Implications of R parity violating Yukawa couplings in $\Delta S = 1$ semileptonic decays of K mesons

Azeem Mir,* Farida Tahir,† Mahnaz Q. Haseeb,‡ and Kamaluddin Ahmed§

Physics Department, COMSATS Institute of Information Technology, Islamabad, Pakistan

(Received 9 May 2007; published 7 December 2007)

We present a class of constraints on products and combinations of Yukawa couplings for R parity violating ($\not{R}_p$) and lepton flavor conserving as well as violating semileptonic decays of K mesons into light pseudoscalar mesons along with two charged leptons at 1σ and 2σ levels. We compare the constraints obtained by semileptonic rare decays with pure leptonic rare decays and find that most of these bounds are now improved over the existing ones. We also study the forward-backward asymmetry in the decays of $K^+ \to \pi^+ l^+ l^-$ ($l = e$ and μ) in the absence of tensor terms. The asymmetry is found to be up to $O(10^{-3})$ ($O(10^{-1})$) for the electron and muon modes, respectively. The asymmetry is found to be as large as $O(10^{-1})$ in the case of $K^+ \to \pi^+ \mu^+ \mu^-$.

DOI: 10.1103/PhysRevD.76.114005 PACS numbers: 13.20.−v, 12.60.Jv, 13.20.Eb

The standard model (SM) gives a well-tested and established theory. However the SM is not a complete gauge theory and leaves some questions unanswered. Supersymmetry (SUSY) provides a framework beyond the SM by assuming the existence of spartners of fermions and bosons. This results in renormalizable and gauge invariant interactions which violate baryon (B) and lepton (L) numbers. Here the scalar fermions are responsible for mediating such interactions. This leads to the dangerous impact on the stability of matter, i.e., there will be fast proton decay. So in order to protect such a decay one imposes a discrete symmetry known as R parity, which is defined as $R = (-1)^{3B+L+2S}$, where B, L, and S are baryon number, lepton number, and spin of a particle [1]. Any R parity conserving Lagrangian which gives a phase (-1) to the sparticles and leaves the standard particles invariant implies two immediate consequences: first, the sparticles should be pair produced; second, the lightest supersymmetric particle (LSP) must be stable which leads to the celebrated missing E_T signature of the SUSY event in high energy detectors and renders the LSP a cold dark matter particle candidate [2]. Although desirable for many reasons the R parity conservation has no well-motivated theoretical grounds. On the other hand relaxing the R parity conservation in a special way gives new insight into long-standing problems of particle physics, in particular, to the neutrino mass problem [3]. It is remarkable that in this framework one neutrino can acquire tree-level supersymmetric mass via tree-level neutrino-neutralino mixing and the other two acquire masses at loop level [4]. This mechanism does not involve physics at large energy scales $O(10^{12}$ GeV) in contrast to the seesaw mechanism but relates the neutrino mass to the weak scale physics accessible for experimental search. Therefore, the models with explicit $\not{R}_\mathbf{p}$ have been considered extensively in literature [3]. In order to search for possible $\not{R}_\mathbf{p}$ in future experiments, we want to know what kind of couplings are constrained by present experimental data. Therefore it is important to determine the constraints on the R parity violating couplings from the present data, especially the data on processes that are forbidden or highly suppressed in the standard model. In models without R parity conserving interaction, the SUSY particles can decay into ordinary particles alone. Therefore, the couplings which violate R parity can be detected by using the usual particle detectors.

In SUSY, the gauge invariant, renormalizable superpotential which explicitly breaks R parity via the lepton and baryon number violating superpotential is [1]

$$W_{\not{R}_\mathrm{p}} = \tfrac{1}{2}\lambda_{ijk}L_iL_jE_k^c + \lambda'_{ijk}L_iQ_jD_k^c + \tfrac{1}{2}\lambda''_{ijk}U_i^CD_j^CD_k^C + \mu_iH_uL_i, \tag{1}$$

where i, j, k are generation indices, L_i and Q_i are the lepton and quark left-handed $SU(2)_L$ doublets, and E^c, D^c are the charge conjugates of the right-handed leptons and quark singlets, respectively. λ_{ijk}, λ'_{ijk}, and λ''_{ijk} are Yukawa couplings. Note that the term proportional to λ_{ijk} is antisymmetric in the first two indices $[i, j]$ and λ''_{ijk} is antisymmetric in the last two indices $[j, k]$, implying $9(\lambda_{ijk}) + 27(\lambda'_{ijk}) + 9(\lambda''_{ijk}) = 45$ independent coupling constants among which 36 are related to the lepton flavor violation (9 from LLE^c and 27 from LQD^c). The last term can be rotated away by a unitary transformation. However this may induce additional terms involving Yukawa couplings, not relevant here [3].

It is worthwhile to mention that rare semileptonic decays are regarded as a source to extrapolate the SM and a testing ground for exploring physics beyond the SM. The data that has been used in calculating results on bounds involving semileptonic decays ($K^+ \to \pi^+ l_i \bar{l}_j$) is given in [5]. These processes can proceed through the simplest model known as the spectator quark model [6]. In this model constituent

*neutrino_79@hotmail.com
†farida_tahir@comsats.edu.pk
ftahir@ku.edu
‡mahnazhaseeb@comsats.edu.pk
§kamal@comsats.edu.pk
kahmed_pk@yahoo.com

quark decays can proceed through single quark subprocess;

$$\bar{q}_p \to \bar{q}'_k l^-_\alpha l^+_\beta,$$

where $\overline{q_k}$ is the constituent of any number of mesons consistent with energy conservation and allowed by the prescribed selection rules. In the SM, the transitions that preserve lepton flavor ($\alpha = \beta$) arise through loop diagrams and are strongly suppressed, while the transitions that violate lepton flavor ($\alpha \neq \beta$), such as $K^+ \to \pi^+ \mu^+ e^-$ are unobservable due to the smallness of neutrino mass.

The processes with the same lepton species are highly suppressed [5]. While the processes like $K^+ \to \pi^- l^+ l^+$ with $l = 1, 2$ are more suppressed by lepton number violation. So we neglect the SM contributions to the processes under consideration. On the other hand, the processes which have different lepton species as a decay product are forbidden due to the conservation of each lepton flavor number. Although the lepton flavor as well as number violating decays are strictly forbidden in the SM, there is a need to study them in current and future experiments to explore the world beyond the SM.

$\not{R}_\mathbf{p}$ interactions contribute to the subprocess $\overline{q_p} \to \overline{q_k}' l^-_\alpha l^+_\beta$ via tree-level sneutrino and up-quark exchange. This allows us to extract significant bounds on the quadratic product of $\not{R}_\mathbf{p}$ coupling from the rare leptonic meson decays.

These rare meson decays lead to a host of bounds on the combination of $\not{R}_\mathbf{p}$ couplings, i.e., $\lambda\lambda'$ or $\lambda'\lambda'$. In the minimal supersymmetric standard model (MSSM) the relevant effective Lagrangian is given by

$$\begin{aligned} L^{\text{eff}}_{\not{R}_\mathbf{p}}(\bar{s} \to \bar{d} + l_\alpha + \bar{l}_\beta) = \frac{4G_F}{\sqrt{2}} [& A^{ds}_{\alpha\beta}(\overline{l_\alpha}\gamma^\mu P_L l_\beta)(\overline{q}_k \gamma_\mu P_R q_P) \\ & - B^{ds}_{\alpha\beta}(\overline{l_\alpha} P_R l_\beta)(\overline{q}_k P_L q_P) \\ & - C^{ds}_{\alpha\beta}(\overline{l_\alpha} P_L l_\beta)(\overline{q}_k P_R q_P)]. \end{aligned} \quad (2)$$

This $L^{\text{eff}}_{\not{R}_\mathbf{p}}$ is the same as obtained from pure leptonic decays of the neutral K mesons [7]. For the K^+ meson, $k = d$ and $p = s$ and $\alpha, \beta = e, \mu$, where α may or may not be equal to β. The process is shown in Fig. 1.

The first term in Eq. (2) comes from the up-squark exchange (where q_p and $\overline{q}_k$ are down-type quarks). We have used the Fierz transformation in order to get the first term while the second and third terms come from the sneutrino exchange. The dimensionless coupling constants $A^{q_k q_p}_{\alpha\beta}$, $B^{q_k q_p}_{\alpha\beta}$, and $C^{q_k q_p}_{\alpha\beta}$ depend on the species of charged leptons, charged mesons, and the K^+ meson. Thus

$$A^{ds}_{\alpha\beta} = \frac{\sqrt{2}}{4G_F} \sum_{m,n,i=1}^{3} \frac{V^\dagger_{ni} V_{im}}{2m^2_{\tilde{u}^c_i}} \lambda'_{\beta n k} \lambda'^*_{\alpha m p}, \quad (3)$$

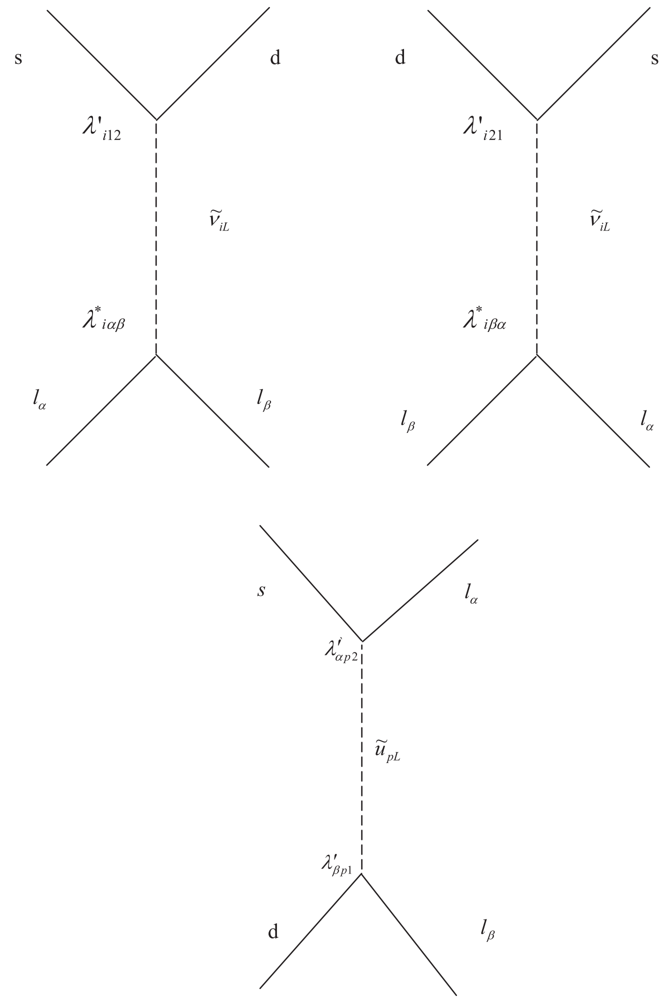

FIG. 1. Tree-level diagrams contributing to charged meson decays. $K^+ \to \pi^+ + l^+{}_\alpha + l^-{}_\beta$, $\overline{s} \to \overline{d} + l^+{}_\alpha + l^-{}_\beta$

$$B^{ds}_{\alpha\beta} = \frac{\sqrt{2}}{4G_F} \sum_{i=1}^{3} \frac{2}{m^2_{\tilde{\nu}_{Li}}} \lambda^*_{i\alpha\beta} \lambda'_{ipk}, \quad (4)$$

$$C^{ds}_{\alpha\beta} = \frac{\sqrt{2}}{4G_F} \sum_{i=1}^{3} \frac{2}{m^2_{\tilde{\nu}_{Li}}} \lambda_{i\beta\alpha} \lambda'^*_{ikp}. \quad (5)$$

Note that in Eqs. (4) and (5) $B^{ds}_{\alpha\beta}$ and $C^{ds}_{\alpha\beta}$ are symmetric under the $\alpha \leftrightarrow \beta$. Thus in the limit of ignorable lepton mass and QCD correction, the decay rate of the processes [8]

$$\Gamma[\bar{s} \to \bar{d} l^+_\alpha l^-_\beta] = \frac{m^5_{K^+}}{192\pi^3} G^2_F \left\{ |A^{ds}_{\alpha\beta}|^2 + \frac{1}{4}(|B^{ds}_{\alpha\beta}|^2 + |C^{ds}_{\alpha\beta}|^2) \right\}. \quad (6)$$

The forward-backward asymmetry (FBA) is related to the asymmetric angular distribution of the dilepton pair with respect to the initial meson direction of momentum in the dilepton rest frame. The FBA is defined as

TABLE I. Limits on the magnitudes of combination constraints $(\lambda'\lambda'^{*})$ and $(\lambda\lambda'^{*})$. In the table we use $L_i = 1/(m_{\tilde{\nu}_{iL}}/100\ \text{GeV})^2$, $Q_i = 1/(m_{\tilde{u}_{iL}}/100\ \text{GeV})^2$. $(1\sigma, 2\sigma)$ errors from the branching fraction, wherever possible, have been incorporated in predicting the bounds.

Process	Combination constrained Q_i	Bound	Combination constrained L_i	Bound
$K^+ \to \pi^+ e^+ e^-$	$\lambda'_{111}\lambda'^*_{112}$, $\lambda'_{121}\lambda'^*_{122}$, $\lambda'_{131}\lambda'^*_{132}$	$1.02 \times 10^{-4}(1\sigma)$ $1.04 \times 10^{-4}(2\sigma)$	$\lambda_{211}\lambda'^*_{212}$, $\lambda_{311}\lambda'^*_{312}$ $\lambda^*_{211}\lambda'_{221}$, $\lambda^*_{311}\lambda'_{321}$	$5.10 \times 10^{-5}(1\sigma)$ $5.20 \times 10^{-5}(2\sigma)$
$K^+ \to \pi^+ \mu^+ \mu^-$	$\lambda'_{211}\lambda'^*_{212}$, $\lambda'_{221}\lambda'^*_{222}$, $\lambda'_{231}\lambda'^*_{232}$	$5.73 \times 10^{-5}(1\sigma)$ $6.13 \times 10^{-5}(2\sigma)$	$\lambda_{122}\lambda'^*_{112}$, $\lambda_{322}\lambda'^*_{312}$ $\lambda^*_{122}\lambda'_{121}$, $\lambda^*_{322}\lambda'_{321}$	$2.86 \times 10^{-5}(1\sigma)$ $3.07 \times 10^{-5}(2\sigma)$
$K^+ \to \pi^+ \mu^+ e^-$	$\lambda'_{211}\lambda'^*_{112}$, $\lambda'_{221}\lambda'^*_{122}$, $\lambda'_{231}\lambda'^*_{132}$	9.83×10^{-7}	$\lambda_{121}\lambda'^*_{112}$, $\lambda_{321}\lambda'^*_{312}$ $\lambda^*_{212}\lambda'_{221}$, $\lambda^*_{312}\lambda'_{321}$	4.91×10^{-7}
$K^+ \to \pi^+ \mu^- e^+$	$\lambda'_{111}\lambda'^*_{212}$, $\lambda'_{121}\lambda'^*_{222}$, $\lambda'_{131}\lambda'^*_{232}$	4.24×10^{-6}	$\lambda_{112}\lambda'^*_{112}$, $\lambda_{312}\lambda'^*_{312}$ $\lambda^*_{221}\lambda'_{221}$, $\lambda^*_{321}\lambda'_{321}$	2.12×10^{-6}

$$A_{\rm FB}(s) = \frac{\int_0^1 d\cos\theta \frac{d^2\Gamma}{dsd\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d^2\Gamma}{dsd\cos\theta}}{\int_0^1 d\cos\theta \frac{d^2\Gamma}{dsd\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d^2\Gamma}{dsd\cos\theta}}. \tag{7}$$

It can be written in the form [9]

$$A_{\rm FB}(s) = \frac{1}{128\pi^3 m_K^3} m_K m_l \beta_l^2 \lambda(s)\,{\rm Re}(F_S F_V^*)\left(\frac{d\Gamma}{ds}\right)^{-1}, \tag{8}$$

where

$$\begin{aligned}\frac{d\Gamma}{ds} = \frac{1}{256\pi^3 m_K^3}\beta_l \lambda^{1/2}(s)\Big\{&|F_S|^2 2(m_K)^2 s\beta_l^2 \\ &+ |F_P|^2 2(m_K)^2 s + |F_V|^2 \frac{1}{3}\lambda(s)\left(1+\frac{2m_l^2}{s}\right) \\ &+ |F_A|^2\left[\frac{1}{3}\lambda(s)\left(1+\frac{2m_l^2}{s}\right) + 8m_K^2 m_l^2\right] \\ &+ {\rm Im}(F_P F_A^*)4m_K m_e(m_\pi^2 - m_K^2 - s)\Big\},\end{aligned} \tag{9}$$

$$\beta_l = \left(1 - \frac{4m_l^2}{s}\right)^{1/2},$$
$$\lambda(s) = m_K^4 + m_\pi^4 + s^2 - 2sm_\pi^2 - 2sm_K^2 - 2m_K^2 m_\pi^2,$$

where s (invariant mass squared of the dilepton system in its rest frame) is bounded as

$$4(m_l)^2 \leq s \leq (m_K - m_\pi)^2.$$

In the SM, the FBA vanishes in $(K^+ \to \pi^+ l\bar{l})$ as the decay amplitude involves no scalar term [9]. Therefore its presence will give a signal of new physics. The possibility of nonzero FBA has been reported by [10] in the case of large $\tan\beta$ for the MSSM. Some authors have explored the possibility of nonzero FBA in the $\not{R}_p$ framework [8,11]. We discuss a similar possibility of nonzero FBA in $(K^+ \to \pi^+ l\bar{l})$ in the $\not{R}_p$ framework. We show that the FBA arises naturally by sneutrino and squark exchange terms. An estimate of the FBA is made from the bounds on Yukawa couplings. The normalized form factors, defined by taking the ratio relative to f_+, are obtained by comparing the $\not{R}_p$ effective Lagrangian with the general invariant amplitude, reproduced in the term of the $\not{R}_p$ framework [9].

$$\begin{aligned} F_V &= \sum_{m,n,i=1}^{3} \frac{V^\dagger_{ni} V_{im}}{2m^2_{\tilde{u}^c_i}} \lambda'_{\beta nk}\lambda'^*_{\alpha mp}, \\ F_A &= -\sum_{i,m,n=1}^{3} \frac{V^\dagger_{ni} V_{im}}{4m^2_{\tilde{u}^c_i}} \lambda'_{\beta nk}\lambda'^*_{\alpha mp}, \\ F_S &= \sum_{i=1}^{3} \frac{m_K}{m_s} \frac{(\lambda^*_{i\alpha\beta}\lambda'_{ipk} + \lambda_{i\beta\alpha}\lambda'^*_{ikp})}{2m^2_{\tilde{\nu}_{Li}}}, \\ F_P &= i\sum_{i=1}^{3} \frac{m_K}{m_s} \frac{(\lambda^*_{i\alpha\beta}\lambda'_{ipk} - \lambda_{i\beta\alpha}\lambda'^*_{ikp})}{2m^2_{\tilde{\nu}_{Li}}}, \end{aligned} \tag{10}$$

where m_s is the strange quark mass.

We have arrived at Table I by using Eqs. (1)–(6). Under the assumption that only one product combination is non-

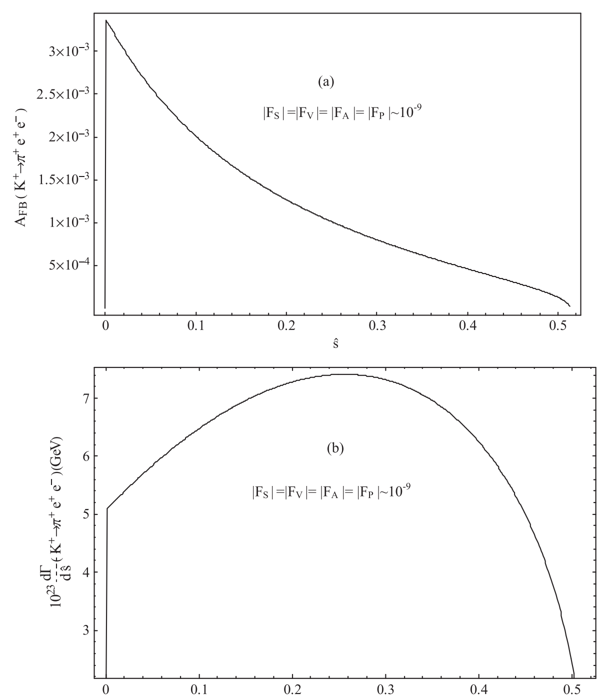

FIG. 2. (a). Forward-backward asymmetry. (b) Differential decay rate for $K^+ \to \pi^+ e^+ e^-$ with functions of $\hat{s} = \frac{s}{(m_K)^2}$.

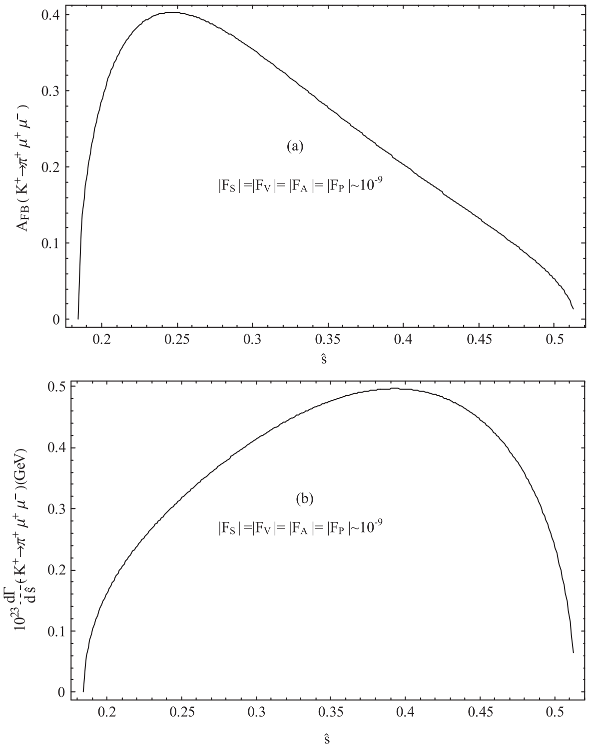

FIG. 3. (a) Forward-backward asymmetry. (b) Differential decay rate for $K^+ \to \pi^+ \mu^+ \mu^-$ with functions of $\hat{s} = \frac{s}{(m_K)^2}$.

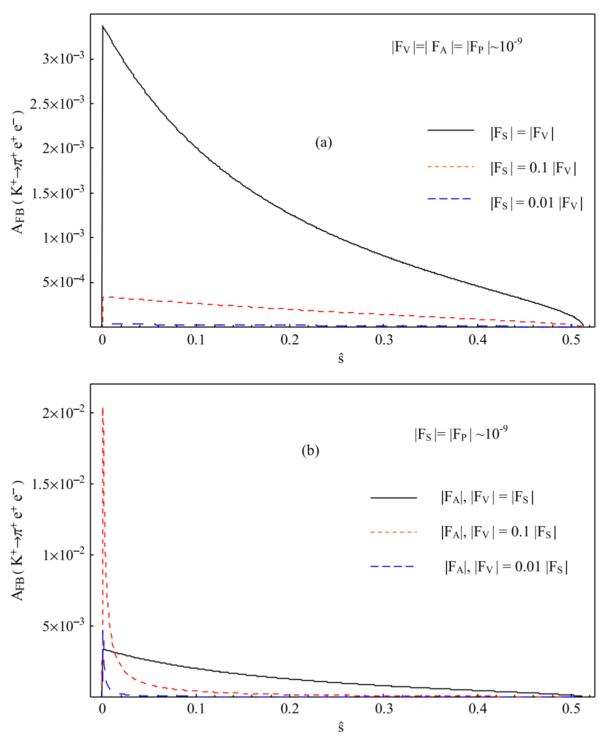

FIG. 4 (color online). Variation of forward-backward asymmetry. (a) Varying F_S, with constant F_V, F_A, and F_P. (b) Varying F_V and F_A with constant F_S and F_P for $K^+ \to \pi^+ e^+ e^-$ with functions of $\hat{s} = \frac{s}{(m_K)^2}$.

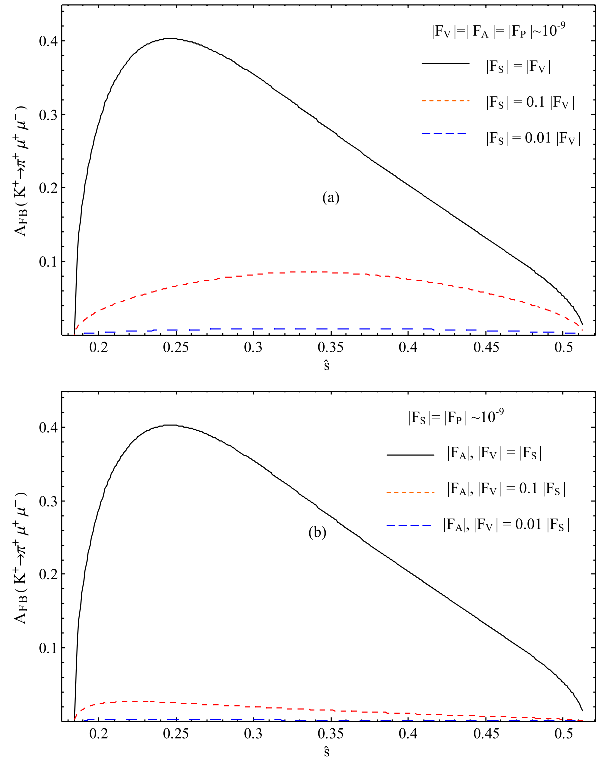

FIG. 5 (color online). Variation of forward-backward asymmetry. (a) Varying F_S, with constant F_V, F_A, and F_P. (b) Varying F_V and F_A with constant F_S and F_P for $K^+ \to \pi^+ \mu^+ \mu^-$ with functions of $\hat{s} = \frac{s}{(m_K)^2}$.

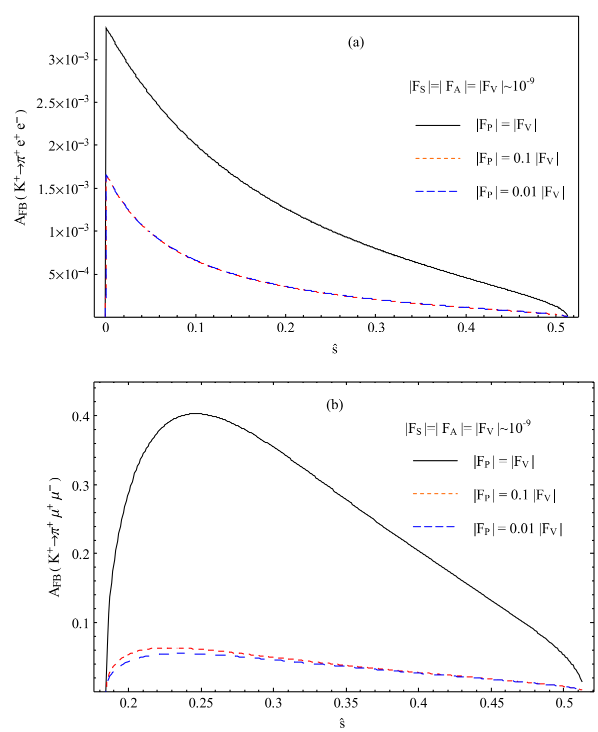

FIG. 6 (color online). Variation of forward-backward asymmetry. Varying F_P, with constant F_V, F_A, and F_S for (a) $K^+ \to \pi^+ e^+ e^-$, (b) $K^+ \to \pi^+ \mu^+ \mu^-$ as functions of $\hat{s} = \frac{s}{(m_K)^2}$.

zero, we obtain the common limit 1.02×10^{-4} (1σ) and 1.04×10^{-4} (2σ) on the magnitude of three coupling products $\lambda'_{1i1}\lambda'^{*}_{1i2}$. We also obtain the common limit 5.10×10^{-5} (1σ) and 5.20×10^{-5} (2σ) on the magnitude of four coupling products $\lambda_{i11}\lambda'^{*}_{i21}$ and $\lambda^{*}_{i11}\lambda'_{i12}$, respectively, for the decay $K^+ \to \pi^+ e^+ e^-$. These limits are improved over those obtained previously for pure leptonic K meson decays [3,7].

A similar situation holds for $K^+ \to \pi^+ \mu^+ \mu^-$, where three coupling products of $\lambda'^{*}_{2i1}\lambda'_{2i2}$ have the common limit 5.73×10^{-5} (1σ), 6.13×10^{-5} (2σ). We further obtain common limits 2.86×10^{-5} (1σ), 3.07×10^{-5} (2σ) on four coupling products $\lambda_{i22}\lambda'^{*}_{i12}$, $\lambda^{*}_{i22}\lambda'_{i21}$, respectively, which are also an improvement over those obtained earlier [3,7]. For $K^+ \to \pi^+ \mu^+ e^-$, we obtain comparable limits 9.83×10^{-7} on $\lambda'_{2i1}\lambda'^{*}_{1i2}$ along with the limit 4.91×10^{-7} on $\lambda_{i21}\lambda'^{*}_{i12}$ which are again improved.

The order of magnitude relation between form factors F_S, F_V, F_P, F_A, $O(10^{-9})$, which has been used in Figs. 2–6, has been estimated from Table I on bounds of Yukawa coupling products. The FBA calculated is of $O(10^{-3})$ for $K^+ \to \pi^+ e^+ e^-$ and $O(10^{-1})$ for $K^+ \to \pi^+ \mu^+ \mu^-$, respectively. Our results for the FBA and differential decay rate are summarized in Figs. 2 and 3. We have used Eqs. (8)–(10) to calculate the FBA and differential decay rate. The vector and axial form factors F_V, $F_A(\lambda'\lambda')$ are induced by squark exchange while the scalar form factor $F_S(\lambda'\lambda)$ is generated by the sneutrino exchange. The FBA for $K^+ \to \pi^+ e^+ e^-$ varies rapidly with respect to $\hat{s}$ while for $K^+ \to \pi^+ \mu^+ \mu^-$, it varies smoothly. In Figs. 4–6, we have shown the variation of the FBA with form factors. The graphs show that the FBA for $K^+ \to \pi^+ \mu^+ \mu^-$ receives a maximum contribution from $\not{R}_p$ SUSY when both squark and sneutrino exchange couplings contribute equally. Any dominance of either couplings will reduce the contribution.

Summarizing, we have obtained some new bounds on the products of Yukawa couplings for the exchange of squarks and sneutrinos in the process $K^+ \to \pi^+ l\bar{l}$ in the mass scale $O(100\ \mathrm{GeV})$. These bounds are deduced at 1σ and 2σ levels. All these limits are obtained under the assumption that only one product combination is nonzero. Additionally, using the exchange of such SUSY particles we predict the FBA in these processes. The estimated FBA is vanishingly small for $K^+ \to \pi^+ e^+ e^-$ due to the smallness of electron mass but is large as $O(10^{-1})$ for $K^+ \to \pi^+ \mu^+ \mu^-$. The authors are not aware of any experiment to measure the FBA in this process.

Azeem Mir is indebted to the Higher Education Commission of Pakistan for financial support and Farida Tahir is grateful to Dr. Douglas W. Mckay, Department of Physics & Astronomy, University of Kansas for discussions.

[1] H. E. Haber and G. Kane, Phys. Rep. **117**, 75 (1985); R. Hagg, J. Lopuszanski, and M. Sohnius, Nucl. Phys. **B88**, 257 (1975); J. Wess and J. Bagger, *Super-symmetry and Super-gravity*, Second edition, Princeton Series in Physics (Princeton University, Princeton, NJ, 1992); G. G. Ross, *Grand Unified Theories* (Addison-Wesley, Reading, MA, 1984); S. P. Martin, arXiv:hep-ph/9709356.

[2] H. E. Haber, arXiv:hep-ph/9306207; S. Dawson, NATO Adv. Study Inst. Ser., Ser. B, Phys. **365**, 33 (1997).

[3] R. Barbier, C. Bérat, M. Besanc, M. Chemtob *et al.*, Phys. Rep. **420**, 1 (2005).

[4] M. Gózdz and W. A. Kaminski, Phys. Rev. D **70**, 095005 (2004); Y. Grossman and S. Rakshit, Phys. Rev. D **69**, 093002 (2004).

[5] http://pdg.lbl.gov/2005/pdg_2005.html.

[6] P. J. Franzini, CERN, Geneva, Switzerland, 1989, Vol. 173, Issue No. 1, p. 1.

[7] F. Tahir, M. Anwar Mughal, and K. Ahmed, Europhys. Lett. **54**, 580 (2001); J. Kim, P. Ko, and D. G. Lee, Phys. Rev. D **56**, 100 (1997); J.-H. Jang, J. K. Kim, and J. S. Lee, Phys. Rev. D **55**, 7296 (1997); J.-H. Jang, Y. G. Kim, and J. S. Lee, Phys. Lett. B **408**, 367 (1997); D. Choudhury and P. Roy, Phys. Lett. B **378**, 153 (1996).

[8] J.-H. Jang, Y. G. Kim, and J. S. Lee, Phys. Rev. D **58**, 035006 (1998).

[9] C.-H. Chen, C. Q. Geng, and I. Lin Ho, Phys. Rev. D **67**, 074029 (2003).

[10] C.-H. Chen, C.-Q. Geng, and A. K. Giri, Phys. Lett. B **621**, 253 (2005).

[11] Y.-G. Xu, R.-M. Wang, and Y.-D. Yang, Phys. Rev. D **74**, 114019 (2006).