

Mixings of four-quark components in light nonsinglet scalar mesons in QCD sum rules

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Mixings of four-quark components in the nonsinglet scalar mesons are studied in the QCD sum rules. We propose a formulation to evaluate the cross correlators of $q\bar{q}$ and $qq\bar{q}\bar{q}$ operators and to define the mixings of different Fock states in the sum rule. It is applied to the nonsinglet scalar mesons, a_0 and K_0^* . It is found that the four-quark operators predict lower masses than the $q\bar{q}$ operators and that the four-quark states occupy about 70%–90% of the lowest mass states.

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I. INTRODUCTION

Hadrons which are not represented by minimal $q\bar{q}$ (meson) or qqq (baryon) configurations are called exotics, and Θ^+ with baryon number $B = 1$ and strangeness $S = +1$ is the most prominent example. Recent developments in hadron spectroscopy are largely enhanced by the discovery of the pentaquark Θ^+ [1]. The LEPS group, who first observed a peak in the NK spectrum produced by the SPring-8 photon beam, has confirmed the resonance in their succeeding experiment [2], while many others have been unsuccessful in producing the resonance in various types of experiments [3]. While experimental efforts are obviously needed now, a large number of theoretical studies have been carried out to explain the mass and the width of Θ^+ , as well as its structure and reactions. At the same time, various new types of exotic hadrons have been proposed and examined. In fact, the spectroscopy of old (ordinary) hadrons has been reexamined, and various possibilities of multiquark components of hadrons have been pointed out. Among them, the possible four-quark structures of the scalar mesons are not new, but rather an old idea [4,5]. The (nonrelativistic) quark model based on $SU(6) \times O(3)$ symmetry does not easily account for the mass pattern of the lowest-lying scalar-meson nonet, [$\sigma(600)$, $a_0(980)$, $f_0(980)$, $K_0^*(800)$]. (K_0^* has been indicated in $K\pi$ final states in J/ψ and D -meson decays, but has not yet been established [6].) The expected mass ordering as $q\bar{q}$ states would be $m(\sigma) \sim m(a_0) < m(f_0)$, if we assume ideal mixing, i.e., $\sigma \sim \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$, $a_0 \sim \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$, and $f_0 \sim s\bar{s}$. This pattern does not agree with the one from experiment. Furthermore, while the scalar mesons are classified as 3P_0 states, their spin-orbit partners $J = 1$ and 2 states are not observed in their vicinity.

A possible solution to this puzzle is to consider four-quark exotic states for the scalar mesons [4]. Suppose that diquarks with flavor 3, color 3, and spin 0, i.e.,

$$\begin{aligned} U &= (\bar{d}\bar{s})_{S=0,C=3,f=3}, & D &= (\bar{s}\bar{u})_{S=0,C=3,f=3}, \\ S &= (\bar{u}\bar{d})_{S=0,C=3,f=3}, \end{aligned} \quad (1)$$

are the building blocks of the scalar mesons. Then the scalar nonet in the ideal mixing may appear as

$$\begin{aligned} \sigma &\sim S\bar{S} \sim (ud)(\bar{u}\bar{d}), \\ a_0 &\sim \frac{1}{\sqrt{2}}(U\bar{U} - D\bar{D}) \sim \frac{1}{\sqrt{2}}[(ds)(\bar{d}\bar{s}) - (su)(\bar{s}\bar{u})], \\ f_0 &\sim \frac{1}{\sqrt{2}}(U\bar{U} + D\bar{D}) \sim \frac{1}{\sqrt{2}}[(ds)(\bar{d}\bar{s}) + (su)(\bar{s}\bar{u})]. \end{aligned}$$

Then one sees that the strange-quark counting successfully predicts the observed mass pattern: $m(\sigma) < m(a_0) \sim m(f_0)$. It also explains why the $J = 0$ state is isolated from $J = 1$ and 2 partners and thus helps to explain anomalies in the scalar-meson nonets. There are other hadrons which are pointed out to have possible exotic multiquark components, such as D_s^* , $X(3872)$, and $\Lambda(1405)$, where the last one may be a baryon resonance composed of 5 quarks [7–11].

Does the QCD dynamics allow such states? We actually have a simple reasoning why the scalar mesons have significant four-quark components. As $q\bar{q}$ states, the scalar mesons, which have positive parity, must contain orbital excitation $L = 1$ (P -wave mesons). It is generally accepted that the orbital excitation in the quark model requires additional 500 MeV, for instance, as is seen from the mass differences between the positive- and the negative-parity baryons; $N(1535) - N(940)$. On the other hand, the four-quark components, $qq\bar{q}\bar{q}$, may be realized in $L = 0$ without orbital excitation and thus have an advantage over the P -wave excited states. In fact, the cost of having an extra $q\bar{q}$ pair may be about 500 MeV; the average mass of π and ρ mesons. In terms of the diquarks, the combination of two scalar diquarks in $L = 0$ is quite preferable in the

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standard quark model and may easily reproduce the observed spectrum of the scalar mesons. Therefore it is quite interesting and important to answer the question whether QCD indeed gives multi-quark states a lower energy and induces mixings of the different Fock components.

Our purpose in this paper is to study the possible mixing of different Fock components in the QCD-sum-rule approach. It is important to establish a well-controlled formulation of the mixings of two (or more) types of operators in producing hadrons from the vacuum. We also examine the definition of the Fock-state-mixing parameters so that we can compare the results with the predictions of the quark models. In this paper, we focus on the four-quark components of the flavor nonsinglet scalar mesons, a_0 and K_0^* , and demonstrate how our formulation works in this case.

In Sec. II, we present the basic ideas of the QCD-sum-rule approach [12,13] and give the interpolating field operators corresponding to the $q\bar{q}$ and $qq\bar{q}\bar{q}$ components of the scalar mesons. In Sec. III, we consider the definition of the mixing parameters of the two Fock components. Two distinct methods of defining the mixing parameters are presented. In Sec. IV, we give the results of the sum rules and study their significances. The conclusion is given in Sec. V.

II. THE QCD SUM RULES

The sum rule is obtained by expressing the two-point function,

$$\Pi(p^2) = i \int d^4x e^{ipx} \langle 0 | T [J(x) J^\dagger(0)] | 0 \rangle, \quad (2)$$

in two ways. One of them is based on the operator product expansion (OPE), where Eq. (2) is calculated in deep Euclidean region ($-p^2 \rightarrow \infty$) and is described in terms of the QCD parameters, such as the quark condensate $\langle \bar{q}q \rangle$, the gluon condensate $\langle \alpha_s \pi^{-1} G^2 \rangle$, the current-quark masses m_q , and so on. The other one is based on a phenomenological parametrization of the spectral function. The spectral function at the physical region ($p^2 > 0$) is assumed to have a sharp-peak resonance at $p^2 = m^2$ and continuum at $p^2 > s_{\text{th}}$:

$$\rho_{\text{phen}}(p^2) = |\lambda|^2 \delta(p^2 - m^2) + \theta(p^2 - s_{\text{th}}) \rho_{\text{OPE}}(p^2). \quad (3)$$

The sum rule is obtained by matching the two expressions so that the mass of the resonance, m , and the other phenomenological parameters can be determined from the QCD parameters. The two expressions of the correlators are connected by a dispersion relation. To simplify the sum rule, we approximate the continuum spectrum as the same form of the spectral function on the OPE side.

We further apply the Borel transformation, which is

$$\mathcal{B} \equiv \lim_{-p^2, n \rightarrow \infty} \frac{(-p^2)^{n+1}}{n!} \left(\frac{d}{dp^2} \right)^n, \quad (4)$$

where the limit is taken with $M^2 \equiv \frac{-p^2}{M^2}$ fixed. It suppresses large- p^2 region by the factor of e^{-p^2/M^2} , and thus the pole contribution is enhanced. Because of the p^2 derivatives, the subtraction terms vanish. The Borel transformation also suppresses the effects of the higher-dimensional terms in the OPE.

We employ the following local operators for a_0^- :

$$\begin{aligned} J_2(x) &= (\bar{u}_a(x) d_a(x)), \\ J_4(x) &= \epsilon_{abc} \epsilon_{dec} (d_a^T(x) C \gamma_5 s_b(x)) (\bar{s}_d(x) \gamma_5 C \bar{u}_e^T(x)), \end{aligned} \quad (5)$$

where a, b, \dots represent the color and $C = i\gamma^2\gamma^0$. The two-quark operator, J_2 , is uniquely determined, while a specific four-quark operator, J_4 , is taken so as to consist of two 0^+ -diquark operators. The flavor structure of J_4 is arranged to be $\bar{U}D$, and thus identical to J_2 . Similarly, the two-quark and the four-quark operators for K_0^* are given by the $SU_f(3)$ rotation from the fields for a_0 , i.e., $u \rightarrow d, d \rightarrow s, s \rightarrow u$ in Eq. (5).

The number of independent four-quark operators is discussed in a previous paper [14]. There are two kinds of flavor-octet four-quark operators: i.e., $\bar{3}_c \times 3_c$ and $6_c \times \bar{6}_c$. Each kind allows five independent operators; thus altogether there are ten possible operators.

It is, however, difficult to find the most suitable operator for the QCD sum rule of the scalar mesons. In Ref. [14], the authors attempt to find the best combination. It is interesting to employ such an operator for the present method, while the computation may be much more complicated.

Instead, as the first attempt, we employ the simplest operator composed of the $\bar{3}_f$ scalar diquarks, $J^P = 0^+$ and $C = \bar{3}$. The scalar diquark is the one that is favored by both perturbative and nonperturbative interactions of quarks in QCD. Perturbative one-gluon exchange gives an attraction to the scalar diquark comparable to that in the color-singlet $\bar{q}q$ system. On the other hand, as a nonperturbative interaction, the instanton induced interaction, which is conjectured to be responsible for spontaneous breaking of chiral symmetry in the QCD vacuum, is strongly attractive in the scalar diquark channel as well. Thus, the operator in Eq. (5) is a suitable choice for the lowest four-quark states.

Our results of the OPE side are listed in the Appendix. The OPE is truncated after the dimension-6 terms for $\Pi_{22}(p^2)$ [see Eq. (A1)]. In order to deal with the same order of power corrections, we expand $\Pi_{24}(p^2)$ [Eq. (A2)] up to dim-9 and $\Pi_{44}(p^2)$ [Eq. (A3)] up to dim-12. High dimensional operators, such as $\langle \bar{q} \bar{q} qq \rangle$ and $\langle \bar{q} \bar{q} qqG \rangle$, are evaluated by the vacuum-saturation approximation.

We consider the diagrams containing the $q\bar{q}$ annihilation in order to take into account the mixing of different Fock states. The quark-pair annihilations are substituted by $\langle \bar{q}q \rangle$ or $\langle \bar{q}g_s \sigma \cdot Gq \rangle$ (see Fig. 1). Because the interpolating fields (5) have the normal ordering, the perturbative part of the $q\bar{q}$ annihilation must disappear. Because the OPE is

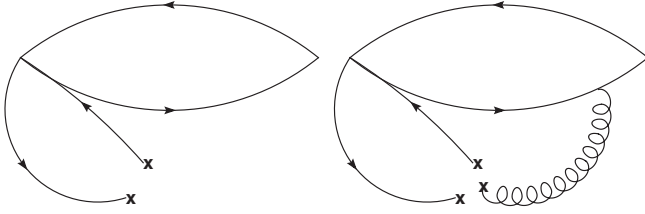


FIG. 1. The annihilation diagrams.

represented as a polynomial in x , only the zeroth order term survives in the $x \rightarrow 0$ limit.

III. MIXING OF FOCK STATES

The main goals of this study are to determine whether the scalar mesons are dominated by $q\bar{q}$ or $qq\bar{q}\bar{q}$, and if they are mixed, to calculate their probabilities. It turns out that such a mixing is not easily quantified. It would be natural to consider the strengths of the couplings of the two-quark and four-quark operators to the physical state and then evaluate the mixing angle. However, such a procedure is strongly dependent on the definition and normalization of the local operators. Indeed, a numerical factor can be easily hidden in the local operators and thus the magnitudes of the coupling strengths are ambiguous.

This problem happens to be more fundamental than just the definition of the operators. In quantum mechanics, mixings of Fock states may occur if the Hamiltonian contains terms which change the number of particles. Then the wave function of an eigenstate can be written uniquely as a linear combination of the Fock components, each of which is normalized properly. In the field theory, however, the Fock-space separation may not be unique as the number of quarks, which is defined by the number of quarks + number of antiquarks, is not a conserved quantum number. One may not be able to “measure” the number of quarks without ambiguity because no conserved charge corresponding to the number of quarks is available. Thus we have to consider the concept of the “number of quarks” in the context of the quantum-mechanical interpretation of the field-theoretical state.

Here, we propose two ways to define the ratio of the Fock-space probability by taking the mixing of $q\bar{q}$ and $qq\bar{q}\bar{q}$ in the scalar (a_0) meson as an example.

In the first approach, we define local operators “normalized” in the context of a full four-quark operator J_4 in Eq. (5). In fact, the four-quark operator J_4 contains a $q\bar{q}$ component, which is obtained by contracting the $\bar{q}q$ pair

by the quark condensate:

$$J_4(x) = J'_4(x) + \underbrace{\frac{1}{6}(\bar{s}s)}_{J'_2} J_2(x). \quad (6)$$

We regard J'_2 and J'_4 as normalized two-quark and four-quark fields, respectively. The quark condensate gives the dimension of normalization. Using $J'_2(x)$ and $J'_4(x)$, we define the mixing parameter, θ , by

$$\begin{aligned} \langle 0|J'_2(x)|a_0\rangle &= \lambda \cos\theta\phi(x), \\ \langle 0|J'_4(x)|a_0\rangle &= \lambda \sin\theta\phi(x). \end{aligned} \quad (7)$$

This definition happens to be equal to defining θ so that $J_a(x) = \cos\theta J'_2(x) + \sin\theta J'_4(x)$ couples to the physical state most strongly:

$$\langle 0|J_a(x)|a_0\rangle = \lambda\phi(x), \quad (8)$$

where $\phi(x)$ denotes the wave function of the center-of-mass motion of a_0 (i.e., a plane wave for a momentum eigenstate).

Then the mixing parameter can be evaluated from the following three correlation functions with an assumption that the poles are at the same position:

$$\begin{aligned} \frac{1}{\pi} \text{Im}i \int d^4x e^{ipx} \langle 0|T[J'_2(x)(J'_2)^\dagger(0)]|0\rangle &= \delta(p^2 - m^2)|\lambda|^2 \cos^2\theta + \text{cont}, \\ \frac{1}{\pi} \text{Im}i \int d^4x e^{ipx} \langle 0|T[J'_4(x)(J'_4)^\dagger(0)]|0\rangle &= \delta(p^2 - m^2)|\lambda|^2 \sin^2\theta + \text{cont}, \\ \frac{1}{\pi} \text{Im}i \int d^4x e^{ipx} \langle 0|T[J'_2(x)(J'_4)^\dagger(0)]|0\rangle &= \delta(p^2 - m^2)|\lambda|^2 \frac{\sin 2\theta}{2} + \text{cont}. \end{aligned} \quad (9)$$

This definition of the mixings is model independent, but it depends on the choice of the local operators. Therefore, it does not necessarily have a direct relation to the mixing parameters employed in the quark models.

In order to define a mixing angle more appropriately to the quark models, one may determine the normalization of the operators using quark-model wave functions. For instance, the local operators in Eq. (5), can be normalized to the wave functions of $q\bar{q}$ or $qq\bar{q}\bar{q}$ states in the MIT bag model:

$$\begin{aligned} |(\bar{q}q)_{P_0}\rangle &= \frac{i}{\sqrt{2}}(|PS\rangle + |SP\rangle) \otimes \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \otimes |\bar{u}d\rangle \otimes \frac{1}{\sqrt{3}}\delta_{ab}|\bar{a}b\rangle, \\ |(qq\bar{q}\bar{q})_{S_0}\rangle &= |SSSS\rangle \otimes \frac{1}{2}(|\uparrow\uparrow\downarrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle) \otimes \frac{1}{2}\epsilon_{\alpha\beta u}\epsilon_{\delta\rho d}|\alpha\beta\bar{\delta}\bar{\rho}\rangle \otimes \frac{1}{2\sqrt{3}}\epsilon_{abc}\epsilon_{dec}|\bar{a}b\bar{d}\bar{e}\rangle, \end{aligned} \quad (10)$$

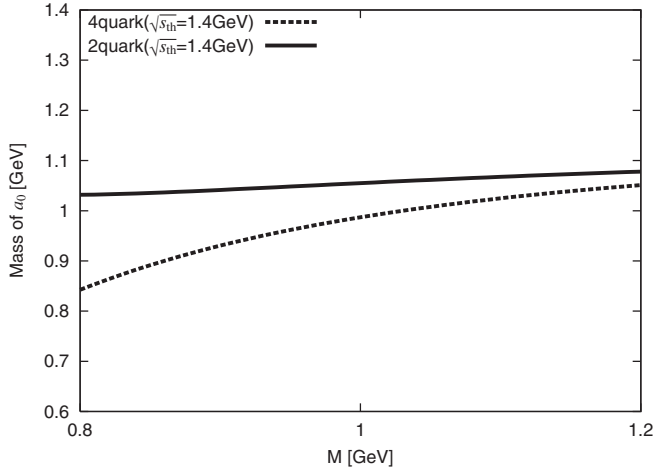


FIG. 2. The masses of a_0 for the pure two-quark and pure four-quark operators as functions of the Borel mass, M . The threshold parameter, $\sqrt{s_{\text{th}}}$, is fixed to 1.4 GeV.

where α, β, \dots represent the flavor and a, b, \dots represent the color. Then we compute the matrix elements,

$$\begin{aligned} \langle 0|J_2(0)|(\bar{q}q)_{P_0}\rangle &= \lambda_2 \phi(0), \\ \langle 0|J_4(0)|(qq\bar{q}\bar{q})_{S_0}\rangle &= \lambda_4 \phi(0), \end{aligned} \quad (11)$$

where $\phi(0)$ denotes the center-of-mass wave function of the bag-model state. Now, assuming the bag-model states (with definite number of quarks) are normalized properly, one can use λ_2 and λ_4 for the normalizations of the operators J_2 and J_4 , respectively.

In calculating the mixing parameter, one needs only the ratio of λ_2 and λ_4 , which is given by

$$\begin{aligned} \frac{\lambda_4}{\lambda_2} &= -\frac{i}{4\pi} \frac{\{\mathcal{N}_4(s_{1/2})\}^4}{\mathcal{N}_2(s_{1/2})\mathcal{N}_2(p_{1/2})} \approx -0.24i \frac{R^3}{R_4^3}, \\ \mathcal{N}_n(S_{1/2}) &= \frac{ER_n}{R_n^{3/2}|j_0(ER_n)|} \frac{1}{\sqrt{2ER_n(-1+ER_n)}}, \\ \mathcal{N}_n(P_{1/2}) &= \frac{ER_n}{R_n^{3/2}|j_1(ER_n)|} \frac{1}{\sqrt{2ER_n(1+ER_n)}}, \end{aligned} \quad (12)$$

and $ER_n = \begin{cases} 2.04 & \text{for } S_{1/2} \\ 3.81 & \text{for } P_{1/2}, \end{cases}$

where R is the bag radius and gives the dimensional scale of the normalizations of the two operators that have different dimensions. We here assume that the bag radius of the $q\bar{q}$ state is the same as that of the $qq\bar{q}\bar{q}$ state. This assumption may be necessary to consider their mixing in the physical state. It is, in fact, not a bad assumption

because the bag radius of the $q\bar{q}$ state must be larger, because of the relative P -wave motion, while that of the four-quark state is large, because it has more quarks.

Thus, the physical state for a_0 is given by the mixings of the two states:

$$\begin{aligned} |a_0\rangle &= i \cos\theta |(\bar{q}q)_{P_0}\rangle + \sin\theta |(qq\bar{q}\bar{q})_{S_0}\rangle \\ &= \cos\theta |a_0(2q)\rangle + \sin\theta |a_0(4q)\rangle. \end{aligned} \quad (13)$$

Note that here the factor i is necessary to keep the phases of the two- and four-quark states in accord so that the mixing parameter can be defined as a real parameter.

In the QCD sum rules, the mixing parameter with the bag-model normalization can be calculated as

$$\frac{\text{Im} \int d^4x e^{ipx} \langle 0|J'_4(x)(J'_4)^\dagger(0)|0\rangle}{\text{Im} \int d^4x e^{ipx} \langle 0|J_2(x)J_2^\dagger(0)|0\rangle} = \left| \frac{\lambda_4}{\lambda_2} \right|^2 \tan^2\theta. \quad (14)$$

This definition is model dependent, but it gives a direct interpretation associated with the quark model.

IV. RESULTS

The mass formula is given by

$$m = \sqrt{\frac{\frac{\partial}{\partial(-1/M^2)} \mathcal{B}[\Pi_{\text{OPE}}(p^2)\theta(s_{\text{th}} - p^2)](M^2)}{\mathcal{B}[\Pi_{\text{OPE}}(p^2)\theta(s_{\text{th}} - p^2)](M^2)}}. \quad (15)$$

We plot the mass of a_0 in the case of pure two-quark and pure four-quark in Fig. 2. We use the values of the QCD parameters given in Table I, where $\langle \bar{u}u \rangle \equiv \langle \bar{d}d \rangle \equiv \langle \bar{q}q \rangle$. We neglect the masses of the u - and the d -quarks. We see that the Borel stability is fairly good. The positions of the poles of the two-quark and the four-quark are close to each other. The four-quark state is slightly lighter.

The mixing parameters from the first-normalization method and the normalized operators according to the bag-model wave functions are plotted for various bag radii in Fig. 3. We observe that the mixing parameter is almost independent of the Borel mass, M . We find that it is independent of threshold parameter, s_{th} , too. The weak M -dependence of the mixing parameter, which is dimensionless, is attributed to the cancellation of the p -dependences (M -dependences) of the two-quark and four-quark correlators. The mixing parameter from the first-normalization method is about 70 degrees; in other words, the four-quark component occupies 90% of a_0 . In the bag model, we take the central value as $R = 4.8 \text{ GeV}^{-1}$, which is the bag radius determined for the $\bar{q}q$ component of a_0 [15]. The mixing parameter for $R = 5.2 \text{ GeV}^{-1}$ is almost the same as the one in the first-

TABLE I. Standard values of the QCD parameters.

m_s	$\langle \bar{q}q \rangle$	$\langle \bar{s}s \rangle$	$m_0^2 \equiv \langle \bar{q}g_s \sigma \cdot Gq \rangle / \langle \bar{q}q \rangle$	$\langle \alpha_s \pi^{-1} G^2 \rangle$	α_s
0.12 GeV	$(-0.23 \text{ GeV})^3$	$0.8 \times \langle \bar{q}q \rangle$	0.8 GeV^2	$(0.33 \text{ GeV})^4$	0.4

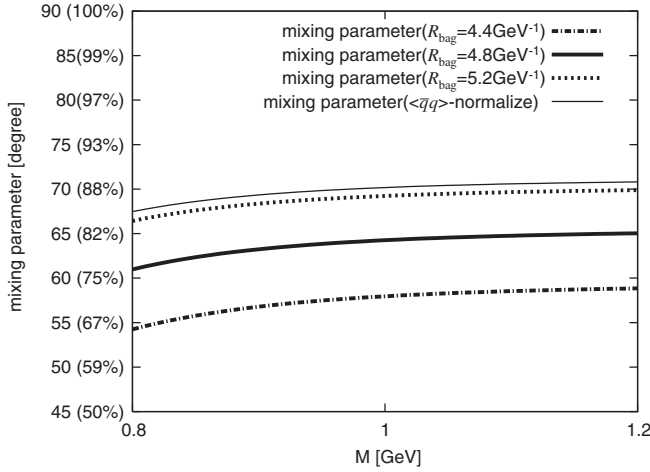


FIG. 3. The mixing parameters of a_0 plotted as functions of the Borel mass, M . The threshold parameter, $\sqrt{s_{\text{th}}}$, is fixed to 1.4 GeV.

normalization method. We see that the dominant component of a_0 (about more than 70%) is the four-quark state for $R > 4.4 \text{ GeV}^{-1}$ and for the first-normalization method.

We plot the mass of the mixed a_0 for various threshold parameter values, s_{th} , in Fig. 4. The predicted mass extracted from the mixed operator, $J_a(x)$, is about 0.9–1.1 GeV, which is similar to the result from the pure-four-quark operator given in Fig. 2.

We examine how well these sum rules work. We plot the pole contribution defined by

$$\frac{\mathcal{B}[\Pi_{\text{OPE}}(p^2)\theta(s_{\text{th}} - p^2)]}{\mathcal{B}[\Pi_{\text{OPE}}(p^2)]}, \quad (16)$$

in Fig. 5. The valid Borel window is taken as the region where the pole contribution is more than 30%. This constraint is weaker than the one usually adopted [13], but we

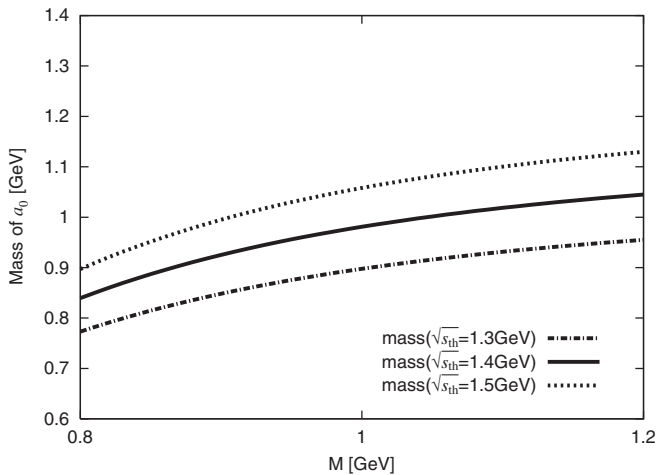


FIG. 4. Masses of a_0 for three choices of the threshold parameter, $\sqrt{s_{\text{th}}} = 1.3, 1.4, \text{ and } 1.5 \text{ GeV}$. The two-quark and four-quark operators are mixed by the first normalization method.

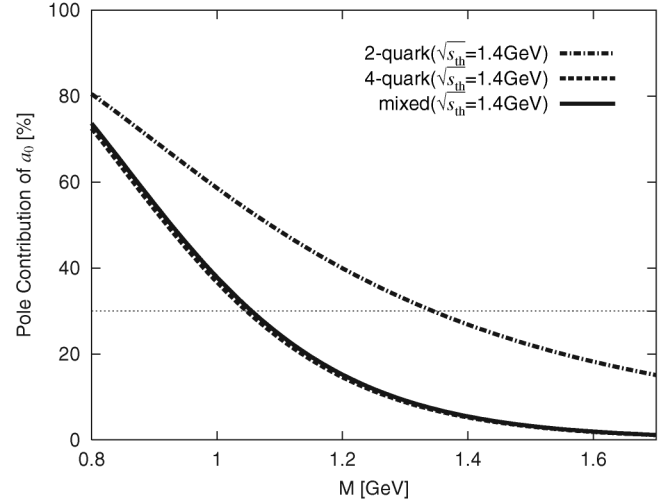


FIG. 5. The pole contribution defined by Eq. (16) for a_0 is plotted as a function of the Borel mass, M , in the cases of pure two-quark components, pure four-quark components and a mixing between them. The threshold parameter, $\sqrt{s_{\text{th}}}$, is fixed to 1.4 GeV.

observe that the results are not sensitive to the choice of this value. In Fig. 5, it is seen that the pole contribution for the pure four-quark correlator is less than the one for the pure two-quark correlator. The reason is that the OPE of the four-quark correlator contains higher powers of p^2 and grows rapidly for large p^2 . The sum rule for the mixed operator shows similar results. We set the Borel window $M < 1.1 \text{ GeV}$. We observe from Fig. 2 that the positions of the poles of the two-quark and four-quark correlators begin to separate in this region. Such Borel mass dependence is considered as a signature of the differences in the couplings of the pole state and the continuum to the two-quark and four-quark operators. The separation at the lower Borel

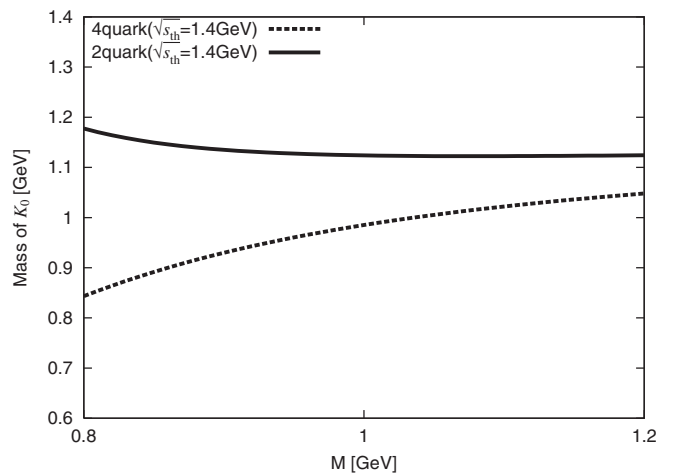


FIG. 6. The masses of K_0^* for the pure two-quark and pure four-quark operators as functions of the Borel mass, M . The threshold parameter, $\sqrt{s_{\text{th}}}$, is fixed to 1.4 GeV.

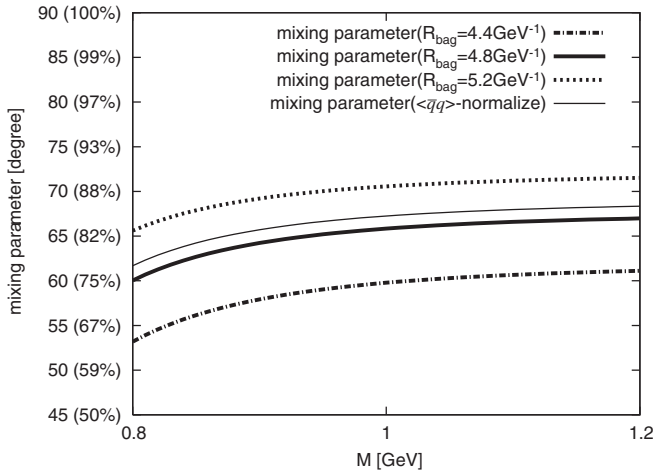


FIG. 7. The mixing parameters of K_0^* plotted as functions of the Borel mass, M . The threshold parameter, $\sqrt{s_{\text{th}}}$, is fixed to 1.4 GeV.

mass indicates that the mixing of the two-quark and four-quark states at the pole position is weak so that the two-quark operator is occupied mainly by higher resonance/continuum states.

The results of K_0^* for pure two-quark and four-quark components are presented in Fig. 6. As compared to the Borel behavior of a_0 in Fig. 2, the mass extracted from the two-quark correlator for K_0^* is heavier, while that of the four-quark components lies at about the same mass as a_0 . The experimental value of the K_0^* mass has been reported as about 0.84 GeV, and it seems to have a large width: about 600 MeV [6]. The scalar-nonet spectrum from our sum rule with the four-quark component shows better agreement with the observed values than that with the two-quark operator. The position of the pole of K_0^* for the two-quark operators starts splitting at low M values from that of the

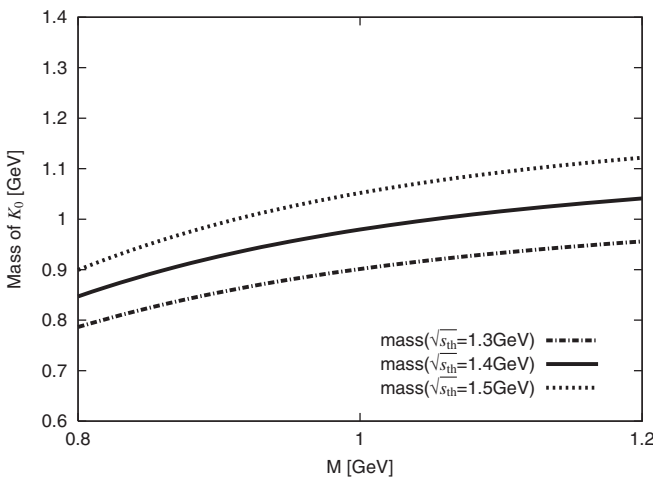


FIG. 8. Masses of K_0^* for three choices of the threshold parameter, $\sqrt{s_{\text{th}}} = 1.3, 1.4,$ and 1.5 GeV. The two-quark and four-quark operators are mixed by the first normalization method.

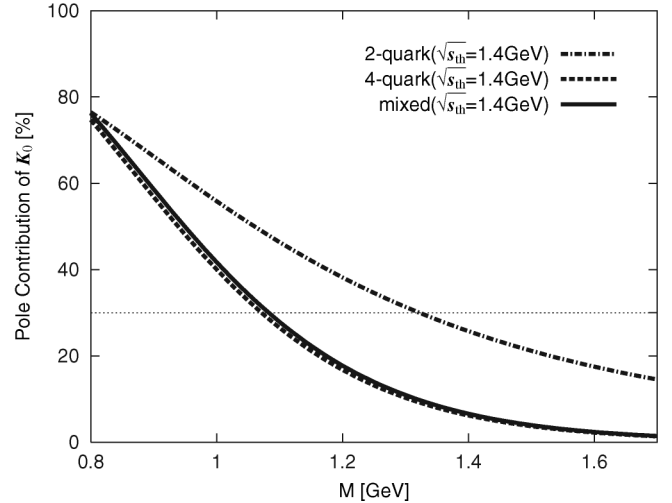


FIG. 9. The pole contribution defined by Eq. (16) for K_0^* is plotted as a function of the Borel mass, M , in the cases of pure two-quark components, pure four-quark components, and a mixing between them. The threshold parameter, $\sqrt{s_{\text{th}}}$, is fixed to 1.4 GeV.

four-quark operators. We, however, attempt to evaluate the mixing parameter by assuming that the positions of the poles are at the same position. The results are shown in Figs. 7 and 8, which correspond to Figs. 3 and 4 in the case of a_0 , respectively. One sees that the four-quark component is dominant, occupying about 70%–90% of K_0^* . These results for K_0^* are similar to those for a_0 .

The pole contribution for K_0^* is shown in Fig. 9. We observe from Figs. 5 and 9 that the behavior is similar to that for a_0 .

V. CONCLUSION

A formulation is proposed to take into account the mixing between different Fock states in the QCD sum rules. In order to quantify the mixing probability, one needs the normalization of the operators. We suggest two ways: One is to define the “normalization” using a multi-quark operator which couples to the ground state most strongly. The other way is to adjust the operators to the normalized wave functions from the MIT bag model.

We apply the formulation to a_0 and K_0^* which are members of the scalar nonet. Our sum rules indicate that 70%–90% of these scalar mesons is composed of the four-quark components. We find that K_0^* is almost degenerate with a_0 .

There exist several other studies of the four-quark scalar nonet in the QCD sum rule. Two of them [14,16] are consistent with our work in reproducing the lighter four-quark mass than two-quark mass. In these analyses, the K_0^* is predicted to have a smaller mass than a_0 . It should, however, be noticed that the predicted masses depend on the threshold parameter rather strongly. Therefore, one

may choose the threshold so that K_0^* is lighter than a_0 reproducing experimental data. K_0^* is a broad resonance which is not completely established. Therefore we do not further examine the mass difference of a_0 and K_0^* . Another work [17] claims that no signal is obtained for the four-quark scalar nonet. We note that their analysis differs from ours in not considering the mixing diagrams, truncating dimension in the OPE, interpolating field, and the definition of the coupling strength of ground state. An article [18] concludes that there is no four-quark scalar nonet, because OPE convergence and pole dominance are not simultaneously fulfilled. We, however, confirm that the OPE tends to converge at the dimension 12. We also note that the pole signal significantly contributes to the spectrum function, although our criterion of the pole dominance (i.e., larger than 30%) is less stringent than the one in the reference (i.e., larger than 50%).

It is extremely interesting that some of the excited hadrons are accounted for by considering exotic multi-quark components. There are several other ‘‘anomalous’’ hadrons which may contain exotic multi-quark components. Among them, the P -wave baryons, in particular $\Lambda(1405)$, are strong candidates to have five-quark components. In another paper [19], we report the results of applying the current method of the Fock-space mixing to the flavor-singlet Λ state. Multi-quark mesons may also be found in heavy quark systems, where newly found states do not fit

well in the $q\bar{q}$ spectrum and thus are suspected to have four-quark components.

It is also important to examine the decay widths and the branching ratios of those multi-quark states. The mechanism for the fall-apart decay in which the multi-quark hadrons dissociate into two color-singlet hadrons without creating $q\bar{q}$ pairs. The widths associated with the fall-apart processes depend strongly on the quantum numbers as well as on the configurations of the multi-quarks. As QCD does not forbid such multi-quark states, the width is the key to understand why we do not see many ‘‘exotic’’ hadrons in nature. Their possibility should be further pursued both experimentally and theoretically.

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APPENDIX: THE RESULT OF OPE

The results of the OPE are summarized as

$$\begin{aligned} \Pi_{22}(p^2) &= i \int d^4x e^{ipx} \langle 0 | T [J_2(x) J_2^\dagger(0)] | 0 \rangle \\ &= -\frac{3}{8\pi^2} p^2 \ln(-p^2) - \frac{1}{2p^2} (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) - \frac{1}{p^2} (m_d \langle \bar{u}u \rangle + m_u \langle \bar{d}d \rangle) - \frac{1}{8p^2} \langle \alpha_s \pi^{-1} G^2 \rangle \\ &\quad - \frac{16\pi\alpha_s}{27(p^2)^2} (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2) - \frac{16\pi\alpha_s}{3(p^2)^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle + \frac{1}{2(p^2)^2} (m_u \langle \bar{d}g_s \sigma \cdot Gd \rangle + m_d \langle \bar{u}g_s \sigma \cdot Gu \rangle), \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \Pi_{24}(p^2) &= i \int d^4x e^{ipx} \langle 0 | T [J_2(x) J_4^\dagger(0)] | 0 \rangle = i \int d^4x e^{ipx} \langle 0 | T [J_4(x) J_2^\dagger(0)] | 0 \rangle \\ &= -\frac{1}{16\pi^2} p^2 \ln(-p^2) \langle \bar{s}s \rangle - \frac{1}{32\pi^2} \ln(-p^2) \langle \bar{s}g_s \sigma \cdot Gs \rangle - \frac{1}{48p^2} \langle \bar{s}s \rangle \langle \alpha_s \pi^{-1} G^2 \rangle - \frac{1}{12p^2} \{ m_u (\langle \bar{u}u \rangle + 2\langle \bar{d}d \rangle) \\ &\quad + m_d (2\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \} \langle \bar{s}s \rangle + \left\{ -\frac{8\pi\alpha_s}{81(p^2)^2} \langle \bar{s}s \rangle (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2) - \frac{8\pi\alpha_s}{9(p^2)^2} \langle \bar{s}s \rangle \langle \bar{u}u \rangle \langle \bar{d}d \rangle \right\} + \frac{4\pi\alpha_s}{81(p^2)^2} (\langle \bar{u}u \rangle \\ &\quad + \langle \bar{d}d \rangle) \langle \bar{s}s \rangle^2 + \frac{1}{12(p^2)^2} \langle \bar{s}s \rangle (m_u \langle \bar{d}g_s \sigma \cdot Gd \rangle + m_d \langle \bar{u}g_s \sigma \cdot Gu \rangle) + \frac{1}{24(p^2)^2} (m_u \langle \bar{d}d \rangle + m_d \langle \bar{u}u \rangle) \langle \bar{s}g_s \sigma \cdot Gs \rangle, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned}
\Pi_{44}(p^2) &= i \int d^4x e^{ipx} \langle 0 | T [J_4(x) J_4^\dagger(0)] | 0 \rangle \\
&= -\frac{12}{4!5!2^8\pi^6} (p^2)^4 \ln(-p^2) - \frac{1}{3!2^6\pi^4} (p^2)^2 \ln(-p^2) \{m_u \langle \bar{u}u \rangle - 2\langle \bar{s}s \rangle\} + m_d \langle \bar{d}d \rangle - 2\langle \bar{s}s \rangle \\
&\quad - 2m_s \langle \bar{u}u \rangle + \langle \bar{d}d \rangle - \langle \bar{s}s \rangle \} - \frac{1}{3!2^8\pi^4} (p^2)^2 \ln(-p^2) \langle \alpha_s \pi^{-1} G^2 \rangle + \frac{1}{96\pi^2} p^2 \ln(-p^2) \langle \bar{s}s \rangle (-\langle \bar{s}s \rangle - 4\langle \bar{u}u \rangle \\
&\quad - 4\langle \bar{d}d \rangle) - \frac{1}{768\pi^4} p^2 \ln(-p^2) (m_u \langle \bar{u}g_s \sigma \cdot Gu \rangle + m_d \langle \bar{d}g_s \sigma \cdot Gd \rangle + 2m_s \langle \bar{s}g_s \sigma \cdot Gs \rangle) \\
&\quad - \frac{1}{96\pi^4} p^2 \ln(-p^2) (m_d \langle \bar{s}g_s \sigma \cdot Gs \rangle + m_s \langle \bar{d}g_s \sigma \cdot Gd \rangle + m_s \langle \bar{u}g_s \sigma \cdot Gu \rangle + m_u \langle \bar{s}g_s \sigma \cdot Gs \rangle) \\
&\quad + \frac{1}{192\pi^2} \ln(-p^2) \{[\langle \bar{u}u \rangle + \langle \bar{d}d \rangle] \langle \bar{s}g_s \sigma \cdot Gs \rangle + \langle \bar{s}s \rangle [\langle \bar{u}g_s \sigma \cdot Gu \rangle + \langle \bar{d}g_s \sigma \cdot Gd \rangle - 2\langle \bar{s}g_s \sigma \cdot Gs \rangle]\} \\
&\quad - \frac{1}{384\pi^2} \ln(-p^2) (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle + 2m_s \langle \bar{s}s \rangle) \langle \alpha_s \pi^{-1} G^2 \rangle + \frac{1}{144\pi^2} \ln(-p^2) (m_d \langle \bar{s}s \rangle + m_s \langle \bar{d}d \rangle + m_s \langle \bar{u}u \rangle \\
&\quad + m_u \langle \bar{s}s \rangle) \langle \alpha_s \pi^{-1} G^2 \rangle - \frac{1}{18p^2} \{ (m_u + m_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \langle \bar{s}s \rangle + m_s \langle \bar{s}s \rangle^2 (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \} \\
&\quad + \frac{1}{9p^2} \{ (m_u \langle \bar{d}d \rangle + m_d \langle \bar{u}u \rangle) \langle \bar{s}s \rangle^2 + 2m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle \langle \bar{s}s \rangle \} - \frac{1}{24p^2} (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle^2 \\
&\quad - \frac{7}{864p^2} (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \langle \bar{s}s \rangle \langle \alpha_s \pi^{-1} G^2 \rangle - \frac{1}{288p^2} \langle \bar{s}s \rangle^2 \langle \alpha_s \pi^{-1} G^2 \rangle \\
&\quad - \frac{41}{2^{10} \cdot 3^2 \pi^2 p^2} (\langle \bar{u}g_s \sigma \cdot Gu \rangle + \langle \bar{d}g_s \sigma \cdot Gd \rangle) \langle \bar{s}g_s \sigma \cdot Gs \rangle - \frac{1}{2^9 \cdot 3 \pi^2 p^2} \langle \bar{s}g_s \sigma \cdot Gs \rangle^2 \\
&\quad + \frac{20\pi\alpha_s}{27(p^2)^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \langle \bar{s}s \rangle^2 - \frac{4\pi\alpha_s}{243(p^2)^2} \langle \bar{s}s \rangle^2 (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2) \\
&\quad - \frac{8\pi\alpha_s}{243(p^2)^2} \langle \bar{s}s \rangle (\langle \bar{s}s \rangle^2 \langle \bar{u}u \rangle + \langle \bar{d}d \rangle^2 \langle \bar{u}u \rangle + \langle \bar{s}s \rangle^2 \langle \bar{d}d \rangle + \langle \bar{u}u \rangle^2 \langle \bar{d}d \rangle) + \frac{8\pi\alpha_s}{243(p^2)^2} \langle \bar{s}s \rangle^3 (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \\
&\quad + \frac{1}{72(p^2)^2} \langle \bar{s}s \rangle^2 (m_u \langle \bar{d}g_s \sigma \cdot Gd \rangle + m_d \langle \bar{u}g_s \sigma \cdot Gu \rangle) \\
&\quad - \frac{1}{36(p^2)^2} \{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle (m_s \langle \bar{d}g_s \sigma \cdot Gd \rangle + m_d \langle \bar{s}g_s \sigma \cdot Gs \rangle) + \langle \bar{d}d \rangle \langle \bar{s}s \rangle (m_u \langle \bar{s}g_s \sigma \cdot Gs \rangle + m_s \langle \bar{u}g_s \sigma \cdot Gu \rangle) \} \\
&\quad + \frac{1}{72(p^2)^2} \langle \bar{s}g_s \sigma \cdot Gs \rangle \langle \bar{s}s \rangle (m_u \langle \bar{d}d \rangle + m_d \langle \bar{u}u \rangle). \tag{A3}
\end{aligned}$$

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