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Just how different are the $SU(2)$ and $SU(3)$ Landau-gauge propagators in the infrared regime?

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The infrared behavior of gluon and ghost propagators in Yang-Mills theories is of central importance for understanding quark and gluon confinement in QCD. While simulations of pure $SU(3)$ gauge theory correspond to the physical case in the limit of infinite quark mass, the $SU(2)$ case (i.e. pure two-color QCD) is usually employed as a simplification, in the hope that qualitative features be the same as for the $SU(3)$ case. Here we carry out the first comparative study of lattice (Landau) propagators for these two gauge groups. Our data were especially produced with equivalent lattice parameters in order to allow a careful comparison of the two cases. We find very good agreement between $SU(2)$ and $SU(3)$ propagators, showing that in the IR limit the equivalence of the two cases is quantitative, at least down to about 1 GeV. Our results suggest that the infrared behavior of these propagators is independent of the gauge group $SU(N_c)$, as predicted by Schwinger-Dyson equations.

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I. INTRODUCTION AND MOTIVATION

Despite recent progress, the infrared structure of Yang-Mills theory is still not fully understood. For QCD, the study of the infrared limit is of central importance for the comprehension of the mechanisms of quark and gluon confinement and of chiral-symmetry breaking. In what concerns confinement, in Landau gauge, the infrared behavior of gluon and ghost propagators is linked with the Gribov-Zwanziger [1,2] and the Kugo-Ojima [3] confinement scenarios. These confinement mechanisms predict, at small momenta, an enhanced ghost propagator and a suppression of the gluon propagator. The strong infrared divergence for the ghost propagator corresponds to a long-range interaction in real space, which may be related to quark confinement. The suppression of the gluon propagator, which should vanish at zero momentum, implies (maximal) violation of reflection positivity and may be viewed as an indication of gluon confinement. Moreover, the interest in the propagators goes beyond the confinement mechanism, as they are inputs for many phenomenological calculations in hadronic physics (see, for example, Refs. [4,5]).

Analytic studies of gluon and ghost propagators using Schwinger-Dyson equations (SDE) [6–8] seem to agree with the above scenarios. (The reader should, however, be aware that, in the literature, there are solutions of the SDE [9,10] that do not comply with the Gribov-Zwanziger or the Kugo-Ojima predictions at small momenta.) Moreover, when dynamic quarks are neglected and assuming that $g^2 \sim 1/N_c$—as suggested by analysis of the large N_c limit [11]—the SDE become independent of the number of color N_c. Thus, they predict that gluon and ghost propagators be independent of N_c.

The Landau-gauge gluon propagator $D(k^2)$ has been investigated with lattice techniques in quenched QCD [i.e. pure $SU(3)$ Yang-Mills theory] [12–30], in pure $SU(2)$ Yang-Mills theory (in 2, 3, and 4 space-time dimensions) [31–41] and in full QCD [42–45]. All lattice studies in 4D suggest a finite nonzero infrared gluon propagator [20,24,26,27,42], in contradiction with the infrared Schwinger-Dyson solution. On the other hand, finite-size effects are very large and not yet well controlled, even in the 3D case [34]. Only in two space-time dimensions [41], using a lattice side L up to about 40 fm, does one find that $D(0)$ extrapolates to zero as L goes to infinity. Let us note that investigation of SDE on a 4-torus [46] suggests that the gluon propagator indeed approaches the infinite-volume limit very slowly, especially for its low-momentum components. On the other hand, even with an infrared-finite propagator, one clearly finds [27,28,33,36,45] that reflection-positivity is violated when sufficiently large lattice volumes are considered. Finally, in the 2D $SU(2)$ case [41] and in the 4D $SU(3)$ case (using asymmetric lattices) [26,30] it was found that the gluon propagator complies with the pure power-law behavior predicted analytically [6,8].

The lattice-Landau-gauge $SU(2)$ and $SU(3)$ ghost propagator $G(k^2)$ has been studied in [22,25,28–31,38–41,43,47–53] and in all cases an enhancement of the propagator compared to the tree-level behavior $1/k^2$ was observed. Concerning the comparison between lattice results and the SDE solution, the two propagators seem to agree only qualitatively. In particular, in three and in four space-time dimensions, the infrared exponent obtained using lattice simulations is always smaller than the one predicted analytically. On the other hand, on the 2D $SU(2)$ case [41], the ghost propagator shows an infrared behavior $1/k^{2.4}$, in agreement with the SDE solution [8].

In summary, for the Landau gauge, the SDE gluon and ghost propagators agree, at least qualitatively, with the lattice propagators. However, while analytic studies using Schwinger-Dyson equations predict *the same* infrared behavior for the $SU(2)$ and $SU(3)$ gauge groups, lattice simulations usually assume that the two cases are different, although their qualitative infrared features may be the same. In this paper, we carry out a comparative study of lattice-Landau-gauge propagators for these two gauge groups. Our data were especially produced by considering equivalent lattice parameters in order to allow a careful comparison of the two cases. We note that we do not assume a power-law behavior for the propagators, but just compare the raw data in the two cases.

II. NUMERICAL SIMULATIONS

We consider four different sets of lattice parameters, with the same lattice size N^4 and the same physical lattice spacing a for the two gauge groups (see Table I). The first three cases are chosen to yield approximately the same physical lattice volume $V = (Na)^4 \approx (1.7\ \text{fm})^4$. This allows a comparison of discretization effects. The fourth case corresponds to a significantly larger physical volume, i.e. $V \approx (3.2\ \text{fm})^4$, in order to study finite-size effects. For all four cases, 50 configurations were generated [55] using the Wilson action. The gluon and the ghost propagators

$$D^{ab}_{\mu\nu}(k^2) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)D(k^2), \tag{1}$$

TABLE I. Lattice setup. The lattice spacing was computed from the string tension, assuming $\sqrt{\sigma} = 440$ MeV. For $SU(3)$, the lattice spacing was taken from [54]. The corresponding β values for $SU(2)$ were computed using the asymptotic scaling analysis discussed in [35].

N^4	a (fm)	Na (fm)	$\beta_{SU(2)}$	$\beta_{SU(3)}$
16^4	0.102	1.632	2.4469	6.0
24^4	0.073	1.752	2.5501	6.2
32^4	0.054	1.728	2.6408	6.4
32^4	0.102	3.264	2.4469	6.0

$$G^{ab}(k^2) = -\delta^{ab}G(k^2) \tag{2}$$

were computed for four different types of momenta: $(k, 0, 0, 0)$, $(k, k, 0, 0)$, $(k, k, k, 0)$, and (k, k, k, k). In the computation of $D(k^2)$ and $G(k^2)$, an average over equivalent momenta and color components was always performed. In this work we use the field definitions and the choice of momenta reported in [35] for the $SU(2)$ case and in [26] for $SU(3)$. In particular, each component k is given (in lattice units) by $k = 2\sin(\pi n/N)$, where n is an integer.

Here, we do not check for possible effects of the breaking of rotational invariance [56]. In particular, we always compare results for the $SU(2)$ and the $SU(3)$ groups using the same type of momenta in the two cases. We also do not consider possible Gribov-copy effects. Indeed, even though they can play an important role in the infrared behavior of the propagators [21,31], with our set of lattice volumes and for the statistics considered here these effects should always be smaller than the statistical error.

The propagators were computed in the minimal Landau gauge, obtained by minimizing the functional

$$S[\Omega] = -\sum_{x,\mu} \mathrm{Tr} U^\Omega_\mu(x), \tag{3}$$

where $U^\Omega_\mu(x) = \Omega(x)U_\mu(x)\Omega^\dagger(x + \hat{e}_\mu)$ is the gauge-transformed link and $\hat{e}_\mu$ is the unit vector along the μ direction. For $SU(2)$ the gauge fixing was performed using a stochastic-overrelaxation algorithm (see [35] for details), while for $SU(3)$ a Fourier-accelerated steepest-descent algorithm was used (see [26] for details).

In what concerns the evaluation of the ghost propagator, in the $SU(2)$ case the Faddeev-Popov matrix was inverted using the method described in [31], while the $SU(3)$ simulation relies on the method discussed in Ref. [47] (considering more than one source). In the calculation of the gluon and of the ghost propagators, the statistical errors were computed with the (single-elimination) jackknife method in the $SU(3)$ case and with the bootstrap method (using 1000 bootstrap samples) in the $SU(2)$ case. We checked that these errors are in agreement with those obtained considering 1 standard deviation.

In order to compare the propagators from the different simulations, the gluon and ghost propagators were renormalized accordingly to

$$D(k^2)|_{k^2=\mu^2} = \frac{1}{\mu^2}, \qquad G(k^2)|_{k^2=\mu^2} = \frac{1}{\mu^2}, \tag{4}$$

using $\mu = 3$ GeV as a renormalization point. The lattice data were interpolated (using splines) to allow the use of such a renormalization point in all the simulations. We have checked that the interpolation reproduces perfectly the lattice data. Let us note that, due to breaking of rotational invariance, the renormalization factors $Z(\mu^2)$ depend, in general, slightly on the type of momenta. Here

we use, for all momenta k, the factor $Z(\mu^2)$ obtained from the on-axis momenta $(k, 0, 0, 0)$.

III. RESULTS

The renormalized $SU(2)$ and $SU(3)$ propagators can be seen for the various lattice setups in Fig. 1 (gluon) and in Fig. 2 (ghost) for the on-axis momenta $(k, 0, 0, 0)$. (Results are similar when considering the other types of momenta.) In all figures we report, on the horizontal axis, the squared magnitude k^2 (in GeV^2) of the four-momentum k. These figures show that, for the set of momenta accessible in our simulations, finite-volume and finite-spacing effects are under control. Moreover, they show that the $SU(2)$ and $SU(3)$ propagators are essentially equal, with slight differences in the low-momentum region. Similar results have been recently presented by Williams [57]. In Figs. 3 (gluon) and 4 (ghost) we show the ratios of $SU(3)$ over $SU(2)$ propagators. The statistical errors were computed assuming Gaussian-error propagation. Note that in the case of the gluon propagator there are momenta for which the discrepancy from 1 for the ratio is about 10% or larger. However, these deviations are not systematic and are probably due to a combination of several effects. These may include breaking of rotational invariance, small statistics, and finite-size effects, such as those related to the global $Z(N_c)$ symmetry of the lattice action [58–61].

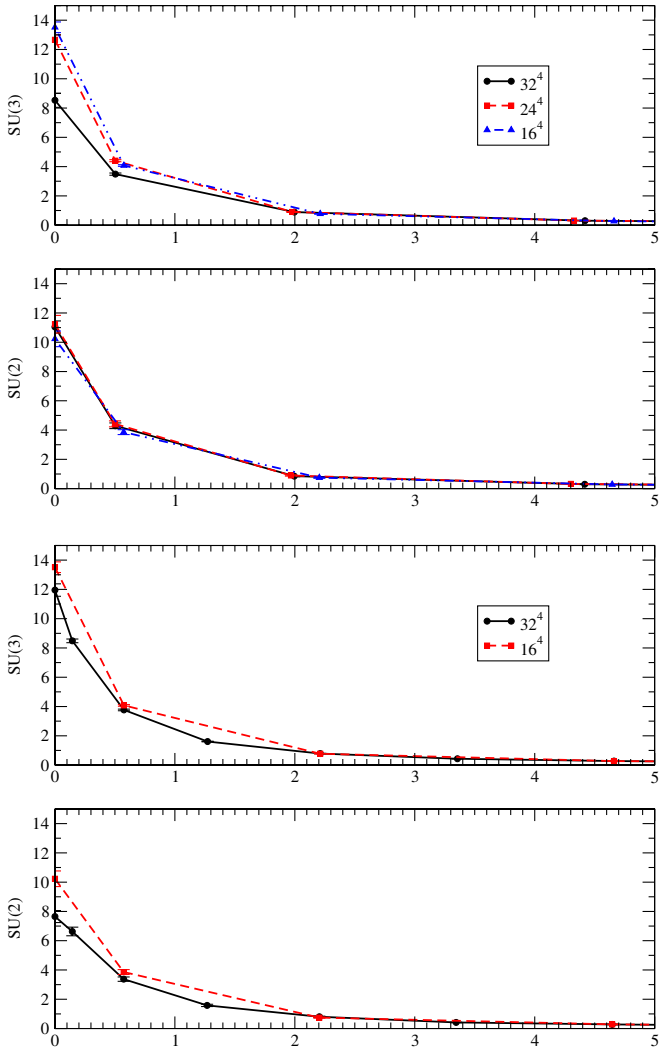

FIG. 1 (color online). Renormalized gluon propagator as a function of the squared magnitude k^2 of the four-momentum k, for on-axis momenta $(k, 0, 0, 0)$. We show results for the $SU(3)$ and the $SU(2)$ cases. In the two top plots we report data for the three sets of lattice parameters with approximately the same physical lattice volume V. In the two bottom plots we report data for the two sets of lattice parameters with $\beta = 6.0$ for $SU(3)$ and $\beta = 2.4469$ for $SU(2)$. In all cases we show only data for $k^2 \leq 5$ GeV^2. For larger momenta, the data using different lattice setups agree well.

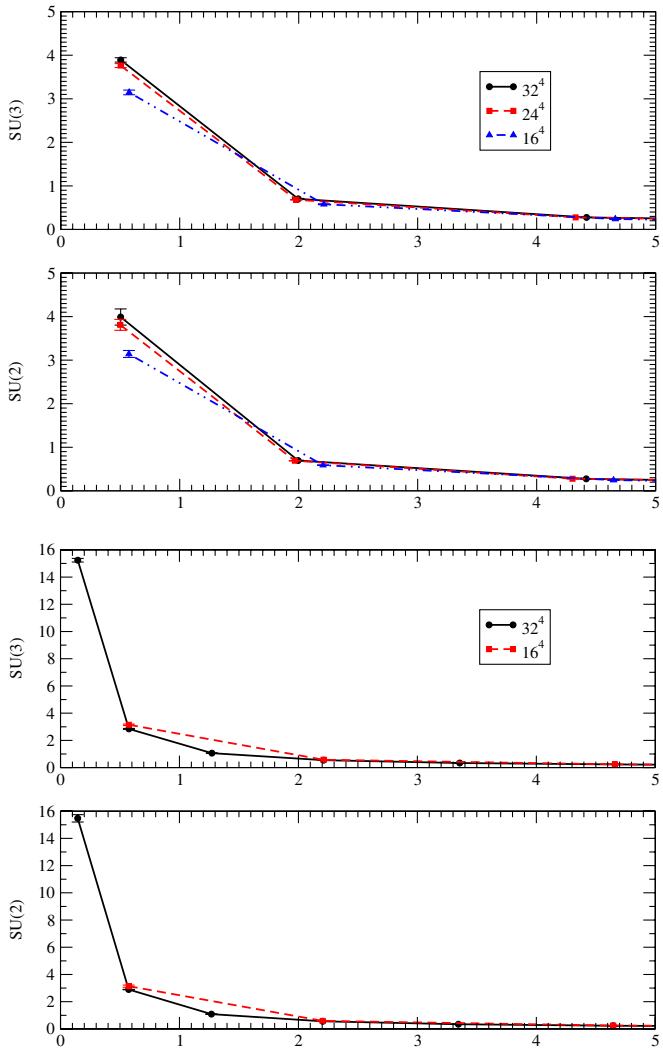

FIG. 2 (color online). Renormalized ghost propagator as a function of the squared magnitude k^2 of the four-momentum k, for on-axis momenta $(k, 0, 0, 0)$. The data are organized as in Fig. 1.

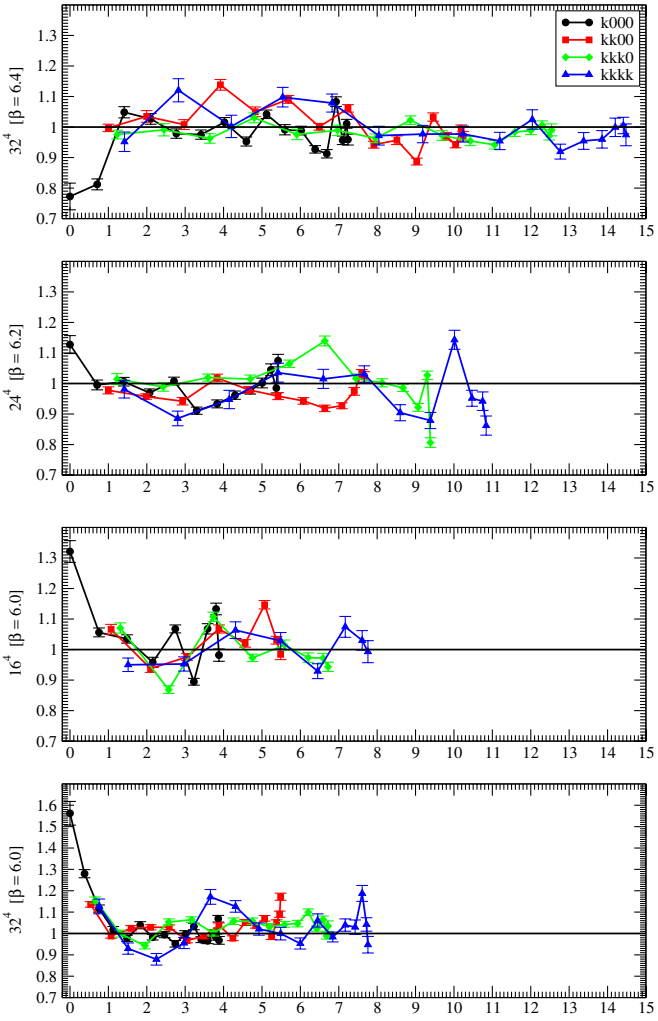

FIG. 3 (color online). Ratios of $SU(3)$ over $SU(2)$ gluon propagators for the four lattice setups considered.

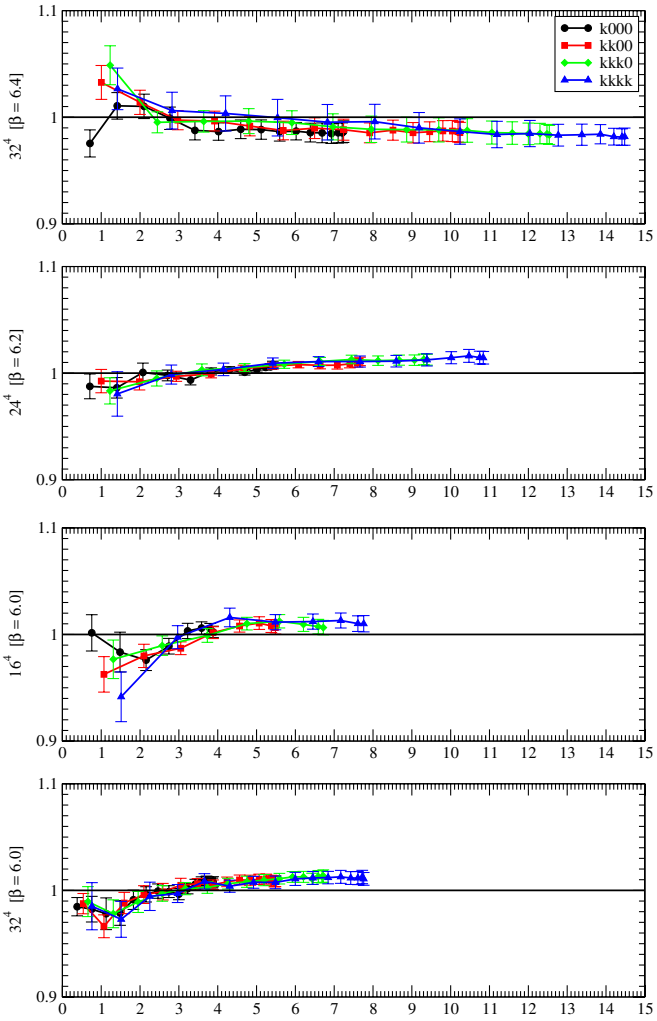

FIG. 4 (color online). Ratios of $SU(3)$ over $SU(2)$ ghost propagators for the four lattice setups considered.

IV. CONCLUSIONS

In summary, considering a careful choice of the lattice parameters, we were able to carry out an unambiguous comparison of the lattice Landau gluon and ghost propagators for $SU(2)$ and $SU(3)$ gauge theories. The data show that the two cases have very similar finite-size and discretization effects. Moreover, we find very good agreement between the two Yang-Mills theories (for our values of momenta larger than 1 GeV), for all lattice parameters and for all types of momenta. Below 1 GeV, the results for the two gauge groups show some differences, especially for the gluon propagator. Note, however, that all ratios are compatible with 1 within 2 standard deviations.

In this sense, our results suggest that the propagators are the same for all $SU(N_c)$ groups in the nonperturbative region, as predicted by Schwinger-Dyson equations. Of course, given the lattice volumes considered, further studies are required before drawing final conclusions about the comparison below 1 GeV. In particular, it will be interesting to investigate if this agreement persists also in the deep-infrared region, where the gluon propagator may show a turnover and a suppression, as predicted in the Gribov-Zwanziger scenario.

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