Lattice renormalization of $\mathcal{O}(a)$ improved heavy-light operators

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The analytical expressions and the numerical values of the renormalization constants of $\mathcal{O}(a)$ improved static-light currents are given at one-loop order of perturbation theory in the framework of heavy quark effective theory: the static quark is described by the HYP action and the light quark is described either with the Clover or the Neuberger action. These factors are relevant to extract from a lattice computation the decay constants f_B, f_{B_s} and the set of bag parameters B_i associated with $B - \bar{B}$ mixing phenomenology in the standard model and beyond.

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I. INTRODUCTION

The extraction of important quantities like V_{ub} or $|V_{ts}/V_{td}|$ needs the nonperturbative calculation of the hadronic form factors that encode the long-distance physics. For example the B meson decay constant f_B has to be precisely known to determine the exclusive V_{ub} from $B \rightarrow \tau\bar{\nu}$ [1]. The detection of physics beyond the standard model in the B_s, $\bar{B}_s$ system is hopeless if the theoretical uncertainty on the bag parameter B_{B_s} associated with the $B_s - \bar{B}_s$ mixing amplitude in the standard model is not reduced [2]. The most satisfying approach to compute such form factors is lattice QCD, as it is only based on first principles of quantum field theory. However, discretization effects induce important systematic errors if $am_Q \geq 1$, where a is the lattice spacing and m_Q is the heavy quark mass. The extrapolation to the continuum limit of physical quantities involving such heavy quarks is difficult, unless the calculation is done on a very fine lattice (e.g. $a \sim 0.02$ fm), which is not possible for the moment because of the too high cost in computation time, or employing the relativistic heavy quark action [3] with properly tuned parameters [4] (see [5] for a recent application of this approach). A way around this problem is the use of heavy quark effective theory (HQET) [6] in which all degrees of freedom of $\mathcal{O}(m_Q)$ are integrated in Wilson coefficients, where $m_Q \gg \Lambda_{\mathrm{QCD}}$. This approach is attractive because the continuum limit exists and results are independent of regularization. A strategy to renormalize nonperturbatively the theory has been proposed and tested for a simple case [7]. A drawback of the standard Eichten-Hill action [8] is the rapid growth of the statistical noise on the correlation functions $C(x_0)$ at $x_0 \sim 1$ fm, making difficult the extraction of hadronic quantities. A method to reduce UV fluctuations is the use of HYP links [9] to build the Wilson line of the static propagator; it has been found that this strategy improves significantly the signal/noise ratio [10]. In this paper we give the analytical expressions and the numerical results of the renormalization constants of static-light bilinear and four-fermion operators at one-loop of perturbation theory when the static quark is described by the HYP action and the light quark is described by the $\mathcal{O}(a)$ improved Clover action or the Neuberger action [11]; in the latter case the extraction of the bag parameters B_i is much safer theoretically because there is no mixing among dimension 6 four-fermion operators of different chirality. This work is an extension to smeared static quark actions of similar computations done with the Eichten-Hill action and with the Clover [12,13] and Neuberger actions [14] respectively. The first of these two new results might be used by the authors of [15] to give the final number of the $\mathrm{N}_f = 2$ P wave static-light decay constant computed with the HYP action. The paper is organized as follows: in Sec. II we will present results obtained by using the tree-level improved static-light operators and in Sec. III we will give renormalization constants of four-fermion operators, leaving the presentation of the numerical result of the bag parameter B_{B_s} to a future paper.

II. TREE LEVEL IMPROVED STATIC-LIGHT CURRENT

A well known approach to reduce the cutoff dependence of matrix elements computed on the lattice is to improve the Wilson light quark action by adding an $\mathcal{O}(a)$ term which is irrelevant in the continuum limit, for example, the Sheikholeslami-Wohlert Clover one [16]. One needs also to improve the inserted operators: in the literature, authors defined rotated fields $\psi' \equiv (1 - a\frac{r}{2}\not{D})\psi$ [17]. We will choose $r = 1$ for the rest of the paper. In principle one could also rotate the static field but it has been shown that it is not necessary in the computation of $\mathcal{O}(a)$ improved on shell matrix elements at tree level [18]. A tree level, the improved bilinear static-light operator will then read

$$O^I_\Gamma \equiv \bar{h}\Gamma\psi' = \bar{h}\Gamma\psi - \frac{a}{2}\bar{h}\Gamma\not{D}\psi, \tag{1}$$

where Γ is any Dirac matrix and we choose the symmetric definition of the covariant derivative $D_\mu\psi(x) = \frac{U_\mu(x)\psi(x+\hat{\mu}) - U^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})}{2a}$. The static quark action reads

$$S^{\mathrm{HQET}} = \sum_n h^\dagger(n)[h(n) - V_4^{\dagger,\mathrm{HYP}}(n-\hat{4})h(n-\hat{4})] + a\delta m h^\dagger(n)h(n), \tag{2}$$

where V_4 is a HYP-smeared link in time direction and δm is a counterterm introduced to cancel the linear divergent part of the static quark self-energy [8]. The light quark action reads

$$S^{\mathrm{Clover}} = S^W - a^4 c_{SW} \sum_{n,\mu,\nu}\left[ig\frac{a}{4}\bar{\psi}(n)\sigma_{\mu\nu}P_{\mu\nu}\psi(n)\right], \tag{3}$$

where $P_{\mu\nu}$ is the discretized strength tensor. The Sheikholeslami-Wohlert coefficient c_{SW} can be fixed at its tree level value $c_{SW}^{\mathrm{tree}} = 1$ to be consistent with a one-loop calculation in perturbation theory. We collect in Table I the Feynman rules which are used. We follow the notations of [19–22] in the rest of the paper and we summarize them in Appendix A.

Note that p' and p are the ingoing and outgoing fermion momenta, respectively. We also introduce an infrared regulator λ for the gluon propagator. We symmetrize the vertex $V^{ab}_{\mu\nu,hhgg}$ by introducing the anticommutator of $SU(3)$ generators, normalized by a factor $\frac{1}{2}$.

At one loop of perturbation theory, a bare matrix element regularized and renormalized in a continuum scheme-for example in the dimension regularization (DR) and in the $\overline{\mathrm{MS}}$ scheme-is written generically in terms of its tree level part

$$\langle O(p,\mu)\rangle^{\mathrm{DR},\overline{\mathrm{MS}}} = \left[1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}(\mu)}{4\pi}\left(\gamma \ln\left(\frac{\mu^2}{p^2}\right) + C_{\mathrm{DR}}\right)\right] \times \langle O(p)\rangle^{\mathrm{tree}}, \tag{4}$$

where γ is the $\mathcal{O}(g^2)$ coefficient of the anomalous dimension of the operator. The same bare matrix element regularized on the lattice reads

$$\langle O(p,a)\rangle^{\mathrm{lat}} = \left[1 + \frac{\alpha_{s0}(a)}{4\pi}(\gamma \ln(a^2p^2) + C_{\mathrm{lat}})\right]\langle O(p)\rangle^{\mathrm{tree}} + \mathcal{O}(a). \tag{5}$$

At this level of perturbation theory one can identify $\alpha_s^{\overline{\mathrm{MS}}}(\mu)$ with the bare coupling $\alpha_{s0}(a)$. One can then write that

$$\langle O\rangle^{\mathrm{DR},\overline{\mathrm{MS}}} = \left[1 - \frac{\alpha_{s0}(a)}{4\pi}(\gamma \ln a^2\mu^2 + C_{\mathrm{lat}} - C_{\mathrm{DR}})\right]\langle O\rangle^{\mathrm{lat}} + \mathcal{O}(a) \equiv Z(a\mu)\langle O\rangle^{\mathrm{lat}} + \mathcal{O}(a). \tag{6}$$

The matching constant between the matrix element renormalized at the scale $\mu = a^{-1}$ in the continuum and the bare matrix element regularized on the lattice is then given by $C_{\mathrm{lat}} - C_{\mathrm{DR}}$. In the following we will be concerned with the static-light currents and discuss C_{lat}.

Let us consider the bare hadronic matrix element regularized on the lattice $\langle H_2|O_\Gamma^I|H_1\rangle^{\mathrm{lat}}$ where H_1 contains the light quark q and H_2 contains the static quark h. It is computed from the ratio

$$R(t,t_1,t_2) = \mathcal{Z}_1\mathcal{Z}_2\frac{C^{(3)}_{J_1,O^I_\Gamma,J_2}(p,p',t,t_1,t_2)}{C^{(2)}_{J_1}(\vec{p},t_1)C^{(2)}_{J_2}(\vec{p}',t_2-t)}$$

where

$$C^{(2)}_{J_i}(\vec{p},t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle J_i(t,\vec{x})J_i^\dagger(0)\rangle$$

is a 2-point correlation function, J_i is an interpolating field of the hadron state H_i containing either the static quark field h or the light quark field q,

$$C^{(3)}_{J_1,O_\Gamma,J_2}(\vec{p},\vec{p}',t,t_1,t_2) = \sum_{\vec{x},\vec{y}} e^{i(\vec{p}\cdot\vec{x}-\vec{p}'\cdot\vec{y})}\langle J_2(t_2,\vec{y}) \times O^I_\Gamma(t)J_1^\dagger(t_1,\vec{x})\rangle$$

is a 3-point correlation function in which the operator O^I_Γ is inserted at time t.

Eventually $\mathcal{Z}_i = \langle H_i^{(0)}|J_i^\dagger|0\rangle$, where $H_i^{(0)}$ is the hadron ground state containing either the static quark h or the light quark q. As usual we determine $\langle H_2^{(0)}|O^I_\Gamma|H_1^{(0)}\rangle^{\mathrm{lat}}$ in the interval of t where $R(t,t_1,t_2)$ is constant (i.e. ground states

TABLE I. Feynman rules.

static quark propagator	$a(1 - e^{-ip_4a} + \epsilon)^{-1}$
vertex $V^a_{\mu,hhg}(p,p')$	$-ig_0T^ah_{4\mu}e^{-i(p_4+p'_4)a/2}$
vertex $V^{ab}_{\mu\nu,hhgg}(p,p')$	$-\frac{1}{2}ag_0^2h_{4\mu}h_{4\nu}\{T^a,T^b\}e^{-i(p_4+p'_4)a/2}$
light quark propagator	$a(i\gamma\cdot\bar{p} + am + \frac{1}{2}\hat{p}^2)^{-1}$
vertex $V^a_{\mu,qqg}(p,p')$	$-igT^a(\gamma_\mu \cos a(p+p')_\mu - i\sin a(p+p')_\mu)$
vertex $V^{ab}_{\mu\nu,qqgg}(p,p')$	$\frac{iag_0^2\delta_{\mu\nu}}{2}\{T^a,T^b\}(\gamma_\mu \sin a(p+p')_\mu + i\cos a(p+p')_\mu)$
improved vertex $V^I_{\mu,qqg}$	$-g_0T^a\frac{r}{2}[\sum_\nu \sigma_{\mu\nu}(\sin a(p-p')_\nu \cos\frac{a}{2}(p-p')_\mu]$
static-light bilinear current O_{Γ_1}	Γ_1
improved static-light bilinear current $O^I_{\Gamma_1,qq}$	$-\frac{i}{2}\Gamma_1\not{\bar{p}}$
improved static-light bilinear current $O^I_{\Gamma_1,qqg}$	$-\frac{aig_0r}{2}\Gamma_1\gamma_\mu\cos a(p+p')_\mu$
gluon propagator in the Feynman gauge	$a^2\delta_{\mu\nu}\delta^{ab}(2W + a^2\lambda^2)^{-1}$

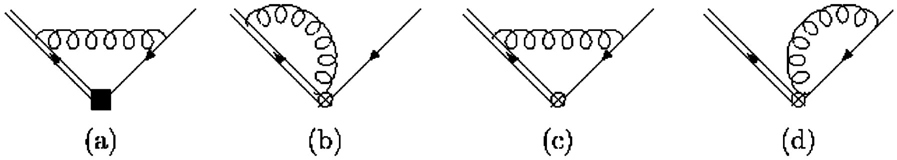

FIG. 1. Diagrams giving the one-loop correction to the $\mathcal{O}(a)$ improved static-light current with the $\mathcal{O}(a)$ improved light quark action.

are safely isolated). As the spectator quark does not play any role in the renormalization of O_Γ^I, one may relate $\langle H_2^{(0)}|O_\Gamma^I|H_1^{(0)}\rangle^{\rm lat}$ to $\langle \bar{h}(p')|O_\Gamma^I|q(p)\rangle^{\rm lat}$. That is why it is justified to compute the matching constants between the currents renormalized in a continuum scheme and the bare currents regularized on the lattice by considering the matrix elements of quarks,[1] which are the only states appropriate to do perturbative calculations. We stress that the mass counterterm δm is canceled in R: thus we will not consider it in our one-loop computations.

At this order of perturbation theory, $\langle \bar{h}(p')|O_\Gamma^I|q(p)\rangle^{\rm lat}$ is given by

$$\begin{aligned}\langle \bar{h}(p')|O_\Gamma^I|q(p)\rangle^{\rm lat} = \sqrt{Z_{2h}}\sqrt{Z_{2l}}\Big\{1 + \frac{\alpha_s}{4\pi}C_F[-\ln(a^2\lambda^2) \\ + d_1 + n - (l+m) \\ + G(d_2 + h - q - 2d^I)]\Big\} \\ \times \langle \bar{h}(p')|O_\Gamma|q(p)\rangle^{\rm tree} \\ \equiv Z_{\rm lat}\langle \bar{h}(p')|O_\Gamma|q(p)\rangle^{\rm tree}, \qquad (7)\end{aligned}$$

where

$$\gamma_0\Gamma\gamma_0 = G\Gamma, \qquad \sqrt{Z_{2h}} = 1 + \frac{\alpha_s}{4\pi}C_F\Big(\frac{e}{2} - \ln(a^2\lambda^2)\Big),$$

$$\sqrt{Z_{2h}} = 1 + \frac{\alpha_s}{4\pi}C_F\Big(\frac{f + f^I + \ln(a^2\lambda^2)}{2}\Big);$$

$d_1 + (d_2 - d^I)G$, hG, $n - (q + d^I)G$ and $-(l+m)$ are contributions given by the 1PI vertex diagrams shown in Fig. 1 and $Z_{2h,l}$ come from the quark self energies. Finally the expression of $C_{\rm lat}$ reads

$$\begin{aligned}C_{\rm lat} = \frac{e + f + f^I}{2} + d_1 + n - (l+m) \\ + G(d_2 + h - q - 2d^I). \qquad (8)\end{aligned}$$

We have collected the numerical values of the various constants in Table II for the HYP parameter sets $\alpha_i = 0$ (corresponding to standard Eichten-Hill action), $\alpha_1 = 1.0$, $\alpha_2 = \alpha_3 = 0$ (corresponding to APE blocking [25]), $\alpha_1 = 0.75$, $\alpha_2 = 0.6$, $\alpha_3 = 0.3$ (HYP1) and $\alpha_1 = 1.0$, $\alpha_2 = 1.0$, $\alpha_3 = 0.5$ (HYP2); their analytical expression is written in Appendix B, while we have collected $C_{\rm lat}$ in terms of α_i for axial and scalar static-light currents in Table III. For the first set of α_i our results agree with [12,13].

We note that the one-loop corrections for the set HYP2 are very small compared to the set $\alpha_i = 0$, confirming the observation that UV fluctuations are strongly suppressed by this action [10], which improves highly the signal/noise ratio. It is particularly impressive on the constant e related to the static field renormalisation. In that case the tadpole contribution is much smaller for HYP2 than for Eichten-Hill (5.96 vs 12.23) and the "sunset" contribution is negative instead of positive (− 9.58 vs 12.25). Another interesting property of the HYP2 action is that the contribution coming from the chiral symmetry breaking term of the light quark action is reduced compared to what is found with the other static quark actions, in particular, HYP1, as

TABLE II. Numerical values of contributions to the correction at one loop of perturbation theory of the $\mathcal{O}(a)$ improved static-light current regularized on the lattice to its tree-level expression; f, f^I, l, and m are extracted from [18] whereas e was computed in [24].

α_i	0	APE	HYP1	HYP2
e	24.48	3.17	2.52	−3.62
d_1	5.46	4.98	4.99	4.72
d_2	−7.22	−3.33	−3.70	−1.87
d^I	−4.14	−2.79	−2.80	−1.99
h	−9.98	−3.40	−4.43	−1.95
n	0.73	−2.33	−1.80	−2.88
q	−2.02	−0.61	−0.78	−0.19
f				13.35
f^I				−3.63
l				−3.42
m				7.35

TABLE III. Lattice contribution to the matching constant between the axial (scalar) static-light current regularized on the lattice and its counterpart renormalized in the continuum. We indicated the contribution $\chi \equiv d_2 + h - q - 2d^I$ coming from the chiral symmetry breaking term of the light quark action.

α_i	0	APE	HYP1	HYP2
$C_{\rm lat}^A$	26.26	5.71	7.13	0.61
$C_{\rm lat}^S$	12.46	4.46	3.63	1.31
χ	−6.90	−0.54	−1.75	0.35

[1]The renormalization constants computed in the MOM scheme are actually extracted numerically on the lattice by considering such matrix elements [23].

indicated in the last row of Table III. The main consequence is that the ratio Z_V/Z_A between the matching constants of the vector and axial static-light currents is closer to 1. Of course this feature is only true at one-loop of perturbation theory and can change at the nonperturbative level.

III. $B_s - \bar{B}_s$ MIXING WITH OVERLAP FERMIONS

In this part we present the results of the computation of the renormalization constants of static-light four-fermion operators with the light quark described by the Neuberger action. The bag parameter B_{B_s} associated with the $B_s - \bar{B}_s$ mixing amplitude in the standard model is defined by

$$B_{B_s} = \frac{\langle \bar{B}_s | (\bar{b}s)_{V-A}(\bar{b}s)_{V-A} | B_s \rangle}{\langle \bar{B}_s | (\bar{b}s)_{V-A}(\bar{b}s)_{V-A} | B_s \rangle_{\text{VSA}}},$$
$$\langle \bar{B}_s | (\bar{b}s)_{V-A}(\bar{b}s)_{V-A} | B_s \rangle_{\text{VSA}} = \langle \bar{B}_s | (\bar{b}s)_{V-A} | 0 \rangle \times \langle 0 | (\bar{b}s)_{V-A} | B_s \rangle. \tag{9}$$

We have to introduce in addition to the operator $O_1 \equiv (\bar{b}s)_{V-A}(\bar{b}s)_{V-A}$ the following operators of the supersymmetric basis:

$$O_2 = (\bar{b}s)_{S-P}(\bar{b}s)_{S-P}, \qquad O_3 = (\bar{b}s)_{V-A}(\bar{b}s)_{V+A},$$
$$O_4 = (\bar{b}s)_{S-P}(\bar{b}s)_{S+P}. \tag{10}$$

Then we define as usual the bag parameters $B_{i=1,\ldots,4}$ in terms of the vacuum saturation approximation matrix elements by

$$\langle \bar{B}_s | O_i | B_s \rangle(\mu) = \langle \bar{B}_s | O_i | B_s \rangle_{\text{VSA}} B_i(\mu).$$

We define the HQET operators $\tilde{O}_{i=1,\ldots,4}$ by

$$\begin{aligned}
\tilde{O}_1 &\equiv \tilde{O}_{VV+AA} = (\bar{h}^{(+)}s)_{V-A}(\bar{h}^{(-)}s)_{V-A},\\
\tilde{O}_2 &\equiv \tilde{O}_{SS+PP} = (\bar{h}^{(+)}s)_{S-P}(\bar{h}^{(-)}s)_{S-P},\\
\tilde{O}_3 &\equiv \tilde{O}_{VV-AA} = (\bar{h}^{(+)}s)_{V-A}(\bar{h}^{(-)}s)_{V+A},\\
\tilde{O}_4 &\equiv \tilde{O}_{SS-PP} = (\bar{h}^{(+)}s)_{S-P}(\bar{h}^{(-)}s)_{S+P},
\end{aligned} \tag{11}$$

and their associated bag parameter $\tilde{B}_i$, $i = 1, 2, 3, 4$.

The extraction of B_{B_s} from our lattice simulation needs the following steps:

(1) $\tilde{B}_i^{\text{lat}}(a)$ are matched onto the continuum $\overline{\text{MS}}$(NDR) scheme at NLO in perturbation theory at the renormalization scale $\mu = 1/a$ [14],

(2) $\tilde{B}_i$ are evolved from $\mu = 1/a$ to $\mu = m_b$ by using the HQET anomalous dimension matrix, known to 2-loop accuracy in perturbation theory [26,27],

(3) $\tilde{B}_i(\mu = m_b)$ are finally matched onto their QCD counterpart, $B_i(m_b)$, in the $\overline{\text{MS}}$(NDR) scheme at NLO [26].

The matching scales are such that neither $\ln(a\mu)$ in step (1) nor $\ln(\mu/m_b)$ in step (3) correct strongly the matching constants. In the following we will concentrate on step (1).

The total lattice fermionic action is $S = S^{\text{HQET}} + S_L^N$ where

$$\begin{aligned}
S_H^{\text{HQET}} &= a^3 \sum_n \{\bar{h}^+(n)[h^+(n) - V_4^{\dagger,\text{HYP}}(n-\hat{4})h^+(n-\hat{4})]\\
&\quad - \bar{h}^-(n)[V_4^{\text{HYP}}(n)h^-(n+\hat{4}) - h^-(n)]\\
&\quad + \delta m[\bar{h}^+(n)h^+(n) + \bar{h}^-(n)h^+(n)]\},\\
S_L^{\text{N}} &= a^3 \sum_n \bar{\psi}(n) D_N(m_0) \psi(n),\\
D_N(m_0) &= \left(1 - \frac{1}{2\rho} a m_0\right) D_N + a m_0,\\
D_N &= \frac{\rho}{a}\left(1 + \frac{X}{\sqrt{X^\dagger X}}\right), \qquad X = D_W - \frac{\rho}{a},\\
0 &< \rho < 2.
\end{aligned} \tag{12}$$

The static quark (antiquark) field satisfies the equation of motion

$$\gamma_0 h^{\pm}(x) = \pm h^{\pm}(x).$$

The HQET action is invariant under the finite heavy quark symmetry (HQS) transformations

$$\bar{h}^{(\pm)}(x) \overset{HQS(i)}{\longrightarrow} -\frac{1}{2}\epsilon^{ijk}\bar{h}^{(\pm)}(x)\gamma_j\gamma_k \qquad (i = 1, 2, 3), \tag{13}$$

and the overlap action is invariant under the infinitesimal chiral transformation [28]

$$\begin{aligned}
\psi &\to \left[1 + i\epsilon\gamma^5\left(1 - \frac{a}{2}D_N\right)\right]\psi,\\
\bar{\psi} &\to \bar{\psi}\left[1 + i\epsilon\left(1 - \frac{a}{2}D_N\right)\gamma^5\right].
\end{aligned} \tag{14}$$

The matching between the operators regularized on the lattice and their counterpart of the continuum needs normally 16 matching constants, as $\tilde{O}_1$ and $\tilde{O}_2$ can mix with $\tilde{O}_3$ and $\tilde{O}_4$:

$$\tilde{O}_i^{\overline{\text{MS}}}(\mu) = Z_{ij}(a\mu)\tilde{O}_j(a), \quad i = 1, \ldots, 4, \quad j = 1, \ldots, 4.$$

However, thanks to heavy quark symmetry, these constants are not all independent. Here we give the details of the proof, as it was not fully presented in [14] or [29] (it was independently presented and generalized in [30]). Under the HQS transformation (13), one has

$$\tilde{O}_{SS+PP} \equiv -\tilde{O}_{(VV+AA)_0}, \qquad \tilde{O}_{VV+AA} \overset{HQS(i)}{\rightarrow} \tilde{O}_{VV+AA},$$

$$\tilde{O}_{SS+PP} \overset{HQS(i)}{\rightarrow} -\tilde{O}_{(VV+AA)_i},$$

$$\tilde{O}_{VV-AA} \overset{HQS(i)}{\rightarrow} \sum_{j=1,3}^{j\neq i} \tilde{O}_{(VV-AA)_j} - (\tilde{O}_{(VV-AA)_i} + \tilde{O}_{(VV-AA)_0}) \equiv (\tilde{O}_{VV-AA})_\perp - (\tilde{O}_{VV-AA})_\parallel,$$

$$\tilde{O}_{SS-PP} \equiv -\tilde{O}_{(VV-AA)_0}, \qquad \tilde{O}_{SS-PP} \overset{HQS(i)}{\rightarrow} \tilde{O}_{(VV-AA)_i}.$$

The different constraints are the following:

$$\begin{aligned}
\langle \tilde{O}_{VV+AA}(\mu)\rangle &= Z_{11}\langle \tilde{O}_{VV+AA}(a)\rangle + Z_{12}\langle \tilde{O}_{SS+PP}(a)\rangle + Z_{13}\langle \tilde{O}_{VV-AA}(a)\rangle + Z_{14}\langle \tilde{O}_{SS-PP}(a)\rangle,\\
\langle \tilde{O}_{VV+AA}(\mu)\rangle &= Z_{11}\langle \tilde{O}_{VV+AA}(a)\rangle - Z_{12}\langle \tilde{O}_{(VV+AA)_i}(a)\rangle + Z_{13}(\langle \tilde{O}_{VV-AA}(a)\rangle_\perp - \langle \tilde{O}_{VV-AA}(a)\rangle_\parallel)\\
&\quad + Z_{14}\langle \tilde{O}_{(VV-AA)_i}(a)\rangle \qquad (\mathrm{HQS}(i)),
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1,3}\langle \tilde{O}_{VV+AA}(\mu)\rangle &\equiv 3\langle \tilde{O}_{VV+AA}(\mu)\rangle\\
&= (3Z_{11} - Z_{12})\langle \tilde{O}_{VV+AA}(a)\rangle - Z_{12}\langle \tilde{O}_{SS+PP}(a)\rangle + (Z_{13} + Z_{14}) \times \langle \tilde{O}_{VV-AA}(a)\rangle + (Z_{14} + 4Z_{13}) \times \langle \tilde{O}_{SS-PP}(a)\rangle,
\end{aligned}$$

implying that

$$Z_{12} = 0, \qquad Z_{14} = 2Z_{13}. \tag{15}$$

$$\begin{aligned}
\langle \tilde{O}_{SS+PP}(\mu)\rangle &= Z_{21}\langle \tilde{O}_{VV+AA}(a)\rangle + Z_{22}\langle \tilde{O}_{SS+PP}(a)\rangle + Z_{23}\langle \tilde{O}_{VV-AA}(a)\rangle + Z_{24}\langle \tilde{O}_{SS-PP}(a)\rangle,\\
-\langle \tilde{O}_{(VV+AA)_i}(\mu)\rangle &= Z_{21}\langle \tilde{O}_{VV+AA}(a)\rangle - Z_{22}\langle \tilde{O}_{(VV+AA)_i}(a)\rangle + Z_{23}(\langle \tilde{O}_{VV-AA}(a)\rangle_\perp - \langle \tilde{O}_{VV-AA}(a)\rangle_\parallel)\\
&\quad + Z_{24}\langle \tilde{O}_{(VV-AA)_i}(a)\rangle \qquad (\mathrm{HQS}(i)),
\end{aligned}$$

$$\begin{aligned}
&-\sum_{i=1,3}\tilde{O}_{(VV+AA)_i}(\mu) \pm \tilde{O}_{(VV+AA)_0}(\mu)\\
&\equiv -\langle \tilde{O}_{SS+PP}(\mu)\rangle - \langle \tilde{O}_{VV+AA}(\mu)\rangle\\
&= (3Z_{21} - Z_{22})\langle \tilde{O}_{VV+AA}(a)\rangle - Z_{22}\langle \tilde{O}_{SS+PP}(a)\rangle + (Z_{23} + Z_{24})\langle \tilde{O}_{VV-AA}(a)\rangle + (Z_{24} + 4Z_{23})\langle \tilde{O}_{SS-PP}(a)\rangle\\
&= -(Z_{11} + Z_{21})\langle \tilde{O}_{VV+AA}(a)\rangle - Z_{22}\langle \tilde{O}_{SS+PP}(a)\rangle - [(Z_{13} + Z_{23})\langle \tilde{O}_{VV-AA}(a)\rangle + (Z_{14} + Z_{24}) \times \langle \tilde{O}_{SS-PP}(a)\rangle],
\end{aligned}$$

giving the constraints

$$Z_{21} = \frac{Z_{22} - Z_{11}}{4}, \qquad Z_{24} = -(Z_{13} + 2Z_{23}). \tag{16}$$

$$\begin{aligned}
\langle \tilde{O}_{VV-AA}(\mu)\rangle &= Z_{31}\langle \tilde{O}_{VV+AA}(a)\rangle + Z_{32}\langle \tilde{O}_{SS+PP}(a)\rangle + Z_{33}\langle \tilde{O}_{VV-AA}(a)\rangle + Z_{34}\langle \tilde{O}_{SS-PP}(a)\rangle,\\
\langle \tilde{O}_{SS-PP}(\mu)\rangle &= Z_{41}\langle \tilde{O}_{VV+AA}(a)\rangle + Z_{42}\langle \tilde{O}_{SS+PP}(a)\rangle + Z_{43}\langle \tilde{O}_{VV-AA}(a)\rangle + Z_{44}\langle \tilde{O}_{SS-PP}(a)\rangle,\\
\langle \tilde{O}_{(VV-AA)_i}(\mu)\rangle &= Z_{41}\langle \tilde{O}_{VV+AA}(a)\rangle - Z_{42}\langle \tilde{O}_{(VV+AA)_i}(a)\rangle + Z_{43}(\langle \tilde{O}_{VV-AA}(a)\rangle_\perp - \langle \tilde{O}_{VV-AA}(a)\rangle_\parallel)\\
&\quad + Z_{44}\langle \tilde{O}_{(VV-AA)_i}(a)\rangle \qquad (\mathrm{HQS}(i)),
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1,3}\tilde{O}_{(VV-AA)_i}(\mu) \pm \tilde{O}_{(VV-AA)_0}(\mu) &\equiv \langle \tilde{O}_{SS-PP}(\mu)\rangle + \langle \tilde{O}_{VV-AA}(\mu)\rangle\\
&= (3Z_{41} - Z_{42})\langle \tilde{O}_{VV+AA}(a)\rangle - Z_{42}\langle \tilde{O}_{SS+PP}(a)\rangle + (Z_{43} + Z_{44})\langle \tilde{O}_{VV-AA}(a)\rangle\\
&\quad + (Z_{44} + 4Z_{43})\langle \tilde{O}_{SS-PP}(a)\rangle\\
&= (Z_{31} + Z_{41})\langle \tilde{O}_{VV+AA}(a)\rangle + (Z_{32} + Z_{42}\langle \tilde{O}_{SS+PP}(a)\rangle + (Z_{33} + Z_{43})\langle \tilde{O}_{VV-AA}(a)\rangle\\
&\quad + (Z_{34} + Z_{44})\langle \tilde{O}_{SS-PP}(a)\rangle.
\end{aligned}$$

One obtains eventually the constraints

$$Z_{44} = Z_{33}, \qquad Z_{42} = -\frac{Z_{32}}{2}, \qquad Z_{41} = \frac{2Z_{31} - Z_{32}}{4}, \qquad Z_{43} = \frac{Z_{34}}{4}. \tag{17}$$

The renormalization matrix has the following structure:

$$Z = \begin{pmatrix} Z_{11} & 0 & Z_{13} & 2Z_{13}\\ \frac{Z_{22}-Z_{11}}{4} & Z_{22} & Z_{23} & -(Z_{13} + 2Z_{23})\\ Z_{31} & Z_{32} & Z_{33} & Z_{34}\\ \frac{2Z_{31}-Z_{32}}{4} & -\frac{Z_{32}}{2} & \frac{Z_{34}}{4} & Z_{33} \end{pmatrix}. \tag{18}$$

Further constraints are obtained thanks to the invariance of the overlap action under the finite chiral transformation

$$\psi \rightarrow i\gamma^5\left(1 - \frac{a}{2}D_N\right)\psi, \qquad \bar{\psi} \rightarrow i\bar{\psi}\left(1 - \frac{a}{2}D_N\right)\gamma^5.$$

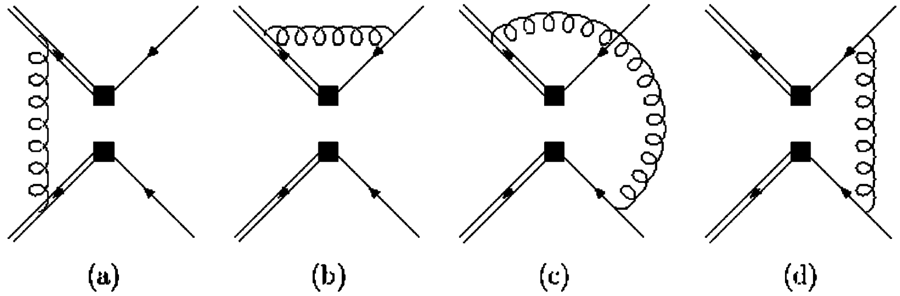

FIG. 2. Diagrams giving the one-loop correction to a static-light four-fermion operator.

Under such a transformation one has

$$\tilde{O}_{VV+AA} \to -\tilde{O}_{VV+AA}, \qquad \tilde{O}_{SS+PP} \to -\tilde{O}_{SS+PP},$$
$$\tilde{O}_{VV-AA} \to +\tilde{O}_{VV-AA}, \qquad \tilde{O}_{SS-PP} \to +\tilde{O}_{SS-PP}.$$

The final result is then

$$Z = \begin{pmatrix} Z_{11} & 0 & 0 & 0 \\ \frac{Z_{22}-Z_{11}}{4} & Z_{22} & 0 & 0 \\ 0 & 0 & Z_{33} & Z_{34} \\ 0 & 0 & \frac{Z_{34}}{4} & Z_{33} \end{pmatrix}. \tag{19}$$

There is no mixing of left-left four-fermion static-light operators regularized on the lattice with dimension 6 operators of different chirality, reducing significantly the systematic error coming from such a spurious mixing when the light quark is described by the Wilson-Clover action: indeed the matching of those operators with their counterpart renormalized in the continuum $\overline{\mathrm{MS}}$ scheme does not need any subtraction.

We recall that the overlap propagator without mass reads[2]

$$S^{ab}_{\text{overlap}}(k) = \delta^{ab}\frac{a}{2\rho}\left(\frac{-i\not{V}}{\omega + b} + 1\right), \qquad b(k) = W(k) - \rho,$$
$$\omega(k) = a(\sqrt{X^\dagger X})_0(k), \tag{20}$$

where X_0 is the free part of the Wilson kernel with a negative mass $-\frac{\rho}{a}$, and the quark-quark-gluon vertex is defined by [31]

$$V^{a,\text{overlap}}_{\mu,qqg}(p,p') = -ig_0T^a\frac{\rho}{\omega(p)+\omega(p')} \times\left[\gamma^\mu c_\mu - is_\mu + \frac{a^2}{\omega(p)\omega(p')}X_0(p') \times(\gamma^\mu c_\mu + is_\mu)X_0(p)\right]. \tag{21}$$

The renormalization constants of dimension 6 static-light four-fermion operators are given at one loop of perturbation theory by the diagrams of Fig. 2.

Following the notations of [14], the matching constants are defined by

$$Z_{11}^{\overline{\mathrm{MS}}} = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\left[\frac{7}{3} + \frac{d_s}{3} - \frac{10d_1}{3} - \frac{c}{3} - \frac{4e}{3} - \frac{4f}{3} + \frac{2d_\xi}{3} + 4\ln(a^2\mu^2)\right],$$
$$Z_{21}^{\overline{\mathrm{MS}}} = \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\left[-\frac{5}{36} - \frac{d_s}{36} - \frac{2d_v}{9} + \frac{d_1}{2} + \frac{c}{4} - \frac{d_\xi}{6} - \frac{2}{3}\ln(a^2\mu^2)\right],$$
$$Z_{22}^{\overline{\mathrm{MS}}} = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\left[\frac{16}{9} + \frac{2d_s}{9} - \frac{8d_v}{9} - \frac{4d_1}{3} + \frac{2c}{3} - \frac{4e}{3} - \frac{4f}{3} + \frac{4}{3}\ln(a^2\mu^2)\right],$$
$$Z_{33}^{\overline{\mathrm{MS}}} = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\left[\frac{41}{12} - \frac{d_v}{6} - \frac{7d_1}{3} + \frac{c}{6} - \frac{4e}{3} - \frac{4f}{3} + \frac{7d_\xi}{6} + \frac{7}{2}\ln(a^2\mu^2)\right],$$
$$Z_{34}^{\overline{\mathrm{MS}}} = \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\left[\frac{1}{2} - d_v + 2d_1 + c - d_\xi - 3\ln(a^2\mu^2)\right],$$
$$Z_{43}^{\overline{\mathrm{MS}}} = \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\left[\frac{1}{8} - \frac{d_v}{4} + \frac{d_1}{2} + \frac{c}{4} - \frac{d_\xi}{4} - \frac{3}{4}\ln(a^2\mu^2)\right],$$
$$Z_{44}^{\overline{\mathrm{MS}}} = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{4\pi}\left[\frac{41}{12} - \frac{d_v}{6} - \frac{7d_1}{3} + \frac{c}{6} - \frac{4e}{3} - \frac{4f}{3} + \frac{7d_\xi}{6} + \frac{7}{2}\ln(a^2\mu^2)\right],$$

[2] We invite the reader to have a look in Appendix A in which the notations used in those equations are made more precise.

TABLE IV. Numerical values of c, $d_1(\rho)$, $f(\rho)$, $d_s(\rho)$, and $d_v(\rho)$ defined in the text.

ρ	1.4	1.6
$f(\rho)$	-17.47	-13.24
$d_s(\rho)$	2.55	3.06
$d_v(\rho)$	0.056	0.068
$d_1(\rho, \alpha_i = 0)$	0.648	0.707
$d_1(\rho, \text{APE})$	0.320	0.346
$d_1(\rho, \text{HYP1})$	0.285	0.306
$d_1(\rho, \text{HYP2})$	0.032	0.026

α_i	0	APE	HYP1	HYP2
c	4.53	-3.63	-3.24	-7.82

where c and d_1 correspond to diagrams 2(a) and 2(b), respectively. The matching constant of the axial static-light current is defined by

$$Z_A^{\overline{\rm MS}} = 1 + \frac{\alpha_s}{12\pi^2}\left[\frac{5}{4} - \frac{e+f}{2} - d_1 + \frac{3}{2}\ln(a^2\mu^2)\right]. \quad (22)$$

We have collected the numerical values of c and d_1 in Table IV and we have given their analytical expression in Appendix C. We agree with the authors of [14] for the analytical expression of $d_1(\alpha_i = 0)$ [32] and for its numerical value. $f(\rho)$, $d_s(\rho)$, and $d_v(\rho)$, involving only light quark legs and computed in [33], are included in the same table for $\rho = 1.4$ and 1.6 that we chose to perform the lattice simulation, and $d_\xi = -4.792\,010$. We obtain for $\rho = 1.4$ and the set HYP1

$$\begin{aligned} Z_{11}^{\overline{\rm MS}}(1/a) &= 1 + \frac{\alpha_s^{\overline{\rm MS}}(1/a)}{4\pi} \times 20.0579, \\ Z_{22}^{\overline{\rm MS}}(1/a) &= 1 + \frac{\alpha_s^{\overline{\rm MS}}(1/a)}{4\pi} \times 19.6915, \\ Z_{A}^{\overline{\rm MS}}(1/a) &= 1 + \frac{\alpha_s^{\overline{\rm MS}}(1/a)}{4\pi} \times 11.2557. \end{aligned} \quad (23)$$

Here we would like to make two remarks.

The first one is that the bag parameters $\tilde{B}^{\overline{\rm MS}}(\mu)_i$ are matched to $\tilde{B}(1/a)_i$ with $\frac{Z_{ij}}{Z_A^2}$: in the ratio the quark self-energies cancel, reducing the corrections.

The second remark concerns the numerical value of the renormalization constants: one needs to define the expansion parameter α_s in terms of the lattice coupling, in order to improve as much as possible the perturbative computation. We decided in our analysis to use the constant $\alpha^V(3.41/a)$, that is related to the average plaquette $\langle 1/3\,\mathrm{Tr}(U_\square)\rangle$ [34], and the ratio $\Lambda_{\overline{\rm MS}}/\Lambda_V$, to compute $\alpha_s^{\overline{\rm MS}}(1/a)$ at two loops of perturbation theory. An alternative approach could have been to choose the scale $\mu = q^*$ between $1/a$ and π/a, as done in [12], and include the spreading in the systematic error as done in [27]. Of course in that case the logarithmic terms appearing in (6) must be taken into account.

IV. CONCLUSION

In this paper we have calculated the one-loop corrections at $O(a)$ of static-light currents $\bar{h}\Gamma q$ and four-fermion operators $(\bar{h}\Gamma q)(\bar{h}\Gamma q)$ in lattice HQET with a hypercubic blocking of the Wilson line which defines the static quark propagator. It determines the renormalization of the operators which are used to compute in the static limit of HQET the decay constant f_B and the bag parameters B_i associated with the $B_s - \bar{B}_s$ mixing amplitude in the standard model and beyond.

In particular we have given values of the renormalization constants of the static-light four-fermion operators when the light quark is described by the overlap action, which is an elegant way to restore on the lattice the chiral symmetry of the continuum but is highly demanding in computation time, so that a nonperturbative renormalization procedure, like the Schrödinger functional scheme [35], is not underway yet. However, a further step could be to compute in this scheme—i.e. nonperturbatively—the matching constants of static-light bilinear currents when the light quark is described in the bulk by the Neuberger operator [36].

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APPENDIX A: NOTATIONS

We give here the notations that appear in the main part of the paper and below in the analytical expressions of matching constants.

$$\int_k \equiv \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4}, \qquad \int_{\vec{k}} \equiv \int_{-\pi}^{\pi} \frac{d^3k}{(2\pi)^3},$$

$$U_\mu(n) = e^{iag_0 A^a_\mu(n)T^a} = 1 + iag_0 A^a_\mu(n)T^a - \frac{a^2g_0^2}{2!}A^a_\mu(n)A^b_\mu(n)T^aT^b + \mathcal{O}(g_0^3),$$

$$U^{\rm HYP}_\mu(n) = e^{iag_0 B^a_\mu(n)T^a} = 1 + iag_0 B^a_\mu(n)T^a - \frac{a^2g^2}{2!}B^a_\mu(n)B^b_\mu(n)T^aT^b + \mathcal{O}(g_0^3),$$

$$A^a_\mu(n) = \int_p e^{ip(n+a/2)}A^a_\mu(p), \qquad B^a_\mu(n) = \int_p e^{ip(n+a/2)}B^a_\mu(p), \qquad F^2 = \sum_{i=1}^4 F_i^2, \qquad \vec{F}^2 = \sum_{i=1}^3 F_i^2$$

$$\Gamma_\lambda = \sin ak_\lambda, \qquad c_\mu = \cos\left(\frac{a(p+p')_\mu}{2}\right), \qquad s_\mu = \sin\left(\frac{a(p+p')_\mu}{2}\right), \qquad M_\mu = \cos\left(\frac{k_\mu}{2}\right),$$

$$N_\mu = \sin\left(\frac{k_\mu}{2}\right), \qquad W = 2N^2, \qquad E^2 = \vec{N}^2 + \frac{a^2\lambda^2}{4}, \qquad E_1^2 = \frac{(\vec{N}^2)^2 + \frac{\vec{\Gamma}^2}{4}}{1+2\vec{N}^2}.$$

$$B^{(1)}_\mu(k) = \sum_\nu h_{\mu\nu}(k)A_\nu(k), \qquad h_{\mu\nu}(k) = \delta_{\mu\nu}D_\mu(k) + (1-\delta_{\mu\nu})G_{\mu\nu}(k),$$

$$D_\mu(k) = 1 - c_1\sum_{\rho\neq\mu}N_\rho^2 + c_2\sum_{\rho<\sigma,\rho,\sigma\neq\mu}N_\rho^2N_\sigma^2 - c_3N_\rho^2N_\sigma^2N_\tau^2,$$

$$G_{\mu\nu}(k) = N_\mu N_\nu\left(c_1 - c_2\frac{N_\rho^2+N_\sigma^2}{2} + c_3\frac{N_\rho^2N_\sigma^2}{3}\right) \equiv N_\mu N_\nu A'_\nu,$$

$$c_1 = (2/3)\alpha_1[1+\alpha_2(1+\alpha_3)], \qquad c_2 = (4/3)\alpha_1\alpha_2(1+2\alpha_3), \qquad c_3 = 8\alpha_1\alpha_2\alpha_3.$$

APPENDIX B: MATCHING CONSTANTS OF $\mathcal{O}(a)$ IMPROVED OPERATORS

Here we give the analytical expressions of the constants d_1, d_2, d^I, n, h, and q.

$$d_1(\alpha_i) = \ln(a^2\lambda^2) + (4\pi)^2\left\{\frac{1}{16}\int_{\vec{k}}\frac{1}{1+2\vec{N}^2}\frac{1}{\sqrt{1+E_1^2}}\frac{1}{E_1}\left(D_4 + \sum_{j=1}^3 A'_jN_j^2\right) + \frac{1}{16}\int_{\vec{k}}\frac{1}{1+2\vec{N}^2}\frac{1}{E_1^2-E^2}\right.$$
$$\left.\times\left[D_4\left(\frac{\sqrt{1+E^2}}{E} - \frac{\sqrt{1+E_1^2}}{E_1}\right) + \sum_{j=1}^3 A'_jN_j^2M_j^2\left(\frac{1}{E\sqrt{1+E^2}} - \frac{1}{E_1\sqrt{1+E_1^2}}\right)\right]\right\}, \tag{B1}$$

$$d_2(\alpha_i) = -\frac{1}{16}\int_{\vec{k}}\frac{D_4}{1+2\vec{N}^2}\frac{1}{E_1^2}, \quad d^I = -\int_{\vec{k}}\frac{D_4}{64}\frac{\vec{\Gamma}^2}{1+2\vec{N}^2}\frac{1}{\vec{N}^2E_1^2}, \tag{B2}$$

$$n = \int_{\vec{k}}\frac{1}{16}\frac{1}{1+2\vec{N}^2}\left[\frac{D_4\sqrt{1+E_1^2}}{E_1} + \frac{\sum_j A'_jN_j^2M_j^2}{E_1\sqrt{1+E_1^2}} + \frac{D_4 + \sum_j A'_jN_j^2}{4(E_1^2-\vec{N}^2)}\left(\frac{4\vec{N}^2(1+\vec{N}^2)-\vec{\Gamma}^2}{\sqrt{\vec{N}^2}\sqrt{1+\vec{N}^2}} - \frac{4E_1^2(1+E_1^2)-\vec{\Gamma}^2}{E_1\sqrt{1+E_1^2}}\right)\right], \tag{B3}$$

$$h = -\int_{\vec{k}}\frac{1}{16}\frac{D_4}{\vec{N}^2}, \qquad q = -\int_{\vec{k}}\frac{1}{64}\frac{D_4\vec{\Gamma}^2}{1+2\vec{N}^2}\frac{1}{E_1^2}. \tag{B4}$$

APPENDIX C: STATIC-LIGHT VERTEX WITH THE OVERLAP ACTION

Here we give the analytical expressions of c and $d_1(\rho)$:

$$c(\alpha_i) = 2\ln(a^2\lambda^2) + (4\pi)^2 \int_{\vec{k}} \frac{D_4^2 - E^2 \sum_{i=1}^3 N_i^2 A_i'^2}{4E^3} \frac{1}{\sqrt{1+E^2}}, \tag{C1}$$

$$\begin{aligned} d_1(\rho) - \ln(a^2\lambda^2) - d_\xi &= -(4\pi)^2 \int_k \frac{\sum_j h_{4j}}{2iN_4 + \epsilon M_4} \frac{1}{2W + a^2\lambda^2} \frac{-i\not{V} + \omega + b}{2\rho(\omega + b)} \frac{\rho}{\omega + \rho} \\ &\quad \times \left[\gamma_j M_j - iN_j - \frac{i\not{V} + b}{\omega}(\gamma_j M_j + iN_j)\right] \\ &= (4\pi)^2 \int_k \frac{1}{2W + a^2\lambda^2} \frac{1}{\omega + b} \frac{1}{\omega + \rho}\left[D_4\left(M_4^2 + \frac{\omega + b}{2}\right) + \sum_j A_j' N_j^2\left(M_j^2 + \frac{\omega + b}{2}\right)\right]. \end{aligned} \tag{C2}$$

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