase in the cross section.

The enhancements of Eq. (27) are constrained by many experiments and many observables, including total cross sections and forward-backward asymmetries. For simplicity, we focus here on total cross sections. Experiments set upper bounds $\Delta\sigma_{\rm exp}$ on new physics contributions to $f_1\bar{f}_1 \to f_2\bar{f}_2$ at center-of-mass energy $\sqrt{s_{\rm exp}}$. This implies the bound

$$\Lambda_4 > \sqrt{s_{\rm exp}}\left[\frac{\pi}{16}v_f\frac{1+v_f^2}{2}|B_d|^2\frac{v^4}{\sqrt{s_{\rm exp}^6}\Delta\sigma_{\rm exp}}\right]^{1/4d}, \tag{28}$$

where we have assumed the minimal set of nonzero couplings to see an effect and taken $e^2|c_4^1c_4^2| = 2\pi$ for consistency with the compositeness literature, which we discuss in Sec. IV B. Lower bounds from a variety of processes and experiments are shown in Fig. 3 and Table I. Note that, since we are considering small enhancements to standard model cross sections, constraints from the Z pole are not significant. The most stringent bounds are from $e\mu$, and these vary from $\Lambda_4 > 2.1$ TeV at $d = 1.1$ to $\Lambda_4 > 460$ GeV at $d = 1.9$. Of course, for a given model, the cross section is enhanced at all energies; these bounds could be improved by combining cross section and $A_{\rm FB}$ data from many different center-of-mass energies and experiments.

The bounds we have derived may also be recast in terms of bounds on the fundamental parameter space of the minimal models discussed in Sec. II B. In Fig. 4 we consider minimal models with $S = 0$, $d_{\rm UV} = 3$, and $c_2 \lesssim 0.01$, and show constraints from $e^+e^- \to \mu^+\mu^-$ in the $(\Lambda_{\mathcal{U}}, M)$ plane for $d = 1.1, 1.5, 1.9$. We see that the constraints have a significant d dependence. For $d = 1.9$, the

TABLE I. Lower bounds on Λ_4 from scalar O interactions, for 4 pairs of fermion species f_1f_2 and 3 representative values of dimension d. These are derived from $\Delta\sigma_{\rm exp}$, the upper bound on new physics contributions to $f_1\bar{f}_1 \to f_2\bar{f}_2$ at center-of-mass energy $\sqrt{s_{\rm exp}}$ at the experiments named.

f_1f_2	Experiment	$\sqrt{s_{\rm exp}}$ [GeV]	$\Delta\sigma_{\rm exp}$ [fb]	Lower bound on Λ_4 [GeV]		
				$d = 1.1$	$d = 1.5$	$d = 1.9$
$e\mu$	LEP/SLC [28]	189	76	2100	670	460
	JADE [29]	34.6	1600	1900	400	220
$e\tau$	LEP/SLC [28]	189	100	1900	640	440
	JADE [29]	34.6	2400	1700	380	200
eq	LEP/SLC [28]	189	240	1600	560	400
	TOPAZ [30]	57.8	4700	1200	340	210
eb	LEP/SLC [28]	189	140	1800	610	430
	VENUS [31]	58.0	3100	1400	360	220

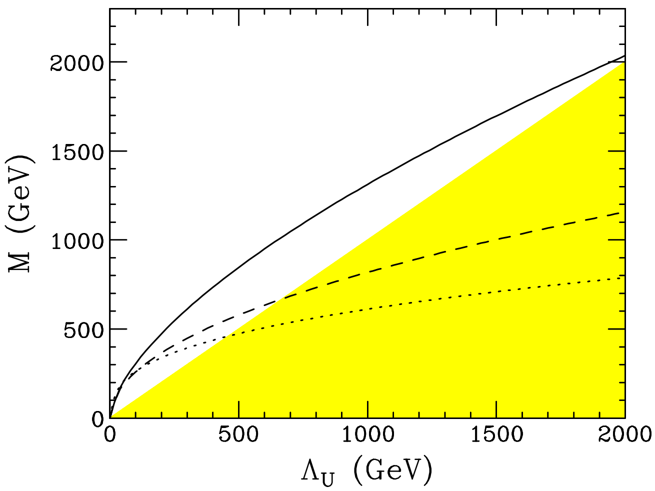

FIG. 4 (color online). Bounds from $e^+e^- \to \mu^+\mu^-$ on the fundamental parameter space $(\Lambda_{\mathcal{U}}, M)$ for a scalar unparticle operator with $d_{\rm UV} = 3$, and $d = 1.1$ (solid), 1.5 (dashed), and 1.9 (dotted). The regions below the contours are excluded. The shaded region is excluded by the requirement $M > \Lambda_{\mathcal{U}}$.

constraints primarily exclude parameter space that is already excluded by the requirement $M > \Lambda_{\mathcal{U}}$. However, for $d = 1.1$, the disfavored region is extended and excludes $M = \Lambda_{\mathcal{U}}$ up to 2 TeV.

IV. VECTOR UNPARTICLES

A. Differential cross sections

For vector unparticles, the leading coupling to fermions is through the interactions

$$\frac{ec_3^{f_{L,R}}}{\Lambda_3^{d-1}} O^\mu \bar{f}_{L,R}\gamma_\mu f_{L,R}. \tag{29}$$

These affect standard model fermion production through virtual O^μ effects, and also through real O^μ production. The new Feynman rules involving O^μ particles are

$$O^\mu \bar{f}_i f_i \text{ vertex: } ie\frac{c_3^{f_i}}{\Lambda_3^{d-1}} P_i \gamma^\mu, \tag{30}$$

$$O^\mu \text{ propagator: } \frac{i}{(q^2)^{2-d}} B_d\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right), \tag{31}$$

where B_d is as defined in Eq. (26), and we have assumed $\partial_\mu O^\mu = 0$.

We focus here on new contributions to $f_i\bar{f}_1 \to f_2\bar{f}_2$ with O^μ unparticles in the s channel, which interfere with the corresponding photon and Z diagrams. Again assuming massless initial state fermions, the resulting total differential cross section is

$$\left.\frac{d\sigma}{dx}\right|_{\rm CM} = N\frac{\pi\alpha^2 s}{2}\sum_{i,j=\gamma,Z,O}\Delta_i\Delta_j^*[X_1^{ij}X_2^{ij}(1+v_f^2x^2) + Y_1^{ij}Y_2^{ij}2v_f x + X_1^{ij}Z_2^{ij}(1-v_f^2)], \tag{32}$$

where

$$X_k^{ij} = Q_{k_L}^i Q_{k_L}^{j*} + Q_{k_R}^i Q_{k_R}^{j*} \tag{33}$$

$$Y_k^{ij} = Q_{k_L}^i Q_{k_L}^{j*} - Q_{k_R}^i Q_{k_R}^{j*} \tag{34}$$

$$Z_k^{ij} = Q_{k_R}^i Q_{k_L}^{j*} + Q_{k_L}^i Q_{k_R}^{j*}. \tag{35}$$

The vertex factors are

$$Q_{f_i}^Z = \frac{I_{f_i} - Q_{f_i}^\gamma \sin^2\theta_W}{\sin\theta_W\cos\theta_W} \qquad Q_{f_i}^O = \frac{c_3^{f_i}}{\Lambda_3^{d-1}}, \tag{36}$$

where $Q_{f_i}^\gamma$ and I_{f_i} are the electric charge and isospin of f_i. The propagator factors are

$$\Delta_\gamma = \frac{1}{q^2} \qquad \Delta_Z = \frac{1}{q^2 - m_Z^2 + im_Z\Gamma_Z} \qquad \Delta_O = \frac{B_d}{(q^2)^{2-d}}. \tag{37}$$

The remaining quantities are as defined below Eq. (27).

B. Bounds from effective contact interactions

As can be seen from the discussion above, for fixed $\sqrt{s}$, the effects of vectorlike O^μ particles are identical to (possibly complex) shifts in γ, Z couplings. Bounds on, say, Z couplings from interactions $f_1\bar{f}_1 \to f_2\bar{f}_2$, are therefore bounds on the scale Λ of O^μ interactions.

A less precise, but more convenient, correspondence is between O^μ interactions and contact interactions. For data collected at a fixed center-of-mass energy $\sqrt{s_{\rm exp}}$, O^μ vertices induce an effective four-fermion contact interaction

$$\frac{e^2c_3^1c_3^2B_d}{\Lambda_3^{2d-2}s_{\rm exp}^{2-d}}\bar{f}_1\gamma^\mu f_1\bar{f}_2\gamma_\mu f_2. \tag{38}$$

This can be compared to the operator

$$\frac{\eta g^2}{2\Lambda_c^2}\bar{f}_1\gamma^\mu f_1\bar{f}_2\gamma_\mu f_2, \tag{39}$$

which has been studied extensively in the context of quark and lepton compositeness. We may therefore derive bounds on O^μ interactions from bounds on the operators of Eq. (39).

The propagator factor B_d is complex. If the $c_3^{f_i}$ coefficients are assumed real, the phase in B_d has interesting consequences. For example, for $d = 1.5$, B_d is imaginary, and so at the Z pole, the O^μ diagram interferes fully with the Z diagram, but not with the γ diagram. This and other

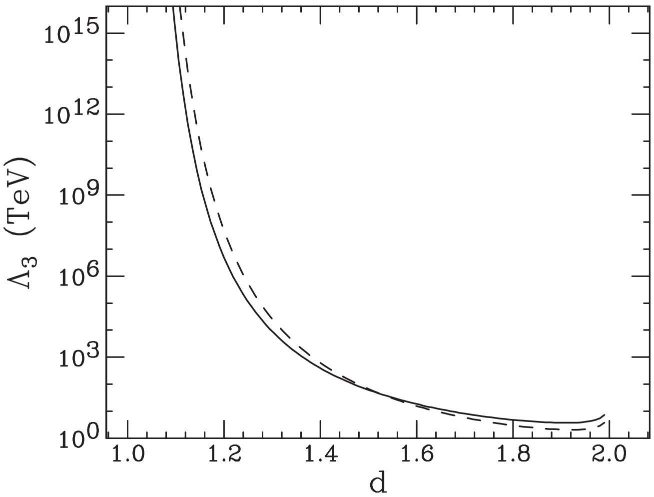

FIG. 5. Lower bounds from L3 [32] (solid) and JADE [29] (dashed) on the vector O^μ interaction scale Λ_3 from processes $e^+e^- \to \mu^+\mu^-$ as a function of the dimension d of the O^μ operator.

interesting consequences of the propagator phase have been discussed previously in Ref. [2].

The coefficients $c_3^{f_i}$ may be complex, however, and so the operators of Eq. (38) have an unknown phase. This ambiguity is not unique to unparticles—the operators of Eq. (39) also have, in principle, complex coefficients. In the compositeness literature, this uncertainty is partially accounted for by deriving bounds for $\eta = \pm 1$, thereby allowing either constructive or destructive interference. In our case, we will derive bounds assuming $c_3^1 c_3^2 B_d$ is real and positive. Bounds for other phases, or incorporating the variation in phase with d, will differ slightly, just as bounds on Eq. (39) depend on the sign of η.

The resulting bound on the scale of O^μ interactions is

$$\Lambda_3 > |B_d|^{1/(2d-2)} \Lambda_{\rm exp} \left(\frac{\Lambda_{\rm exp}}{\sqrt{s_{\rm exp}}} \right)^{(2-d)/(d-1)}, \qquad (40)$$

where $\Lambda_{\rm exp}$ is the bound on the compositeness scale Λ_c in Eq. (39) resulting from data taken at center-of-mass energy $\sqrt{s_{\rm exp}}$, and we have followed the conventions of the fermion compositeness literature [26,27] in assuming the minimal set of nonzero couplings to see an effect and setting $e^2 |c_3^1 c_3^2| = g^2/2\ (= 2\pi)$.

Equation (40) has several interesting features. First, given $\Lambda_{\rm exp} > \sqrt{s_{\rm exp}}$, the bound becomes increasingly stringent as $d \to 1$. This is as it should be—in this limit, the operator of Eq. (29) becomes almost renormalizable, and so less sensitive to Λ_3. Second, for equivalent $\Lambda_{\rm exp}$, the bound is more stringent for *lower* $\sqrt{s_{\rm exp}}$. As a result, constraints from experiments now far from the energy frontier, but still above $\Lambda_{\mathcal{U}}$, the scale of conformal symmetry breaking, are in some cases the leading constraints.

We present bounds on Λ_3 in Fig. 5 and Table II. In every case, we conservatively choose $\sqrt{s_{\rm exp}}$ to be the maximum center-of-mass energy at which the relevant data were taken. This is a conservative assumption, but the bounds are not very sensitive to it, especially for d near 2, where the constraints on Λ_3 are least stringent. For example, at $d = 1.9$, taking $\sqrt{s_{\rm exp}} = 136$ GeV instead of 189 GeV for LEP2 bounds strengthens the bound on Λ_3 by only 4%.

The results given in Fig. 5 and Table II illustrate the features noted above. For low d, the bounds on Λ rise quickly, and are in some cases above the Planck scale for $d = 1.1$. At the same time, even for d near 2, the lower bound on Λ_3 is at least 2 TeV in all channels considered. We also see that many of the leading bounds for low d arise from data at $\sqrt{s_{\rm exp}} \sim 50$ GeV, far from LEP2 energies.

As in Sec. III, we also present the bounds of Table II as constraints in the fundamental parameter space of minimal models in Fig. 6. Bounds in the $(\Lambda_{\mathcal{U}}, M)$ plane are not singular as $d \to 1$. The singularity in Λ_3 is artificial—the physically relevant quantity is Λ_3^{d-1} not Λ_3—and this is removed by considering the fundamental parameters. In contrast to the scalar case, we find that the bounds are far stronger than the consistency requirement $M > \Lambda_{\mathcal{U}}$.

TABLE II. Lower bounds on Λ_3, the scale of vector O^μ interactions, for 4 pairs of fermion species $f_1 f_2$ and 3 representative values of dimension d. These are derived from $\Lambda_{\rm exp}$, the lower bound on the scale of four-fermion contact interactions derived from data with maximum center-of-mass energy $\sqrt{s_{\rm exp}}$ at the experiments named.

$f_1 f_2$	Experiment	$\sqrt{s_{\rm exp}}$ [GeV]	$\Lambda_{\rm exp}$ [TeV]	Lower bound on Λ_3 [TeV]		
				$d = 1.1$	$d = 1.5$	$d = 1.9$
$e_L \mu_L$	L3 [32]	189	8.5	9.1×10^{14}	61	3.7
	JADE [29]	46.8	4.4	3.6×10^{17}	66	2.1
$e_L \tau_L$	L3 [32]	189	5.4	9.7×10^{12}	25	2.2
	JADE [29]	46.8	2.2	3.5×10^{14}	16	1.0
$e_L q_L$	OPAL [33]	207	8.2	2.8×10^{14}	52	3.5
	TOPAZ [34]	57.9	1.2	1.2×10^{11}	4.0	0.48
$e_L b_L$	ALEPH [35]	183	5.6	1.9×10^{13}	27	2.3
	CELLO [36]	43.0	1.1	7.3×10^{11}	4.5	0.45

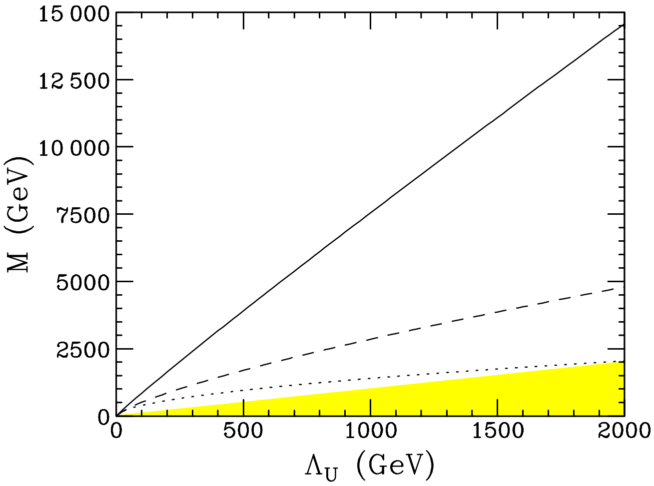

FIG. 6 (color online). Bounds from $e^+e^- \to \mu^+\mu^-$ on the fundamental parameter space $(\Lambda_{\mathcal{U}}, M)$ for a vector unparticle operator with $d_{\rm UV} = 3$, and $d = 1.1$ (solid), 1.5 (dashed), and 1.9 (dotted). The regions below the contours are excluded. The shaded region is excluded by the requirement $M > \Lambda_{\mathcal{U}}$.

Vector unparticle effects are enhanced by interference with the standard model, in contrast to the scalar unparticle case.

We have given only a sampling of possible bounds, corresponding to compositeness bounds with $\eta = 1$ with LL couplings only. Bounds for $\eta = -1$ and different chiralities are simple to derive. These results may also be extended to other $f_1 f_2$ pairs, although the definition of $\sqrt{s}_{\rm exp}$ is less well-defined for bounds from hadron-hadron or lepton-hadron interactions and also for $e_{L,R}e_{L,R}$, where t-channel effects are present.

V. CONCLUSIONS

Unparticles from conformal hidden sectors provide qualitatively new signals for new physics. We have considered the theoretical framework of minimal models, both to include the constraint of low conformal symmetry breaking and to clarify the relationship of the various scales that enter phenomenological analyses.

We considered couplings of scalar unparticles to standard model operators of dimensions 2 and 4 and vector unparticles to dimension 3 operators. The mass scales Λ_n of these couplings are related to the onset of scale invariance $\Lambda_{\mathcal{U}}$ by specific powers of a common factor $r \leq 1$. Results are presented for two values $r = 0.5$ and $r = 0.1$. We find that the requirement of a significant conformal window places strong constraints on models.

These considerations suggest that the most robust probes of unparticle effects must come from high energies. With this in mind, we then considered some of the most promising probes of unparticle effects. We derived bounds on the scales for both scalar and vector unparticles from precision e^+e^- data at center-of-mass energies $\sqrt{s} \approx 30$ to 200 GeV. These bounds were determined for a number of representative channels and presented both in terms of the phenomenological parameters Λ_n and in terms of constraints on the fundamental mass parameters M and $\Lambda_{\mathcal{U}}$.

The analysis of Sec. II A implies that $\Lambda_4 < \Lambda_3$, that is, that the characteristic mass scale for unparticle couplings to standard model gauge bosons is lower than for unparticle couplings to standard model fermions. This suggests that stronger bounds than the ones found here may be derived from gluon-gluon processes at hadron colliders such as the Tevatron and LHC, or possibly from enhancements to ultrahigh energy cosmic ray and cosmic neutrino cross sections. These analyses are left to future work.

We likewise noted an exotic effect of scalar unparticles on gauge coupling evolution. Application to the running of the fine structure constant to M_Z resulted in an unnaturally small value of the coefficient of the coupling of dimension 2 operators, H^2, to scalar unparticles. This disfavors scalar unparticles and suggests that only vector ones couple to the standard model.

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