Confusing sterile neutrinos with deviation from tribimaximal mixing at neutrino telescopes

Ram Lal Awasthi[*] and Sandhya Choubey[†]

Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211019, India
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We expound on the impact of extra sterile species on the ultra high energy neutrino fluxes in neutrino telescopes. We use three types of well-known flux ratios and compare the values of these flux ratios in the presence of sterile neutrinos, with those predicted by deviation from the tribimaximal mixing scheme. We show that in the upcoming neutrino telescopes, it is easy to confuse the signature of sterile neutrinos with that of the deviation from tribimaximal mixing. We also show that if the measured flux ratios acquire a value well outside the range predicted by the standard scenario with three active neutrinos only, it might be possible to tell the presence of extra sterile neutrinos by observing ultra high energy neutrinos in future neutrino telescopes.

I. INTRODUCTION

Cumulative effort through a series of experiments spanning more than four decades has helped augment our understanding of the properties of neutrinos. That neutrinos have mass, mixing, and hence flavor oscillations has been firmly established. The picture emerging from the combined results of solar [1], atmospheric [2], reactor [3,4], and accelerator [5,6] neutrino experiments is seen to be consistent with $\Delta m_{21}^2 = 8 \times 10^{-5}$ eV^2 and $\Delta m_{31}^2 = 2.5 \times 10^{-3}$ eV^{2},[1] and the so-called "tribimaximal mixing" [7] pattern for the neutrinos [8,9]. In the tribimaximal (TBM) mixing scheme the mixing matrix has the form

$$U^{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \qquad (1)$$

In this scenario, the atmospheric neutrino mixing angle θ_{23} is maximal ($\sin^2\theta_{23} = 0.5$), the so-called Chooz mixing angle $\theta_{13} = 0$, while the solar neutrino mixing angle is such that $\sin^2\theta_{12} = 1/3$, all consistent with the current global data set. Maximal θ_{23} and zero θ_{13} can be easily obtained by imposing discrete family symmetries such as the $\mu - \tau$ symmetry [10] or the $L_\mu - L_\tau$ symmetry [11]. While $\mu - \tau$ symmetry does not predict the neutrino masses and the solar mixing angle, a phenomenologically viable scenario which satisfies all neutrino data can be obtained within an approximate $L_\mu - L_\tau$ scheme [11].

The TBM mixing scheme will be put to a stringent test with the next generation experiments involving neutrinos from the sun, atmosphere, reactors, and accelerators. The mixing angle $\sin^2\theta_{12}$ can be measured to less than 16% accuracy at 3σ using solar neutrinos and to 6% accuracy

from a future ~ 60 km baseline survival probability minimum (SPMIN) reactor experiment [12]. Another prospect for accurate determination of $\sin^2\theta_{12}$ is to dope the Super-Kamiokande or even megaton water detectors with gadolinium [13]. The small and hitherto undetermined mixing angle θ_{13} will be probed in the upcoming reactor experiments [14], as well as in accelerator based experiments using beams from conventional sources [15], beams produced by decay of accelerated radioactive ions stored in rings ("beta-beams") [16]or beams from decay of accelerated muons stored in rings ("neutrino factory") [17]. In principle, one of the best determinations of θ_{13} could be possible from the neutrino signal of a future galactic supernova [18]. The deviation of θ_{23} from maximality and the sign of $D_{23} \equiv 0.5 - \sin^2\theta_{23}$ can be experimentally checked in atmospheric neutrino experiments [19].

A very well-known and interesting feature of the TBM mixing scheme arises in the context of the flavor ratios of ultra high energy neutrinos arriving on Earth. Ultra high energy neutrinos are created mainly through decay of high energy pions produced in pp and $p\gamma$ collisions. Hence their relative flavor content at the source is expected to be

$$\{\phi_e^0 : \phi_\mu^0 : \phi_\tau^0\} \equiv \{1:2:0\}, \qquad (2)$$

where ϕ_α^0 ($\alpha \equiv e, \mu, \tau$) are the fluxes at the source. Under the TBM scheme, due to the inherent $\mu - \tau$ symmetry, $\theta_{23} = 45°$ and $\theta_{13} = 0$, and therefore one obtains the flux ratio at Earth as [20]

$$\{\phi_e : \phi_\mu : \phi_\tau\} \equiv \{1:1:1\}. \qquad (3)$$

Next generation km^3 neutrino telescopes such as the IceCube in Antarctica [21] and Km3NET in the Mediterranean [22] are being especially designed and built to detect ultra high energy neutrinos coming from astrophysical sources. A lot of interest has been recently generated on the potential of using the observed flavor ratios of the ultra high energy neutrinos in neutrino telescopes in deciphering the predictions or deviation from tribimaximal mixing [23]. The key idea is that the flavor ratios are

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[*]ramlal@mri.ernet.in
[†]sandhya@mri.ernet.in
[1]We follow the convention where $\Delta m_{ij}^2 = m_i^2 - m_j^2$.

predicted to be one, as given by Eq. (3), only if $\theta_{13} = 0$ and $\theta_{23} = 45°$, a prediction of TBM mixing. If the mixing matrix was to deviate slightly from the TBM prediction through a change in θ_{23} or θ_{13}, we would see a difference in the flavor ratios, which could be used to pin down the extent of this deviation.

Here we study how the flavor ratios of ultra high energy neutrinos get affected if we have extra sterile neutrinos mixed with the three active ones. Sterile neutrinos have been the subject of much discussion recently, following the much-awaited MiniBooNE results [24]. The MiniBooNE experiment was designed to specifically test the observed signal at the LSND experiment [25], which can be explained most convincingly by neutrino oscillations. The MiniBooNE data seem to contradict the LSND signal, as they do not see the kind of electron excess predicted by the oscillation explanation of the LSND data sample. Since the claimed flavor oscillation observed by LSND demands a $\Delta m^2 \sim$ eV^2, it cannot be accommodated along with the solar and atmospheric neutrino observations within a three-generation framework. Three separate Δm^2 scales can be possible if we have at least 4 neutrino states. Since the number of light active neutrinos are restricted to three from the decay width of the Z bosons at CERN LEP, the extra neutrinos have to be sterile. The most economical scenario, with only one extra sterile neutrino, leads to the so-called $2 + 2$ and $3 + 1$ mass spectra for the neutrinos [26]. The $2 + 2$ spectrum is heavily disfavored by the solar and atmospheric neutrino data [9]. The $3 + 1$ mass spectrum, though comparatively less disfavored before the MiniBooNE results, suffers from a tension in explaining simultaneously the observed oscillation signal in the LSND experiment and the null signals in the other short baseline experiments [27]. The full analysis of all short baseline data including the MiniBooNE results further disfavors the $3 + 1$ scheme [28]. Adding two sterile neutrinos turns out to be the next option to reconcile the LSND observations with the rest of the world neutrino data. In this so-called $3 + 2$ scheme [29] the tension between the LSND and MiniBooNE data is reduced if one allows for *CP* violation, and this mass spectra is found to be consistent with global neutrino oscillation results [28].

In this paper we look at the signatures of extra sterile neutrinos on the observed flavor ratios of ultra high energy neutrinos in neutrino telescopes. We look at the impact of sterile neutrinos on the flavor ratios *vis-a-vis* the impact due to deviation from TBM mixing. First, for small active-sterile mixing, we elucidate the confusion between the two cases and identify the range of the mixing angles between the active and sterile neutrinos which would give flavor ratios close to the ones predicted by deviated TBM mixing scenarios. Next we show that large values of hitherto unconstrained mixing angles between the active and sterile neutrino species could lead to extreme values for flavor ratios, providing a smoking gun signal for the existence of

light sterile neutrinos which are heavily mixed with the active neutrinos. Thus the signal from neutrino telescopes could provide an independent check of the results from the short baseline experiments in general and LSND and MiniBooNE, in particular.

We present all results in this paper in the framework of a $3 + 1$ scenario *just for simplicity*. It is absolutely straightforward to extend our results and conclusions to the $3 + 2$ scenario. Here we have chosen not to work with the $3 + 2$ mass spectra because that would entail many extra mixing angles, making the results look complicated. Therefore for the sake of illustration, we allow only two active-sterile mixing angles to be nonzero in this work. In Sec. II we give a brief discussion of the ultra high energy neutrino fluxes and the corresponding flux/flavor ratios. In Sec. III the prediction from TBM mixing and the impact of deviation from TBM mixing in the framework of just three active neutrinos is reviewed. Section IV gives the oscillation probabilities for three active and one extra sterile neutrino, while Sec. V gives the corresponding flux ratios. In Sec. VI we present our numerical results. We end in Sec. VII with our conclusions.

II. ULTRA HIGH ENERGY NEUTRINO FLUXES

Ultra high energy neutrinos are predicted from a number of astrophysical sources. The *fireball* model, which has been put forward as the most plausible source for gamma ray bursts [30], is expected to produce ultra high energy neutrinos as well. Protons moving in the jets of the *fireball* are accelerated to very high energies. These highly accelerated protons undergo pp (or pn) and $p\gamma$ collisions, producing pions. The pions produce neutrinos through their decay channels

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \text{followed by} \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \tag{4}$$

and

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad \text{followed by} \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu. \tag{5}$$

It is expected that the $p\gamma$ reaction will predominantly produce π^+, while both π^+ and π^- are expected from the pp and pn collisions. Therefore, for the pp and pn collisions we expect the flavor ratio

$$\{\phi^0_{\nu_e} : \phi^0_{\bar{\nu}_e} : \phi^0_{\nu_\mu} : \phi^0_{\bar{\nu}_\mu} : \phi^0_{\nu_\tau} : \phi^0_{\bar{\nu}_\tau} :\} \equiv \{1 : 1 : 2 : 2 : 0 : 0\}, \tag{6}$$

where $\phi^0_{\nu_\alpha}$ and $\phi^0_{\bar{\nu}_\alpha}$ are the neutrino and antineutrino fluxes, respectively, of flavor α. For $p\gamma$ collisions on the other hand, we expect

$$\{\phi^0_{\nu_e} : \phi^0_{\bar{\nu}_e} : \phi^0_{\nu_\mu} : \phi^0_{\bar{\nu}_\mu} : \phi^0_{\nu_\tau} : \phi^0_{\bar{\nu}_\tau} :\} \equiv \{1 : 0 : 1 : 1 : 0 : 0\}. \tag{7}$$

Though apparently it looks that the flux ratio for the two dominant channels of ultra high energy neutrinos are different, the neutrino telescopes will not have sensitivity to

the charge of the resultant leptons and hence in general are not expected to be able to distinguish the neutrino of a given flavor from its antineutrino. Only when the Glashow resonance ($\bar{\nu}_e e \rightarrow W^- \rightarrow$ anything) is the detection channel is $\bar{\nu}_e$ unambiguously detected and it is possible to measure the ν_e and $\bar{\nu}_e$ signal separately. However, this occurs only over a small energy window $E = m_W^2/2m_e = 6.3$ PeV, where m_W and m_e are the W and e mass, respectively. In what follows, we will consider the sum of the neutrino and antineutrino signal at the neutrino telescope and in general refer to the flavor ratio as

$$\{\phi_e^0 : \phi_\mu^0 : \phi_\tau^0\} \equiv \{1{:}2{:}0\}, \tag{8}$$

where $\phi_\alpha \equiv \phi_{\nu_\alpha} + \phi_{\bar{\nu}_\alpha}$. If the ultra high energy neutrinos come from a neutron source, then the flux ratio is expected to be

$$\{\phi_{\nu_e}^0 : \phi_{\bar{\nu}_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\bar{\nu}_\mu}^0 : \phi_{\nu_\tau}^0 : \phi_{\bar{\nu}_\tau}^0\} \equiv \{0{:}1{:}0{:}0{:}0{:}0\}, \tag{9}$$

while for a muon damped source where the secondary muon is trapped and hence does not decay, it would be

$$\{\phi_{\nu_e}^0 : \phi_{\bar{\nu}_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\bar{\nu}_\mu}^0 : \phi_{\nu_\tau}^0 : \phi_{\bar{\nu}_\tau}^0\} \equiv \{0{:}0{:}1{:}1{:}0{:}0\}. \tag{10}$$

However, in this paper we will assume that only pions are the sources of ultra high energy neutrinos and work with the flavor ratio given in Eq. (8).

Since the absolute flux predictions for the ultra high energy neutrinos could be uncertain by a huge amount, it is better to work with ratios of the fluxes. In this paper we exemplify the predictions of deviation from TBM mixing, and the impact of an additional sterile neutrino, using the following flux ratios:

$$R_e = \frac{\phi_e}{\phi_\mu + \phi_\tau}, \qquad R_\mu = \frac{\phi_\mu}{\phi_e + \phi_\tau},$$
$$R_\tau = \frac{\phi_\tau}{\phi_e + \phi_\mu}, \tag{11}$$

where ϕ_e, ϕ_μ, and ϕ_τ, respectively, are the $\nu_e + \bar{\nu}_e$, $\nu_\mu + \bar{\nu}_\mu$, and $\nu_\tau + \bar{\nu}_\tau$ fluxes at Earth after oscillations. Written in terms of the oscillation probabilities,

$$R_e = \frac{P_{ee} + 2P_{e\mu}}{2P_{\mu\mu} + P_{e\mu} + P_{e\tau} + 2P_{\mu\tau}},$$
$$R_\mu = \frac{2P_{\mu\mu} + P_{e\mu}}{P_{ee} + 2P_{e\mu} + P_{e\tau} + 2P_{\mu\tau}}, \tag{12}$$
$$R_\tau = \frac{2P_{\mu\tau} + P_{e\tau}}{P_{ee} + 2P_{e\mu} + 2P_{\mu\mu} + P_{e\mu}},$$

where we have used $P_{e\mu} = P_{\mu e}$, since there is no chance of *CP* violation here. For the case for three active neutrinos only the flux ratios are given by

$$R_e = \frac{1 + (P_{e\mu} - P_{e\tau})}{2 - (P_{e\mu} - P_{e\tau})}, \qquad R_\mu = \frac{2 - (P_{e\mu} + 2P_{\mu\tau})}{1 + (P_{e\mu} + 2P_{\mu\tau})},$$
$$R_\tau = \frac{P_{e\tau} + 2P_{\mu\tau}}{3 - (P_{e\tau} + 2P_{\mu\tau})}, \tag{13}$$

where $P_{\alpha\beta}$ is the $\nu_\alpha \rightarrow \nu_\beta$ oscillation probability discussed in the following sections. For the case where we have one extra sterile neutrino in addition to the three active ones, the flux ratios can be written as

$$R_e = \frac{1 + (P_{e\mu} - P_{e\tau}) - P_{es}}{2 - (P_{e\mu} - P_{e\tau}) - 2P_{\mu s}},$$
$$R_\mu = \frac{2 - (P_{e\mu} + 2P_{\mu\tau}) - 2P_{\mu s}}{1 + (P_{e\mu} + 2P_{\mu\tau}) - P_{es}},$$
$$R_\tau = \frac{P_{e\tau} + 2P_{\mu\tau}}{3 - (P_{e\tau} + 2P_{\mu\tau}) - P_{es} - 2P_{\mu s}},$$

where P_{es} and $P_{\mu s}$ are the $\nu_e \rightarrow \nu_s$ and $\nu_\mu \rightarrow \nu_s$ oscillation probabilities, respectively. However, we prefer to express the flux ratios in the sterile case by the most general form given by Eq. (12).

III. OSCILLATION PROBABILITIES AND FLUX RATIOS IN TBM MIXING

For neutrinos coming from astrophysical sources the oscillation probabilities can be written in general as

$$P_{\alpha\beta} = \delta_{\alpha\beta} - \sum_{i \neq j} U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2, \tag{14}$$

where i, j and α, β are the generation indices of the mass basis and flavor basis, respectively. The oscillatory terms, $\sin^2(\Delta m_{ij}^2 L/4E)$, have averaged out to $1/2$ and hence the probabilities depend only on the elements of the mixing matrix. The neutrino mixing matrix for three active neutrinos can be parametrized as [31]

$$U = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}e^{i\delta} & C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13}e^{i\delta} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} & C_{23}C_{13} \end{pmatrix}, \tag{15}$$

where $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$, and δ is the CP phase. Under the tribimaximal mixing ansatz, $S_{12}^2 = 1/3$, $S_{23}^2 = 1/2$, and $S_{13}^2 = 0$, hence Eq. (15) reduces to the form given by Eq. (1). The oscillation probabilities therefore have an extremely simple form and are given as

$$P_{ee}^{\text{TBM}} = \frac{5}{9}, \qquad P_{\mu\mu}^{\text{TBM}} = \frac{7}{18}, \qquad P_{e\mu}^{\text{TBM}} = \frac{2}{9},$$
$$P_{e\tau}^{\text{TBM}} = \frac{2}{9}, \qquad P_{\mu\tau}^{\text{TBM}} = \frac{7}{18}, \qquad P_{\tau\tau}^{\text{TBM}} = \frac{7}{18}. \tag{16}$$

The effect of the $\mu - \tau$ symmetry which is inbuilt in TBM mixing is clearly reflected by $P_{\mu\mu}^{\text{TBM}} = P_{\mu\tau}^{\text{TBM}} = P_{\tau\tau}^{\text{TBM}}$ and $P_{e\mu}^{\text{TBM}} = P_{e\tau}^{\text{TBM}}$ in Eq. (16). Therefore, the flux ratios defined in the previous section are

$$R_e^{\text{TBM}} = R_\mu^{\text{TBM}} = R_\tau^{\text{TBM}} = \frac{1}{2}. \tag{17}$$

Any deviation from TBM mixing in the three active neutrino case will be reflected in these flux ratios, which will deviate from $1/2$. We calculate the probabilities using Eqs. (14) and (15) for the case where mixing angles are different from those predicted by TBM mixing and present the results for the flux ratios using Eq. (13).

IV. OSCILLATION PROBABILITIES WITH STERILE NEUTRINOS

In what follows, we will work in a framework where we include one extra sterile neutrino in addition to the three active ones and parametrize the 4×4 mixing matrix as

$$U_s = R(\theta_{34})R(\theta_{24})R(\theta_{23})R(\theta_{14})R(\theta_{13})R(\theta_{12}), \tag{18}$$

where $R(\theta_{ij})$ are the rotation matrices and θ_{ij} the mixing angle. Note that we have put all phases to zero in Eq. (18). In general for the $3 + 1$ scenario there are 3 CP-violating Dirac phases. However, in this paper we have neglected the effect of the CP-violating phases for simplicity.

A. TBM mixing with sterile neutrinos

If we assume that the mixing angles θ_{12}, θ_{13}, and θ_{23} follow the same values as in TBM mixing, i.e., $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{13} = 0$, and $\sin^2\theta_{23} = 1/2$, and we further assume that the mixing angle $\theta_{14} = 0$, then the 4×4 mixing matrix U_s is given by

$$U_s = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 & 0 \\ -\sqrt{\frac{1}{6}}C_{24} & \sqrt{\frac{1}{3}}C_{24} & \sqrt{\frac{1}{2}}C_{24} & S_{24} \\ \sqrt{\frac{1}{6}}[C_{34} + S_{24}S_{34}] & -\sqrt{\frac{1}{3}}[C_{34} + S_{24}S_{34}] & \sqrt{\frac{1}{2}}[C_{34} - S_{24}S_{34}] & C_{24}S_{34} \\ \sqrt{\frac{1}{6}}[C_{34}S_{24} - S_{34}] & -\sqrt{\frac{1}{3}}[C_{34}S_{24} - S_{34}] & -\sqrt{\frac{1}{2}}[C_{34}S_{24} + S_{34}] & C_{24}C_{34} \end{pmatrix}. \tag{19}$$

Using Eq. (14) we get the oscillation probabilities

$$P_{ee} = 1 - 2U_{e2}^2 U_{e1}^2 = \frac{5}{9}, \tag{20}$$

$$P_{e\mu} = -2U_{e2}U_{e1}U_{\mu2}U_{\mu1} = \frac{2}{9}C_{24}^2, \tag{21}$$

$$P_{\mu\mu} = 1 - 2U_{\mu2}^2 U_{\mu1}^2 - 2U_{\mu3}^2(U_{\mu2}^2 + U_{\mu1}^2)$$
$$\qquad - 2U_{\mu4}^2(1 - U_{\mu4}^2)$$
$$= 1 - 2C_{24}^2 + \frac{25}{18}C_{24}^4, \tag{22}$$

$$P_{e\tau} = -2U_{e2}U_{e1}U_{\tau2}U_{\tau1} = \frac{2}{9}(C_{34} + S_{24}S_{34})^2, \tag{23}$$

$$P_{\mu\tau} = -2U_{\mu2}U_{\mu1}U_{\tau2}U_{\tau1}$$
$$\qquad - 2U_{\mu3}U_{\tau3}(U_{\mu2}U_{\tau2} + U_{\mu1}U_{\tau1}) + 2U_{\mu4}^2 U_{\tau4}^2$$
$$= \frac{1}{18}(7C_{24}^2 C_{34}^2 + 25C_{24}^2 S_{24}^2 S_{34}^2 - 4C_{24}^2 C_{34}S_{24}S_{34}). \tag{24}$$

B. Deviation from TBM mixing and sterile neutrinos

If the mixing angles θ_{12}, θ_{13}, and θ_{23} are different from that predicted by the exact TBM ansatz, then one has to use the expression for the full U_s given by Eq. (18). For simplicity we use just one case where the mixing in the 3×3 active sector could deviate from TBM, viz., the case where the mixing angle θ_{23} is nonmaximal. For this case the matrix U_s is given by

$$U_s = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 & 0 \\ -\sqrt{\frac{1}{3}}C_{23}C_{24} & \sqrt{\frac{2}{3}}C_{23}C_{24} & S_{23}C_{24} & S_{24} \\ \sqrt{\frac{1}{3}}[S_{23}C_{34} + C_{23}S_{24}S_{34}] & -\sqrt{\frac{2}{3}}[S_{23}C_{34} + C_{23}S_{24}S_{34}] & [C_{23}C_{34} - S_{23}S_{24}S_{34}] & C_{24}S_{34} \\ \sqrt{\frac{1}{3}}[C_{23}C_{34}S_{24} - S_{23}S_{34}] & -\sqrt{\frac{2}{3}}[C_{23}C_{34}S_{24} - S_{23}S_{34}] & -[C_{34}S_{23}S_{24} + C_{23}S_{34}] & C_{24}C_{34} \end{pmatrix}, \tag{25}$$

and the oscillation probabilities are

$$P_{ee} = 5/9, \tag{26}$$

$$P_{e\mu} = \frac{4}{9}C_{23}^2 C_{24}^2, \tag{27}$$

$$P_{\mu\mu} = 1 - \frac{4}{9}C_{23}^4 C_{24}^4 - 2C_{24}^4 S_{23}^2 C_{23}^2 - 2S_{24}^2 C_{24}^2, \tag{28}$$

$$P_{e\tau} = \frac{4}{9}(C_{34}S_{23} + S_{24}S_{34}C_{23})^2, \tag{29}$$

$$P_{\mu\tau} = \frac{1}{36}C_{24}^2 [56C_{23}^2 C_{34}^2 S_{23}^2 \\ + S_{24}\{(57 - 8C2_{23} + 7C4_{23})S_{24}S_{34}^2 \\ + 2(-2 + 7C2_{23})S2_{23}S2_{34}\}], \tag{30}$$

where $C2_{ij} = \cos 2\theta_{ij}$ and $S2_{ij} = \sin 2\theta_{ij}$. In this paper we will mainly discuss the deviation from TBM mixing by changing $\sin^2\theta_{23}$ from its maximal value. The TBM mixing ansatz is also violated if either $\sin^2\theta_{13} \neq 0$ or $\sin^2\theta_{12} \neq 1/3$. We will present one plot where we show the flux ratios as a function of $\sin^2\theta_{13}$. We will see that the effect of the deviation of the flux ratios from the TBM predicted value of $1/2$ due to $\sin^2\theta_{13}$ is very small compared to the change due to $\sin^2\theta_{23}$. We therefore keep $\sin^2\theta_{13}$ fixed at zero for the results predicted by the sterile case in this paper.[2] Therefore for all results with sterile neutrinos, we will keep $\sin^2\theta_{13} = 0$ and $\sin^2\theta_{12} = 1/3$ for simplicity and allow only $\sin^2\theta_{23}$ to deviate from maximality.

V. FLUX RATIOS WITH STERILE NEUTRINOS

As discussed before, flux ratios for ultra high energy neutrinos are the best model independent probes for understanding neutrino properties. If we consider the case where the only deviation for TBM mixing comes from θ_{23} being nonmaximal, while $\sin^2\theta_{12} = 1/3$ and $\theta_{13} = 0$, and continue to work under the approximation that $\theta_{14} = 0$, then the flux ratios R_e, R_μ, and R_τ are given as

$$R_e = \frac{5 + 8C_{24}^2 C_{23}^2}{R_e^{\text{denom}}}, \tag{31}$$

where

$$R_e^{\text{denom}} = (18 + 4C_{24}^2 C_{23}^2 - 8C_{23}^4 C_{24}^4 - 36C_{24}^4 S_{23}^2 C_{23}^2 \\ - 36S_{24}^2 C_{24}^2 + 4(C_{34}S_{23} + S_{24}S_{34}C_{23})^2 \\ - 8C_{24}^2 C_{23}^2(C_{34}S_{23} + S_{34}S_{24}C_{23})^2 \\ + 36C_{24}^2 S_{23}C_{23}(C_{23}C_{34} - S_{23}S_{24}S_{34})(S_{24}S_{34}C_{23} \\ + S_{23}C_{34}) + 36S_{24}^2 S_{34}^2 C_{24}^2), \tag{32}$$

$$R_\mu = \frac{18 + 4C_{24}^2 C_{23}^2 - 8C_{23}^4 C_{24}^4 - 9C_{24}^4 S2_{23}^2 - 9S2_{24}^2}{R_\mu^{\text{denom}}}, \tag{33}$$

where

$$R_\mu^{\text{denom}} = 5 + 8C_{24}^2 C_{23}^2 + 4(C_{34}S_{23} + S_{24}S_{34}C_{23})^2 \\ + 9S_{34}^2 S2_{24}^2 - 8C_{24}^2 C_{23}^2(C_{34}S_{23} + S_{24}S_{34}C_{23})^2 \\ + 18C_{24}^2 S2_{23}(C_{34}C_{23} - S_{23}S_{24}S_{34})(S_{24}S_{34}C_{23} \\ + S_{23}C_{34}), \tag{34}$$

and

$$R_\tau = \frac{R_\tau^{\text{num}}}{23 + 12C_{23}^2 C_{24}^2 - 8C_{24}^4 C_{23}^4 - 9C_{24}^4 S2_{23}^2 - 9S2_{24}^2}, \tag{35}$$

where

$$R_\tau^{\text{num}} = 4(C_{34}S_{23} + S_{34}S_{24}C_{23})^2 + 9S_{34}^2 S2_{24}^2 \\ - 8C_{24}^2 C_{23}^2(C_{34}S_{23} + S_{34}S_{24}C_{23})^2 \\ + 18C_{24}^2 S2_{23}(C_{34}C_{23} - S_{23}S_{24}S_{34})(S_{24}S_{34}C_{23} \\ + S_{23}C_{34}). \tag{36}$$

A. Flux ratios when $\theta_{24} = 0$

If we work in a further simplified scenario where the only nonzero sterile mixing angle is the angle θ_{34} and put both $\theta_{14} = 0$ and $\theta_{24} = 0$, then the flux ratios are given as

$$R_e = \frac{5 + 8C_{23}^2}{22 - 4S_{23}^2 S_{34}^2 - 8C_{23}^4 - 36S_{23}^2 C_{23}^2 + 28S_{23}^2 C_{23}^2 C_{34}^2}, \tag{37}$$

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[2]Since $\sin^2\theta_{13} = 0$ the effect of the normal *CP* phase is also completely absent.

$$R_\mu = \frac{18 + 4C_{23}^2 - 8C_{23}^4 - 9S2_{23}^2}{5 + 8C_{23}^2 + 4C_{34}^2 S_{23}^2 + 7S2_{23}^2 C_{34}^2}, \quad (38)$$

$$R_\tau = \frac{4C_{34}^2 S_{23}^2 + 28C_{23}^2 S_{23}^2 C_{34}^2}{23 - 24C_{23}^2 S_{23}^2 + 4C_{23}^4}. \quad (39)$$

B. Flux ratios when $\theta_{34} = 0$

If instead of putting $\theta_{24} = 0$, we put $\theta_{34} = 0$ along with $\theta_{14} = 0$ then the flux ratios are

$$R_e = \frac{5 + 8C_{24}^2 C_{23}^2}{22 - 4C_{23}^2 - 36C_{24}^2 S_{24}^2 + 32C_{24}^2 C_{23}^2 - 28C_{24}^2 S_{24}^2 C_{23}^4 - 36C_{24}^4 C_{23}^2}, \quad (40)$$

$$R_\mu = \frac{18 + 4C_{24}^2 C_{23}^2 - 8C_{23}^4 C_{24}^4 - 9C_{24}^4 S2_{23}^2 - 9S2_{24}^2}{9 - 28C_{24}^2 C_{23}^4 - 4C_{23}^2 + 36C_{24}^2 C_{23}^2}, \quad (41)$$

$$R_\tau = \frac{4S_{23}^2 + 28C_{24}^2 C_{23}^2 S_{23}^2}{23 + 12C_{23}^2 C_{24}^2 - 8C_{24}^4 C_{23}^4 - 9C_{24}^4 S2_{23}^2 - 9S2_{24}^2}. \quad (42)$$

Since the constraints from the short baseline experiments restrict θ_{24} to very small values, we can expand the flux ratios in a Taylor series and keep only the first order terms in $\sin^2\theta_{24}$, giving

$$R_e \simeq \frac{5 + 8C_{24}^2 C_{23}^2}{22 - 8C_{23}^2 - 36S_{24}^2 + 12C_{23}^4 S_{24}^2 + 10S2_{23}^2 S_{24}^2}, \quad (43)$$

$$R_\mu \simeq \frac{(18 - 32C_{23}^2 + 28C_{23}^4) - 4S_{24}^2(9 - 17C_{23}^2 + 14C_{23}^4)}{9 - 28C_{24}^2 C_{23}^4 - 4C_{23}^2 + 36C_{24}^2 C_{23}^2}, \quad (44)$$

$$R_\tau \simeq \frac{4S_{23}^2 + 28C_{24}^2 C_{23}^2 S_{23}^2}{(23 - 24C_{23}^2 + 28C_{23}^4) - 4S_{24}^2(9 - 15C_{23}^2 + 14C_{23}^4)}. \quad (45)$$

VI. RESULTS

The mixing angles for both active-active and active-sterile mixing are constrained by the results of the neutrino oscillation experiments [1–6,25,27]. The current 3σ limits on θ_{12}, θ_{23}, and θ_{13} are [8,9]

$$0.25 < \sin^2\theta_{12} < 0.39, \quad (46)$$

$$\sin^2 2\theta_{23} > 0.9, \quad (47)$$

$$\sin^2\theta_{13} < 0.044. \quad (48)$$

For the parametrization given by Eq. (18) which we use for the mixing matrix corresponding to the $3 + 1$ scenario, the

mixing angles governing the active-sterile mixing are θ_{14}, θ_{24}, and θ_{34}. The mixing angles θ_{14} and θ_{24} are severely constrained by LSND [25] and other short baseline data [24,27]. We refer the reader to [28] for the most up-to-date analysis of the short baseline experiments. In what follows, we will present all our results for

$$\sin^2\theta_{14} = 0 \quad (49)$$

and for very small values of $\sin^2\theta_{24}$. Both these mixing angles are severely constrained by the current data. However, the mixing angle θ_{34} does not appear in either P_{ee} or $P_{e\mu}$ which are probed by most of the short baseline experiments like Bugey, CDHS, KARMEN, LSND, and MiniBooNE. This mixing angle appears in the $P_{\mu\tau}$ and $P_{e\tau}$ channels. These channels were probed by the Chorus and NOMAD experiments at CERN. However, in the mass square difference range relevant for LSND, the constraints on the mixing angle θ_{34} are extremely weak. We will therefore allow $\sin^2\theta_{34}$ to take all possible values. We reiterate that the $3 + 1$ mass spectrum for the neutrinos is now comprehensively disfavored after the release of the MiniBooNE results. Our choice of still using it as an exemplary case stems purely from keeping the discussion simple. Our choice of taking $\theta_{14} = 0$ is also due to the same reason. In fact, we would have a similar situation even if we had taken the $3 + 2$ mass scheme and kept all angles except θ_{24} and θ_{34} nonzero.

A. Three active neutrinos and deviation from TBM mixing

In Fig. 1 we show the flux ratios as a function of $\sin^2\theta_{23}$ for three active neutrinos. The dashed lines show the case where $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ are kept fixed at their TBM values of $1/3$ and 0, respectively. We see that for this case, R_e varies between [0.437–0.569] as $\sin^2\theta_{23}$ changes from [0.4–0.6]. Correspondingly, R_μ and R_τ vary between [0.488–0.558] and [0.446–0.511], respectively. We note that R_e depends sharply on $\sin^2\theta_{23}$ and decreases linearly as the value of $\sin^2\theta_{23}$ increases. Both R_μ and R_τ show a nonlinear increase as $\sin^2\theta_{23}$ increases. R_μ increases faster when $\sin^2\theta_{23} > 0.5$, while R_τ shows a sharper dependence when $\sin^2\theta_{23} < 0.5$. This means that R_e is a good probe of
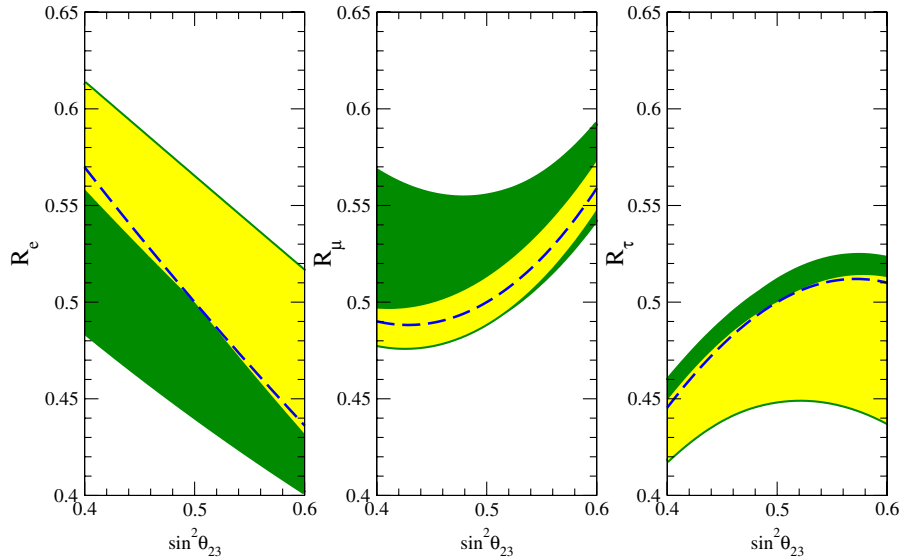
FIG. 1 (color online). The flux ratios R_e, R_μ, and R_τ versus $\sin^2\theta_{23}$, for three active neutrinos. The light bands correspond to allowed ranges of the flux ratios corresponding to all currently 3σ allowed values of $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ and where $\cos\delta$ is kept fixed as 1. The dark bands show the corresponding allowed values where $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ vary in their current 3σ allowed ranges and $\cos\delta$ is allowed to vary between -1 and 1. The upper (lower) boundaries of the light and dark bands almost coincide with each other in the left (middle and right) panels.

the deviation of θ_{23} from maximality and its octant for all values of $\sin^2\theta_{23}$, while R_μ is good only for $\sin^2\theta_{23} > 0.5$ and R_τ for $\sin^2\theta_{23} < 0.5$. If we allow even $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ to deviate from their TBM mixing values and vary freely within their current 3σ ranges keeping $\delta = 0$ fixed, then we obtain the range of flux ratios shown by the light bands in Fig. 1. If the CP phase δ is also allowed to take all possible values then the results obtained are shown by the dark bands in Fig. 1. We see that the allowed ranges of R_e, R_μ, and R_τ for the currently allowed 3σ range of $\sin^2\theta_{23}$

between $[0.34-0.66]$ are, respectively,

$$0.378 \leq R_e \leq 0.645, \qquad 0.476 \leq R_\mu \leq 0.644,$$
$$0.379 \leq R_\tau \leq 0.524, \qquad (50)$$

where we allow $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ to vary within their current 3σ allowed values and δ over its full range. If we had three active neutrinos only, since we know that $\sin^2 2\theta_{23} > 0.9$ at 3σ, the flux ratios *must* be restricted within the bounds given by Eq. (50). If we measure a
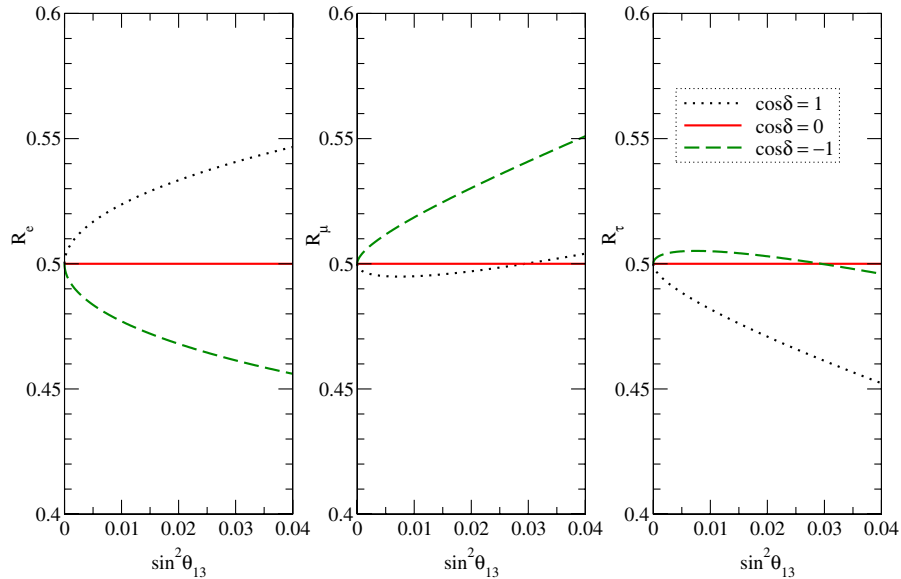

FIG. 2 (color online). The flux ratios R_e, R_μ, and R_τ versus $\sin^2\theta_{13}$, for three active neutrinos and when $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$. The dotted lines correspond to $\cos\delta = 1$, the solid lines to $\cos\delta = 0$, and the dashed lines to $\cos\delta = -1$.
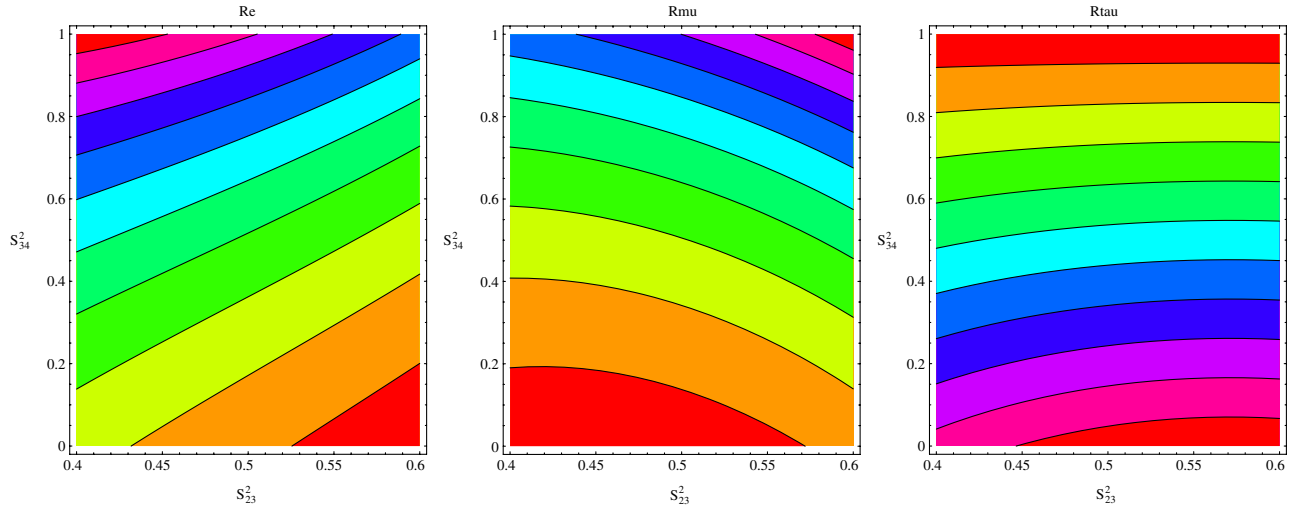
FIG. 3 (color online).   Contour bands of the flux ratios R_e, R_μ, and R_τ in the $\sin^2\theta_{23} - \sin^2\theta_{34}$ plane. The R_e bands increase from 0.44 (lower right-hand corner) to 1.1 (upper left-hand corner) in steps of 0.066, the R_μ bands increase from 0.49 (lower left-hand corner) to 1.18 (upper right-hand corner) in steps of 0.069, while the R_τ bands increase from 0.00 (bottom) to 0.51 (up) in steps of 0.051. We have kept $\theta_{14} = 0 = \theta_{24}$ and all other mixing angles at their TBM mixing values.

flux ratio outside these ranges in the future neutrino telescopes, then that would be a certain indication of new physics and, in particular, as we will see in the next subsection, of the presence of extra sterile neutrinos mixed with the active ones.

In Fig. 2 we show the flux ratios as a function of $\sin^2\theta_{13}$, keeping $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ at their TBM values. The dotted lines in this figure show the case for $\cos\delta = 1$, the solid lines for $\cos\delta = 0$ and the dashed lines for $\cos\delta = -1$. A comparison of Fig. 2 with Fig. 1 shows that the impact of changing $\sin^2\theta_{23}$ from its TBM value of $1/2$ is much larger on the flux ratios than changing $\sin^2\theta_{13}$ from its TBM value of 0. Therefore, in the rest of the paper we only show the effect of the deviation from TBM mixing by changing $\sin^2\theta_{23}$ from $1/2$ and keep $\sin^2\theta_{13}$ fixed at 0.

B. Impact of sterile neutrinos

For the sake of illustration we begin by presenting results for the case where the active-sterile mixing angles $\sin^2\theta_{14} = 0$ and $\sin^2\theta_{24} = 0$. For the active-active mixing angles we take $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ at their TBM values of $1/3$ and 0, respectively, and allow only $\sin^2\theta_{23}$ to vary on both sides of its TBM value of $1/2$. In other words, deviation from TBM mixing is caused by a nonmaximal θ_{23}, while the presence of the sterile neutrino and its impact comes through the nonzero mixing angle θ_{34}.

In Fig. 3 we show the contour bands of the flux ratios R_e, R_μ and R_τ in the $\sin^2\theta_{23}$-$\sin^2\theta_{34}$ plane. The R_e bands increase from 0.44 (lower right-hand corner) to 1.1 (upper left-hand corner) in steps of 0.066, the R_μ bands increase from 0.49 (lower left-hand corner) to 1.18 (upper right-hand corner) in steps of 0.069, while the R_τ bands increase from 0.00 (top) to 0.51 (bottom) in steps of 0.051. As noted in the previous subsection, we see from the figure that R_e

has a very sharp dependence on $\sin^2\theta_{23}$ and its value decreases as $\sin^2\theta_{23}$ increases. R_μ shows a milder dependence and its value increases with $\sin^2\theta_{23}$, while R_τ is almost independent of the value of $\sin^2\theta_{23}$ for $\sin^2\theta_{23} > 0.5$ and shows an extremely mild dependence for $\sin^2\theta_{23} < 0.5$. All flux ratios depend strongly on the value of $\sin^2\theta_{34}$. The flux ratios R_e and R_μ increase as $\sin^2\theta_{34}$ increases, while R_τ decreases. This can be seen also from Eqs. (37)– (39), where $\sin^2\theta_{34}$ decreases the denominators of R_e and R_μ, while it decreases the numerator of R_τ. The figure also shows that the dependence of R_τ on $\sin^2\theta_{34}$ is linear, while that of R_e and R_μ are nonlinear. This is corroborated by the Eqs. (39), (37), and (38), respectively. The most important thing we can see from this figure is the following: the same value of R_e, R_μ, and R_τ can be obtained for different sets of values of $\sin^2\theta_{23}$ and $\sin^2\theta_{34}$. This gives us degenerate solutions in the $\sin^2\theta_{23}$-$\sin^2\theta_{34}$ plane. This will therefore lead to confusion[3] between the case where there are only three active neutrinos with deviations from TBM mixing and the case where TBM holds in the three active regime but there are extra sterile neutrinos which are mixed with the active neutrinos. This degeneracy is obviously more pronounced when the correlation between $\sin^2\theta_{23}$ and $\sin^2\theta_{34}$ is greater. Therefore, the largest effect is expected in R_e and smallest in R_τ.

To further illustrate the confusion between deviation from TBM mixing and the presence of sterile neutrinos we present Fig. 4(a), which shows in the $\sin^2 2\theta_{23}$-$\sin^2\theta_{34}$ plane the contours for the condition

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[3]This is just one example in which we show this confusion. This confusion is expected even in the most general case where there might be more sterile neutrinos, with all mixing angles and CP phases nonzero.
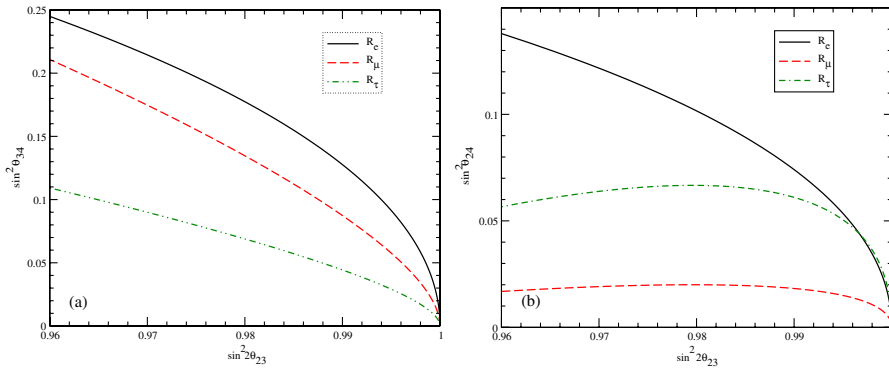
FIG. 4 (color online). Panel (a) shows the contours in the $\sin^2 2\theta_{23} - \sin^2\theta_{34}$ plane which satisfy the corresponding condition given by Eq. (51). Each point on a given line gives the value of $\sin^2\theta_{34}$ which along with TBM in active part would give the same flux ratio as three active neutrinos but with that value for $\sin^2 2\theta_{23}$. Panel (b) gives the corresponding contours in the $\sin^2 2\theta_{23} - \sin^2\theta_{24}$ plane, showing the condition Eq. (55).

$$R_\alpha(\sin^2\theta_{23} \neq 0.5, \sin^2\theta_{34} = 0)$$
$$= R_\alpha(\sin^2\theta_{23} = 0.5, \sin^2\theta_{34} \neq 0), \qquad (51)$$

where $\alpha = e$, μ, or τ. The left-hand side of Eq. (51) gives the flux ratios predicted by deviation from TBM mixing for three active neutrinos only, while the right-hand side corresponds to TBM mixing in the active part but with extra sterile neutrinos. We remind the reader that $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{13} = 0$, $\sin^2\theta_{14} = 0$, and $\sin^2\theta_{24} = 0$ on both sides of Eq. (51). Using Eq. (37)–(39) condition (51) leads to

$$S_{34}^2 = \frac{12 - 24S_{23}^2}{13 - 8S_{23}^2} \qquad (52)$$

for $\alpha = e$,

$$S_{34}^2 = \frac{15 - 72S_{23}^2 + 84S_{23}^4}{14 - 24S_{23}^2 + 28S_{23}^4} \qquad (53)$$

for $\alpha = \mu$, and

$$S_{34}^2 = \frac{27 - 96S_{23}^2 + 84S_{23}^4}{27 - 32S_{23}^2 + 28S_{23}^4} \qquad (54)$$

for $\alpha = \tau$. We have plotted Eqs. (52)–(54) in Fig. 4(a), where the solid line corresponds to the condition for R_e, the dashed line for R_μ and the dot-dashed line for R_τ. Any point on a given line in this figure gives the value of $\sin^2\theta_{23}$ and $\sin^2\theta_{34}$ such that Eq. (51) is satisfied. For instance, the case with three active neutrinos only and with $\sin^2 2\theta_{23} = 0.96$ predicts a value of R_e which can be obtained with TBM mixing in the active sector plus an extra sterile neutrino with $\sin^2\theta_{34} = 0.245$. We note that for the same $\sin^2 2\theta_{23}$, the values of $\sin^2\theta_{34}$ for the sterile case which can also simultaneously satisfy the condition for R_μ and R_τ are different, the corresponding values of $\sin^2\theta_{34}$ being 0.21 and 0.11, respectively. This means that in principle if we knew what is the true value of $\sin^2\theta_{34}$, we would be able to

distinguish the results for the sterile case with that for active neutrinos only with deviation from TBM mixing, if we measured all the three flux ratios simultaneously. However, in practice we do not have any precise information on this mixing angle. Therefore, this confusion will be hard to solve even if we had measurements on all the three flux ratios at neutrino telescopes.

The second very important aspect evident from Fig. 3 is that the values of the flux ratios can easily exceed the range given in Eq. (50). For instance, we see that values of $R_e >$ 0.645 is possible for $\sin^2\theta_{23} > 0.4$ and $\sin^2\theta_{34} > 0.25$. Note that we have kept $\sin^2\theta_{13} = 0$, $\sin^2\theta_{12} = 1/3$, and all CP phases to 0. In fact for this case, when $\sin^2\theta_{34} = 0$ the range of R_e corresponds to [0.437–0.569], as in Fig. 1 for $\delta = 0$. We can draw similar conclusions from R_μ and R_τ using the middle and right panels of Fig. 3. If the measured flux ratios correspond to such extreme values, then this could be a signature for extra sterile neutrinos.

In Fig. 5 we show the contours of constant flux ratios in the $\sin^2\theta_{23}$-$\sin^2\theta_{24}$ plane, keeping $\theta_{34} = 0$, $\theta_{14} = 0$, $\theta_{13} = 0$, and $\sin^2\theta_{12} = 1/3$. The R_e bands increase from 0.436 (lower right-hand corner) to 0.610 (upper left-hand corner) in steps of 0.017, the R_μ bands increase from 0.447 (upper left-hand side) to 0.559 (lower right-hand corner) in steps of 0.011, while the R_τ bands increase from 0.445 (left-hand side) to 0.526 (right-hand side) in steps of 0.008. Note that even though we have shown results up to $\sin^2\theta_{24} = 0.2$, this mixing angle is severely constrained and is expected to be $\lesssim 0.05$. We note from the figure that R_e increases with $\sin^2\theta_{24}$, as the denominator in Eq. (43) decreases much faster with $\sin^2\theta_{24}$ than the numerator. On the other hand R_μ decreases with $\sin^2\theta_{24}$ because the numerator in Eq. (44) decreases with $\sin^2\theta_{24}$ faster than the denominator. For R_τ the dependence is more complicated. Figure 4(b) illustrates the confusion that could arise between a three-generation scheme with deviation from TBM mixing with the one with sterile neutrinos. In panel (b) of this figure we show the contours corresponding to the
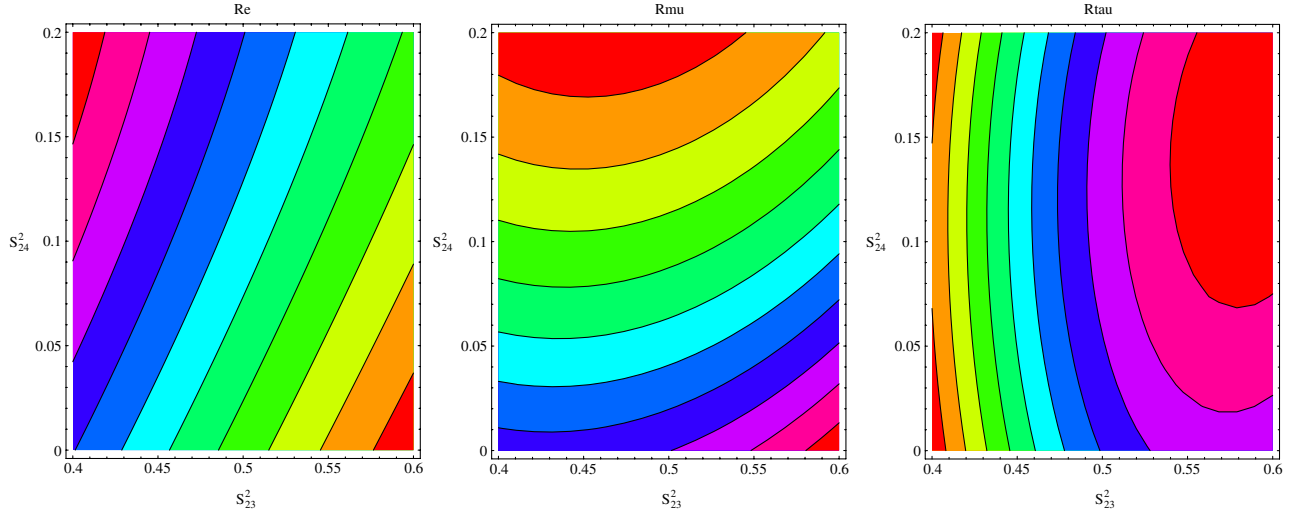
FIG. 5 (color online). Contour bands of the flux ratios R_e, R_μ, and R_τ in the $\sin^2\theta_{23} - \sin^2\theta_{24}$ plane. The R_e bands increase from 0.436 (lower right-hand corner) to 0.610 (upper left-hand corner) in steps of 0.017, the R_μ bands increase from 0.447 (upper left-hand side) to 0.559 (lower right-hand corner) in steps of 0.011, while the R_τ bands increase from 0.445 (left-hand side) to 0.526 (right-hand side) in steps of 0.008. We have kept $\theta_{14} = 0 = \theta_{34}$ and all other mixing angles at their TBM mixing values.

condition

$$R_\alpha(\sin^2\theta_{23} \neq 0.5, \sin^2\theta_{24} = 0)$$
$$= R_\alpha(\sin^2\theta_{23} = 0.5, \sin^2\theta_{24} \neq 0), \qquad (55)$$

where $\alpha = e$, μ, or τ. As before, the left-hand side of Eq. (55) gives the flux ratios predicted by deviation from TBM mixing for three active neutrinos only, while the right-hand side corresponds to TBM mixing in the active part but with extra sterile neutrinos. We reiterate that $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{13} = 0$, $\sin^2\theta_{14} = 0$, and $\sin^2\theta_{34} = 0$ on both sides of Eq. (55). From Eqs. (43)–(45) we get the conditions

$$S_{24}^2 = \frac{4 - 8S_{23}^2}{9 - 8S_{23}^2} \qquad (56)$$

for $\alpha = e$,

$$S_{24}^2 = \frac{5 - 24S_{23}^2 + 28S_{23}^4}{28S_{23}^4 - 24S_{23}^2 - 2} \qquad (57)$$

for $\alpha = \mu$, and

$$S_{24}^2 = \frac{9 - 32S_{23}^2 + 28S_{23}^4}{7 - 32S_{23}^2 + 28S_{23}^4} \qquad (58)$$

for $\alpha = \tau$. These conditions are plotted in Fig. 4(b).

In the general scenario of course we expect both $\sin^2\theta_{24}$ and $\sin^2\theta_{34}$ to be nonzero. However, we still keep $\sin^2\theta_{14}$ to be zero for simplicity. In Fig. 6 we present three-dimensional plots showing the flux ratios as a function of $\sin^2\theta_{24}$ and $\sin^2\theta_{34}$ for $\sin^2\theta_{23} = 0$. The left panel shows R_e, the middle panel shows R_μ, and the right panel shows R_τ. We have kept $\theta_{13} = 0$ and $\sin^2\theta_{12} = 1/3$.
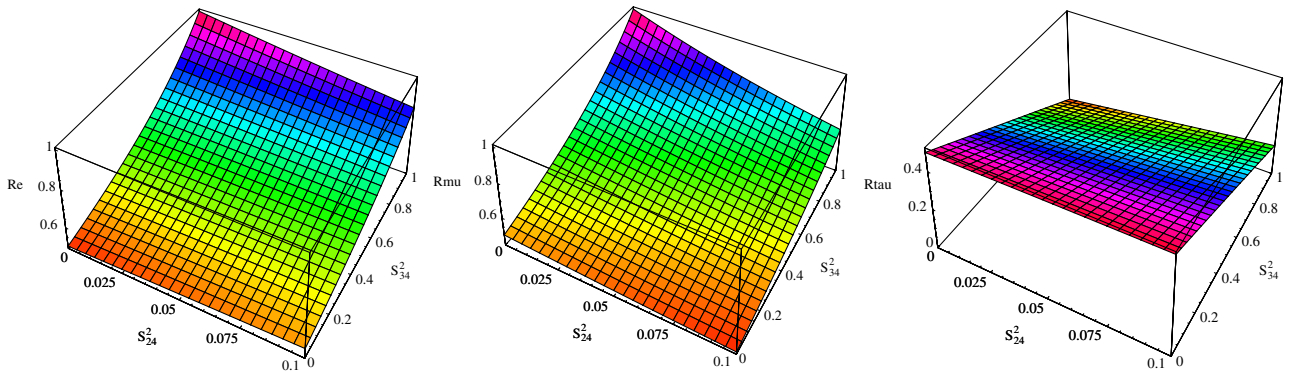

FIG. 6 (color online). Three-dimensional plots showing the flux ratios R_e (left panel), R_μ (middle panel), and R_τ (right panel), as a function of $\sin^2\theta_{24}$ and $\sin^2\theta_{34}$. The angles $\theta_{13} = 0 = \theta_{14}$, $\sin^2\theta_{12} = 1/3$, and $\sin^2\theta_{23} = 0.5$.

TABLE I. Comparison of the flavor ratios predicted by the different new physics scenarios considered in the literature with the standard three flavor picture (first row) and presence of extra sterile neutrinos (bottom rows). Note that for the $CPTV$ + decay models, the flavor ratios correspond to antineutrinos only. For pseudo-Dirac neutrinos we show only two cases, one where only the oscillations due to the second mass splitting is averaged out and another where oscillations due to both second and third splitting are averaged out. We refer the reader to [34] for the complete list. Note that we have used mixing angles corresponding to TBM mixing for the ratios given in this table. This is why they might differ slightly from the ones given in [32–34].

Model	$\sin^2\theta_{24}$	$\sin^2\theta_{34}$	Flavor at Earth
Standard	$\cdots$	$\cdots$	1:1:1
Decay (NH)	$\cdots$	$\cdots$	4:1:1
Decay (IH)	$\cdots$	$\cdots$	0:1:1
$CPT[\frac{1}{2}(\Phi_{\text{std}} + \bar{\Phi}_{CPTV})]$	$\cdots$	$\cdots$	1.07:1:0.79
CPT + Decay (NH) $[\frac{1}{2}(\bar{\Phi}_{CPTV})$ only]	$\cdots$	$\cdots$	2.43:1:0.14
CPT + Decay (IH) $[\frac{1}{2}(\bar{\Phi}_{CPTV})$ only]	$\cdots$	$\cdots$	0.04:1:3.30
Pseudo-Dirac (only 1)	$\cdots$	$\cdots$	0.73:1:1
Pseudo-Dirac (only 2)	$\cdots$	$\cdots$	1:1:1
Pseudo-Dirac (2 and 3)	$\cdots$	$\cdots$	1.43:1:1
Sterile	0.04	0.0	1.05:1:1.04
Sterile	0.04	1.0	1.05:1:0.12
Sterile	0.1	0.0	1.12:1:1.08
Sterile	0.1	1.0	1.12:1:0.32

VII. DISCUSSIONS

In this paper we have considered the presence of sterile neutrinos as the only signature of physics beyond the standard paradigm. Other forms of new physics could also manifest themselves in the observed flux ratios of the ultra high energy neutrinos. In particular, neutrino decay predicts spectacular results for the ratios, where one expects the flavor ratio 4:1:1 for the normal hierarchy[4] and 0:1:1 for the inverted hierarchy [32]. The authors of [33] considered the effect of CPT-violation on the flavor ratios, for both stable as well as unstable neutrinos. In [34] the prospects of probing the existence of pseudo-Dirac neutrinos was discussed. In Table I we show the predicted flavor ratios for all these new physics scenarios and compare them with that expected for sterile neutrinos. We note that the largest deviation for ϕ_e/ϕ_μ comes for the neutrino decay model, both with normal (NH) and inverted (IH) hierarchy. The ratio ϕ_μ/ϕ_τ, however, for this model stays at 1, just like in the standard neutrino case. This ratio is predicted to be very different for the case where we have CPT-violation (with and without neutrino decay) and where we have sterile neutrinos with large $\sin^2\theta_{34}$. For sterile neutrinos, this ratio decreases almost linearly with $\sin^2\theta_{34}$, the slope being determined by $\sin^2\theta_{24}$. We can see that for very large $\sin^2\theta_{34}$, the ϕ_μ/ϕ_τ ratio predicted by sterile neutrinos is very similar to that expected for antineutrinos with CPT-violation plus decay and NH. In fact,

since ϕ_μ/ϕ_τ for sterile neutrinos starts from a large value and decreases to a small number, for some range of values of $\sin^2\theta_{34}$ one would get a ϕ_μ/ϕ_τ similar to the one predicted by CPT-violation with stable neutrinos as well. Therefore, we see that there is ample scope of confusing sterile neutrino signatures in ultra high energy flavor ratio with that of CPT-violation. We also note that the ϕ_e/ϕ_μ ratio predicted by sterile neutrinos could be close to some of the cases for pseudo-Dirac neutrinos.

So far we have presented all results for ratios defined in terms of the ultra high energy neutrino fluxes arriving at Earth. However, one should bear in mind that what is physically relevant as far as the measurement is concerned is the ratio of the corresponding number of events in the neutrino telescope. The number of events expected in the ν_μ, ν_e, and ν_τ channels for an assumed normalization for the ultra high energy neutrino flux was performed in [35]. While ν_μ are easiest to detect as the resultant muons leave distinct tracks in the detector, the ν_e are detected by observing the electromagnetic showers the electrons create. The efficiency of detecting electron events is generally lower than that for muons and it decreases as the neutrino energy increases. The threshold for muon events is expected to be about 10^2 GeV while that for electrons would be about 10^3 GeV. The ν_τ are detectable through the so-called double-bang and lollipop events [36] only at relatively higher energies of about 10^6 GeV. Their detection efficiency would become comparable to that for electrons for energies larger than 10^7 GeV [35]. In all our results we had tacitly given equal weight to all three flavors. However, from the discussion above, it is clear that the behavior of

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[4]The authors of [32] get 6:1:1 because they take $\sin^2\theta_{12} = 1/4$. The ratio given above is obtained for TBM mixing for which $\sin^2\theta_{12} = 1/3$.

the flavor ratios would change, once we used the event ratios rather than the flux ratios, at least for neutrino energies $\ll 10^7$ GeV. In fact, for neutrino energies below 10^6 GeV, we should not include the ν_τ flux at all and work with just the ϕ_e/ϕ_μ ratio. Above energies of 10^7 GeV we could in principle work with all three flavors, however the neutrino flux falls steeply with energy and we expect only a handful of events at these very high energies. We have seen in Table I that sterile components cause the largest deviation from the standard model prediction for ϕ_τ. To see this we would necessarily need observations at higher energies, where statistical uncertainty could be the biggest challenge. A detector much larger than IceCube would probably be required for getting an unambiguous signature of sterile species in ultra high energy neutrinos. This constraint is applicable to most cases which rely on flavor ratios involving the ν_τ. However, we reiterate that even though an unambiguous *signal* for sterile neutrinos in a km^3 detector such as IceCube might prove to be difficult, sterile neutrinos will definitely cause confusion with the signal for deviation from TBM mixing, and this conclusion is valid for all neutrino telescopes.

VIII. SUMMARY AND CONCLUSIONS

Neutrino telescopes offer a possible way of constraining the mixing angles. In particular, considerable interest has been generated of late in using the flux/flavor ratios of ultra high energy neutrinos in probing the deviation of the mixing matrix from the TBM mixing ansatz. Since it has been observed that the effect of deviating $\sin^2\theta_{12}$ from $1/3$ on the flux ratios is not large, we kept $\sin^2\theta_{12}$ fixed at its TBM value of $1/3$. Deviation from TBM mixing can then be caused either by $\sin^2\theta_{13} \neq 0$ or $\sin^2\theta_{23} \neq 1/2$. We showed that the effect of changing $\sin^2\theta_{13}$ from zero is small on the flux ratios. We argued that the maximum impact on the flux ratios arises when we change the mixing angle $\sin^2\theta_{23}$ from $1/2$. Therefore, we presented most results by varying the value of $\sin^2\theta_{23}$ on both sides of its TBM value.

We presented the expressions for the flux ratios assuming that there was one extra sterile neutrino and the mass spectrum followed a $3 + 1$ pattern. Even though we are aware that this mass spectrum is now disfavored following the recently declared MiniBooNE results and the mass spectrum allowed is the $3 + 2$ scheme, we used the $3 + 1$

scheme as an exemplary case for simplicity. Our formalism can be easily extended to the $3 + 2$ scenario, only then one has to contend with many more parameters. We showed that even in a very simplified picture where only one active-sterile mixing angle $\sin^2\theta_{34}$ (or $\sin^2\theta_{24}$) was kept nonzero, it was possible to easily confuse the predicted flux ratios for the deviation from TBM case with three active neutrinos with the case where TBM holds in the three active sector but we have one extra sterile neutrino. We also showed that for very large values of the mixing angle $\sin^2\theta_{34}$, which as of now is almost unconstrained, we get predictions for the flux ratios that are completely out of the possible range predicted by three active neutrinos. If the measured flux ratio really conformed with such extreme values, this would then be a smoking gun signal for sterile neutrinos which are heavily mixed with active neutrinos. We also presented results where we took $\sin^2\theta_{24}$ to be the only nonzero active-sterile mixing angle. Finally, we showed results for two nonzero active-sterile mixing angles, $\sin^2\theta_{34}$ and $\sin^2\theta_{24}$.

We stress that our results would be valid even if a sterile neutrino is not needed to explain the LSND data. Since for ultra high energy neutrinos coming from astrophysical sources, the oscillatory terms anyway average out to zero irrespective of the value of Δm^2, a sterile neutrino with any value for Δm^2 would give such signatures in the neutrino telescopes if the mixing angle such as θ_{34}, which has not yet been checked in terrestrial experiments, was nonzero and large.

In conclusion, if the values of the measured flux ratios in the future neutrino telescopes turn out be different from their TBM prediction of $1/2$, it would still not be foolproof evidence for deviation from TBM mixing. Sterile neutrinos with reasonably small mixing with active neutrinos could mimic a similar response, even when the active sector conforms to TBM mixing. One will never be able to tell one scenario from the other by measuring the flux ratios of ultra high energy neutrinos, if the measured flux ratios were within the range predicted by the standard picture with three active neutrinos. However, if the measured ratios turn out to be clearly outside the range predicted by the three active case, we would have a signal for the existence of sterile neutrinos. One probably needs a larger detector with sufficient statistics for ν_τ events for an unambiguous signal.

[1] B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); J. N. Abdurashitov *et al.* (SAGE Collaboration), Zh. Eksp. Teor. Fiz. **122**, 211 (2002) [J. Exp. Theor. Phys. **95**, 181 (2002)]; W. Hampel *et al.* (GALLEX Collaboration), Phys. Lett. B **447**, 127 (1999); S. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Lett. B **539**, 179 (2002); B. Aharmim *et al.* (SNO Collaboration), Phys. Rev. C **72**, 055502 (2005).

[2] Y. Ashie *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **71**, 112005 (2005).

[3] K. Eguchi *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **90**, 021802 (2003); T. Araki *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **94**, 081801 (2005).

[4] M. Apollonio *et al.*, Eur. Phys. J. C **27**, 331 (2003).

[5] E. Aliu *et al.* (K2K Collaboration), Phys. Rev. Lett. **94**, 081802 (2005).

[6] D. G. Michael *et al.* (MINOS Collaboration), Phys. Rev. Lett. **97**, 191801 (2006).

[7] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. B **535**, 163 (2002); Z. z. Xing, Phys. Lett. B **533**, 85 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. B **547**, 219 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003); P. F. Harrison and W. G. Scott, arXiv:hep-ph/0402006; P. F. Harrison and W. G. Scott, Phys. Lett. B **594**, 324 (2004).

[8] S. Choubey, Phys. At. Nucl. **69**, 1930 (2006); S. Goswami, Int. J. Mod. Phys. A **21**, 1901 (2006); S. Goswami, A. Bandyopadhyay, and S. Choubey, Nucl. Phys. B, Proc. Suppl. **143**, 121 (2005); A. Bandyopadhyay *et al.*, Phys. Lett. B **608**, 115 (2005); G. L. Fogli *et al.*, Prog. Part. Nucl. Phys. **57**, 742 (2006).

[9] M. Maltoni *et al.*, New J. Phys. **6**, 122 (2004); M. C. Gonzalez-Garcia and M. Maltoni, arXiv:hep-ph0704.1800.

[10] T. Fukuyama and H. Nishiura, hep-ph/9702253; R. N. Mohapatra and S. Nussinov, Phys. Rev. D **60**, 013002 (1999); E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001); C. S. Lam, Phys. Lett. B **507**, 214 (2001); P. F. Harrison and W. G. Scott, Phys. Lett. B **547**, 219 (2002); T. Kitabayashi and M. Yasue, Phys. Rev. D **67**, 015006 (2003); W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003); J. Phys. G **30**, 73 (2004); Y. Koide, Phys. Rev. D **69**, 093001 (2004); A. Ghosal, arXiv:hep-ph/0304090; W. Grimus *et al.*, Nucl. Phys. **B713**, 151 (2005); R. N. Mohapatra, J. High Energy Phys. 10 (2004) 027; A. de Gouvea, Phys. Rev. D **69**, 093007 (2004); R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72**, 053001 (2005); R. N. Mohapatra and S. Nasri, Phys. Rev. D **71**, 033001 (2005); R. N. Mohapatra, S. Nasri, and H. B. Yu, Phys. Lett. B **615**, 231 (2005); Phys. Rev. D **72**, 033007 (2005); Y. H. Ahn *et al.*, Phys. Rev. D **73**, 093005 (2006); B. Brahmachari and S. Choubey, Phys. Lett. B **642**, 495 (2006); K. Fuki and M. Yasue, Nucl. Phys. **B783**, 31 (2007).

[11] S. Choubey and W. Rodejohann, Eur. Phys. J. C **40**, 259 (2005); B. Adhikary, Phys. Rev. D **74**, 033002 (2006); T. Ota and W. Rodejohann, Phys. Lett. B **639**, 322 (2006).

[12] A. Bandyopadhyay, S. Choubey, and S. Goswami, Phys. Rev. D **67**, 113011 (2003); A. Bandyopadhyay *et al.*, Phys. Rev. D **72**, 033013 (2005); H. Minakata, H. Nunokawa, W. J. C. Teves, and R. Zukanovich Funchal, Phys. Rev. D **71**, 013005 (2005).

[13] S. Choubey and S. T. Petcov, Phys. Lett. B **594**, 333 (2004); S. T. Petcov and T. Schwetz, Phys. Lett. B **642**, 487 (2006).

[14] K. Anderson *et al.*, arXiv:hep-ex/0402041.

[15] Y. Itow *et al.*, arXiv:hep-ex/0106019; D. S. Ayres *et al.* (NOvA Collaboration), arXiv:hep-ex/0503053.

[16] C. Volpe, J. Phys. G **34**, R1 (2007); S. K. Agarwalla *et al.*, Phys. Rev. D **75**, 097302 (2007); S. K. Agarwalla, S. Choubey, and A. Raychaudhuri, Nucl. Phys. **B771**, 1 (2007), and references therein.

[17] C. Albright *et al.* (Neutrino Factory/Muon Collider Collaboration), arXiv:physics/0411123.

[18] A. S. Dighe and A. Y. Smirnov, Phys. Rev. D **62**, 033007 (2000); C. Lunardini and A. Y. Smirnov, J. Cosmol. Astropart. Phys. 06 (2003) 009; C. Lunardini and A. Y. Smirnov, Nucl. Phys. **B616**, 307 (2001); S. Choubey, D. Majumdar, and K. Kar, J. Phys. G **25**, 1001 (1999); G. Dutta *et al.*, Phys. Rev. D **61**, 013009 (1999); A. S. Dighe *et al.*, J. Cosmol. Astropart. Phys. 01 (2004) 004; H. Minakata *et al.*, Phys. Lett. B **542**, 239 (2002); A. S. Dighe, M. T. Keil, and G. G. Raffelt, J. Cosmol. Astropart. Phys. 06 (2003) 006; A. Bandyopadhyay *et al.*, arXiv:hep-ph/0312315; T. Marrodan Undagoitia *et al.*, J. Phys. Conf. Ser. **39**, 287 (2006); A. S. Dighe, M. T. Keil, and G. G. Raffelt, J. Cosmol. Astropart. Phys. 06 (2003) 005; S. Choubey, N. P. Harries, and G. G. Ross Phys. Rev. D **74**, 053010 (2006); Phys. Rev. D **76**, 073013 (2007).

[19] S. Choubey and P. Roy, Phys. Rev. D **73**, 013006 (2006); M. C. Gonzalez-Garcia, M. Maltoni, and A. Y. Smirnov, Phys. Rev. D **70**, 093005 (2004).

[20] H. Athar, M. Jezabek, and O. Yasuda, Phys. Rev. D **62**, 103007 (2000).

[21] A. Achterberg *et al.* (IceCube Collaboration), Astropart. Phys. **26**, 155 (2006); S. R. Klein (for the IceCube Collaboration), AIP Conf. Proc. **842**, 971 (2006); K. Rawlins *et al.* (IceCube Collaboration), "IceCube: A Multipurpose Neutrino Telescope," http://icecube.wisc.edu.

[22] U. F. Katz, Nucl. Instrum. Methods Phys. Res., Sect. A **567**, 457 (2006).

[23] P. Bhattacharjee and N. Gupta, arXiv:hep-ph/0501191; P. D. Serpico and M. Kachelriess, Phys. Rev. Lett. **94**, 211102 (2005); P. D. Serpico, Phys. Rev. D **73**, 047301 (2006); Z. Z. Xing and S. Zhou, Phys. Rev. D **74**, 013010 (2006); W. Winter, Phys. Rev. D **74**, 033015 (2006); Z. z. Xing, Phys. Rev. D **74**, 013009 (2006); D. Majumdar and A. Ghosal, Phys. Rev. D **75**, 113004 (2007); W. Rodejohann, J. Cosmol. Astropart. Phys. 01 (2007) 029.

[24] A. A. Aguilar-Arevalo *et al.* (The MiniBooNE Collaboration), Phys. Rev. Lett. **98**, 231801 (2007).

[25] C. Athanassopoulos *et al.* (The LSND Collaboration) Phys. Rev. Lett. **77**, 3082 (1996); C. Athanassopoulos *et al.* (The LSND Collaboration) Phys. Rev. Lett. **81**, 1774 (1998).

[26] J. J. Gomez-Cadenas and M. C. Gonzalez-Garcia, Z. Phys. C **71**, 443 (1996); S. Goswami, Phys. Rev. D **55**, 2931 (1997); S. M. Bilenky, C. Giunti, and W. Grimus, Eur. Phys. J. C **1**, 247 (1998).

[27] Y. Declais *et al.*, Phys. Lett. B **338**, 383 (1994); F. Dydak *et al.*, Phys. Lett. **134B**, 281 (1984); I. E. Stockdale *et al.*, Phys. Rev. Lett. **52**, 1384 (1984); B. Armbruster *et al.* (KARMEN Collaboration), Phys. Rev. D **65**, 112001 (2002); P. Astier *et al.* (NOMAD Collaboration), Phys. Lett. B **570**, 19 (2003).

[28] M. Maltoni and T. Schwetz, Phys. Rev. D **76**, 093005 (2007).

[29] M. Sorel, J. M. Conrad, and M. H. Shaevitz, Phys. Rev. D **70**, 073004 (2004).

[30] E. Waxman and J. N. Bahcall, Phys. Rev. Lett. **78**, 2292 (1997); E. Waxman and J. N. Bahcall, Phys. Rev. D **59**, 023002 (1998).

[31] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957); **34**, 247 (1957); Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[32] J. F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, and T. J. Weiler, Phys. Rev. Lett. **90**, 181301 (2003); J. F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, and T. J. Weiler, Phys. Rev. D **69**, 017303 (2004).

[33] G. Barenboim and C. Quigg, Phys. Rev. D **67**, 073024 (2003).

[34] J. F. Beacom, N. F. Bell, D. Hooper, J. G. Learned, S. Pakvasa, and T. J. Weiler, Phys. Rev. Lett. **92**, 011101 (2004); P. Keranen, J. Maalampi, M. Myyrylainen, and J. Riittinen, Phys. Lett. B **574**, 162 (2003).

[35] J. F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, and T. J. Weiler, Phys. Rev. D **68**, 093005 (2003); [**72**, 019901(E) (2005)].

[36] J. G. Learned and S. Pakvasa, Astropart. Phys. **3**, 267 (1995).